Modelling Topological Materials with D-branes

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NCSR "Demokritos" and National & Kapodistrian University of Athens

INPP Annual Meeting

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References & Collaborators

- In collaboration with M. de Leeuw and C. Kristjansen:

[1]. M. de Leeuw, C. Kristjansen, and G. Linardopoulos. "One-point Functions of Non-protected Operators in the $SO(5)$ Symmetric D3-D7 dCFT". *J. Phys.* **A50** (2017), 254001. arXiv: 1612.06236 [hep-th]

- In collaboration with J. Hoppe and T. Turgut:

[2]. J. Hoppe, G. Linardopoulos, and T.O. Turgut. "New Minimal Hypersurfaces in $\mathbb{R}^{(k+1)(2k+1)}$ and S^{2k^2+3k}". *Mathematische Nachrichten* (2016). arXiv: 1602.09101 [math.DG]

- In collaboration with M. Axenides and E. Floratos:

[3]. M. Axenides, E. Floratos, and G. Linardopoulos. "M2-brane Dynamics in the Classical Limit of the BMN Matrix Model". *Phys. Lett.* **B773** (2017), 265. arXiv: 1707.02878 [hep-th]

[4]. M. Axenides, E. Floratos, and G. Linardopoulos. "The Omega-Infinity Limit of Single Spikes". *Nucl. Phys.* **B907** (2016), 323. arXiv: 1511.03587 [hep-th]

[5]. M. Axenides, E. Floratos, and G. Linardopoulos. "Stringy Membranes in AdS/CFT". *JHEP* **08** (2013), 089. arXiv: 1306.0220 [hep-th]

- In collaboration with E. Floratos and G. Georgiou:

[6]. E. Floratos, G. Georgiou, and G. Linardopoulos. "Large-Spin Expansions of GKP Strings". *JHEP* **03** (2014), 018. arXiv: 1311.5800 [hep-th]

- In collaboration with E. Floratos:

[7]. G. Linardopoulos. "Classical Strings and Membranes in the AdS/CFT Correspondence". PhD thesis. National and Kapodistrian University of Athens, 2015

[8]. G. Linardopoulos. "Large-Spin Expansions of Giant Magnons". *PoS* (CORFU2014), 154. arXiv: 1502.01630 [hep-th]

[9]. E. Floratos and G. Linardopoulos. "Large-Spin and Large-Winding Expansions of Giant Magnons and Single Spikes". *Nucl.Phys.* **B897** (2015), 229. arXiv: 1406.0796 [hep-th]

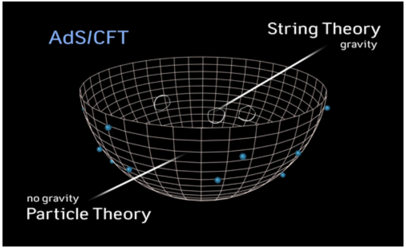
Section 1

Topological Materials and Holography

AdS/CFT Holography

- The AdS/CFT correspondence has changed the way we think about physics

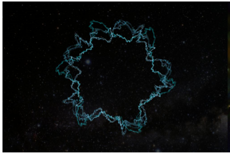
- A gravitational theory may provide clues about a non-gravitational theory and vice-versa



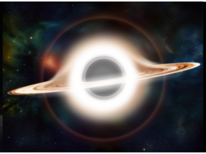
AdS/CFT Holography

- The AdS/CFT correspondence has changed the way we think about physics

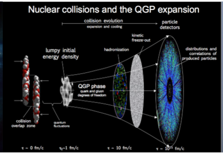
- A wealth of applications...



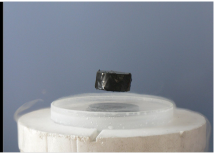
String Theory



Black Holes

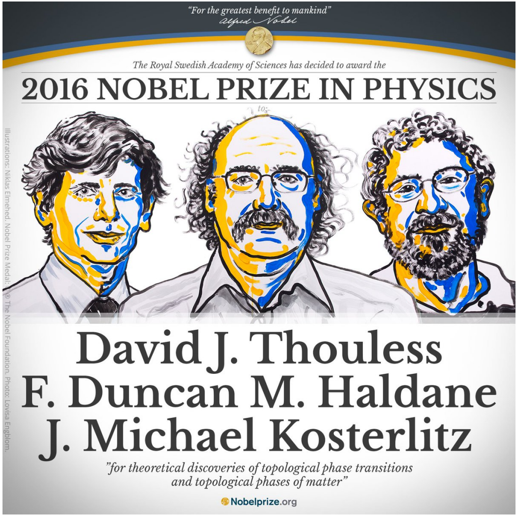


AdS/QCD



AdS/CMT

Topological materials



Topological materials

The Classical Hall Effect (E. Hall, 1879)

- Drude model:

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - e\mathbf{v}\times\mathbf{B} - \frac{m\mathbf{v}}{\tau}$$

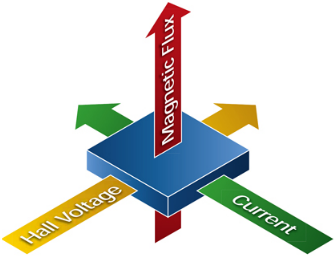
- In equilibrium $\mathbf{J} = -ne\mathbf{v}$ and we obtain Ohm's law:

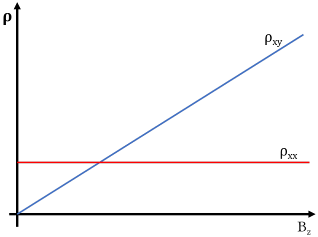
$$\mathbf{J} = \boldsymbol{\sigma}\cdot\mathbf{E},$$

where the Hall resistivity is

$$\boldsymbol{\rho} = \boldsymbol{\sigma}^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix} = \frac{1}{\sigma_{\rm dc}}\begin{pmatrix} 1 & \omega_{\rm B}\tau \\ -\omega_{\rm B}\tau & 1 \end{pmatrix}$$

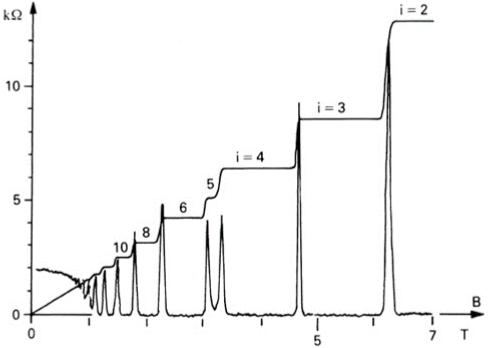
$$\sigma_{dc} \equiv \frac{e^2 n\tau}{m} \quad \text{(dc conductivity)}, \quad \omega_{\rm B} \equiv \frac{eB_z}{m} \quad \text{(Cyclotron frequency)}$$





Topological materials

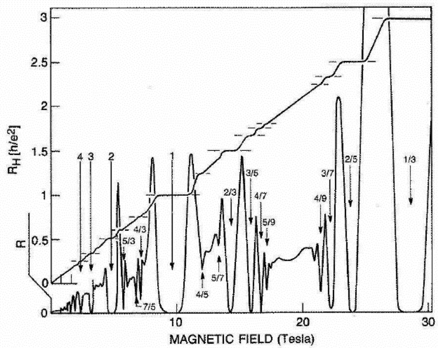
IQHE



$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu}, \qquad \nu \in \mathbb{Z}$$

von Klitzing, 1981 (NP 1985)

FQHE



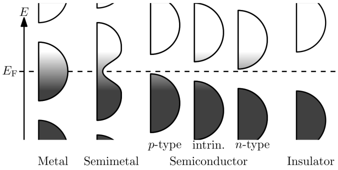
$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu}, \qquad \nu \in \mathbb{Q}$$

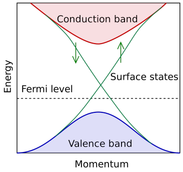
Tsui,Störmer 1982 (NP with Laughlin 1998)

Topological materials

Topological insulators

- A topological insulator is a material that insulates in its bulk and conducts on its surface.





- Metallic edge states were predicted to occur in quantum wells in 1987 (observed in 2007).
- 3d topological insulators were discovered in bismuth-antimony (BiSb) alloys in 2008.
- Potentially useful as spintronic (memory) devices in Quantum Computers.

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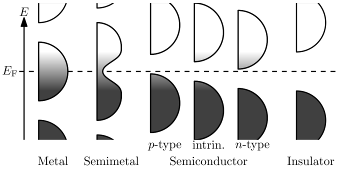

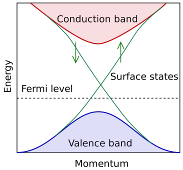

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- 3d topological insulators were discovered in bismuth-antimony (BiSb) alloys in 2008.
- Potentially useful as spintronic (memory) devices in Quantum Computers.
- Models of 2d topological insulators can be studied in the context of AdS/CFT holography.

Section 2

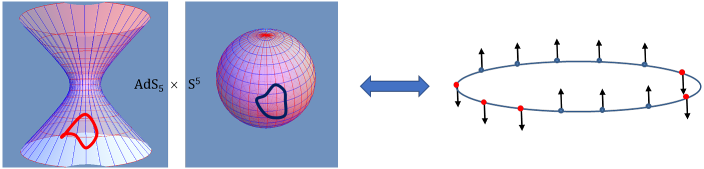
Intersecting Branes

The D3-D7 system

Let us recall the original formulation AdS/CFT correspondence:

$$\left\{\text{Type IIB superstring theory on } \mathrm{AdS}_5 \times \mathrm{S}^5\right\} = \left\{\mathcal{N}=4,\ SU(N) \text{ SYM theory in } 3+1 \text{ dimensions}\right\}$$

(J. Maldacena, 1998)



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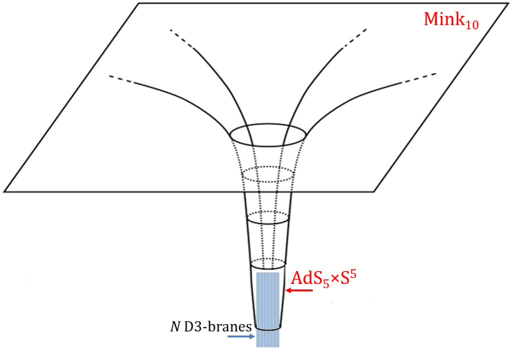

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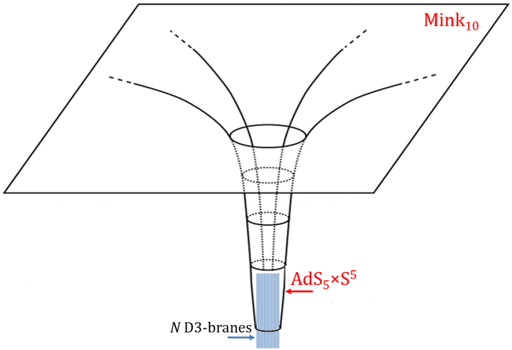

The D3-branes extend along x_1, x_2, x_3...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						

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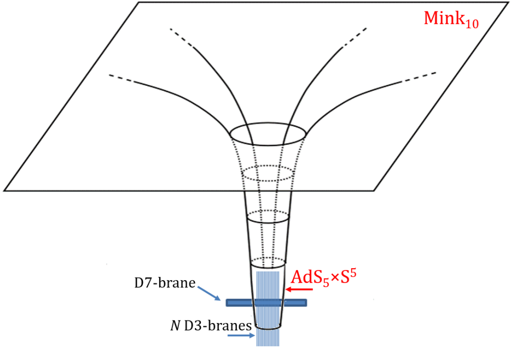
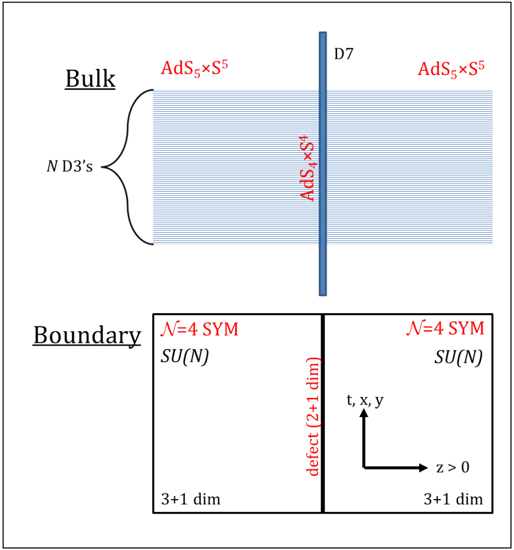


The D3-branes extend along x_1, x_2, x_3. Now insert a single (probe) D7-brane at $x_3 = x_9 = 0$...

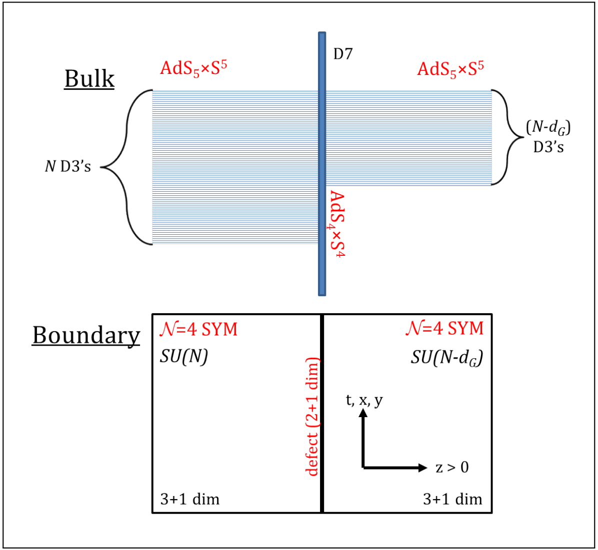
	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	●	●	●	●						
D7	●	●	●		●	●	●	●	●	

The D3-D7 system: description



- In the bulk, the D3-D7 system describes IIB superstring theory on $AdS_5 \times S^5$ bisected by a D7-brane with worldvolume geometry $AdS_4 \times S^4$.
- The dual field theory is still $SU(N)$, $\mathcal{N} = 4$ SYM in $3+1$ dimensions, that interacts with a CFT living on the $2+1$ dimensional defect:
$$S = S_{\mathcal{N}=4} + S_{2+1}.$$
- Due to the presence of the defect, the total bosonic symmetry of the system is reduced from $SO(4,2) \times SO(6)$ to $SO(3,2) \times SO(5)$.
- The relative co-dimension of the branes is $\#ND = 6 \rightarrow$ no unbroken supersymmetry.
- Tachyonic instability...

The $(\text{D3-D7})_{d_G}$ system



- To stabilize the system, add an instanton bundle on the S^4 component of the $\text{AdS}_4 \times S^4$ D7-brane, with instanton number $d_G = (n+1)(n+2)(n+3)/6$.
  (Myers-Wapler, 2008)
- Then exactly d_G of the N D3-branes ($N \gg d_G$) will end on the D7-brane.
- On the dual SCFT side, the gauge group $SU(N) \times SU(N)$ breaks to $SU(N) \times SU(N - d_G)$.
- Equivalently, the fields of $\mathcal{N} = 4$ SYM develop nonzero vevs...
  (Karch-Randall, 2001b)

A fractional topological insulator

Other ways of dealing with the tachyonic instability:

- Embed the D7-brane into the full D3-brane geometry...
  Davis-Kraus-Shah (2008) & Kristjansen-Semenoff (2016)

- Introduce an AdS cutoff while sending the unstable mode towards the BF bound...
  Kutasov-Lin-Parnachev (2011) & Mezzalira-Parnachev (2015)

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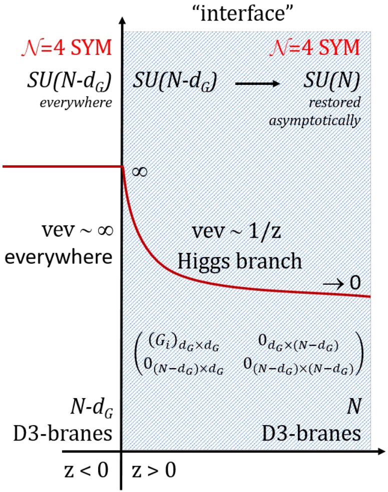
$$S = S_{\mathcal{N}=4} + S_{2+1},$$

- S_{2+1} is the action of relativistic fermions that live on the 2+1 dimensional defect... (Rey, 2008)
- There are also fermion mass terms that break P and T symmetry... (Davis-Kraus-Shah, 2008)
- S_{2+1} then contains a Chern-Simons term... Hall conductivities ($T = 0$):

$$\sigma_{xx} = 0 \qquad \& \qquad \sigma_{xy} = \frac{Ne^2}{4\pi} \cdot \mathrm{sgn}(m)$$

Modelling the (D3-D7)$_{d_G}$ interface



- An interface is a wall between two (different/same) QFTs
- It can be described by means of classical solutions that are known as "fuzzy-funnel" solutions
  Constable-Myers-Tafjord, 1999 & 2001
- Here, we need an interface to separate the $SU(N)$ and $SU(N-d_G)$ regions of (D3-D7)$_{d_G}$ dCFT..
- A manifestly $SO(5) \subset SO(3,2) \times SO(5)$ symmetric solution is given by:

$$\Phi_i(z) = \frac{G_i \oplus 0_{(N-d_G)\times(N-d_G)}}{\sqrt{8}\, z}, \quad i = 1, \ldots, 5, \qquad \Phi_6 = 0\,,$$

Kristjansen-Semenoff-Young, 2012

where the five d_G-dimensional matrices G_i are known as "fuzzy" S^4 matrices or simply as G-matrices.

Section 3

One-point Functions

M. de Leeuw, C. Kristjansen, G. Linardopoulos, *One-Point Functions of Non-protected Operators in the $SO(5)$ Symmetric D3-D7 dCFT*. J.Phys. A:Math.Theor., **50** (2017) 254001, [arXiv:1612.06236]

One-point functions

One-point functions are the most important observables in a dCFT. From them and the conformal data (Δ's, C_{ijk}'s, etc.) one can determine all the correlators of the theory (and the theory itself) by using the OPE.

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- Our dCFT is dual to the $SO(5)$ symmetric $(\text{D3-D7})_{d_G}$ probe brane system.
- Our goal is to calculate the one-point functions of $\mathfrak{so}(6)$ highest-weight eigenstates:

$$\langle \mathcal{O}(z, \mathbf{x}) \rangle = \frac{C}{z^{\Delta}}, \qquad C = \frac{1}{\sqrt{L}} \left(\frac{\pi^2}{\lambda} \right)^{L/2} \cdot \frac{\langle \text{MPS} | \Psi \rangle}{\langle \Psi | \Psi \rangle^{\frac{1}{2}}}, \qquad d_G \ll N \to \infty,$$

where

$$\langle \text{MPS} | \Psi \rangle = z^M \cdot \sum_{1 \leq x_k \leq L} \psi(x_k) \cdot \text{Tr} \left[G_5^{x_1 - 1} \mathcal{W} G_5^{x_2 - x_1 - 1} \mathcal{Y} G_5^{x_3 - x_2 - 1} \overline{\mathcal{W}} G_5^{x_4 - x_3 - 1} \overline{\mathcal{Y}} \ldots \right]$$

and Ψ is an eigenstate of the $\mathfrak{so}(6)$ Hamiltonian, with

$$\langle \Psi | \Psi \rangle^{\frac{1}{2}} = \sqrt{\sum_{1 \leq x_k \leq L} \psi^2(x_k)}.$$

Vacuum state

For the vacuum overlap we have found:

$$\langle \mathrm{MPS}|0\rangle = \mathrm{Tr}\left[G_5^L\right] = \sum_{j=1}^{n+1}\left[j\,(n-j+2)\,(n-2j+2)^L\right].$$

Changing variables $j \leftrightarrow (n+2-j)$, an overall factor $(-1)^L$ comes out, leading the vacuum overlap to zero for L odd. Equivalently, we may write

$$\langle \mathrm{MPS}|0\rangle = \begin{cases} 0, & L \text{ odd} \\ 2^L \cdot \left[\frac{2}{L+3}\, B_{L+3}\left(-\frac{n}{2}\right) - \frac{(n+2)^2}{2(L+1)}\, B_{L+1}\left(-\frac{n}{2}\right)\right], & L \text{ even}, \end{cases}$$

by using the relationship between the Hurwitz zeta function and the Bernoulli polynomials $B_m(x)$.

In the large-n limit we find:

$$\langle \mathrm{MPS}|0\rangle \sim \frac{n^{L+3}}{2\,(L+1)\,(L+3)} + O\left(n^{L+2}\right), \qquad n \to \infty.$$

Non-protected operators

- The overlaps $\langle \text{MPS}|\Psi\rangle$ of all the highest-weight eigenstates vanish unless:

$$L = \text{ even} \qquad \& \qquad \#\mathcal{W} = \#\overline{\mathcal{W}}, \qquad \#\mathcal{Y} = \#\overline{\mathcal{Y}}.$$

Therefore the only $\mathfrak{so}(6)$ eigenstates that have nonzero one-point functions are those with:

$$N_1 = 2N_2 = 2N_3 \equiv M \;\; (\text{even}).$$

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- More generally, we can consider eigenstates with $N_1 = 2$, $N_2 = N_3 = 1$ and arbitrary L:

$$|p\rangle = \sum_{x_1<x_2} \left(e^{ip(x_1-x_2)} + e^{ip(x_2-x_1+1)}\right) \cdot |\ldots \underset{x_1}{\mathcal{X}} \ldots \underset{x_2}{\overline{\mathcal{X}}} \ldots\rangle - 2\sum_{x_3} \left(1 + e^{ip}\right) \cdot |\ldots \underset{x_3}{\overline{\mathcal{Z}}} \ldots\rangle,$$

  where the dots stand for $\mathcal{Z}$, and $\mathcal{X}$ is any of the complex scalars $\mathcal{W}$, $\overline{\mathcal{W}}$, $\mathcal{Y}$, $\overline{\mathcal{Y}}$.

- Here's the one-loop energy of the $L211$ eigenstates:

$$E = L + \frac{\lambda}{\pi^2} \sin^2\left[\frac{2m\pi}{L+1}\right] + \ldots, \qquad m = 1, \ldots, L+1$$

$L211$ Bethe eigenstates

- The corresponding one-point function for all n is given in terms of the $n=1$ one:

$$\langle \mathcal{O}_{L211} \rangle = \left[\frac{u^2}{u^2 - 1/2} \sum_{n \bmod 2}^{n} j^L \cdot \frac{(n+2)^2 - j^2}{8} \cdot \frac{[u^2 + \frac{(n+2)j+1}{4}][u^2 - \frac{(n+2)j-1}{4}]}{[u^2 + (\frac{j+1}{2})^2][u^2 + (\frac{j-1}{2})^2]} \right] \cdot \langle \mathcal{O}_{L211}^{n=1} \rangle,$$

where

$$\langle \mathcal{O}_{L211}^{n=1} \rangle = 8 \sqrt{\frac{L}{L+1} \frac{u^2 - \frac{1}{2}}{u^2 + \frac{1}{4}}} \sqrt{\frac{u^2 + \frac{1}{4}}{u^2}}, \qquad u \equiv \frac{1}{2} \cot \frac{p}{2}.$$

- The results fully reproduce the numerical values (given in units of $(\pi^2/\lambda)^{L/2}/\sqrt{L}$):

L	$N_{1/2/3}$	eigenvalue γ	n=1	n=2	n=3	n=4
2	2 1 1	6	$20\sqrt{\frac{2}{3}}$	$40\sqrt{6}$	$140\sqrt{6}$	$1120\sqrt{\frac{2}{3}}$
4	2 1 1	$5+\sqrt{5}$	$20+\frac{44}{\sqrt{5}}$	$\frac{96}{5}\left(15+\sqrt{5}\right)$	$84\left(21-\sqrt{5}\right)$	$\frac{3584}{5}\left(10-\sqrt{5}\right)$
4	2 1 1	$5-\sqrt{5}$	$20-\frac{44}{\sqrt{5}}$	$288-\frac{96}{\sqrt{5}}$	$84\left(21+\sqrt{5}\right)$	$\frac{3584}{5}\left(10+\sqrt{5}\right)$
6	2 1 1	1.50604	3.57792	324.178	11338.3	98726
6	2 1 1	4.89008	9.90466	1724.55	19513.8	120347
6	2 1 1	7.60388	61.6252	1044.86	8830.95	49114.4

Outlook

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- Higher loop orders, Wilson loops, 2-point functions à la D3-D5?

Outlook

- Closed formula for all $\mathfrak{so}(6)$ Bethe eigenstates? Integrability? (work in progress)
- Extend current methods to the $\mathfrak{su}(3)$ and $\mathfrak{so}(6)$ sectors of the D3-D5 system (work in progress)
- Integrability of β-deformed version of D3-D5 (work in progress)
- Thermodynamic limit ($M, L \to \infty$, $M/L =$const), string theory/strong coupling side ($\lambda \to \infty$)...
- Move on to the $SU(2) \times SU(2)$ solution of D3-D7 and its β-deformation...
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Thank you!