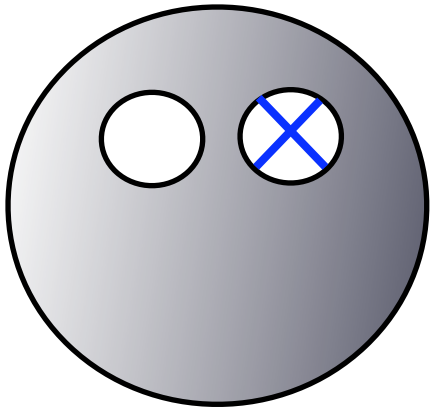
Introduction to Orientifolds

Marcus Berg, CoPS, Fysikum



Talk posted at
http://www.physto.se/~mberg

Overview

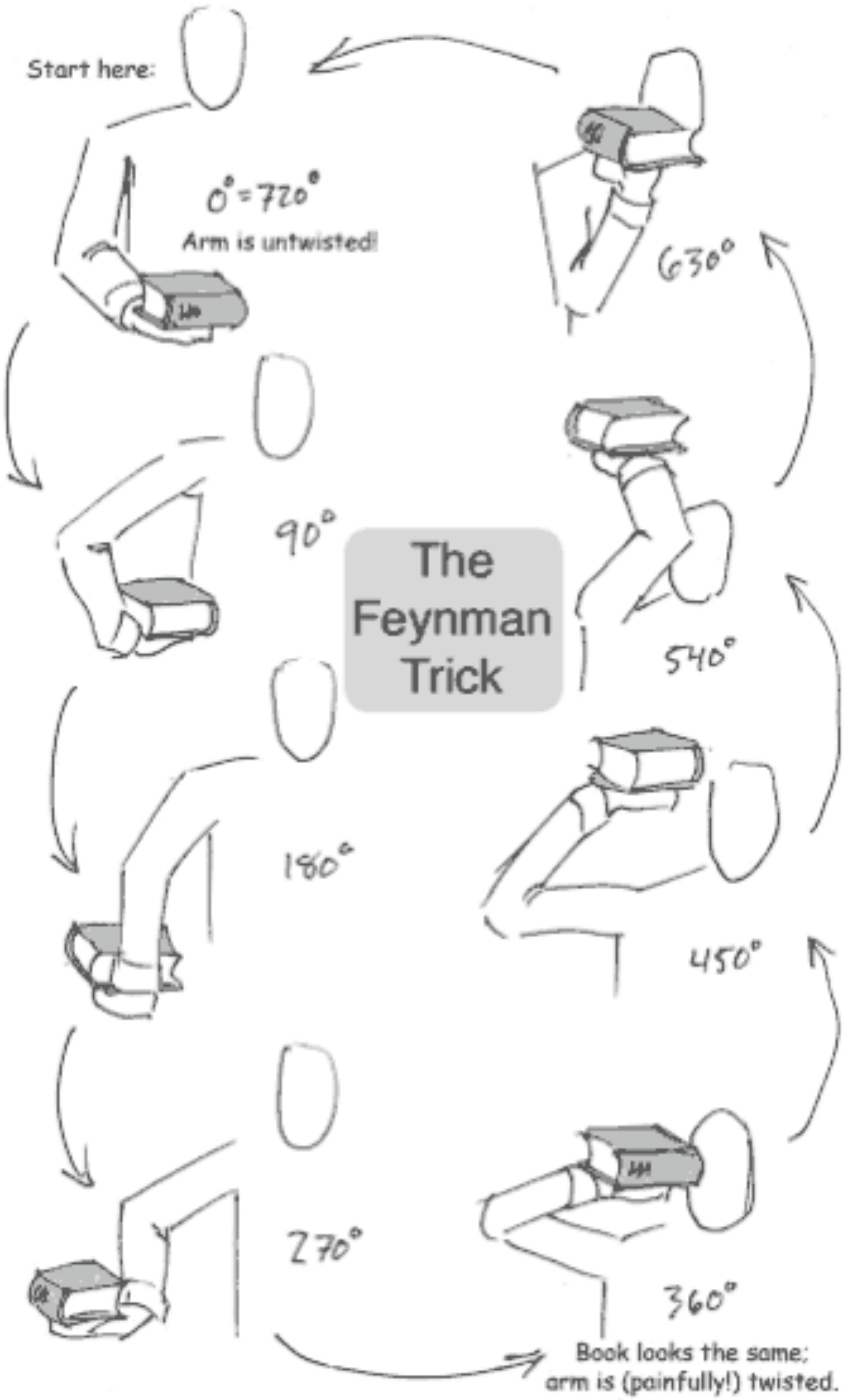
- Orientability in QFT
- Tadpoles and anomalies (QFT, strings)
- What is an orientifold plane?
- Applications: particle physics, cosmology
- Brief statement about condensed matter

Will try to argue that this line of argument
is not specific to string theory as we know it
(but is specific to theories of extended objects!)

Orientability in Quantum Field Theory: spinors

START

T. Jackson, '03

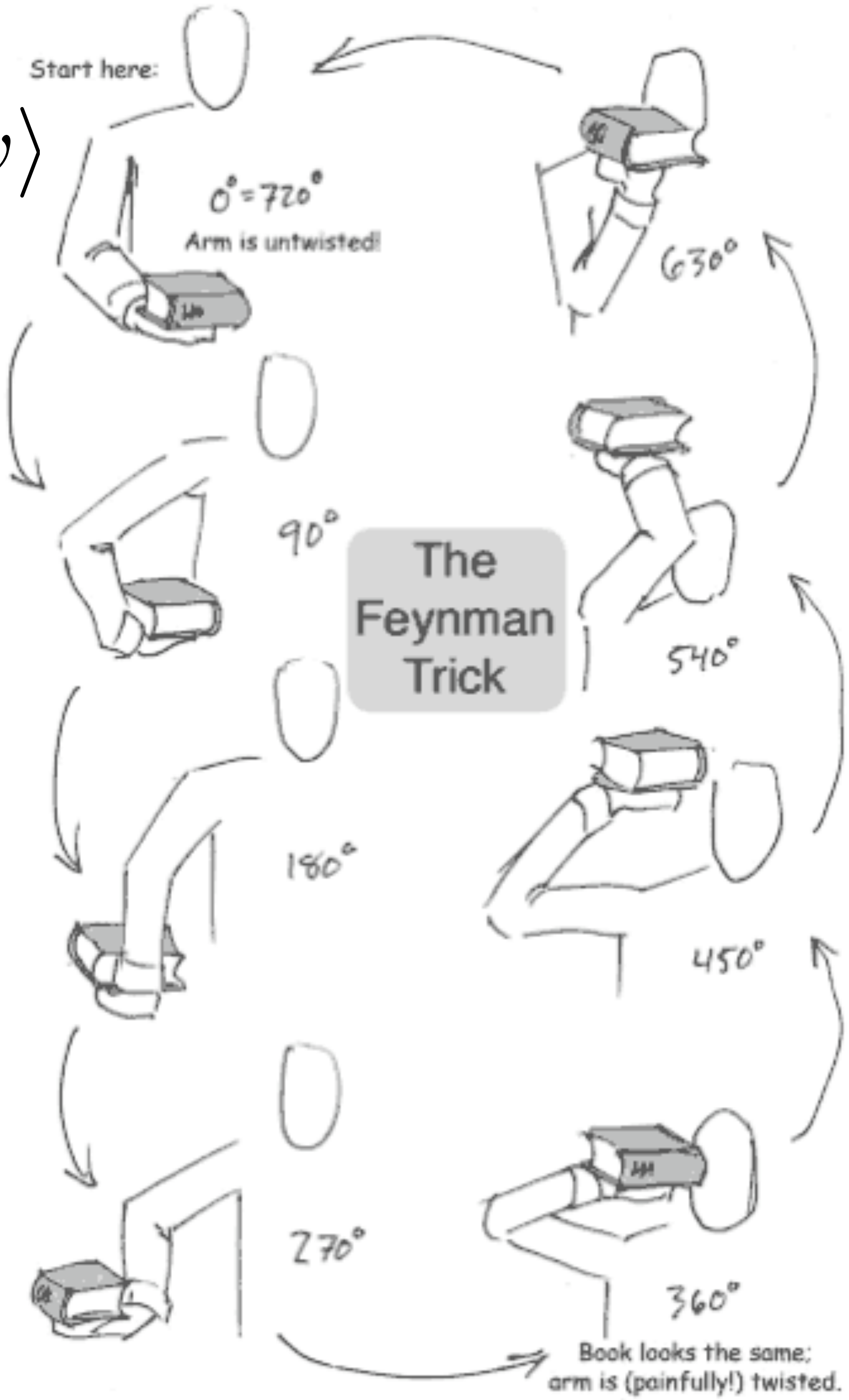


$$S_{R(2\pi)}\,|\psi\rangle = -|\psi\rangle$$

Orientability in Quantum Field Theory: spinors

$$(S_{R(2\pi)})^2\,|\psi\rangle = +|\psi\rangle$$

T. Jackson, '03



Well-known:
e.g. Sakurai, p. 162

$$S_{R(2\pi)}\,|\psi\rangle = -|\psi\rangle$$

Orientability in Quantum Field Theory: spinors

$$S_\Lambda \gamma^\mu S_\Lambda^{-1} = \Lambda_\nu{}^\mu \gamma^\nu$$

- Orientation preserving Lorentz transformation $\Lambda_\nu{}^\mu$: even number of gamma matrices, e.g. $S_{R(2\pi)}$
- Orientation reversing Lorentz transformation $\Lambda_\nu{}^\mu$: odd number of gamma matrices, e.g. S_P ← $\vec{x} \to -\vec{x}$

2π rotation → $S_{R(2\pi)}$; spinor → $|\psi\rangle$

$$S_{R(2\pi)} |\psi\rangle = - |\psi\rangle$$

$$(S_{R(2\pi)})^2 |\psi\rangle = + |\psi\rangle$$

Parity operation → S_P

$$S_P |\psi\rangle = p |\psi\rangle$$

$$(S_P)^2 |\psi\rangle = p^2 |\psi\rangle \qquad p^2 = ?$$

Orientability in Quantum Field Theory: spinors

Lesson: nothing is really "obvious"when it comes to spinors
(avoid unnecessary assumptions)

In particular, parity does not have to square to one.

$$S_P^2 = \pm 1$$

e.g. M.B., DeWitt-Morette, Gwo & Kramer '00
(one among many)

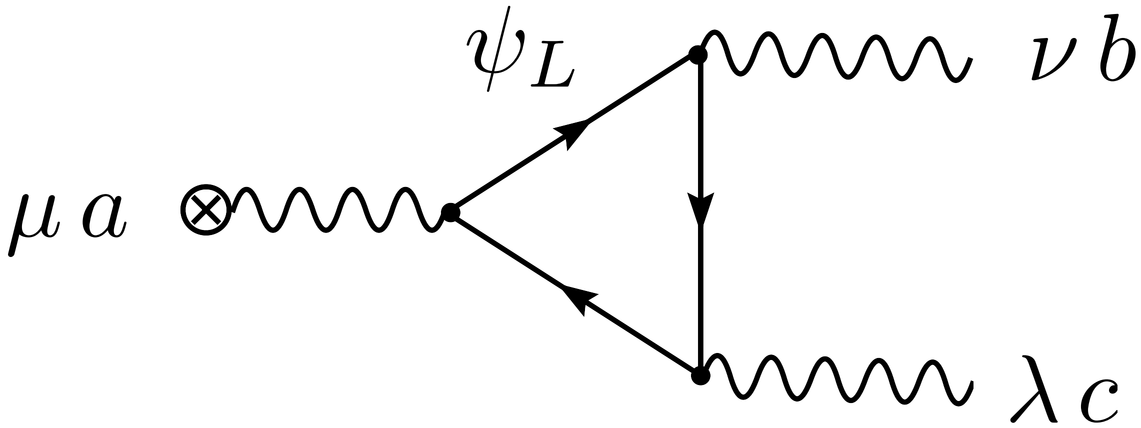
"What is the intrinsic parity of a left-handed electron?"

$$\begin{pmatrix} \psi_{\mathrm{L}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_{\mathrm{R}} \end{pmatrix} = \psi_{\mathrm{Dirac}}$$

Perturbation theory anomalies

Ex: gauge fields, group G, coupled to left-handed fermions ψ_L

Peskin & Schroeder, p. 680



gauge symmetry current

$$\langle p,\nu,b;k,\lambda,c|\partial_\mu j^{\mu a}|0\rangle = \frac{g^2}{8\pi^2}\epsilon^{\alpha\beta\mu\nu}p_\alpha k_\beta \mathcal{A}^{abc}$$

SM:

$SU(2):\ \mathcal{A}$ vanishes for any number of fermions

$U(1):\ \mathcal{A} = 3\cdot\left(\frac{2}{3}-\frac{1}{3}\right)+(0-1)=0$

Perturbation theory anomalies

Ex: gauge fields, group G, coupled to left-handed fermions ψ_L

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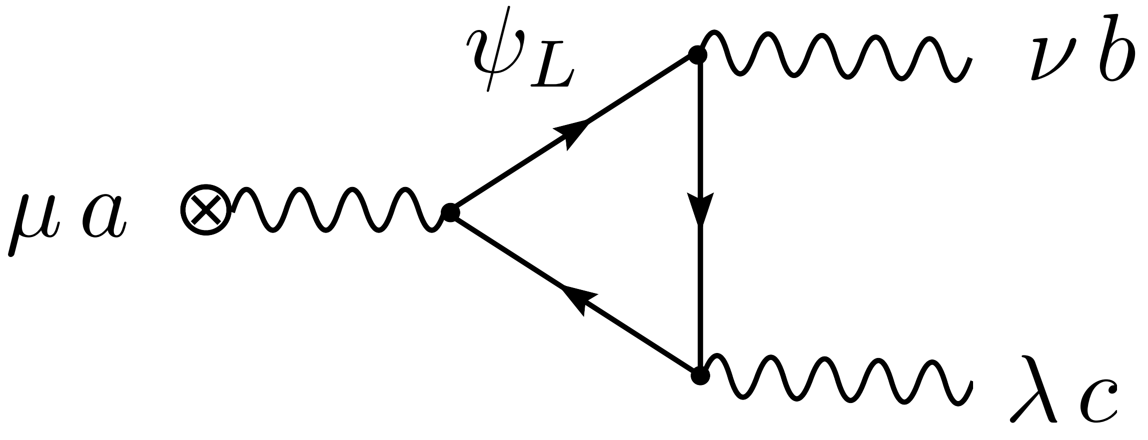


gauge symmetry current

$$\langle p, \nu, b; k, \lambda, c | \partial_\mu j^{\mu a} | 0 \rangle = \frac{g^2}{8\pi^2} \epsilon^{\alpha\nu\beta\lambda} p_\alpha k_\beta \mathcal{A}^{abc}$$

SM:

$SU(2)$: $\mathcal{A}$ vanishes for any number of fermions

$U(1)$: $\mathcal{A} = 3 \cdot (\frac{2}{3} - \frac{1}{3}) + (0 - 1) = 0$

G has potential anomalies if (except $U(1)$) : $\pi_5(G) \neq 0$

Orientability and extended objects

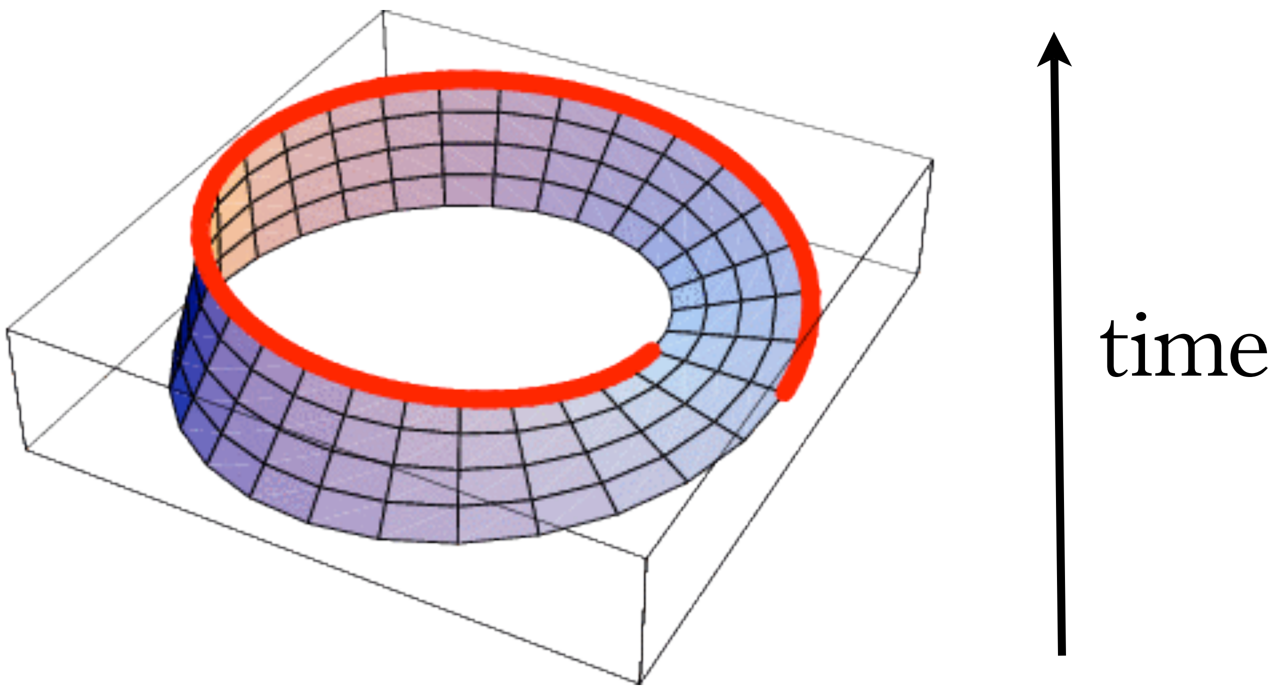
Ex. one-dimensional objects: strings!

Worldsheet = surface swept out by string in time

- Unorientable string worldsheets?

would expect similar issues for "more extended" objects, like membranes ... more some day?

Orientability and open strings



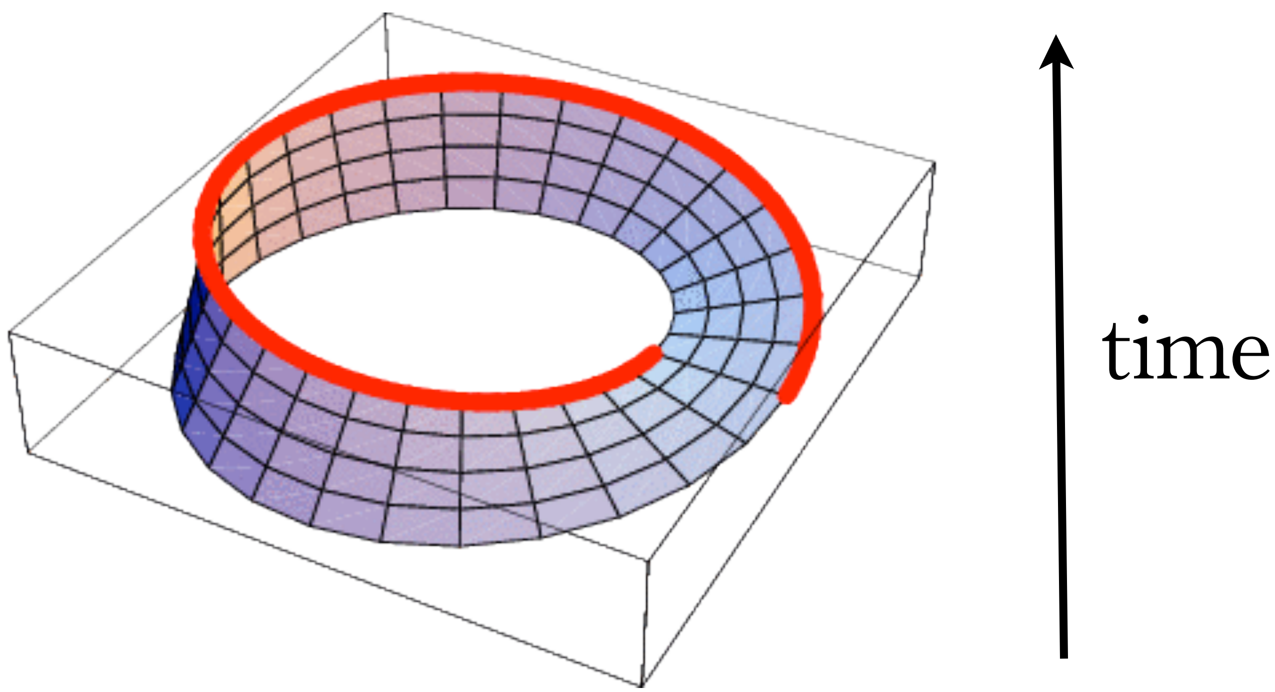
Sagnotti '87
...
Gimon, Polchinski '96

Orientability and open strings

Sagnotti '87

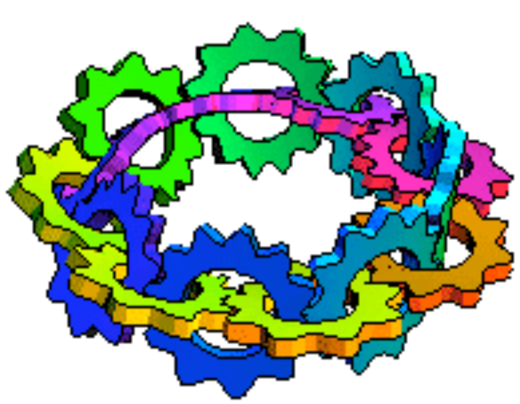
...

Gimon, Polchinski '96

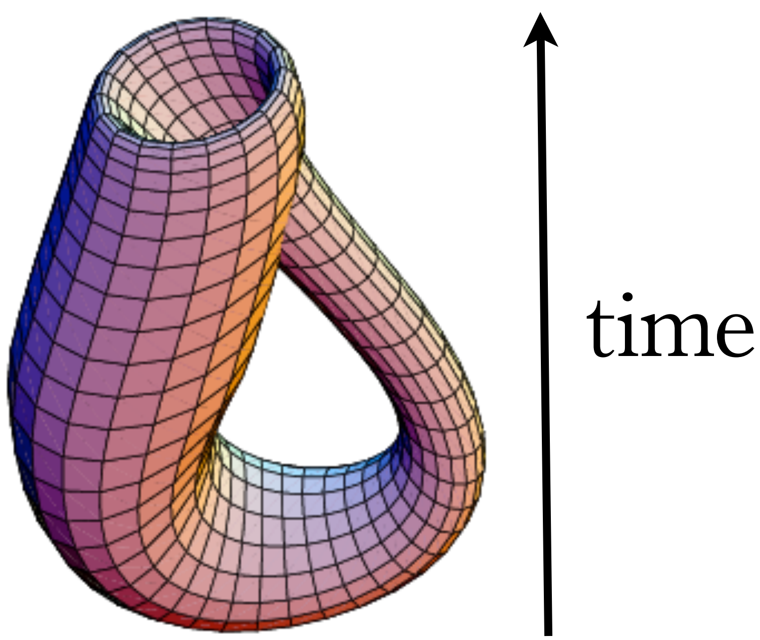


mathworld.wolfram.com

Möbius strip

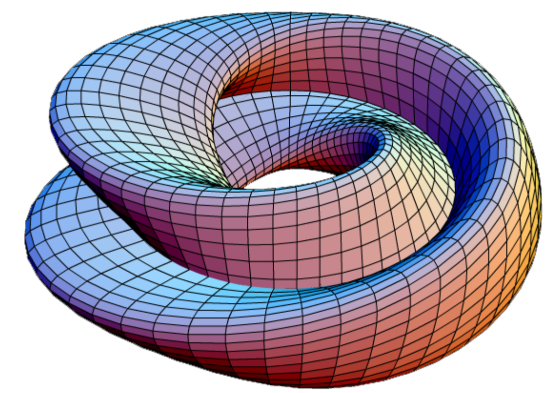


Orientability and closed strings



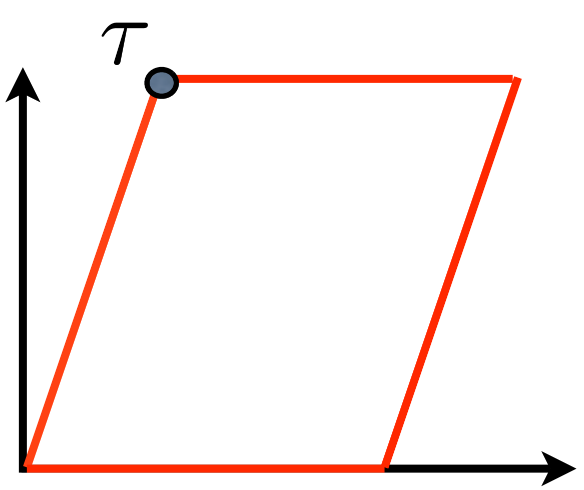
Klein bottle

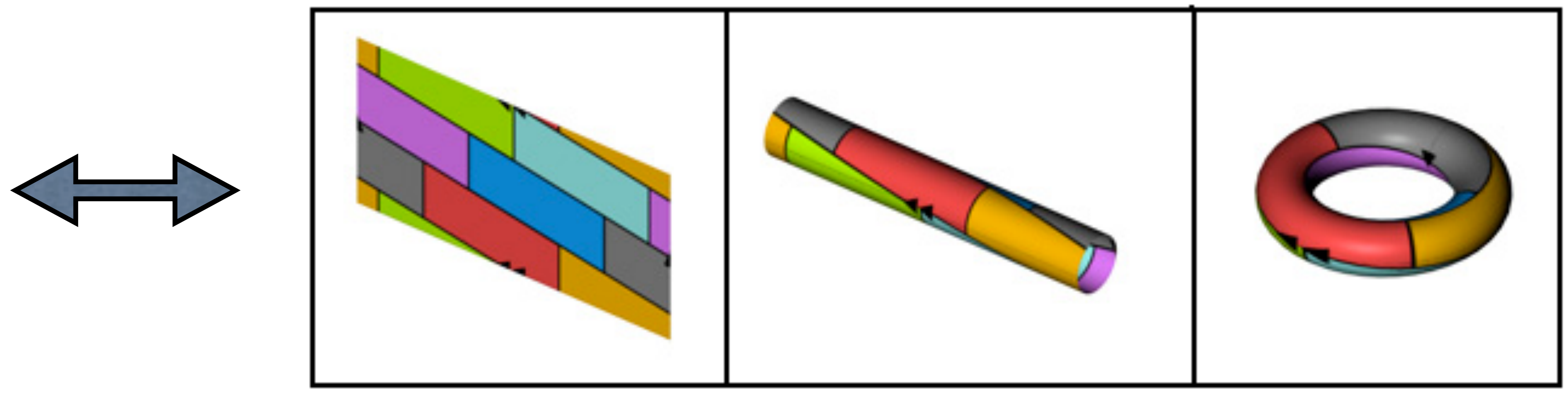
or:



More tractable:

Involutions of the (worldsheet) torus

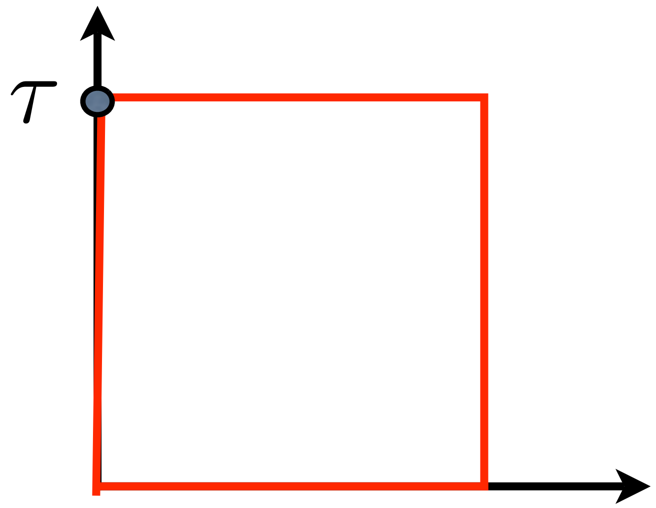


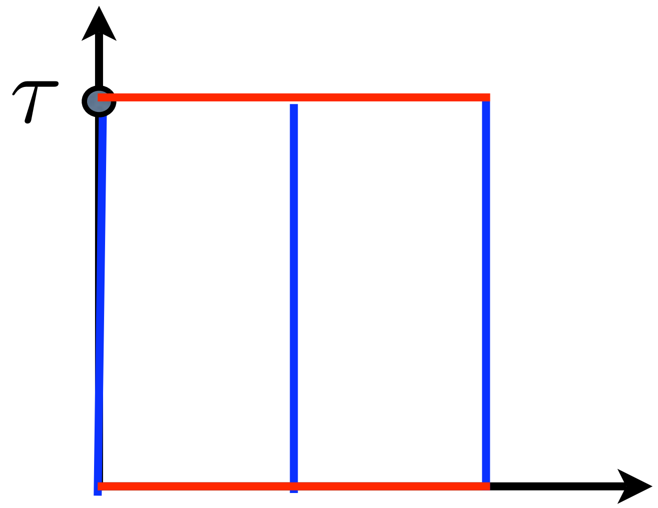


Make a cylinder from a torus:

$\tau = it, \quad t \in \mathbb{R}$

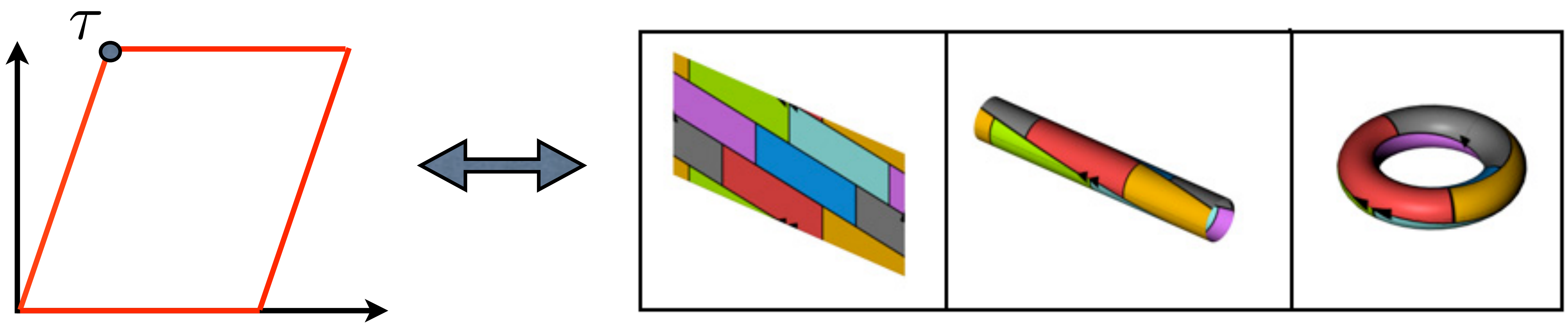
Identify under $I(z) = 1 - \bar{z}$





fixed lines = boundaries

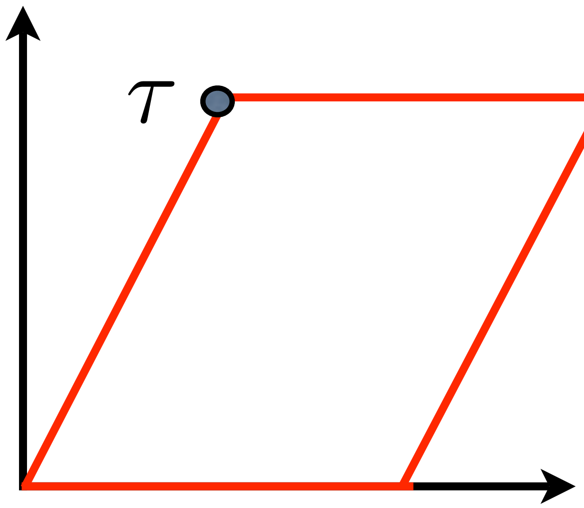
Involutions of the (worldsheet) torus

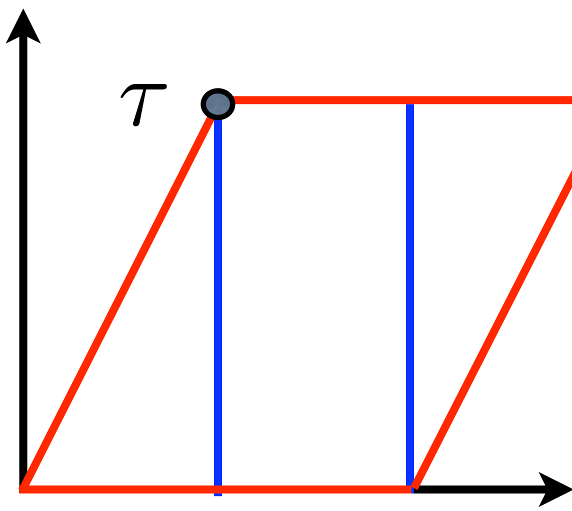


Make a Möbius strip:

$\tau = \frac{1}{2} + it, \quad t \in \mathbb{R}$

Identify under $I(z) = 1 - \bar{z}$



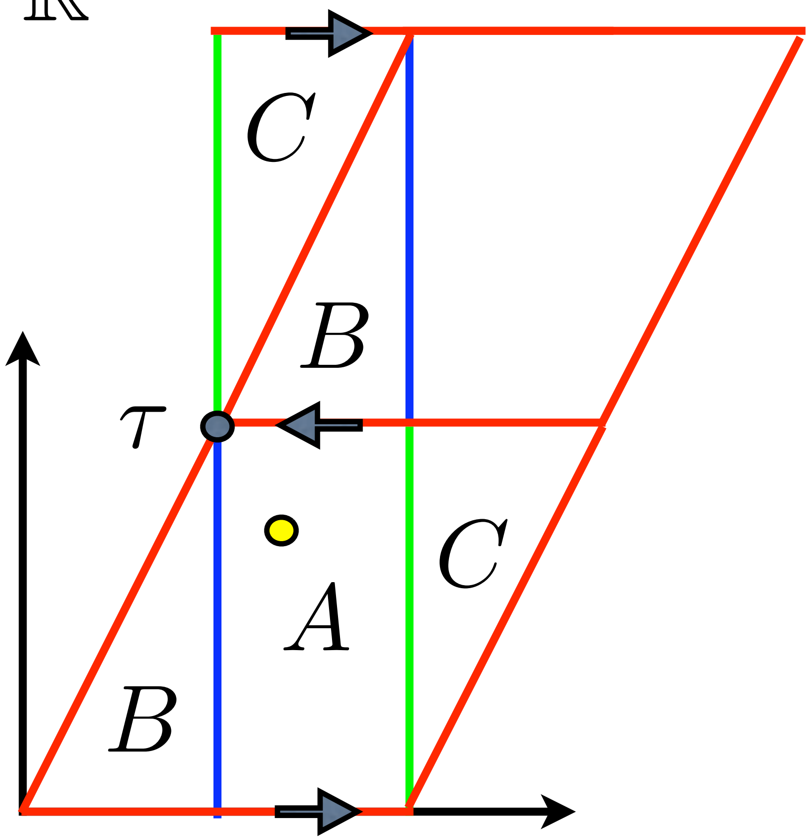


fixed lines = boundaries

Involutions of the (worldsheet) torus

e.g. Angelantonj, Sagnotti '02

$\tau = \frac{1}{2} + it, \quad t \in \mathbb{R}$

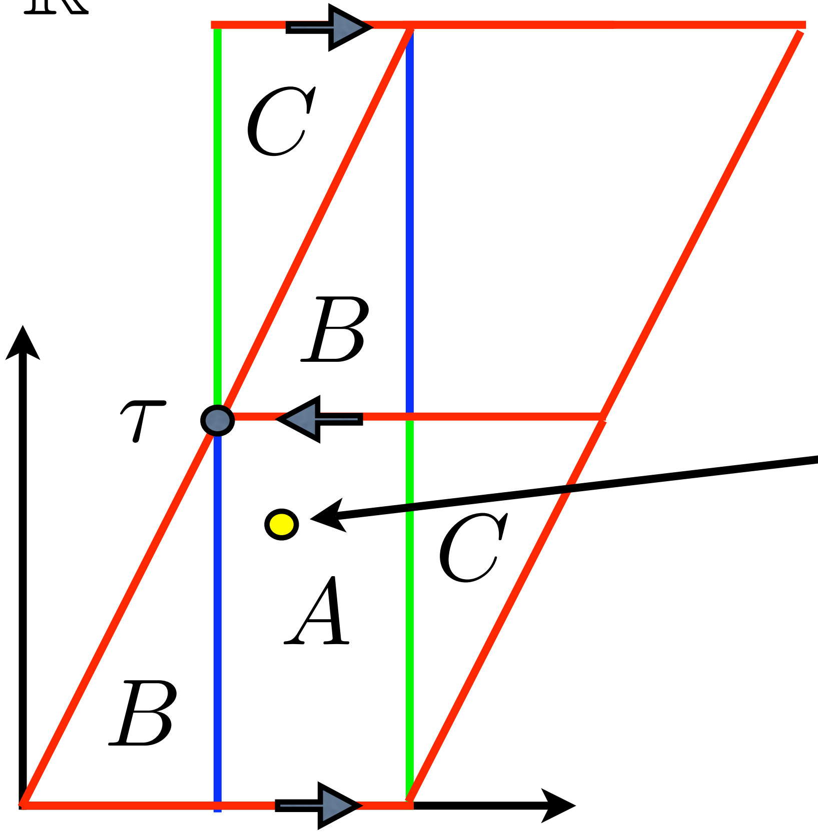


Indeed, this only has one boundary,
and left and right change as we go up

Involutions of the (worldsheet) torus

e.g. Angelantonj, Sagnotti '02

$\tau = \frac{1}{2} + it, \quad t \in \mathbb{R}$

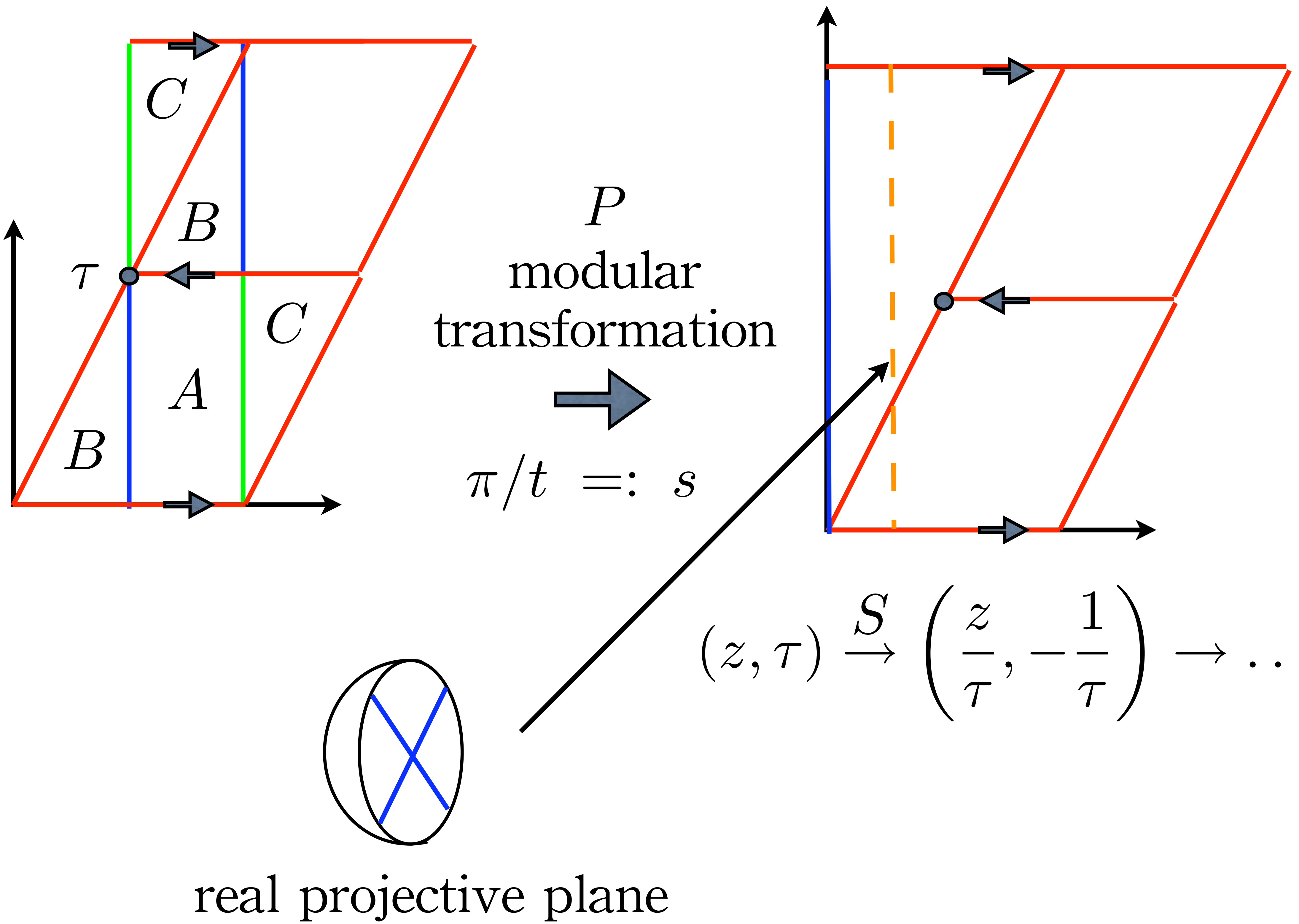


marked point z (think: worldsheet operator)

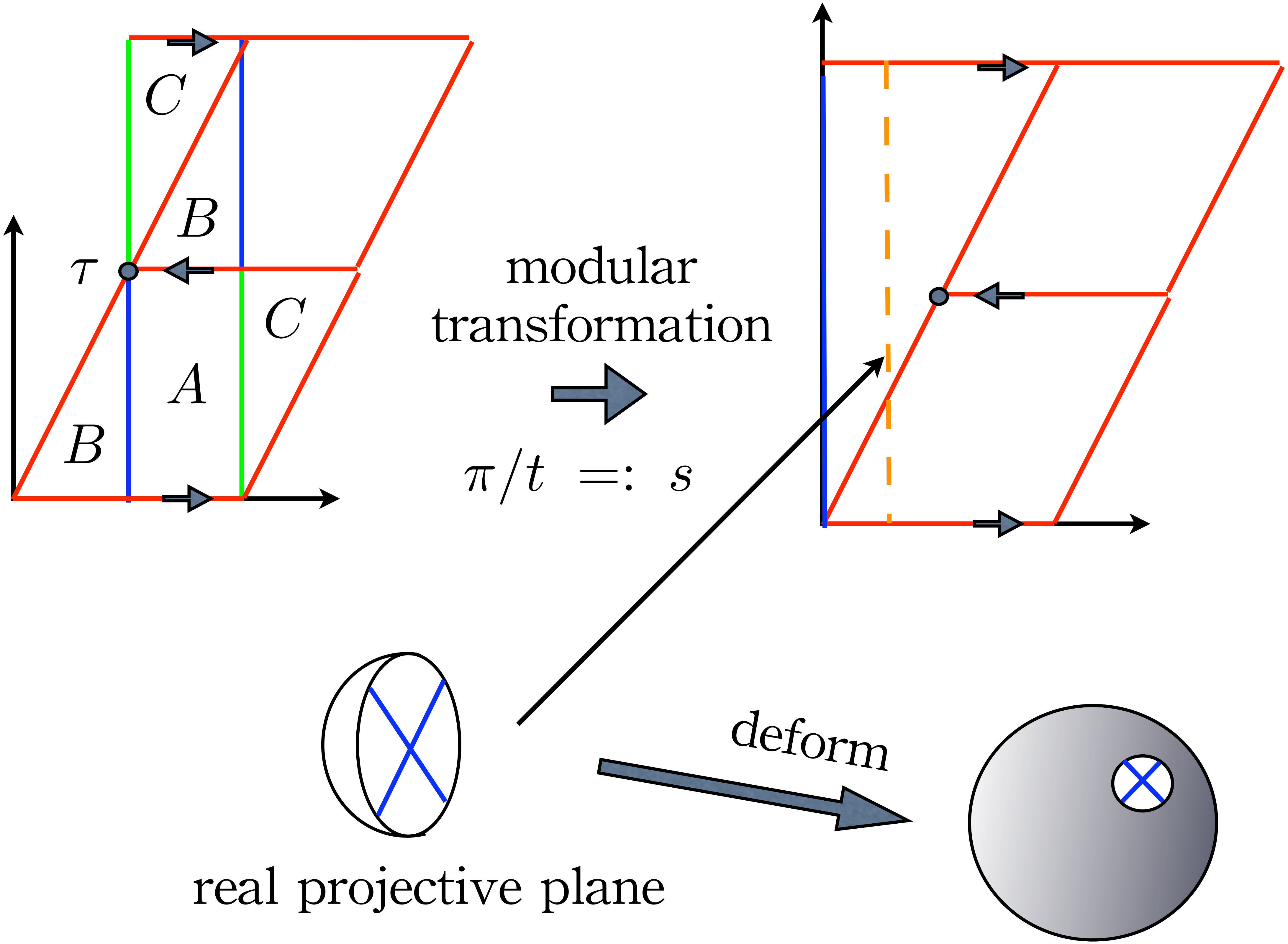
Indeed, this only has one boundary,
and left and right change as we go up

now: $(z, \tau) \xrightarrow{S} \left(\frac{z}{\tau}, -\frac{1}{\tau} \right)$

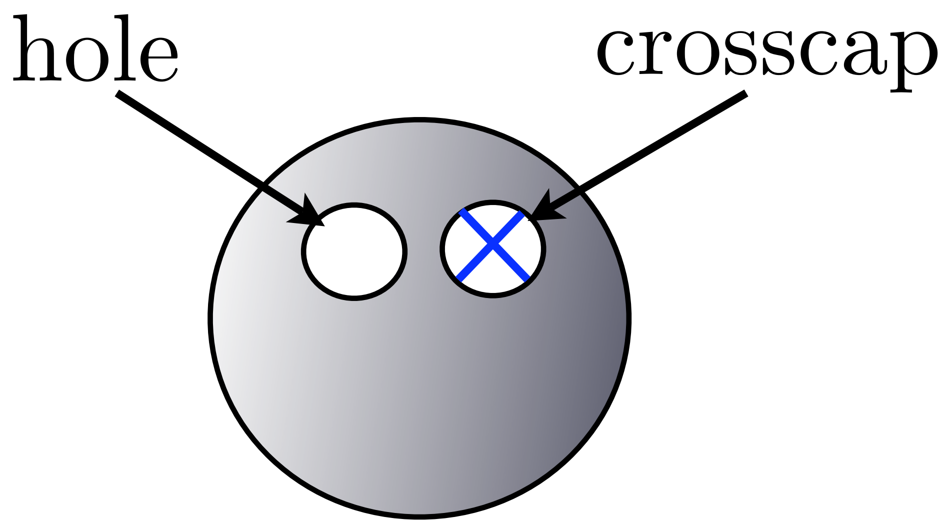
The "crosscap"



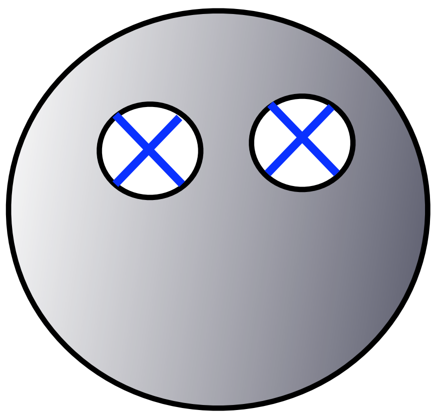
The "crosscap"



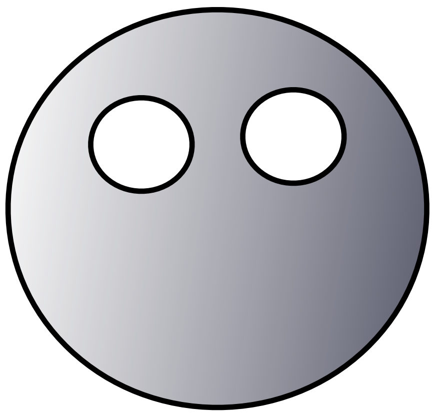
Topology I (rubber)



Möbius strip



Klein bottle



Cylinder (annulus)

organized by

$$\chi = 2 - 2h - b - c$$

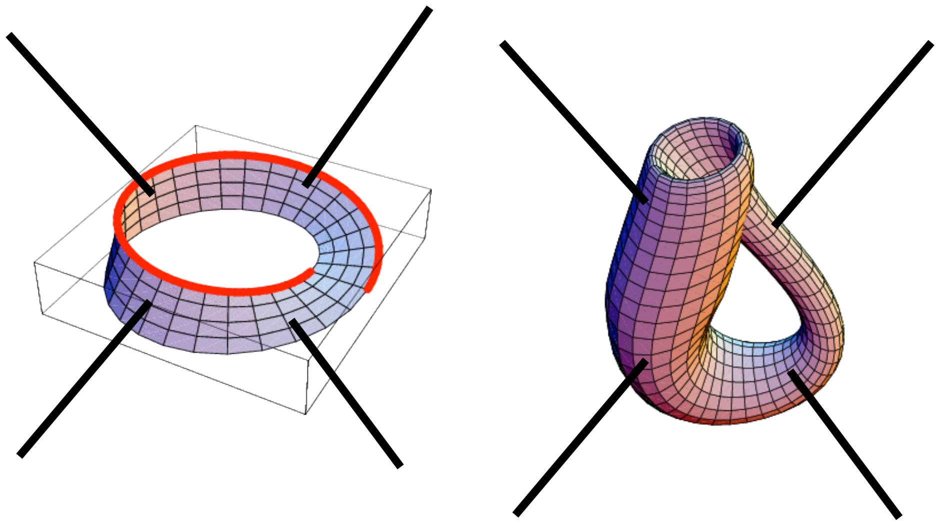
handles

boundaries

crosscaps

Now let's get more serious:

S-matrix of unoriented strings



weighted by ${g_\mathrm{s}}^{-\chi}$!

How do these "unoriented string loop amplitudes" contribute to the string S-matrix; what is the one-loop effective action?

e.g. $\dfrac{1}{g^2_{\mathrm{YM}}(\phi)}\,\mathrm{tr}\,F^2$

First question: interactions of open string endpoints

String endpoints and gauge charges

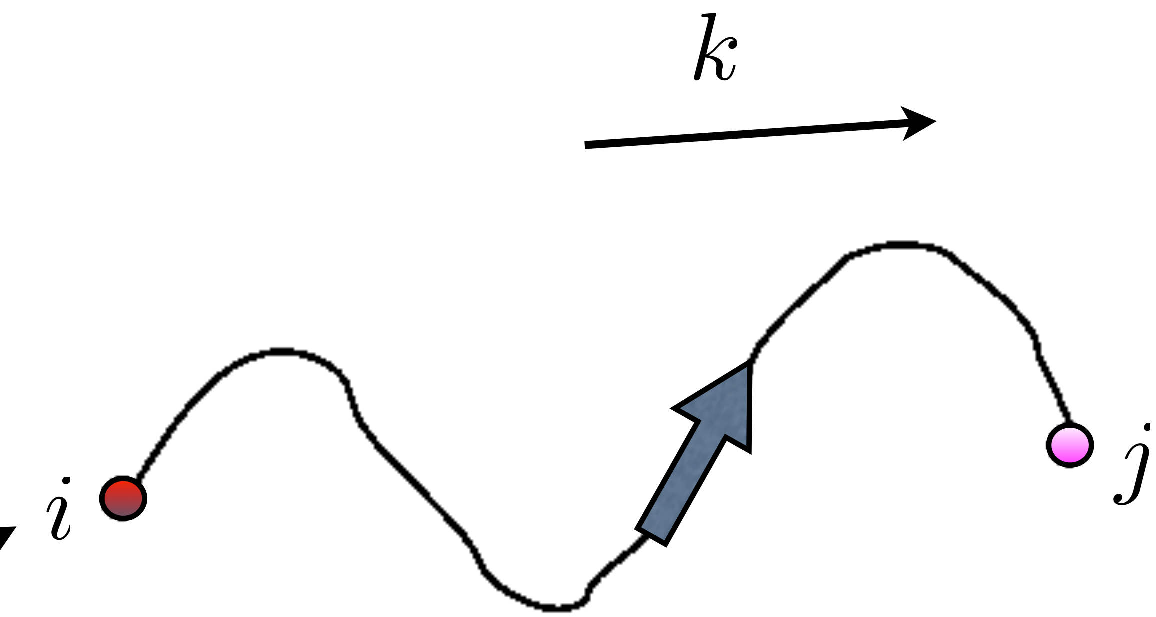
Chan, Paton '69

spacetime momentum

endpoints!

Oriented
open string state: $|N; k; ij\rangle$

number of excited oscillators



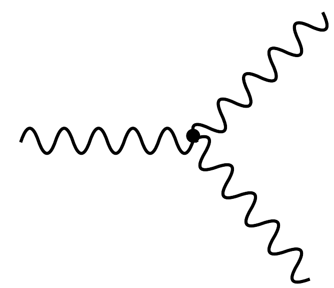
n states
(think of quark in fundamental
representation of gauge group)

$N_3 = 1$

String endpoints and gauge charges

Parity of string states without endpoints: phase

$$\Omega|N;k\rangle = \omega_N|N;k\rangle \qquad \omega_N = (-1)^{1+\alpha' m^2}$$



$= 0$ "Furry's theorem"

- Important: parity ω_N preserved by interactions

Parity of string states with endpoints: same phase?

$$\Omega|N;k;\,ij\rangle = \omega_N|N;k;\,ji\rangle \quad ?$$

Not necessarily, can even move around: matrix phases

$$\Omega|N;k;\,ij\rangle = \omega_N \gamma_{jj'}|N;k;\,j'i'\rangle\gamma^{-1}_{i'i}$$

Gauge group from parity constraint

we saw: $\Omega|N;k;\,ij\rangle = \omega_N \gamma_{jj'}|N;k;\,j'i'\rangle\gamma^{-1}_{i'i} \qquad i,j = 1\ldots n$

if we restrict to $\omega = +1 \quad \Rightarrow \quad \gamma^{\mathrm{T}} = \pm\gamma$

Gauge algebra of open string endpoints:
antisymmetric or symmetric matrices

$SO(n)$ or $Sp(n)$

Will now argue: in 10 dimensions, n is uniquely fixed!

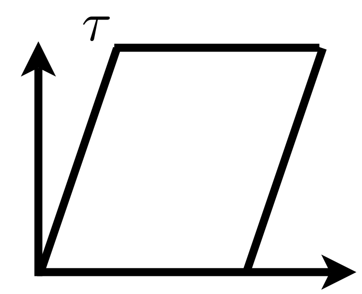
First step to S-matrix: partition function
(0-point amplitude)

Partition functions

Hamiltonian

Virasoro generator

String partition functions $\quad Z(q) \propto \mathrm{Tr}\,(q^{L_0}\bar{q}^{\tilde{L}_0})$



Ex: bosonic string

$$q = e^{2\pi i\tau}$$

$$\sum_{n=0}^{\infty} q^{nN} = \frac{1}{1-q^N}$$

Polchinski p.209

$$\prod_{N=0}^{\infty}\left(\sum_{n=0}^{\infty} q^{nN}\right) = \frac{1}{\prod_{N=0}^{\infty}(1-q^N)} = \frac{1}{q^{-1/24}\eta(q)}$$

Dedekind η function

Partition functions

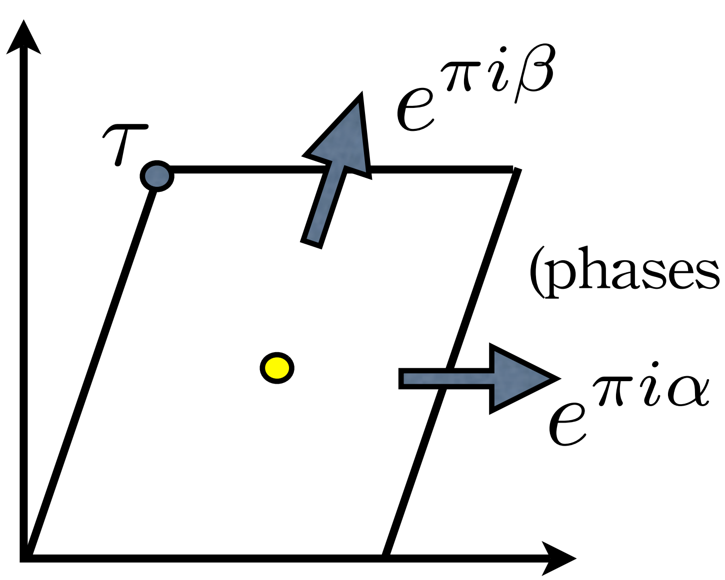
String partition functions $Z(q) \propto \mathrm{Tr}\,(q^{L_0}\bar{q}^{\tilde{L}_0})$

Ex: superstring

$q = e^{2\pi i\tau}$

Jacobi ϑ function

$$\mathrm{Tr}_{\alpha,\beta}(q^{L_0}) = \frac{\vartheta[{}^{\alpha/2}_{\beta/2}](0,\tau)}{\eta(\tau)} =: Z^{\alpha}_{\beta}(\tau)$$



(phases for Green's functions)

Polchinski Vol 2, p.32

Tadpoles

$SO(n)$ or $Sp(n)$

$$Z_{C,0} \quad \propto \quad n^2 \int_0^\infty ds \; \underbrace{\eta(is/\pi)^{-8} \left[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4 \right]}_{\stackrel{s\to\infty}{\longrightarrow} \; 16 + \mathcal{O}(e^{-2s})}$$

Cylinder

Length of cylinder

$\pi/t \; =: \; s$

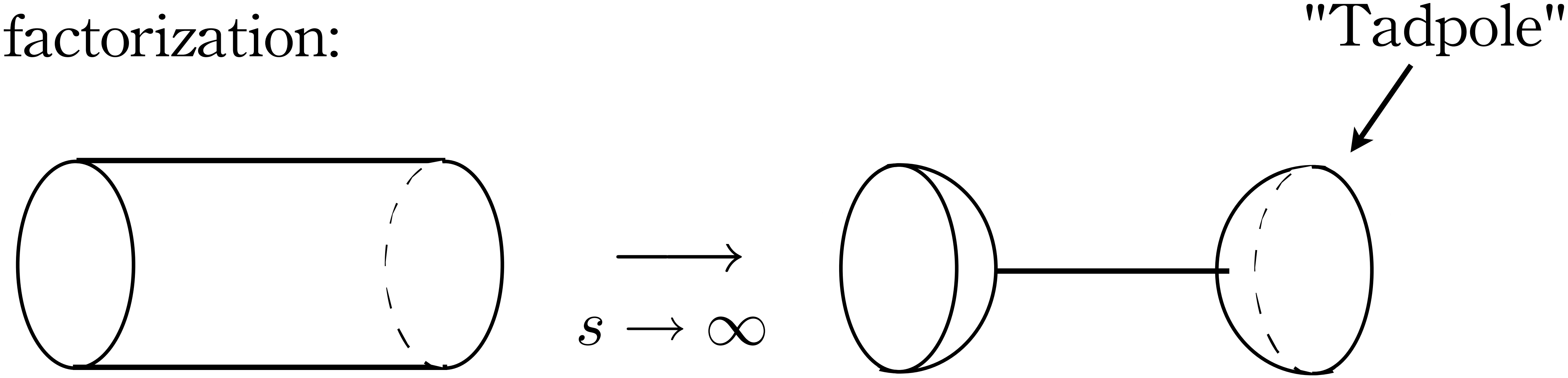
Tadpoles

$$Z_{C,0} \quad \propto \quad n^2 \int_0^\infty ds \underbrace{\eta(is/\pi)^{-8} \left[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4 \right]}$$

$$\stackrel{s \to \infty}{\longrightarrow} \quad 16 + \mathcal{O}(e^{-2s})$$

$$\frac{1}{p^2} = \int_0^\infty ds \;\; e^{-sp^2}$$

factorization:



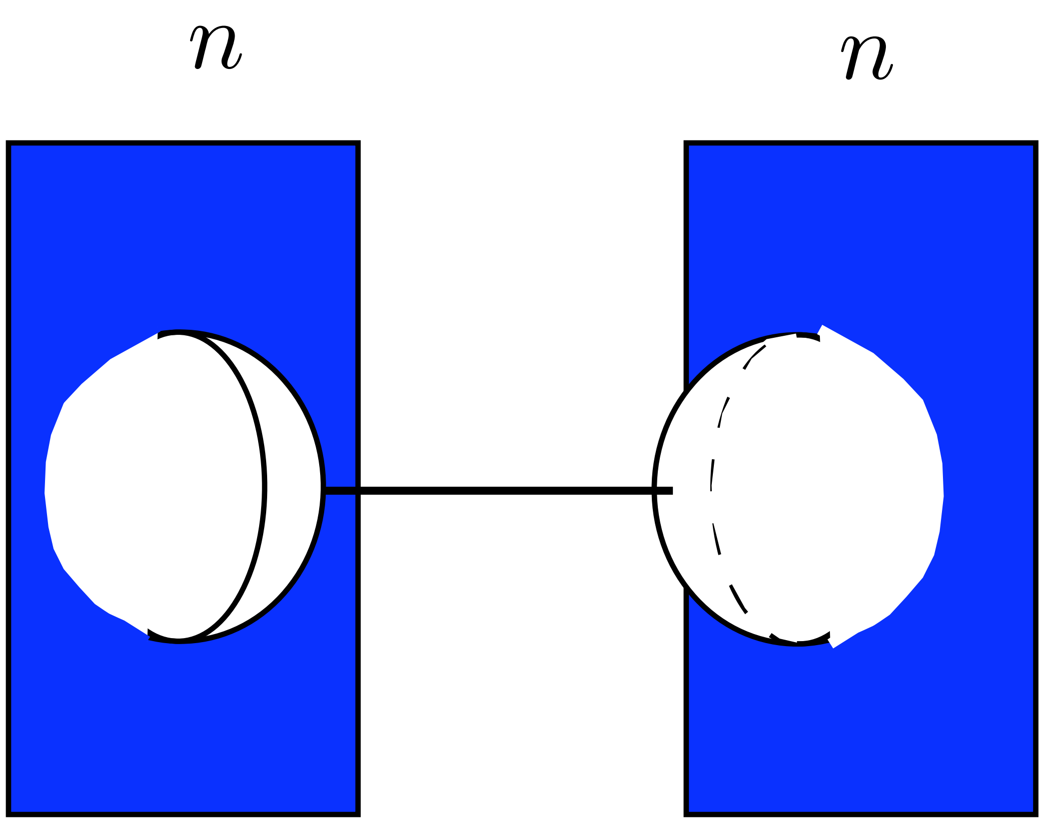
$$\frac{1}{p^2} \; \rightarrow \; \frac{1}{0} = \infty$$

Tadpoles

$SO(n)$

$Z_{C,0} \quad \propto \quad n^2$

"n D-branes emitting and reabsorbing closed strings"



Tadpoles

$SO(n)$

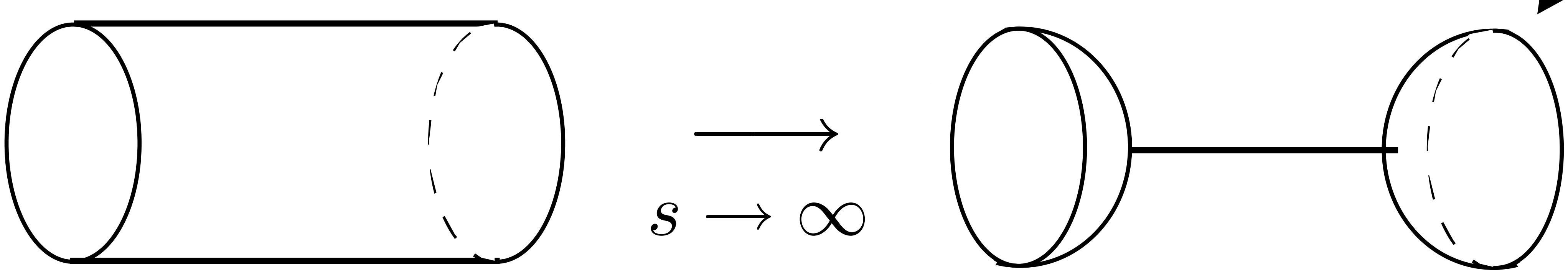
$$Z_{C,0} \quad \propto \quad n^2 \int_0^\infty ds \, \underbrace{\eta(is/\pi)^{-8} \left[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4 \right]}$$

$$\xrightarrow{s \to \infty} \; 16 + \mathcal{O}(e^{-2s})$$

$$\frac{1}{p^2} = \int_0^\infty ds \;\; e^{-sp^2}$$

factorization:

"Tadpole"



$$\frac{1}{p^2} \;\to\; \frac{1}{0} = \infty$$

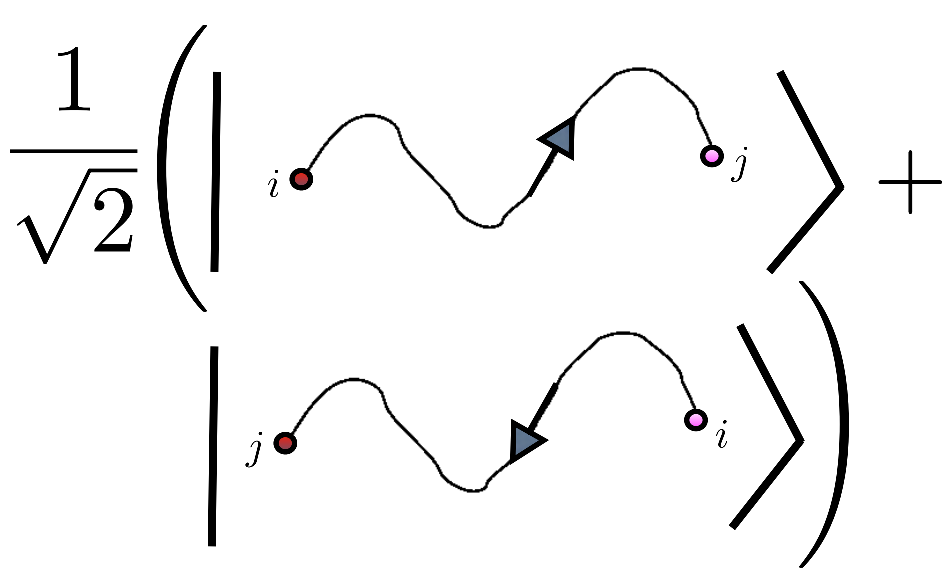
Unoriented partition functions

Closed strings symmetric w.r.t. left- and right-moving oscillations

Can project onto states with e.g. $\Omega = +1$

$$
\begin{aligned}
Z(q) &\propto \mathrm{Tr}\,(q^{L_0}\bar{q}^{\tilde{L}_0}) \\
&\rightarrow \mathrm{Tr}\,(\frac{1+\Omega}{2} q^{L_0}\bar{q}^{\tilde{L}_0}) \\
&= \frac{1}{2}\mathrm{Tr}\,(q^{L_0}\bar{q}^{\tilde{L}_0}) + \frac{1}{2}\mathrm{Tr}\,(\Omega q^{L_0}\bar{q}^{\tilde{L}_0})
\end{aligned}
$$

cf. left-handed electron:

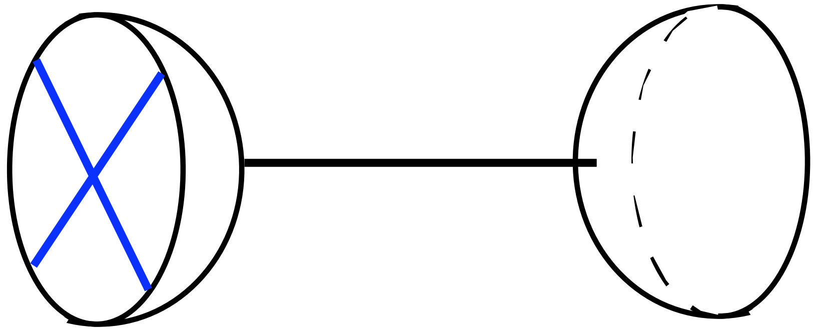


noun: "An orientifold (theory)"
verb: "To orientifold a theory"

Tadpoles

$$Z_{M,1} \overset{s\to\infty}{\longrightarrow} \pm 2^5 \cdot 2n \int_0^\infty ds\,(16 + \mathcal{O}(e^{-2s}))$$

$\mathrm{Tr}(\Omega q^{L_0} \ldots)$



real projective plane

$$\frac{1}{p^2} \to \frac{1}{0} = \infty$$

For SO, comes with different sign (minus)
than previous (cylinder) amplitude!

Tadpole cancellation

$$Z_{C,0} \quad \propto \quad n^2 \int_0^\infty ds \, \underbrace{\eta(is/\pi)^{-8} \left[Z_0^0(is/\pi)^4 - Z_1^0(is/\pi)^4 \right]}$$

$$\xrightarrow{s\to\infty} \; 16 + \mathcal{O}(e^{-2s})$$

$$Z_{M,1} \quad \to \quad \pm 2^5 \cdot 2n \int_0^\infty ds \, (16 + \mathcal{O}(e^{-2s}))$$

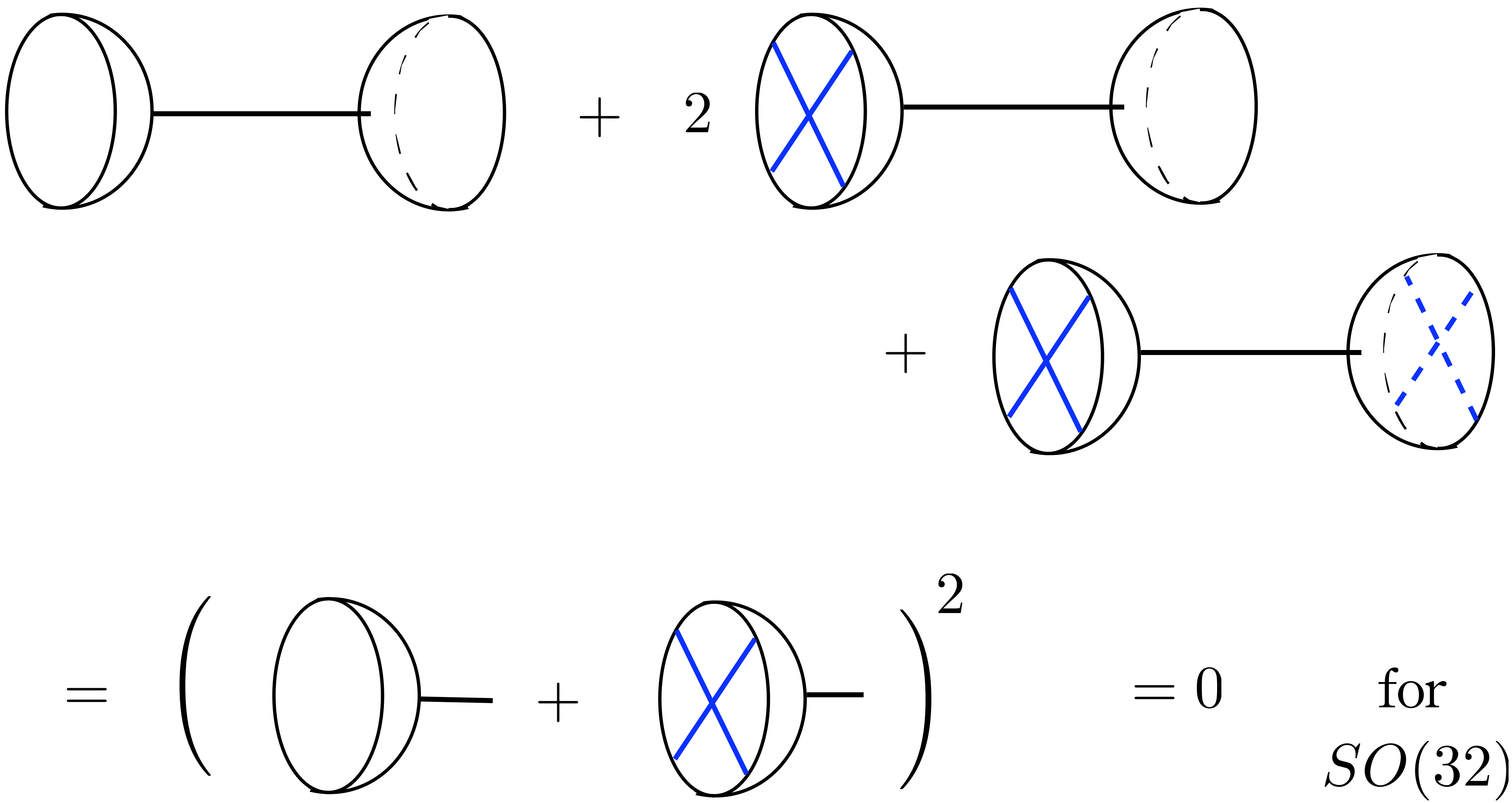
$$Z_{K,0} \quad \to \quad 2^{10} \int_0^\infty ds \, (16 + \mathcal{O}(e^{-2s}))$$

Total coefficient of divergence:

$$n^2 + 2^{10} \pm 2^5 \cdot 2n = (n \pm 32)^2$$

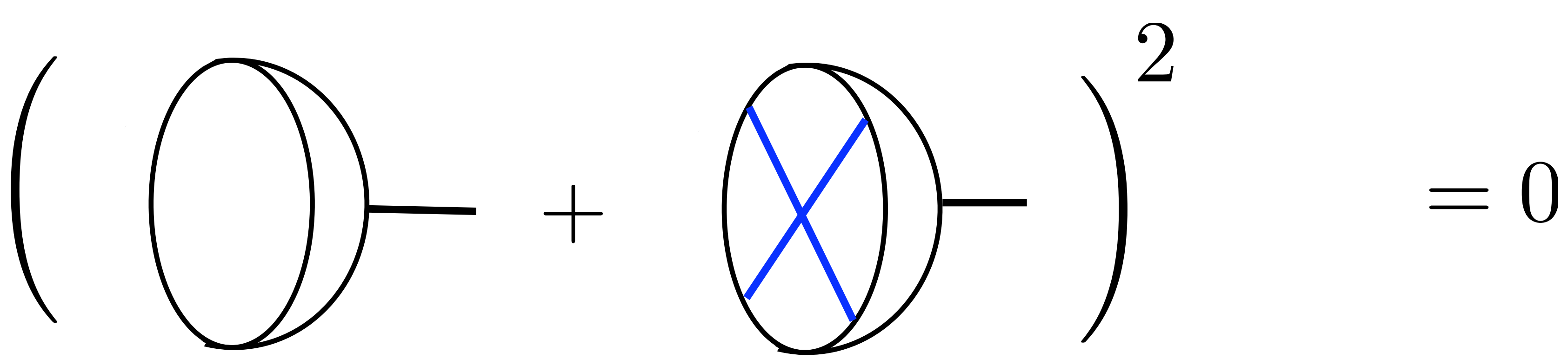
Tadpole cancellation

Divergences, schematically:



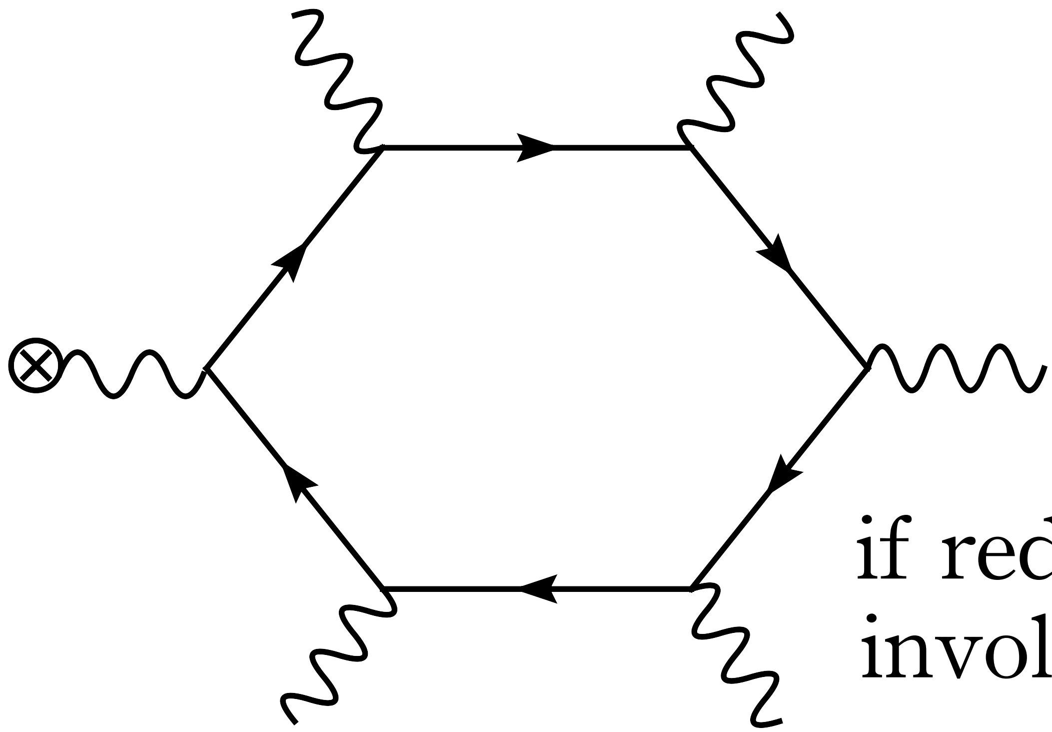
Tadpole cancellation

Note that for oriented open strings, we had no chance!



Tadpole cancellation and anomalies

e.g. Green-Schwarz-Witten, vol. 2, p. 148



QFT anomaly in 10d:
need hexagon diagram

if reducible, can cancel against lower traces involving other fields (Chern-Simons terms)

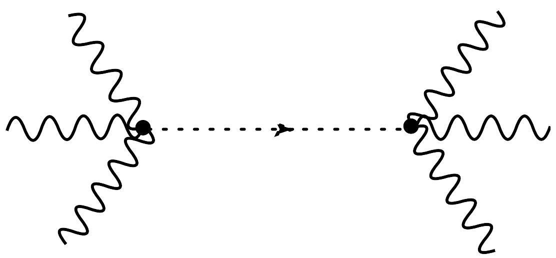
$$\mathrm{Tr}_{\mathrm{a}}(t^6) = (n-32)\mathrm{Tr}_{\mathrm{v}}(t^6) + 15\mathrm{Tr}_{\mathrm{v}}(t^2)\mathrm{Tr}_{\mathrm{v}}(t^4)$$

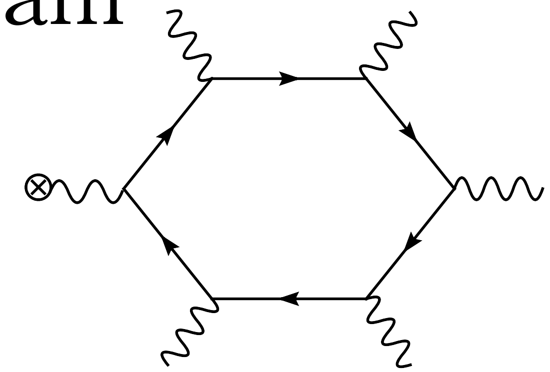
$SO(32)$ here too! What is going on?

Tadpole cancellation $\Rightarrow$ anomaly cancellation!

Green, Schwarz '84

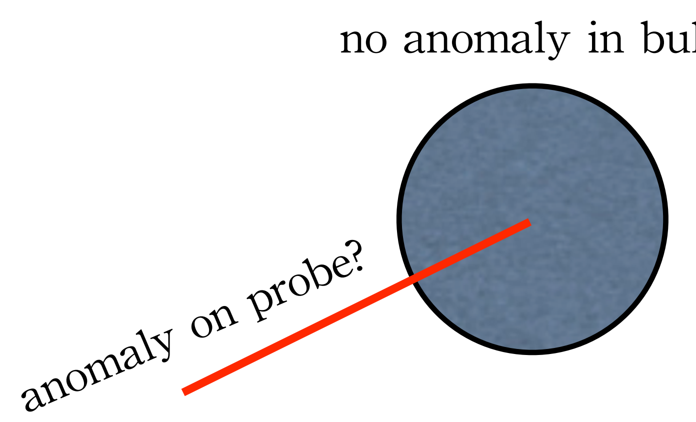
Chern-Simons cancellation terms come from different degeneration limits of a single string diagram





tadpole cancellation implies even more in dimensions below 10....
"twisted tadpole cancellation": "K-Theory"

Witten '98



So far: worldsheet parity: "everywhere in spacetime"
is that all there is to orientifold geometry?

T-Duality

Ex: boson on a circle,
Z has symmetry:

$$R \leftrightarrow \frac{\alpha'}{R}$$

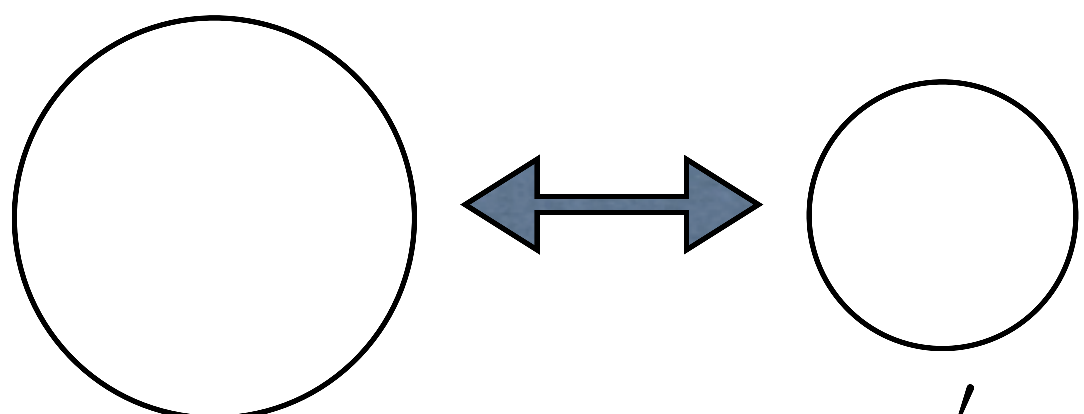
← (String length)^2

$$n \leftrightarrow w$$

← Winding number

Momentum
in compact direction
(quantized!)

Polchinski, p. 247



R $\quad$ $\frac{\alpha'}{R}$

T-Duality

Take one embedding coordinate (= map worldsheet to 1D):

$$X(z,\bar{z}) = X_L(z) + X_R(\bar{z})$$

in a compact direction:

$$p_{\mathrm{L}} = \frac{n}{R} + \frac{wR}{\alpha'}$$

$$p_{\mathrm{R}} = \frac{n}{R} - \frac{wR}{\alpha'}$$

Polchinski p. 236

T-dual: $X'(z,\bar{z}) = X_L(z) - X_R(\bar{z})$

Same theory in another variable!

Worldsheet parity / spacetime parity

now:

$$X'(z,\bar{z}) = X_L(z) - X_R(\bar{z})$$

means the "T-dual worldsheet parity" acts as

$$\Omega' : \quad X'(z,\bar{z}) \longleftrightarrow -X'(\bar{z},z)$$

so in the T-dual theory

$$\Omega' = \Omega P$$

also: Hassan '99

Def: orientifold plane in spacetime = fixed locus of Ω'

Worldsheet parity / spacetime parity

now:

$$X'(z, \bar{z}) = X_L(z) - X_R(\bar{z})$$

means the "T-dual worldsheet parity" acts as

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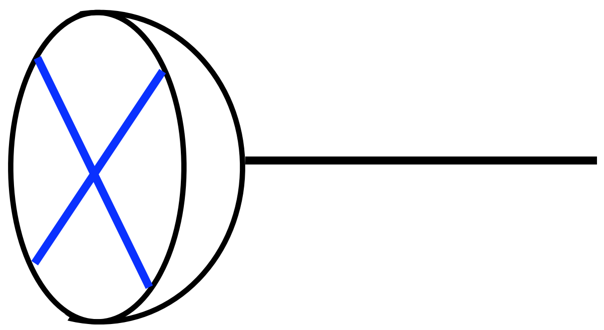
also: Hassan '99

Def: orientifold plane in spacetime = fixed locus of Ω'

in other words, not just "O9-planes",
but also "O8-planes", "O7-planes", ...

What is an orientifold plane?

Well, what does "what" mean?



Effective descriptions

"summarize"

Ex: Elementary mechanics
Angular momentum effective potential

"summarize"

Ex: Quantum field theory
Coleman-Weinberg effective potential

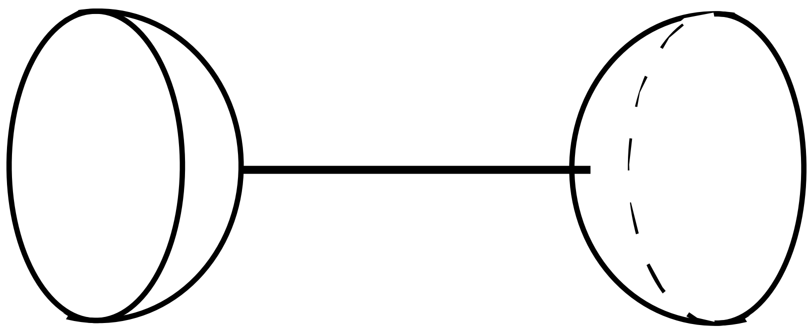
Effective descriptions: D-branes

- Logic: effective descriptions for each probe,
  each parameter range

- For extended probes, there may not be a simple description in terms of ordinary geometry!

The Dp-brane potential

Polchinski, Vol.2, p. 157



$\Longrightarrow V(\phi)$

now: physical separation ϕ

Gauge theory <u>on</u> D-brane: interpret moduli space geometrically

Orientifold theories: full D-brane $V_{\rm eff}$ not known

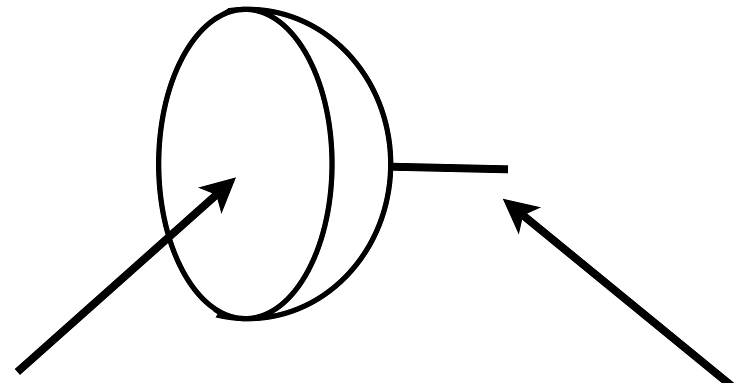
(typically only minimally supersymmetric)

Effective descriptions: D-branes

D-branes have tension (energy density)

$N \gg 1$ D-branes $\Rightarrow$ appreciable gravitational field

Then, makes sense to ask about Dp-brane metric (any p)



$$
\begin{aligned}
ds^2 &= Z(r)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z(r)^{1/2}dx^m dx^m \\
e^{2\Phi} &= Z(r)^{(3-p)/2}
\end{aligned}
$$

$$Z(r) = 1 + \frac{\rho^{7-p}}{r^{7-p}}\ ;\ r^2 = x^m x^m$$

$\rho \propto gN$

Effective descriptions: D-branes

D-branes have tension (energy density)

$N \gg 1$ D-branes $\Rightarrow$ appreciable gravitational field

Then, makes sense to ask about Dp-brane metric (any p)

Point: for <u>some</u> extended objects, several tried and tested effective descriptions exist

Effective descriptions: O-planes themselves

Potential? (don't move!)
Metric? (can't put arbitrarily many!)

D2-brane probes near an O6-plane:
Atiyah-Hitchin metric!

$\sim$ moduli space of gauge theory on D2-brane

Atiyah-Hitchin metric

e.g. Hanany, Pioline '00

Bianchi IX:

SO(3)-invariant one-forms

$$ds^2 = (abc)^2 dt^2 + a^2\sigma_1^2 + b^2\sigma_2^2 + c^2\sigma_3^2$$

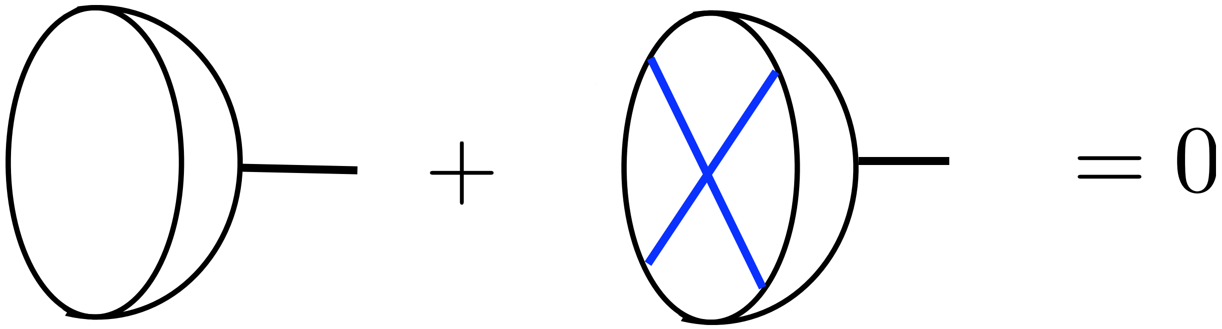
$$bc =: w_1,\ ca =: w_2,\ ab =: w_3$$

$$w_3(t) = -\frac{\pi}{6}\left(E_2(t) + \vartheta_3^4(t) + \vartheta_4^4(t)\right) \quad , \ldots$$

$$t \to 0 \implies \text{Taub-NUT of mass } -1$$

- Metric can be obtained from Toda field equation
- Not really "the orientifold metric" (only O6 probed by D2)

Negative Tension in Quantum Gravity?



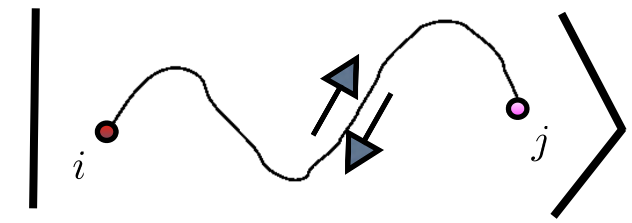
Ford '97

- Negative energy density is generically hard to make sense of in gravitational theories (cf. "Ford criterion")

- Seems to work here because there are D-branes (positive tension) "somewhere else"

- "Phenomenological" approach sometimes not very meaningful here, need full string theory

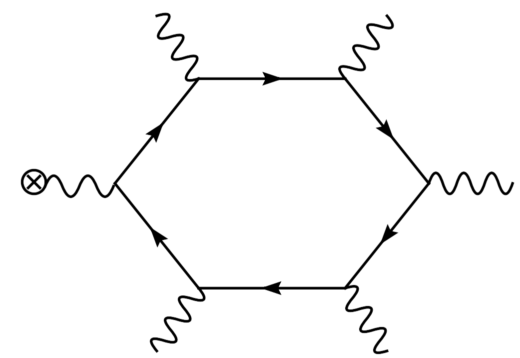
Summary so far



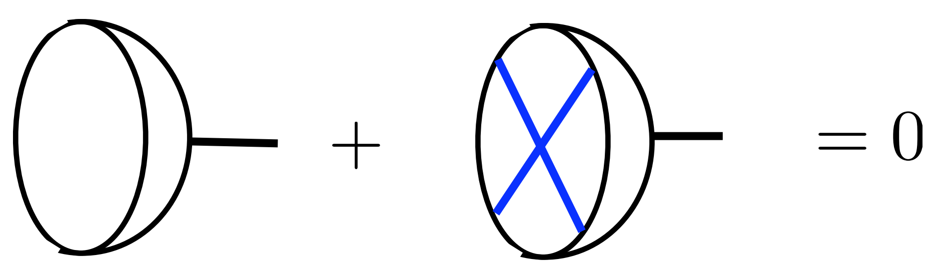
- Orientifold theories are obtained from theories with some orientation-reversing symmetry (e.g. closed strings)

- Tadpole cancellation is a powerful constraint on these theories

- It implies anomaly cancellation (and more)



- Total charge and tension cancelled between D-branes and O-planes



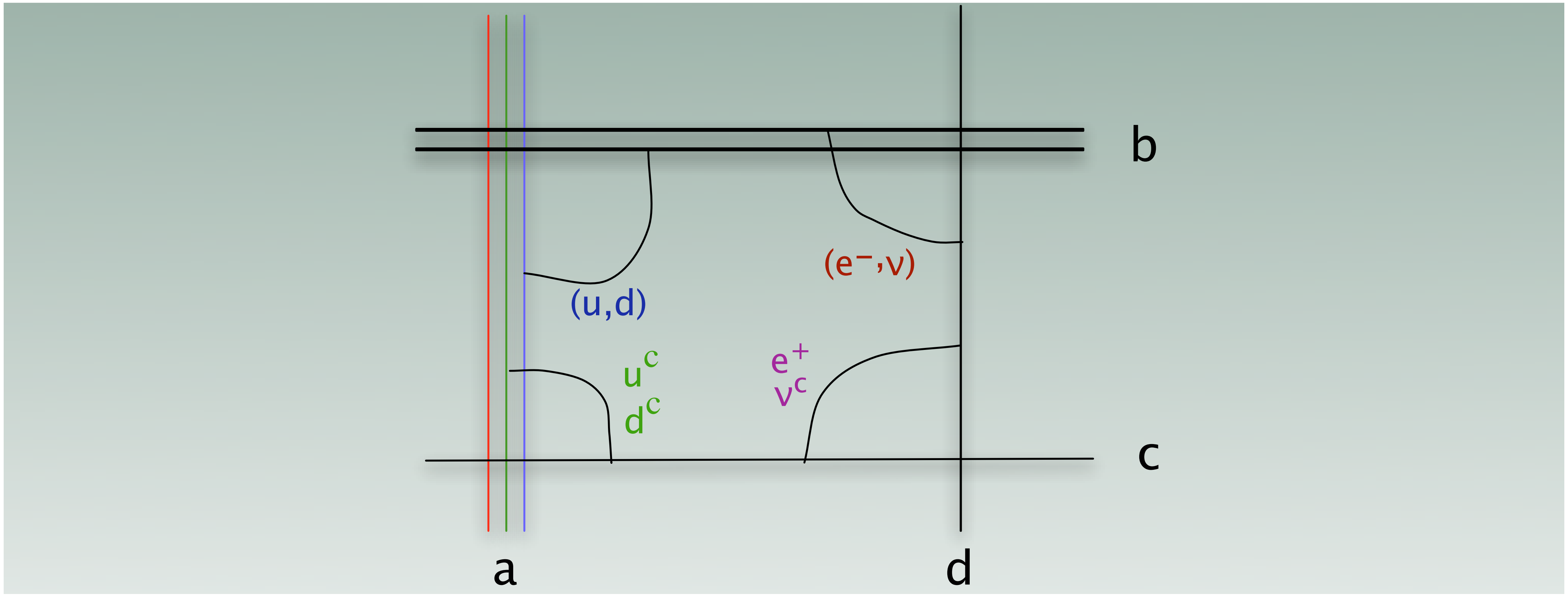
Now: a few applications

Application I: MSSM orientifolds

Model-building: simplest D-brane models never contain only three leptonic doublets

e.g. Zwiebach '03

Schellekens



Orientifolds do!

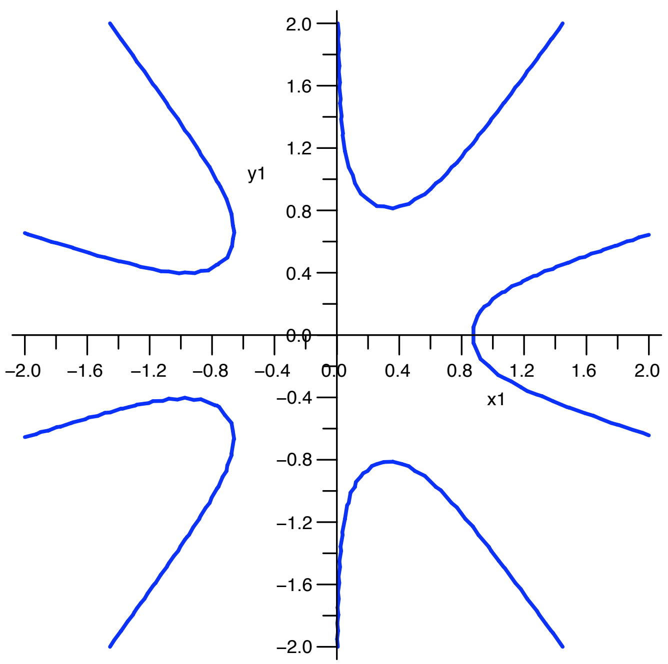
Ibanez, Marchesano, Rabadan '01

Denef, Douglas, Florea '04
...
...
Blumenhagen, Moster, Plauschinn,
Nov 21, '07

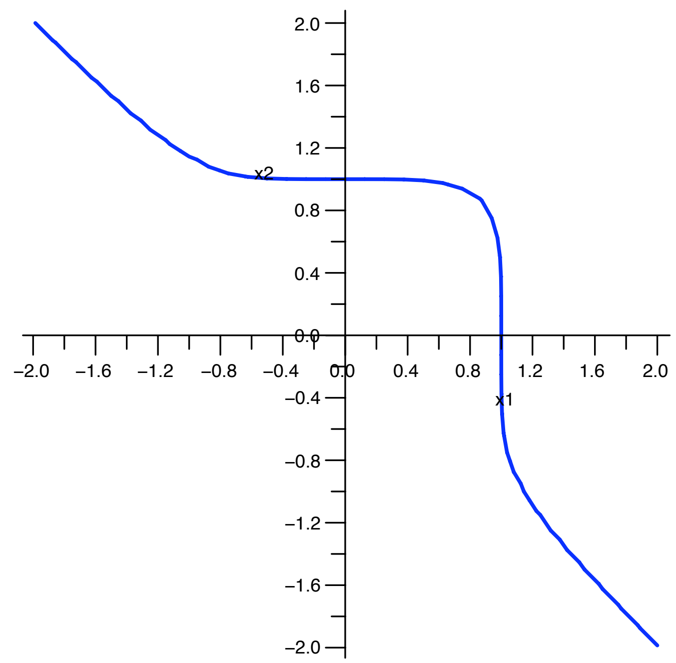
Application II: "KKLT" orientifold

nontrivial real slices through complex manifolds, e.g. quintic

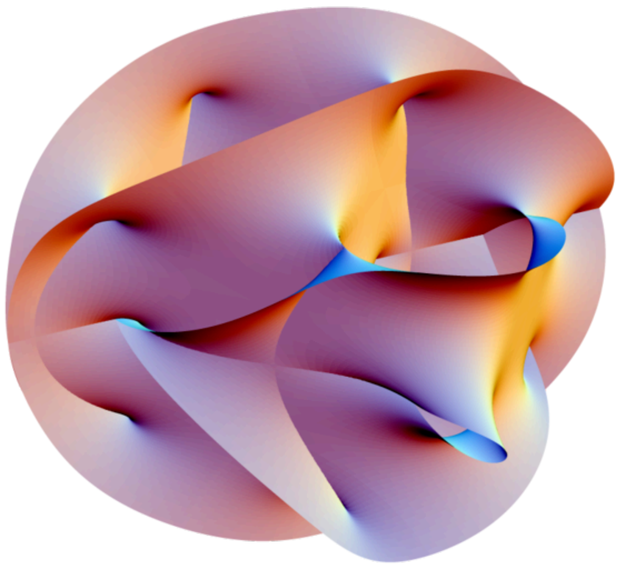
$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = c$$



oriented strings



unoriented strings



"generalization of manifolds to the
point of view of
unoriented open strings probing them"

cancelled tension = no cosmological constant at tree level!
("no-scale model")

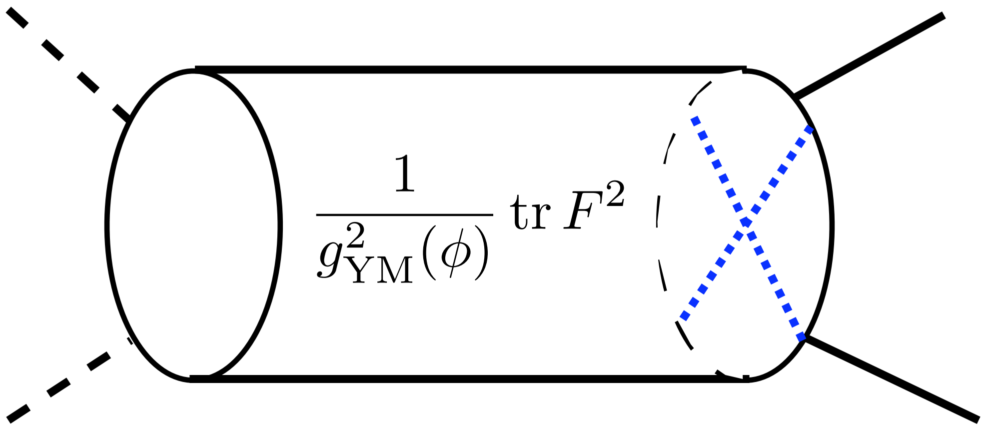
Application III: Cosmological singularity in string theory

- Many attempts at modelling very early universe in simple string models
- Most run into problems — e.g. Horowitz, Polchinski '02
- Lorentzian orientifolds seem promising — Cornalba, Costa, Kounnas '02; Cornalba, Costa '03; ...

non-orientifolds: Boost, Shifted–boost, Null–boost

Orbifold	b	$\mathcal{E}$	Result
Boost	$\sqrt{\frac{2p^-}{p^+}}\,\lvert C^+\rvert$	$\sqrt{2p^+p^-}\,e^{\pi\Delta n}$	Unstable
Shifted–boost	$2\pi R n$	$\sqrt{2p^+p^-}\,e^{\pi\Delta n}$	Unstable
Null–boost	$2C$	$\lvert p^-\rvert 2\pi\Delta n$	Unstable
O–plane	$\frac{2}{3}(\pi n)^2\Delta R$	$\lvert p^-\rvert 2\pi\Delta n$	Stable

Future work: Green's function method



loop amplitudes
in non-toroidal orientifolds

in some models: inflaton

Sum over eigenfunctions of Laplacian, integrate
$\Rightarrow$ V_{eff} of D-brane probe

- Has been done for conifold (cone over $T^{1,1}$)
- Compact case: K3 manifold?
  Atiyah-Hitchin?

M. B, Haack, Kors '04
Giddings, Maharana '05
Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan '06

Condensed matter	**"Applied string theory"**
(several attempts in the past)	
here → Conductivity	Holography (extensions of AdS/CFT)
	↕ close connection
Field theories (e.g. graphene)	Direct D-brane constructions

Prerequisite: my previous KoF talk on AdS/CFT!

Ohm's law at strong coupling?

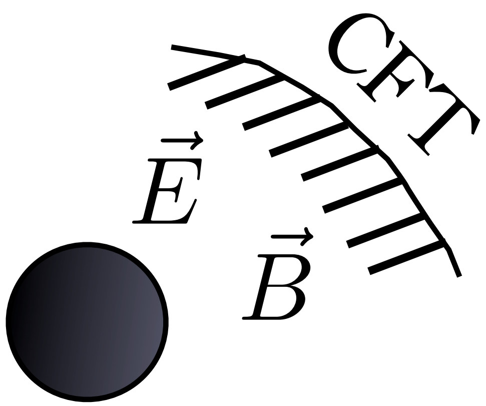
Hartnoll, Kovtun, Mueller, Sachdev '07

Hartnoll, Herzog '07

Dyonic black hole in AdS_4

$\Updownarrow$ AdS/CFT duality

2+1 CFT with charge density
and background magnetic field



S duality acting on conductivity $\quad \sigma \rightarrow -\dfrac{1}{\sigma}$

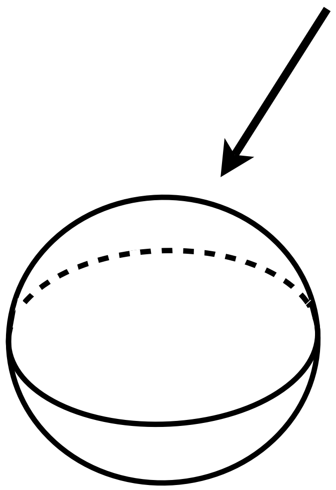
Underlying R-symmetry: $SO(8)$

History: symmetry reduction in AdS/CFT

In AdS/CFT (and more generally): e.g. Klebanov, Strassler '00

- Start with maximal symmetry (SUSY, conformal, ...)
- Find various ways to reduce it (e.g. Calabi-Yau)

"geometrize" symmetries of boundary theory (e.g. R-symmetry)



e.g. $AdS_5 \quad \times \quad S^5 \qquad (SO(6))$

Symmetry reduction in AdS/CFT

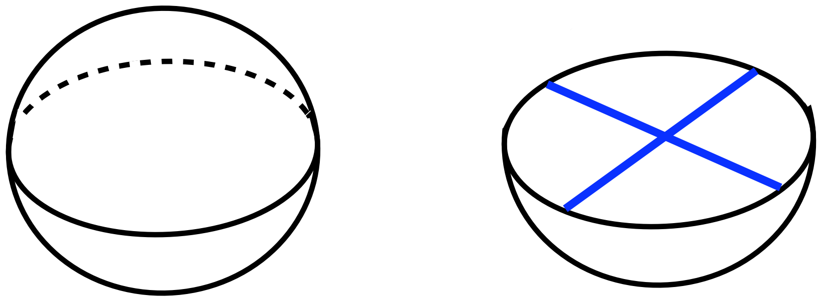
In AdS/CFT (and more generally):

- Start with maximal symmetry (SUSY, conformal, ...)
- Find various ways to reduce it (e.g. Calabi-Yau)

Fayyazuddin and Spalinski '98
Witten '98

Orientifold the extra dimensions

1/N expansion involves nonorientable surfaces (odd powers of 1/N !)



$$S^5 \quad \rightarrow \quad RP^5$$

Like this, could e.g. orientifold the 3+1 dyonic black hole times 7-sphere

Summary

- Orientifold theories are obtained from theories with some orientation-reversing symmetry (e.g. closed strings)
- Tadpole cancellation is a powerful constraint on these theories
- It implies anomaly cancellation (and more)
- Cancelling tensions gives "no-scale" model at string tree-level
- It might be interesting to symmetry-reduce existing condensed matter "applications"

Summary

Orientifolds are interesting
... and there is much left to be understood!