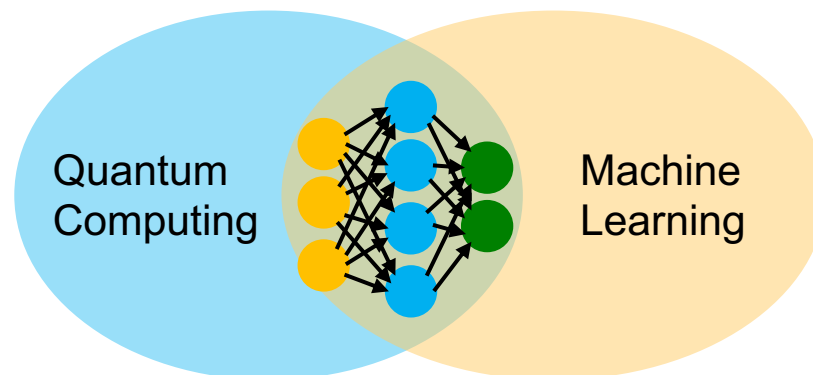




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Quantum Machine Learning on NISQ hardware



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Dipartimento di Fisica, Università di Pavia (IT)

**INT online program on «Scientific Quantum Computing
and Simulation on Near-Term Devices» – 26/10/2020**

The NISQ era

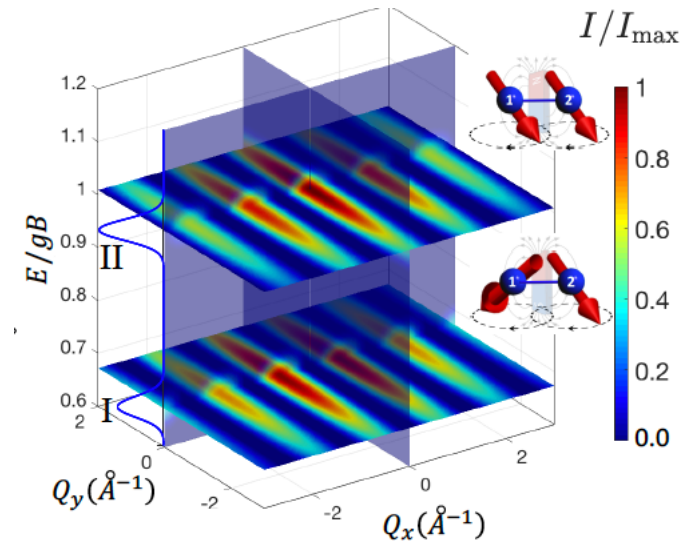
What can we do with near-term quantum computing hardware?

→ Any practical quantum advantage to be expected?

➤ Quantum Simulations

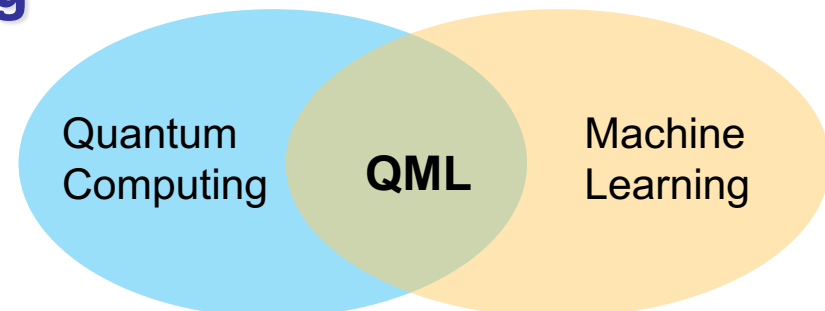
E.g.: inelastic neutron scattering cross section from magnetic molecules simulated on IBM-Q

Chiesa et al., *Nature Physics* **15**, 455 (2019)



➤ Quantum Machine Learning

new knowledge at the forefront of classical and quantum computing
→ potential for *practical* advantage



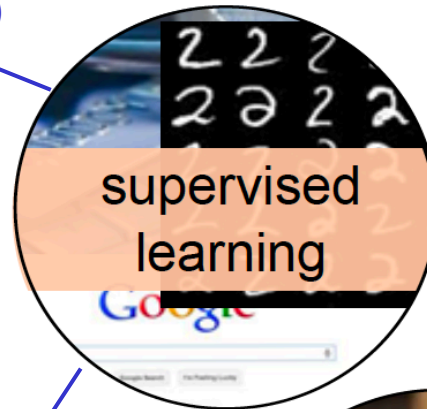
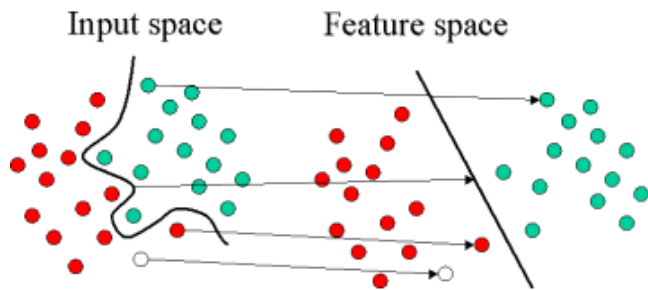
Machine Learning

ML is based on finding suitable mathematical models (functions) mapping **input data** into **output predictions**

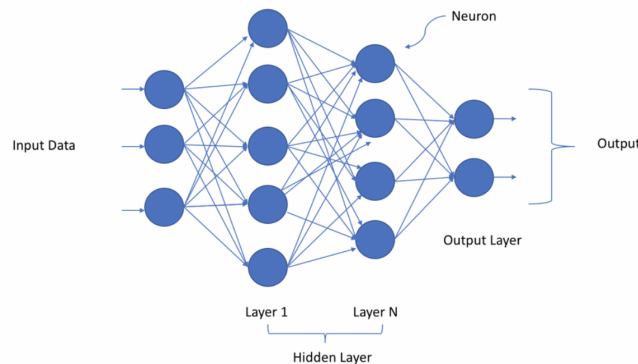


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E.g., support vector machines (SVM)



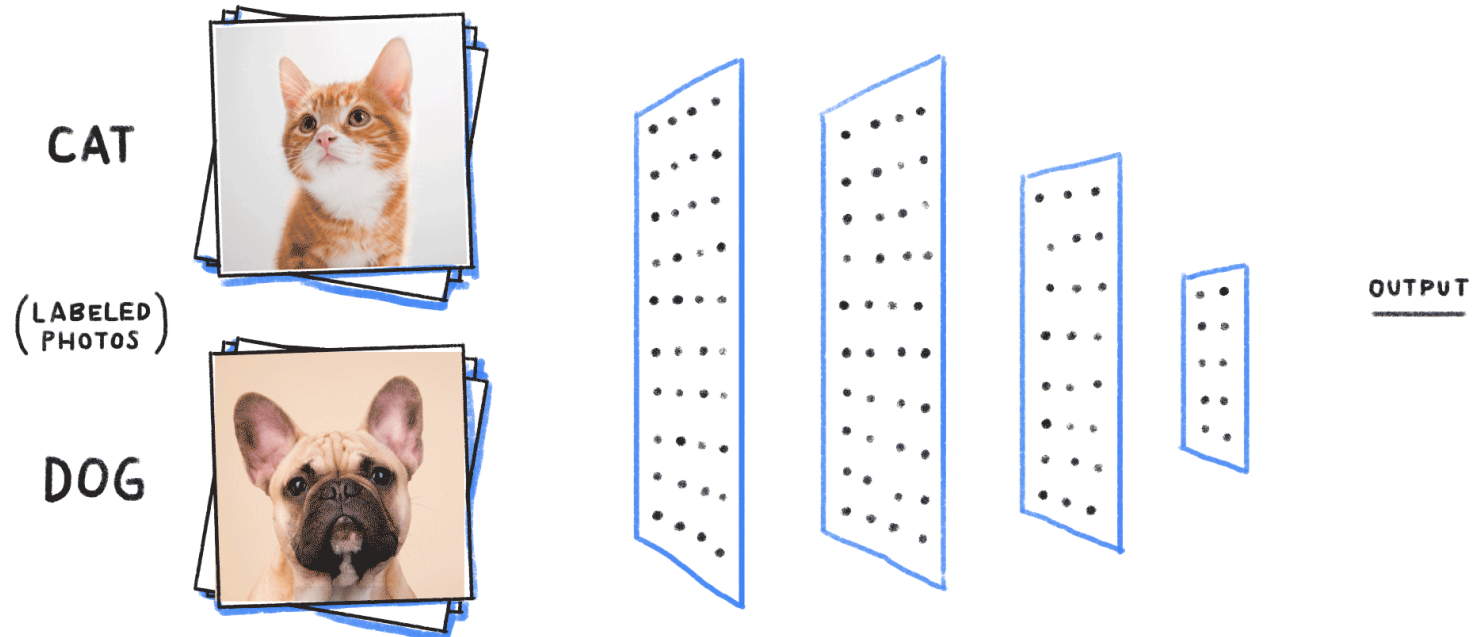
E.g., deep neural networks (DNN)



D. Silver et al.,
Nature **550**, 354 (2017)

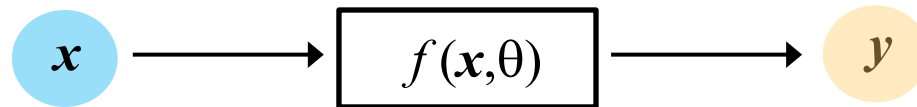
An example of supervised learning

- Image classification through training of a DNN



Credits: becominghuman.ai/building-an-image-classifier-using-deep-learning-in-python-totally-from-a-beginners-perspective-be8dbaf22dd8

- In general:



ML task is to learn how f maps x into y , on varying θ , such that the algorithm can correctly predict y upon being fed with previously unknown x



Quantum Machine Learning

Applying quantum computing resources to ML tasks



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		Type of Algorithm	
		<i>classical</i>	<i>quantum</i>
Type of Data	<i>classical</i>	CC	CQ
	<i>quantum</i>	QC	QQ

Schuld & Petruccione, *Supervised learning with Quantum Computers* (Springer, 2018)

Biamonte et al., *Quantum Machine Learning*, Nature **549**, 195 (2017)

Kernel methods



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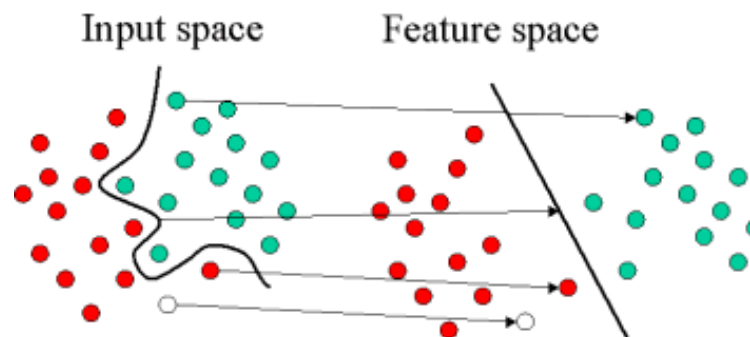
- Map data into a much larger ‘feature’ space

$$\mathbf{x} \longrightarrow \phi(\mathbf{x})$$

- Define the kernel (inner product)

$$k(x, x') \doteq \langle \phi(\mathbf{x}), \phi(\mathbf{x}') \rangle_{\mathcal{F}}$$

- Find the best separating hyperplane in the higher dimensional feature space (through the given kernel)



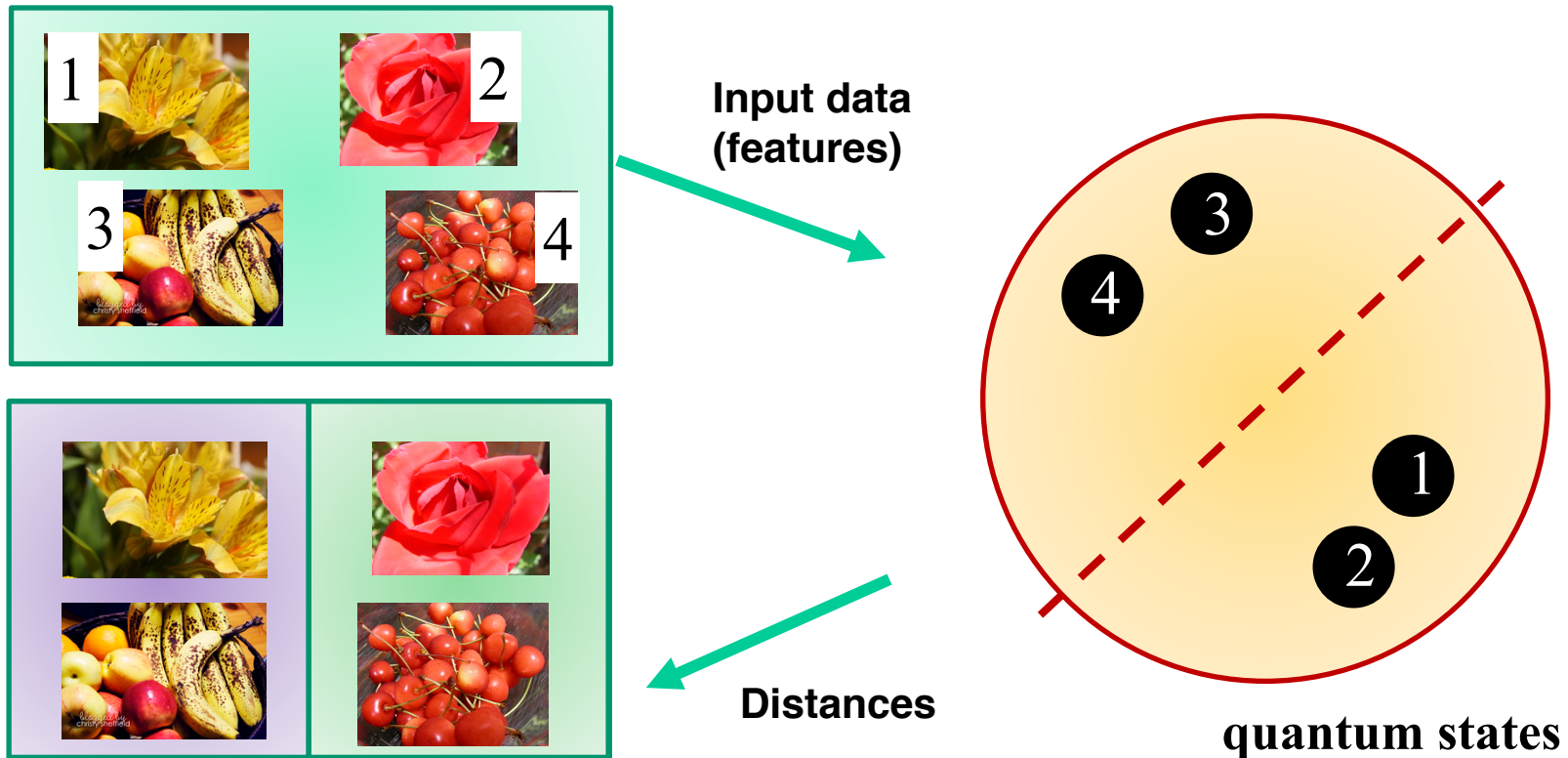
- Support vector machines (SVM) belong to this class of algorithms

Quantum SVM



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Mapping classical data to a (exp large) feature space and finding distances is what a quantum computer can do best



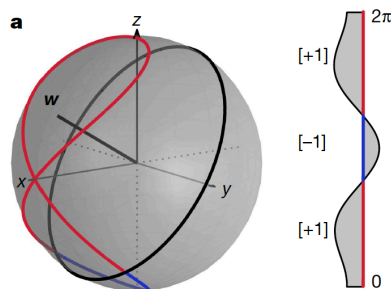
Schuld & Killoran, *Quantum machine learning in feature Hilbert spaces*,
Phys. Rev. Lett. **122**, 040504 (2019)

Schuld, *Machine learning in quantum spaces*,
Nature **567**, 179 (2019)

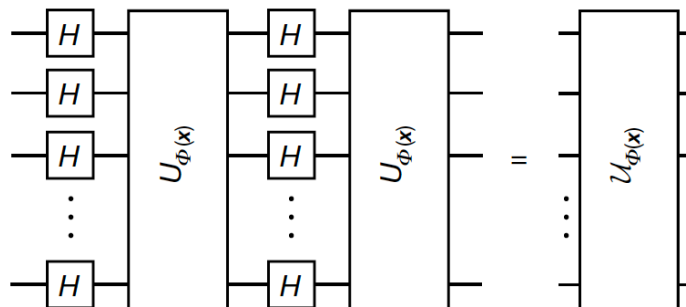
The IBM result on NISQ hardware



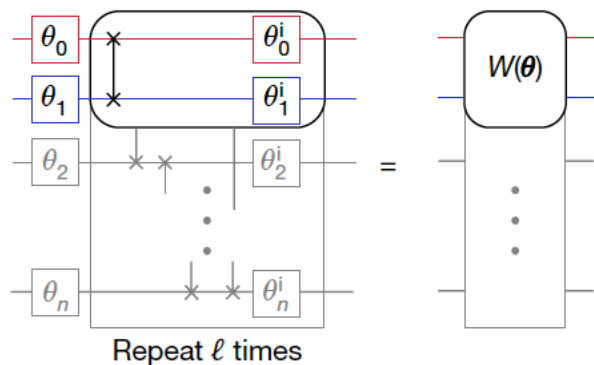
1- Binary data



2- Quantum feature map $x \rightarrow |\Phi(x)\rangle$



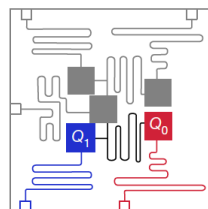
3 - Parametrized variational circuit



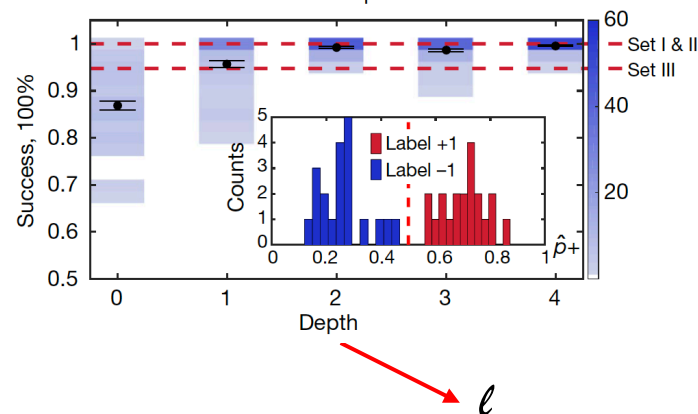
4 - measurement with binary output

$$p_y(\hat{x}) = \langle \Phi(x) | W^\dagger(\theta) M_y \hat{W}(\theta) | \Phi(x) \rangle$$

θ is trained through classical optimization algorithm



classification after training \rightarrow

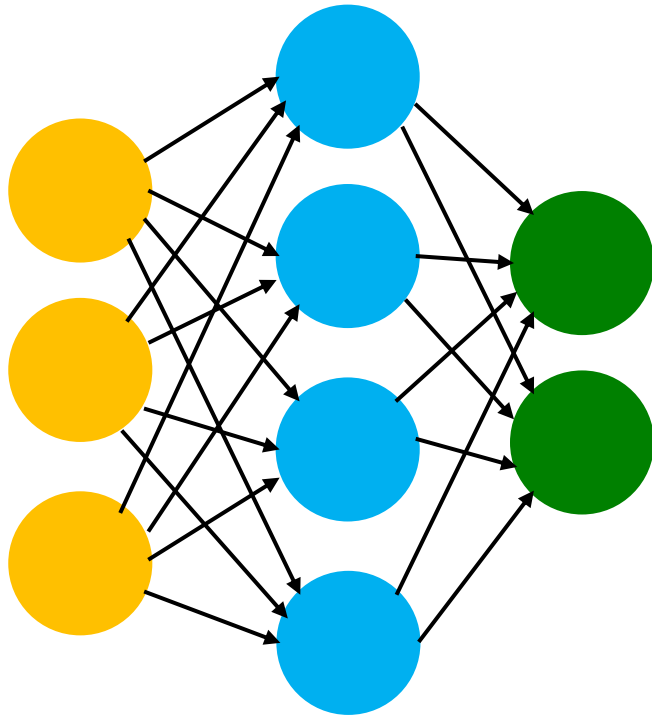


Havlicek et al.,
Nature **567**, 209 (2019)

Artificial neural networks (ANN)

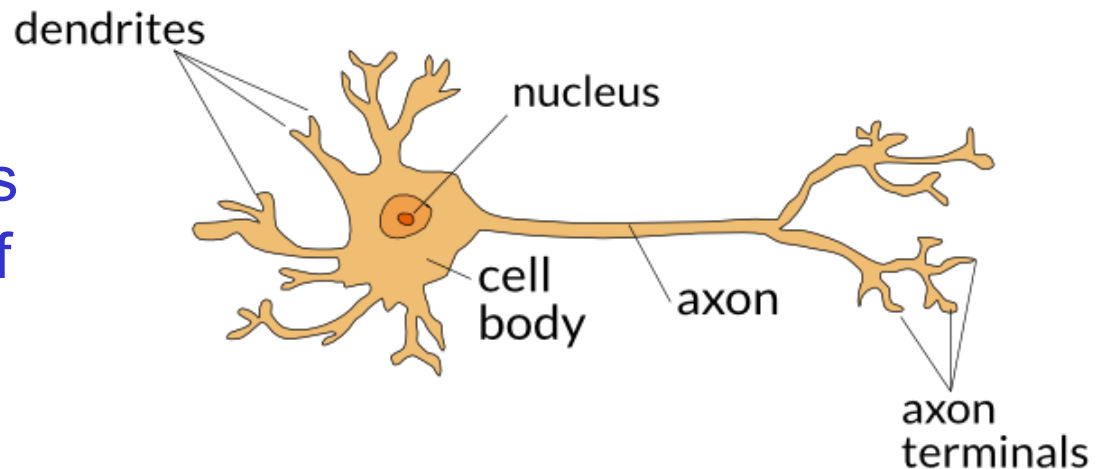


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- Basis for several AI algorithms
- applications in pattern recognition, speech recognition, classification, ...

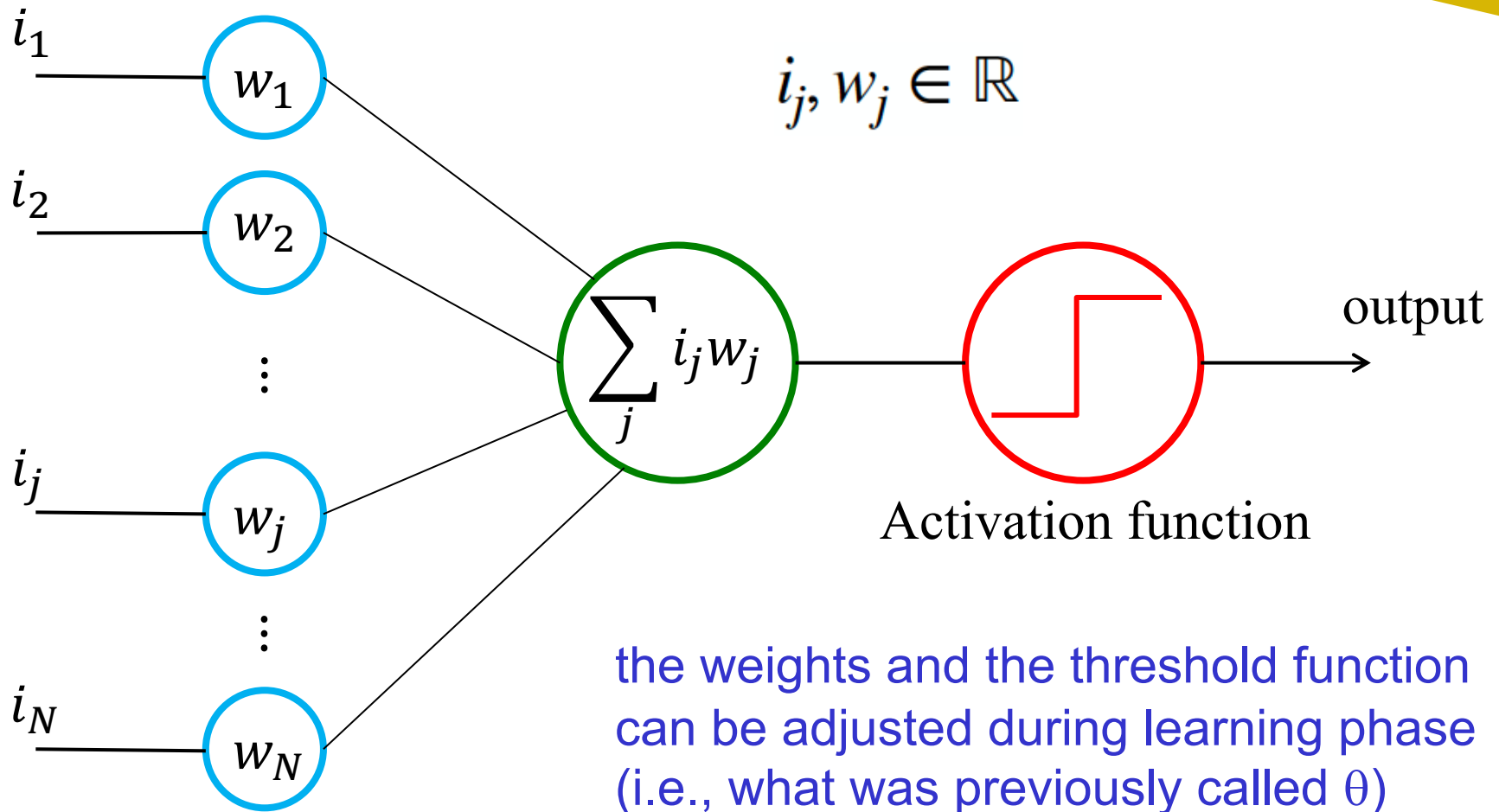
Each node mimics the functionality of a single neuron



The classical perceptron as a model of artificial neuron



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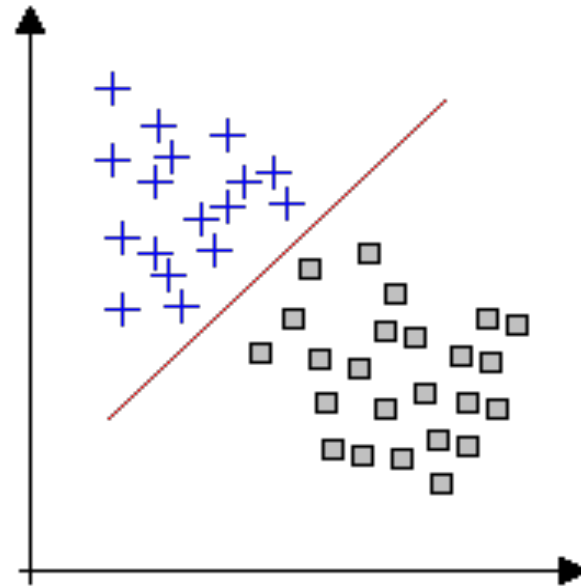


Linear classifier



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The classical perceptron is
the simplest linear classifier



It requires extension to a multilayer structure to be
able to perform nonlinear tasks

Quantum neural network models

➤ Quantum perceptrons

Schuld et al., Phys. Lett. A **7**, 660 (2015)

N. Wiebe, A. Kapoor and K. M. Svore, arXiv:1602.04799 (2016)

Y. Cao, G. G. Guerreschi and A. Aspuru-Guzik, arXiv:1711.11240 (2017)

Torrontegui et al., EPL (Europhysics Letters) **125** (2019)

...

➤ Quantum algorithms for artificial neural networks

Schuld et al., EPL **119**, 60002 (2017)

Wan et al., npj Quant Info **3**, 36 (2017)

E. Farhi and H. Neven, arXiv:1802.06002 (2018)

Rebentrost et al., Phys. Rev. A **98**, 042308 (2018)

Grant et al., npj Quant Info **4**, 65(2018)

Killoran et al., Phys. Rev. Research **1**, 033063 (2019)

Cong et al., Nature Physics (2019)

Mari et al., Quantum **4**, 340 (2020)

...

McCulloch-Pitts neurons on a quantum computer



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The key function

$$\vec{i} \cdot \vec{w} = \sum_j i_j w_j$$


➤ Encoding input and weights


$$\vec{i} = \begin{pmatrix} i_0 \\ i_1 \\ \vdots \\ i_{2^N-1} \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_{2^N-1} \end{pmatrix}$$

McCulloch-Pitts
neuron model

$$i_j, w_j = -1, +1$$


$$|\psi_i\rangle = \frac{1}{\sqrt{2^N}} \sum_{j=0}^{2^N-1} i_j |j\rangle$$


$$|\psi_w\rangle = \frac{1}{\sqrt{2^N}} \sum_{j=0}^{2^N-1} w_j |j\rangle$$

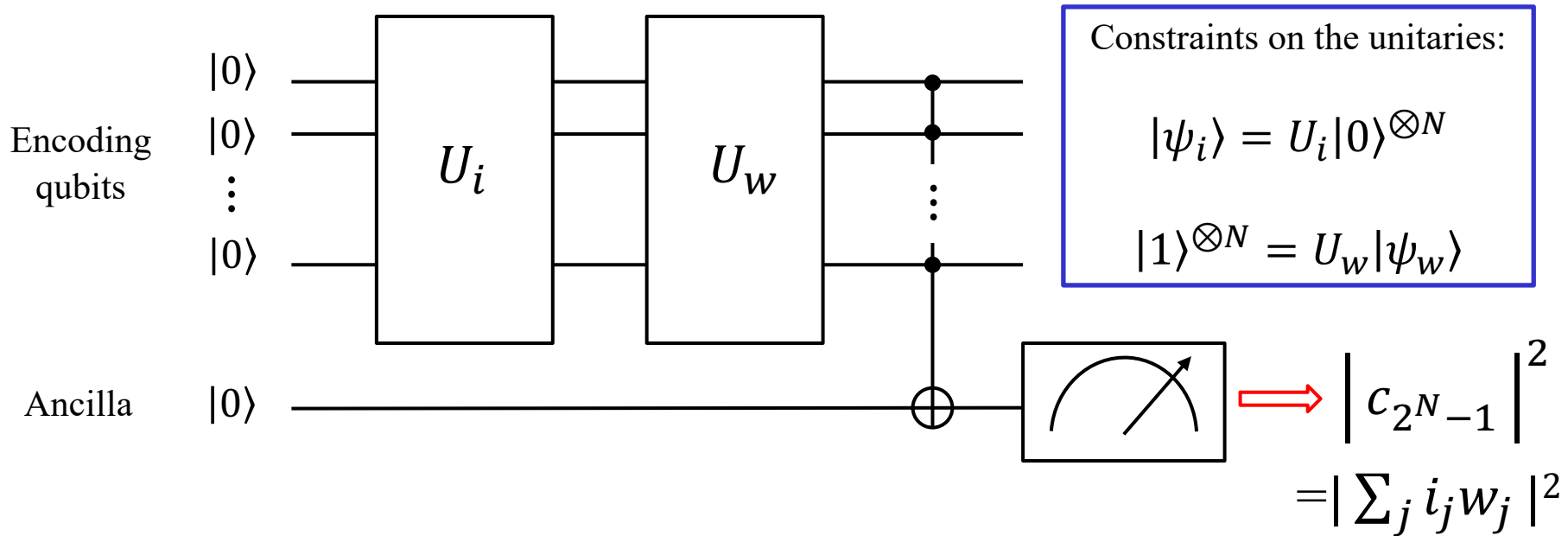
$$|j\rangle \in \{ |000\dots 00\rangle, |000\dots 01\rangle, \dots |111\dots 11\rangle \}$$

McCulloch-Pitts neurons on a quantum computer



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The quantum algorithm: a circuit model



$$|0\rangle^{\otimes N}|0\rangle_a \longrightarrow \sum_{j=0}^{2^N-2} c_j |j\rangle|0\rangle_a + c_{2^N-1} |2^N-1\rangle|1\rangle_a \quad \text{with } c_{2^N-1} = \langle \psi_i | \psi_w \rangle$$

Elementary pattern recognition



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$$N = 2$$

$$\vec{i} = \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

i_0	i_1
i_2	i_3

+1 = white

-1 = black

$$\vec{i} = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \square & \blacksquare \\ \hline \end{array}$$

$$\vec{w} = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \square & \blacksquare \\ \hline \end{array}$$

$$|\langle \psi_i | \psi_w \rangle|^2 = 1$$

It's me!

$$\vec{i} = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \square & \blacksquare \\ \hline \end{array}$$

$$\vec{w} = \begin{array}{|c|c|} \hline \square & \blacksquare \\ \hline \blacksquare & \square \\ \hline \end{array}$$

$$|\langle \psi_i | \psi_w \rangle|^2 = 1$$

It's still me!
(in negative colors)

$$\vec{i} = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \square & \blacksquare \\ \hline \end{array}$$

$$\vec{w} = \begin{array}{|c|c|} \hline \blacksquare & \blacksquare \\ \hline \square & \square \\ \hline \end{array}$$

$$|\langle \psi_i | \psi_w \rangle|^2 = 0$$

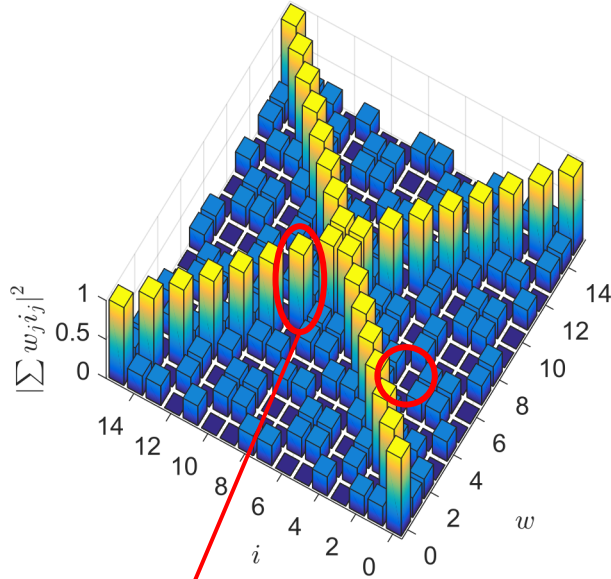
It's not me!

Running the algorithm on NISQ-hardware

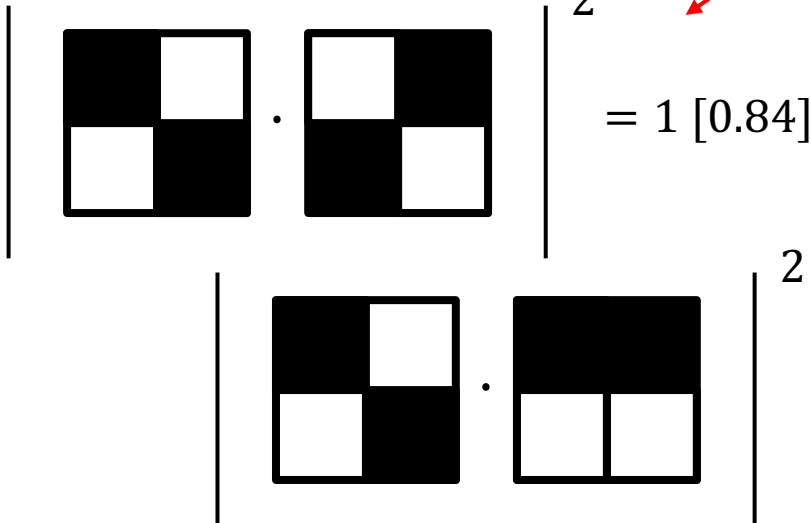
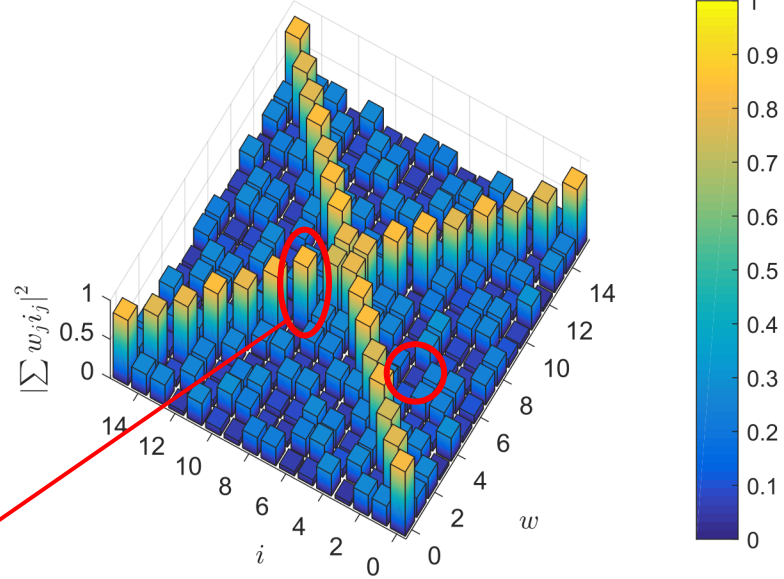


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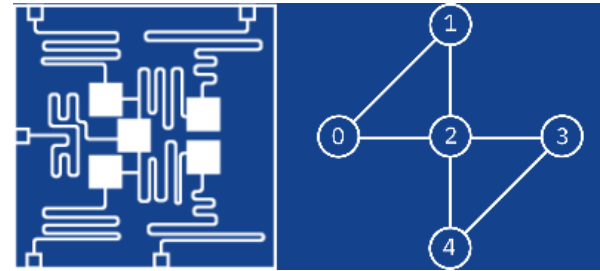
Exact result ($N = 2$)



Experiment ($N = 2 + 1$ ancilla)



IBM-Q Experience 5-qubit 'Tenerife' processor



Tacchino et al., npj Quant. Info **5**, 26 (2019)

Can we train it?

YES! → simple perceptron update rule

Chosen a target \vec{w}_t , build a training set by assigning positive (negative) labels to few inputs \vec{i} for which $\vec{i} \cdot \vec{w}_t > \theta$ ($\vec{i} \cdot \vec{w}_t < \theta$), then randomly initialize \vec{w} to be trained and:

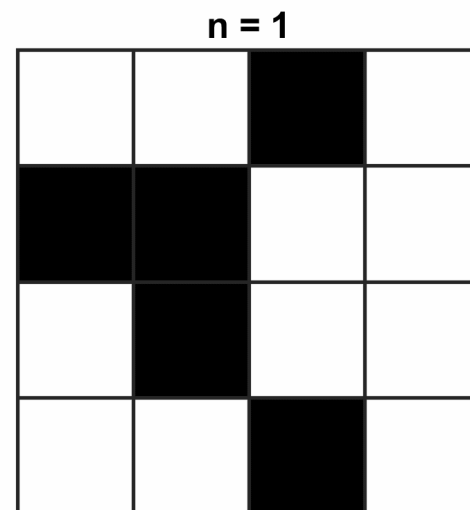
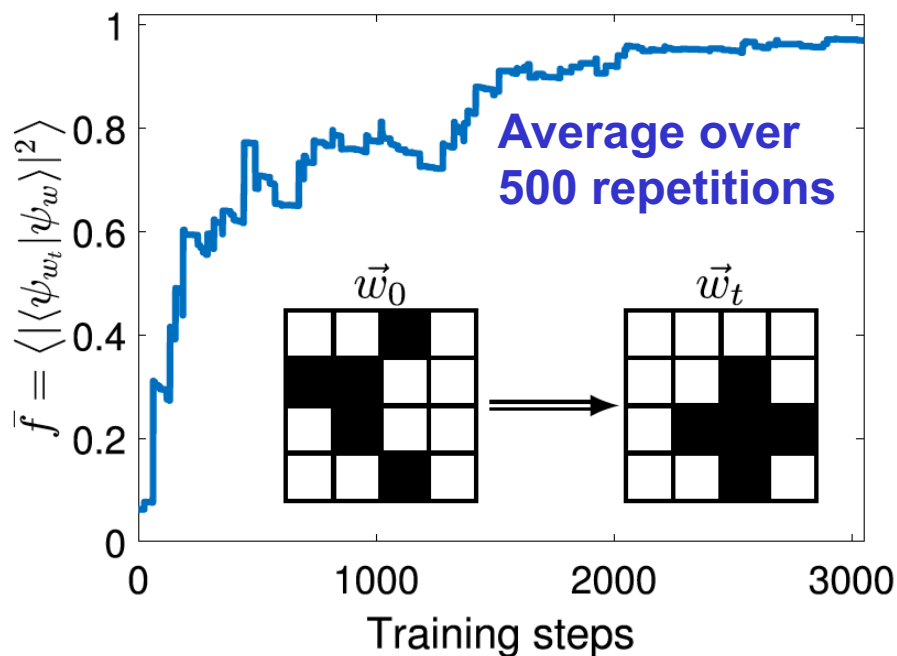
- If $\vec{i} \cdot \vec{w}$ is positive but should be negative (κ common entries), randomly flip $\eta\kappa$ signs ($0 < \eta < 1$ 'learning' rate)
- If $\vec{i} \cdot \vec{w}$ is negative but should be positive (κ opposite entries), randomly flip $\eta\kappa$ different signs
- If $\vec{i} \cdot \vec{w}$ is correct, do nothing



Elementary training on IBM simulator



- Theoretical simulation of the algorithm for $N=4$ qubits + 1 ancilla (**NOT** on real quantum hardware, yet)
- Recognize a cross (or its negative) out of a training set of input vectors (e.g., 50 positive, 3000 negative)

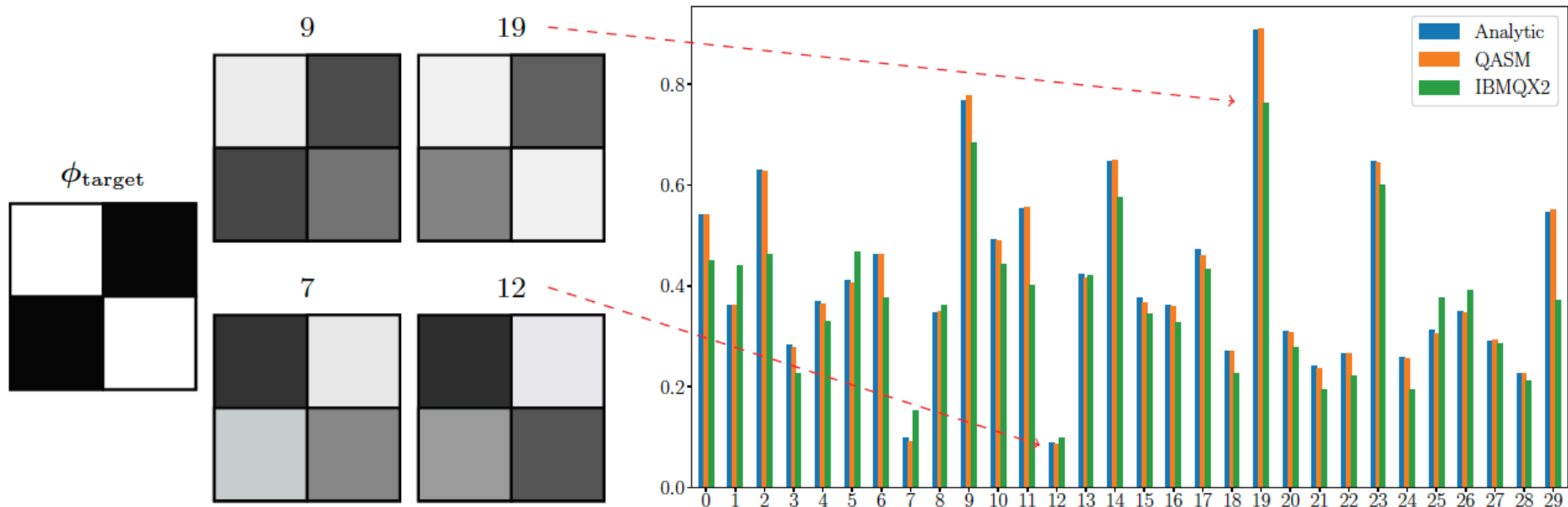


Recently extended: continuous valued input data

An array of real-valued input $\theta = (\theta_0, \dots, \theta_{N-1})$ with $\theta_i \in [0, \pi]$

Can be encoded as $\vec{i} = (e^{i\theta_0}, e^{i\theta_1}, \dots, e^{i\theta_{N-1}}) \longrightarrow |\psi_i\rangle = \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} i_k |k\rangle$

Allows to classify grey scale images without increasing the number of qubits

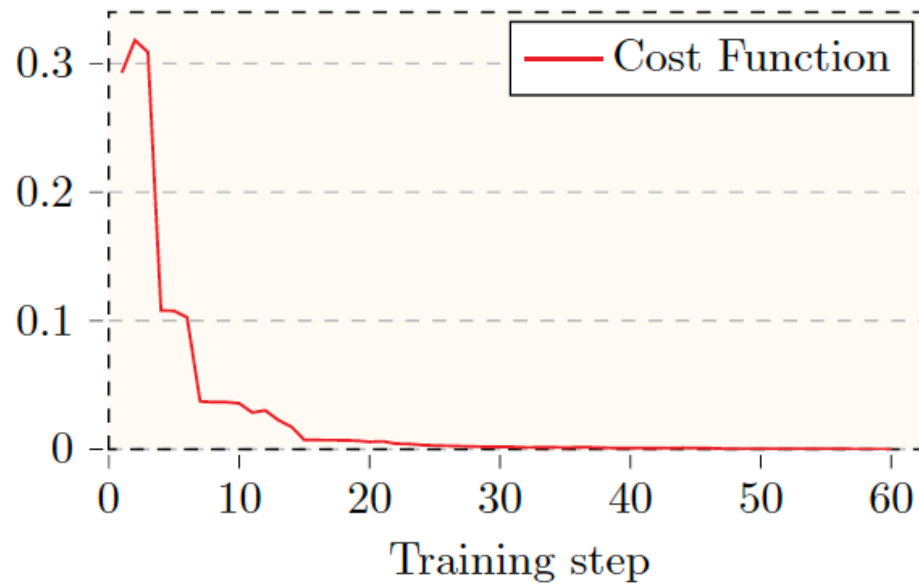


Hybrid quantum/classical learning

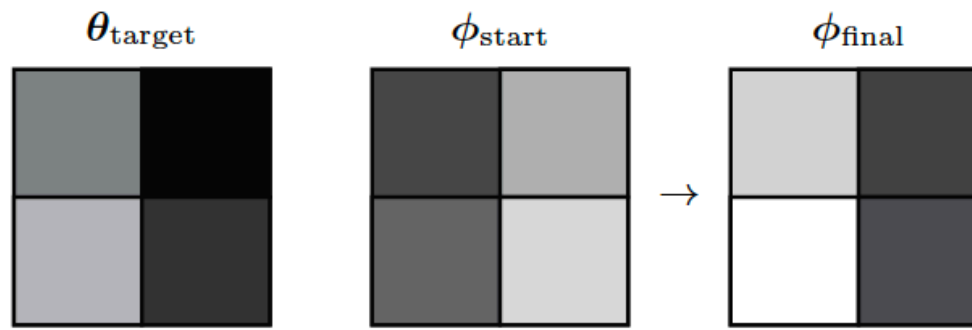
Quantum algorithm trained through classical backpropagation



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(a)



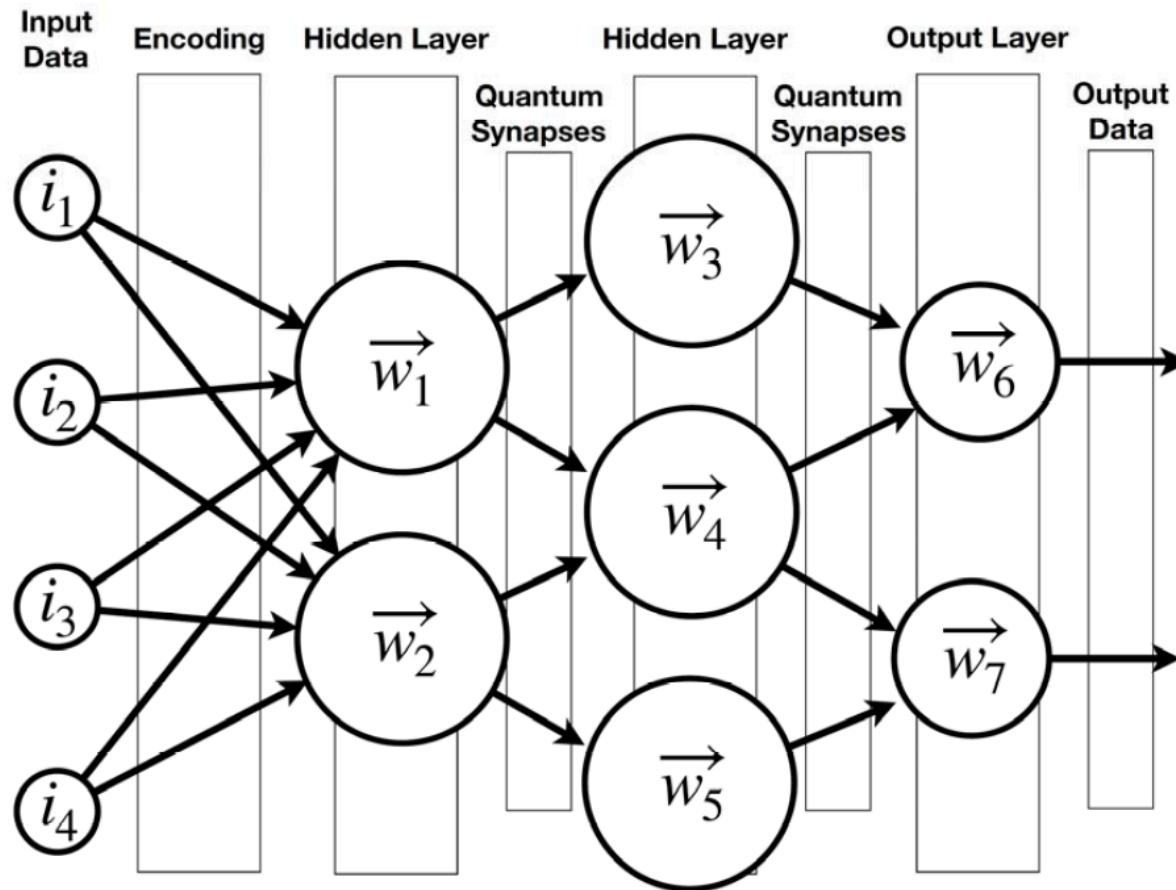
(b)

Quantum ANN

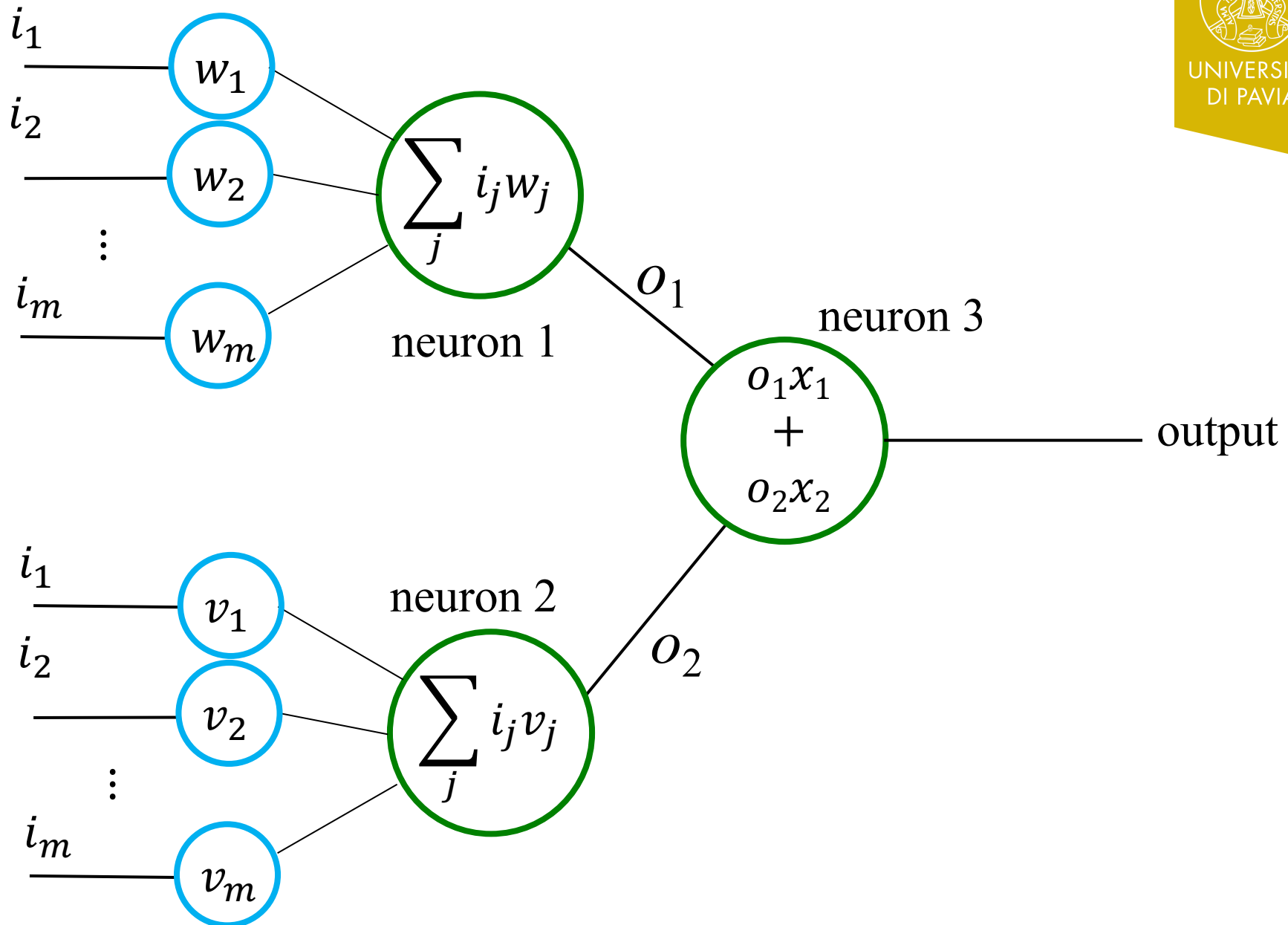
A deep neural network is required to perform more complex ML tasks



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Example (still runs on NISQ hardware)

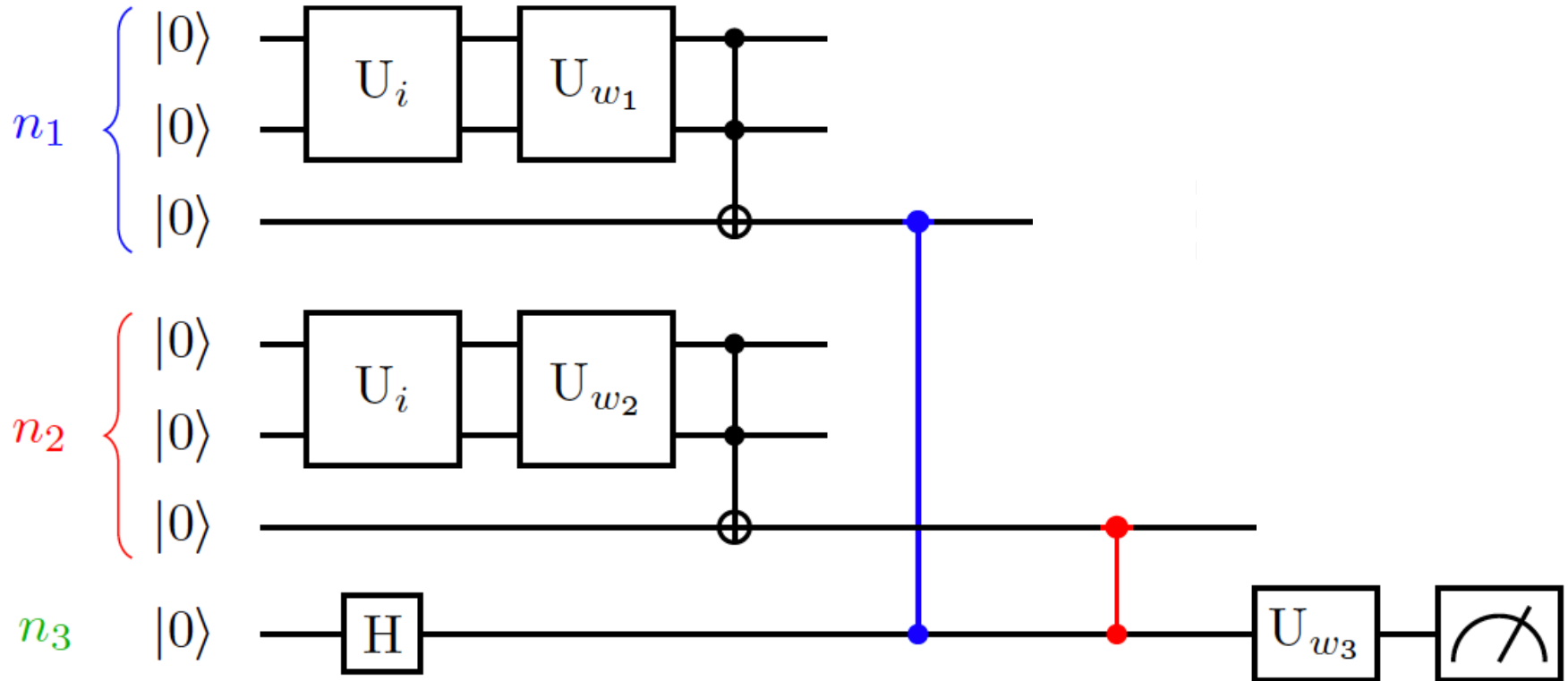


The corresponding quantum circuit

Quantum synapses \rightarrow multi-controlled operations



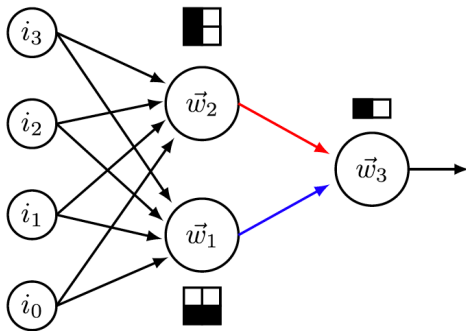
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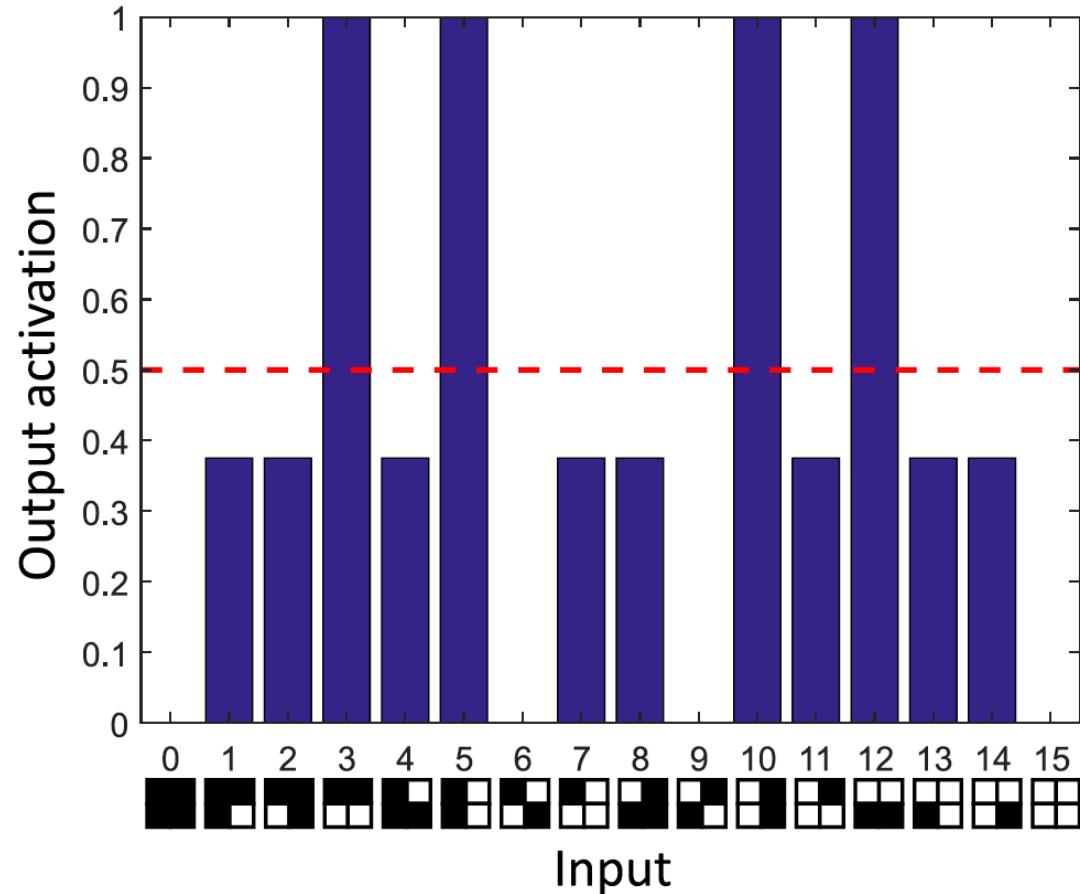
A classification task that is impossible to a single perceptron



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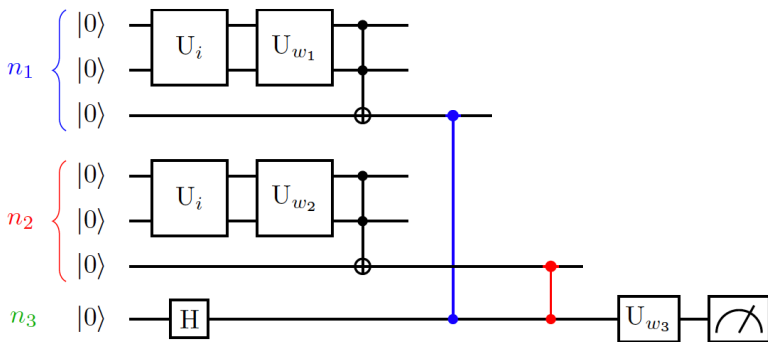
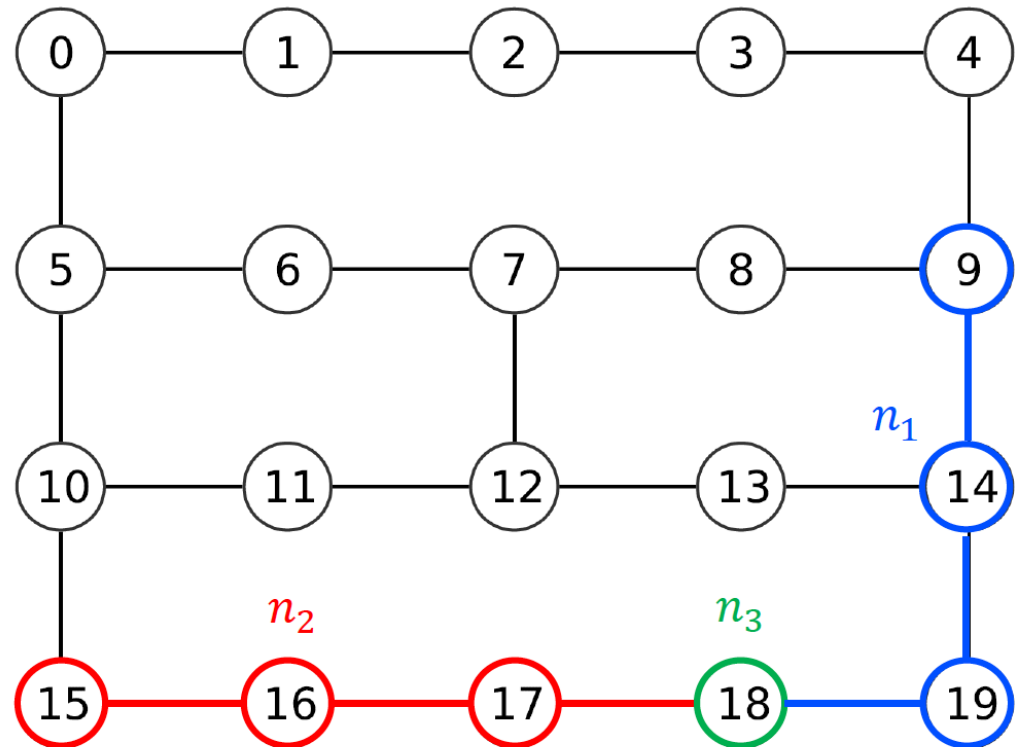
Ideal outcome
of the ANN



Implementation on IBM - Q



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Layout of the IBM Q – Poughkeepsie NISQ processor

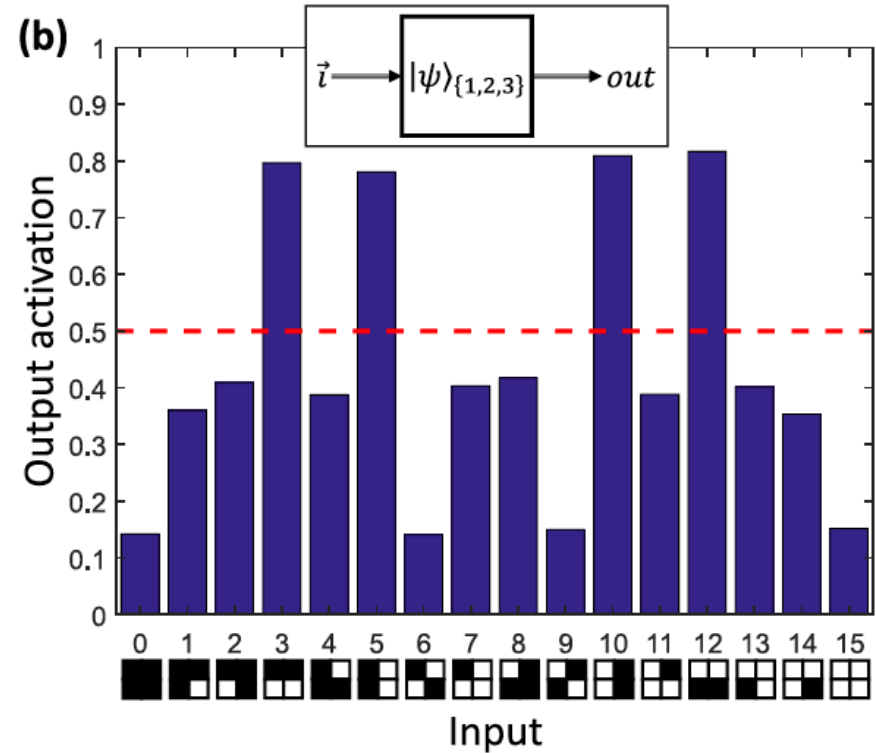
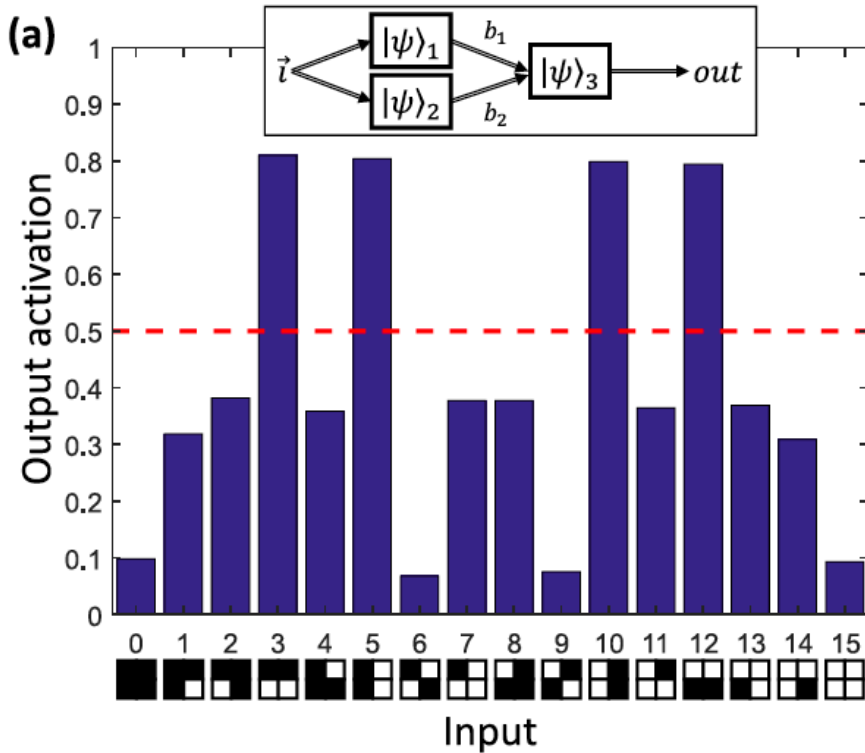
Results on IBM Q - Poughkeepsie



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Hybrid
configuration

Fully coherent
configuration



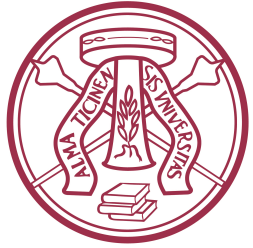
with activation threshold at 0.5, the quantum hardware is able to fully classify these patterns with 100% success

Open questions and challenges

- How does it scale?
- How efficient is it?
- Quantum training?
- Test on real hardware based on different technologies (e.g., trapped ions)
- Test with larger input data (possible use cases)
- Input quantum states (QQ)



People



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F. Tacchino*



S. Mangini



D. Bajoni



C. Macchiavello



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IBM Research | Zurich



P. Barkoutsos



I. Tavernelli

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