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The Proper Treatment of Quantification in Ordinary English

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The aim of this paper is to present in a rigorous way the syntax and semantics of a certain fragment of a certain dialect of English. For expository purposes the fragment has been made as simple and restricted as it can be while accommodating all the more puzzling cases of quantification and reference with which I am acquainted.[1]

Patrick Suppes claims, in a paper prepared for the present workshop [the 1970 Stanford Workshop on Grammar and Semantics], that "at the present time the semantics of natural languages are less satisfactorily formulated than the grammars ... [and] a complete grammar for any significant fragment of natural language is yet to be written." This claim would of course be accurate if restricted in its application to the attempts emanating from the Massachusetts Institute of Technology, but fails to take into account the syntactic and semantic treatments proposed in Montague (1970a, b). Thus the present paper cannot claim to present the *first* complete syntax (or grammar, in Suppes' terminology) and semantics for a significant fragment of natural language; and it is perhaps not inappropriate to sketch relations between the earlier proposals and the one given below.

Montague (1970b) contains a general theory of languages, their interpretations, and the inducing of interpretations by translation. The treatment given below, as well as that in Montague (1970a) and the treatment of a fragment of English proposed at the end of Montague (1970b), can all easily be construed as special cases of that general theory. The fragment in Montague (1970a) was considerably more restricted in scope than those in Montague (1970b) or the present paper, in that although it admitted indirect discourse, it failed to accommodate a number of more complex intensional locutions, for instance, those involving *intensional verbs* (that is, verbs like **seeks, worships, conceives**). The fragment in Montague (1970b) did indeed include intensional verbs but excluded certain intensional locutions involving pronouns (for instance, the sentence **John wishes to catch a fish and eat it**, to which a number of linguists have recently drawn attention). The present treatment is capable of accounting for such examples, as well as a number of other heretofore unattempted puzzles, for instance, Professor Partee's **the temperature is ninety but it is rising** and the problem of intensional prepositions. On the other hand, the present treatment, unlike that in Montague (1970b), will not directly accommodate such sentences as J. M. E. Moravcsik's **a unicorn appears to be approaching**, in which an indefinite term *in subject position* would have a nonreferential reading, but must treat them indirectly as paraphrases (of, in this case, **it appears that a unicorn is approaching** or **that a unicorn is approaching appears to be true**).

On their common domain of applicability the three treatments essentially agree in the truth and entailment conditions imposed on sentences.[2] Further, when only *declarative* sentences come into consideration, it is the construction of such conditions that (Suppes notwithstanding) should count as the central concern of syntax and semantics.[3] Nevertheless, the details of the present development possess certain aesthetic merits, of coherence and conceptual simplicity, not to be found in the treatment of English in Montague (1970b). (It is in order to preserve these merits that I here forgo a direct account of such sentences as Moravcsik's.)

1 The Syntax of a Fragment of English

Let e and t be two fixed objects (0 and 1, say) that are distinct and neither ordered pairs nor ordered triples. Then *Cat*, or the set of *categories* of English, is to be the smallest set X such that (1) e and t are in X, and (2) whenever A and B are in X, A/B and $A//B$ (that is, $\langle 0, A, B\rangle$ and $\langle 1, A, B\rangle$ respectively) are also in X.

It should be pointed out that our categories are not sets of expressions but will instead serve as *indices* of such sets. We regard e and t as the categories of entity expressions (or individual expressions) and truth value expressions (or declarative sentences) respectively. We shall regard the categories A/B and $A//B$ as playing the same semantical but different syntactical roles. An expression of either category is to be such that when it is combined (in some as yet unspecified way, and indeed in different ways for the two categories) with an expression of category B, an expression of category A is produced. (The precise character of the categories A/B and $A//B$ is unimportant; we require only two different kinds of ordered pair.)

It will be observed that our syntactic categories diverge from those of Ajdukiewicz (1960) only in our introduction of two compound categories (A/B and $A//B$) where Ajdukiewicz would have had just one. The fact that we need only two copies is merely an accident of English or perhaps of our limited fragment; in connection with other languages it is quite conceivable that a larger number would be required.[4]

Keeping in mind the intuitive roles described above, we may single out as follows certain traditional syntactic categories.

IV, or the category of intransitive verb phrases, is to be t/e.
T, or the category of terms, is to be t/IV.
TV, or the category of transitive verb phrases, is to be IV/T.
IAV, or the category of IV-modifying adverbs, is to be IV/IV.
CN, or the category of common noun phrases, is to be t//e.

The following categories will also be exemplified in our fragment although no special symbol will be introduced for them.

t/t is the category of sentence-modifying adverbs.
IAV/T is the category of IAV-making prepositions.

IV/t is the category of sentence-taking verb phrases.
IV//IV is the category of IV-taking verb phrases.

By B_A is understood the set of *basic expressions* of the category A; the notion is characterized as follows.

$$
\begin{aligned}
B_{IV} &= \{\textbf{run, walk, talk, rise, change}\}\\
B_{T} &= \{\textbf{John, Mary, Bill, ninety, he}_0, \textbf{he}_1, \textbf{he}_2, \ldots\}\\
B_{TV} &= \{\textbf{find, lose, eat, love, date, be, seek, conceive}\}\\
B_{IAV} &= \{\textbf{rapidly, slowly, voluntarily, allegedly}\}\\
B_{CN} &= \{\textbf{man, woman, park, fish, pen, unicorn, price, temperature}\}\\
B_{t/t} &= \{\textbf{necessarily}\}\\
B_{IAV/T} &= \{\textbf{in, about}\}\\
B_{IV/t} &= \{\textbf{believe that, assert that}\}\\
B_{IV//IV} &= \{\textbf{try to, wish to}\}
\end{aligned}
$$

$B_A = \Lambda$ (that is, the empty set) if A is any category other than those mentioned above. (In particular, the sets $\mathbf{B}_e$ of basic entity expressions and $\mathbf{B}_t$ of basic declarative sentences are empty.)

By a *basic expression* of the present fragment is understood a member of $\bigcup_{A \in Cat} B_A$.
By P_A is understood the set of *phrases* of the category A. (We may read "P_{CN}", "P_{TV}", and the like as "the set of common noun phrases", "the set of transitive verb phrases", and so on.) These sets are introduced, in a sense to be made precise below, by the following rules, S1–S17.

Syntactic rules

Basic rules

S1. $B_A \subseteq P_A$ for every category A.

S2. If $\zeta \in P_{CN}$, then $F_0(\zeta)$, $F_1(\zeta)$, $F_2(\zeta) \in P_T$,
where $F_0(\zeta) = \textbf{every } \zeta$,
$F_1(\zeta) = \textbf{the } \zeta$,
$F_2(\zeta)$ is **a** ζ or **an** ζ according as the first word in ζ takes **a** or **an**.

S3. If $\zeta \in P_{CN}$ and $\phi \in P_t$, then $F_{3,n}(\zeta, \phi) \in P_{CN}$, where $F_{3,n}(\zeta, \phi) = \zeta$ **such that** ϕ'; and ϕ' comes from ϕ by replacing each occurrence of $\textbf{he}_\textbf{n}$ or $\textbf{him}_\textbf{n}$ by $\left\{\begin{matrix}\textbf{he}\\ \textbf{she}\\ \textbf{it}\end{matrix}\right\}$ or $\left\{\begin{matrix}\textbf{him}\\ \textbf{her}\\ \textbf{it}\end{matrix}\right\}$ respectively, according as the first B_{CN} in ζ is of $\left\{\begin{matrix}\text{masc.}\\ \text{fem.}\\ \text{neuter}\end{matrix}\right\}$ gender.

Rules of functional application

S4. If $\alpha \in P_{t/IV}$ and $\delta \in P_{IV}$, then $F_4(\alpha, \delta) \in P_t$, where $F_4(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first *verb* (i.e., member of B_{IV}, B_{TV}, $B_{IV/t}$, or $B_{IV//IV}$) in δ by its third person singular present.

S5. If $\delta \in P_{IV/T}$ and $\beta \in P_T$, then $F_5(\delta, \beta) \in P_{IV}$, where $F_5(\delta, \beta) = \delta\beta$ if β does not have the form $\mathbf{he}_n$ and $F_5(\delta, \mathbf{he}_n) = \delta\ \mathbf{him}_n$.
S6. If $\delta \in P_{IAV/T}$ and $\beta \in P_T$, then $F_5(\delta, \beta) \in P_{IAV}$.
S7. If $\delta \in P_{IV/t}$ and $\beta \in P_t$, then $F_6(\delta, \beta) \in P_{IV}$, where $F_6(\delta, \beta) = \delta\beta$.
S8. If $\delta \in P_{IV//IV}$ and $\beta \in P_{IV}$, then $F_6(\delta, \beta) \in P_{IV}$.
S9. If $\delta \in P_{t/t}$ and $\beta \in P_t$, then $F_6(\delta, \beta) \in P_t$.
S10. If $\delta \in P_{IV/IV}$ and $\beta \in P_{IV}$, then $F_7(\delta, \beta) \in P_{IV}$, where $F_7(\delta, \beta) = \beta\delta$.

Rules of conjunction and disjunction

S11. If $\phi, \psi \in P_t$, then $F_8(\phi, \psi), F_9(\phi, \psi) \in P_t$, where $F_8(\phi, \psi) = \phi$ **and** ψ, $F_9(\phi, \psi) = \phi$ **or** ψ.
S12. If $\gamma, \delta \in P_{IV}$, then $F_8(\gamma, \delta), F_9(\gamma, \delta) \in P_{IV}$.
S13. If $\alpha, \beta \in P_T$, then $F_9(\alpha, \beta) \in P_T$.

Rules of quantification

S14. If $\alpha \in P_T$ and $\phi \in P_t$, then $F_{10,n}(\alpha, \phi) \in P_t$, where either (i) α does not have the form $\mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing the first occurrence of $\mathbf{he}_n$ or $\mathbf{him}_n$ by α and all other occurrences of $\mathbf{he}_n$ or $\mathbf{him}_n$ by $\left\{\begin{matrix}\mathbf{he}\\ \mathbf{she}\\ \mathbf{it}\end{matrix}\right\}$ or $\left\{\begin{matrix}\mathbf{him}\\ \mathbf{her}\\ \mathbf{it}\end{matrix}\right\}$ respectively, according as the gender of the first B_{CN} or B_T in α is $\left\{\begin{matrix}\text{masc.}\\ \text{fem.}\\ \text{neuter}\end{matrix}\right\}$, or
(ii) $\alpha = \mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing all occurrences of $\mathbf{he}_n$ or $\mathbf{him}_n$ by $\mathbf{he}_k$ or $\mathbf{him}_k$ respectively.
S15. If $\alpha \in P_T$ and $\zeta \in P_{CN}$, then $F_{10,n}(\alpha, \zeta) \in P_{CN}$.
S16. If $\alpha \in P_T$ and $\delta \in P_{IV}$, then $F_{10,n}(\alpha, \delta) \in P_{IV}$.

Rules of tense and sign

S17. If $\alpha \in P_T$ and $\delta \in P_{IV}$, then $F_{11}(\alpha, \delta), F_{12}(\alpha, \delta), F_{13}(\alpha, \delta), F_{14}(\alpha, \delta), F_{15}(\alpha, \delta) \in P_t$, where:
$F_{11}(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first verb in δ by its negative third person singular present;
$F_{12}(\alpha, \delta) = \alpha\delta''$ and δ'' is the result of replacing the first verb in δ by its third person singular future;
$F_{13}(\alpha, \delta) = \alpha\delta'''$ and δ''' is the result of replacing the first verb in δ by its negative third person singular future;
$F_{14}(\alpha, \delta) = \alpha\delta''''$ and δ'''' is the result of replacing the first verb in δ by its third person singular present perfect; and finally,
$F_{15}(\alpha, \delta) = \alpha\delta'''''$ and δ''''' is the result of replacing the first verb in δ by its negative third person singular present perfect.

The precise characterization of the sets P_A, for A a category, is accomplished as follows. We first define the auxiliary notions occurring in the rules above in an obvious and traditional way: the *gender* of an arbitrary member of $B_T \cup B_{CN}$, the *indefinite article taken* by an arbitrary basic expression, and the *third person singular present*, the *negative third person singular present*, the *third person singular future*, the *negative third person singular future*, the *third person singular present perfect*, and the *negative third person singular present perfect* of an arbitrary verb. Then we may regard S1–S17 as constituting a simultaneous inductive definition of the sets P_A. Since, however, inductive definitions of this form are somewhat unusual, it is perhaps in order to state a corresponding explicit definition: the sets P_A (for $A \in \mathrm{Cat}$) are the smallest sets satisfying S1–S17; that is to say, $\langle P_A \rangle_{A \in \mathrm{Cat}}$ is the unique family of sets indexed by Cat such that (1) $\langle P_A \rangle_{A \in \mathrm{Cat}}$ satisfies S1–S17, and (2) whenever $\langle P'_A \rangle_{A \in \mathrm{Cat}}$ is a family of sets indexed by Cat, if $\langle P'_A \rangle_{A \in \mathrm{Cat}}$ satisfies S1–S17, then $P_A \subseteq P'_A$ for all $A \in \mathrm{Cat}$. (It is easily shown, using an idea I believe to have originated with Dr. Perry Smith, that there is exactly one family of sets satisfying these conditions.)

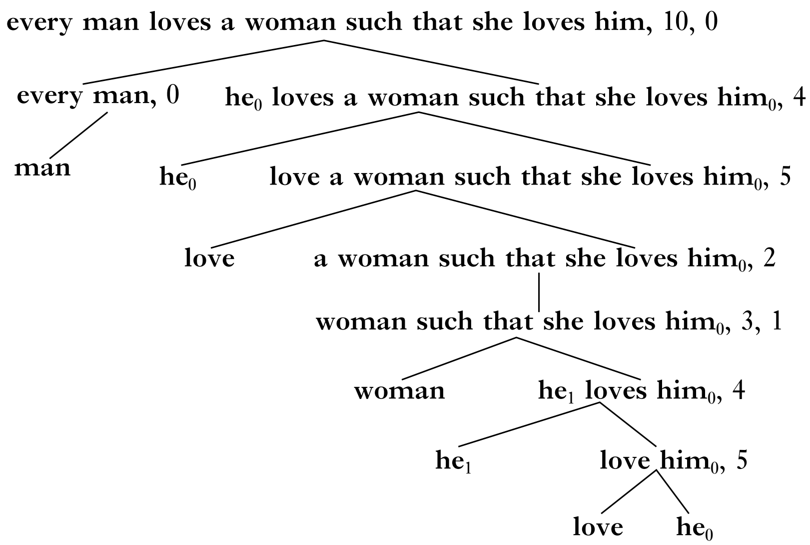
By a *meaningful expression* of the present fragment of English we may understand a member of any of the sets P_A for $A \in \mathrm{Cat}$.

As an example, let us show that

every man loves a woman such that she loves him

is a declarative sentence (that is, member of P_t). By S1, **love** $\in P_{TV}$ and **he**$_0 \in P_T$. Hence, by S5, **love him**$_0 \in P_{IV}$. Therefore, by S1 and S4, **he**$_1$ **loves him**$_0 \in P_t$. Thus, by S1 and S3, **woman such that she loves him**$_0 \in P_{CN}$. Therefore, by S2, **a woman such that she loves him**$_0 \in P_T$. Hence, by S1 and S5, **love a woman such that she loves him**$_0 \in P_{IV}$. Therefore, by S1 and S4, **he**$_0$ **loves a woman such that she loves him**$_0 \in P_t$. Also, by S1 and S2, **every man** $\in P_T$; and hence, by S14, **every man loves a woman such that she loves him** $\in P_t$.

We may indicate the way in which this sentence has just been constructed by means of the following *analysis tree*:



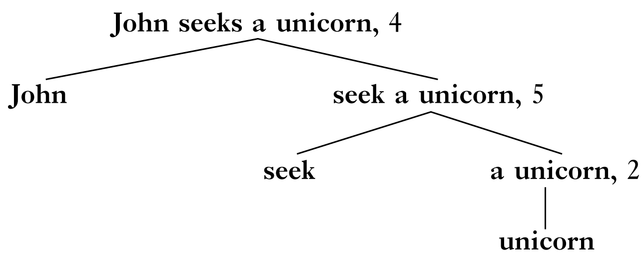
To each node we attach a meaningful expression, together, in case that expression is not basic, with the index of that structural operation among $F_0 - F_2, F_{3,0}, F_{3,1}, \ldots,$ $F_4 - F_9, F_{10,0}, F_{10,1}, \ldots, F_{11} - F_{15}$ (as characterized above, within S1–S17) which we understand as having been applied in obtaining the expression in question; the nodes dominated by any node are to be occupied by the expressions to which the structural operation is understood as having been applied in obtaining the expression occupying the superior node. (For example, the numbers 10,0 attached to the top node of the tree above indicate that the expression attached to that node is regarded as the value of the operation $F_{10,0}$ as applied to certain arguments; and the nodes beneath indicate that those arguments are understood to be the expressions **every man** and **he$_0$ loves a woman such that she loves him$_0$**.) A precise characterization of an *analysis tree* in the sense of these remarks would be routine and will not be given here; for such a characterization in an analogous context the reader might consult Montague (1970a).

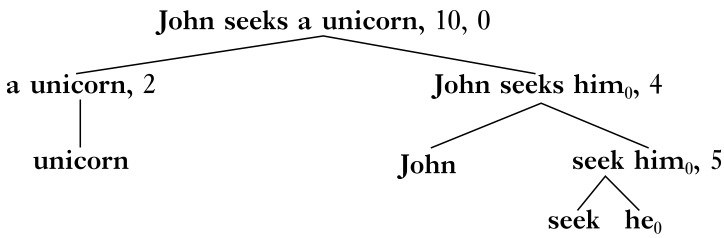
Now there are other ways of constructing the sentence under consideration, and hence other analysis trees for it; indeed, it can be shown that every declarative sentence of our fragment has infinitely many analysis trees. But in the case considered, the various analysis will differ only inessentially; that is to say, they will all lead to the same semantical results.

There are other cases, however, of which this cannot be said. For instance, the sentence

John seeks a unicorn

has two essentially different analyses, represented by the following two trees:





As we shall see, the first of these trees corresponds to the *de dicto* (or nonreferential) reading of the sentence, and the second to the *de re* (or referential) reading.

Thus our fragment admits genuinely (that is, semantically) ambiguous sentences. If it were desired to construct a corresponding unambiguous language, it would be convenient to take the analysis trees themselves as the expressions of that language; it would then be obvious how to characterize (in keeping with Montague (1970b)) the structural operations of that language and the correspondence relation between its expressions and those of ordinary English.[5] For present purposes, however, no such construction is necessary.

2 Intensional Logic

We could (as in Montague (1970a)) introduce the semantics of our fragment directly; but it is probably more perspicuous to proceed indirectly, by (1) setting up a certain simple artificial language, that of tensed intensional logic, (2) giving the semantics of that language, and (3) interpreting English indirectly by showing in a rigorous way how to translate it into the artificial language. This is the procedure we shall adopt; accordingly, I shall now present the syntax and semantics of a tensed variant of the intensional logic I have discussed on earlier occasions.[6]

Let s be a fixed object (2, say) distinct from e and t and not an ordered pair or triple. Then *Type*, or the set of *types*, is the smallest set Y such that (1) $e,t \in Y$, (2) whenever $a,b \in Y$, $\langle a,b \rangle \in Y$, and (3) whenever $a \in Y$, $\langle s,a \rangle \in Y$.

We shall employ denumerably many variables and infinitely many constants of each type. In particular, if n is any natural number and $a \in Type$, we understand by $v_{n,a}$ the n^{th} variable of type a, and by Con_a the set of constants of type a. (The precise cardinality of Con_a need not concern us, provided only that it be infinite.)

By ME_a is understood the set of *meaningful expressions* of type a; this notion has the following recursive definition:

1 Every variable and constant of type a is in ME_a.
2 If $\alpha \in \mathrm{ME}_a$ and u is a variable of type b, then $\lambda u\alpha \in \mathrm{ME}_{\langle b, a\rangle}$.
3 If $\alpha \in \mathrm{ME}_{\langle a,b\rangle}$ and $\beta \in \mathrm{ME}_a$, then $\alpha(\beta) \in \mathrm{ME}_b$.
4 If $\alpha, \beta \in \mathrm{ME}_a$, then $\alpha = \beta \in \mathrm{ME}_t$.
5 If $\phi, \psi \in \mathrm{ME}_t$ and u is a variable, then $\neg\phi, [\phi \wedge \psi], [\phi \vee \psi], [\phi \rightarrow \psi], [\phi \leftrightarrow \psi], \bigvee u\phi, \bigwedge u\phi, \Box\phi, W\phi, H\phi \in \mathrm{ME}_t$.
6 If $\alpha \in \mathrm{ME}_a$, then $[\hat{}\alpha] \in \mathrm{ME}_{\langle s,a\rangle}$.
7 If $\alpha \in \mathrm{ME}_{\langle s, a\rangle}$, then $[\check{}\alpha] \in \mathrm{ME}_a$.
8 Nothing is in any set ME_a except as required by 1–7.[7]

By a *meaningful expression* of intensional logic is understood a member of $\bigcup_{a\in Type} \mathrm{ME}_a$.

If u is a variable of type a, then $\lambda u\alpha$ is understood as denoting that function from objects of type a which takes as value, for any such object x, the object denoted by α when u is understood as denoting x. The expression $\alpha(\beta)$ is as usual understood as denoting the value of the function denoted by α for the argument denoted by β. The equality symbol $=$, the negation symbol $\neg$, the conjunction symbol $\wedge$, the disjunction symbol $\vee$, the conditional symbol $\rightarrow$, the biconditional symbol $\leftrightarrow$, the existential quantifier $\bigvee$, and the universal quantifier $\bigwedge$ are all understood in the usual way. The symbols $\Box$, W, H may be read "it is necessary that," "it will be the case that," "it has been the case that," respectively. The expression $[\hat{}\alpha]$ is regarded as denoting (or having as its *extension*) the *intension* of the expression α. The expression $[{}^{\vee}\alpha]$ is meaningful only if α is an expression that denotes an intension or sense; in such a case $[{}^{\vee}\alpha]$ denotes the corresponding extension.

We could have done with a much smaller stock of primitive symbols, as in Montague (1970b); but there is no point in considering here the relevant reductions.

In the presentation of actual expressions of intensional logic square brackets will sometimes for perspicuity be omitted, and sometimes gratuitously inserted.

Let A, I, J be any sets, which we may for the moment regard as the set of entities (or individuals[8]), the set of possible worlds, and the set of moments of time respectively. In addition, let a be a type. Then $D_{a,A,I,J}$, or the set of *possible denotations* of type a corresponding to A, I, J, may be introduced by the following recursive definition. (If X and Y are any sets, then as usual we understand by X^{Y} the set of all functions with domain Y and values in X, and by $X \times Y$ the *Cartesian product* of X and Y (that is, the set of all ordered pairs $\langle x,y \rangle$ such that $x \in X$ and $y \in Y$). Further, we identify the truth values falsehood and truth with the numbers 0 and 1 respectively.)

$$D_{e,A,I,J} = A,$$
$$D_{t,A,I,J} = \{0,1\},$$
$$D_{\langle a,b\rangle,A,I,J} = \mathbf{D}_{b,A,I,J}{}^{D_{a,A,I,J}},$$
$$D_{\langle s,a\rangle,A,I,J} = D_{a,A,I,J}{}^{I\times J}.$$

By $S_{a,A,I,J}$, or the set of *senses* of type a corresponding to A, I, J, is understood $D_{\langle s,a\rangle,A,I,J}$, that is, $D^{I\times J}_{a,A,I,J}$.

By an *interpretation* (or intensional model) is understood a quintuple $\langle A,I,J,\le,F\rangle$ such that (1) A,I,J are nonempty sets, (2) $\le$ is a simple (that is, linear) ordering having J as its field, (3) F is a function having as its domain the set of all constants, and (4) whenever $a \in$ *Type* and $\alpha \in \mathrm{Con}_a$, $F(\alpha) \in S_{a,A,I,J}$.

Suppose that $\mathfrak{A}$ is an interpretation having the form $\langle A,I,J,\le,F\rangle$. Suppose also that g is an $\mathfrak{A}$-*assignment* (of values to variables), that is, a function having as its domain the set of all variables and such that $g(u) \in D_{a,A,I,J}$ whenever u is a variable of type a. If α is a meaningful expression, we shall understand by $\alpha^{\mathfrak{A},g}$ the *intension* of α with respect to $\mathfrak{A}$ and g; and if $\langle i,j\rangle \in I \times J$ then $\alpha^{\mathfrak{A},i,j,g}$ is to be the *extension* of α with respect to $\mathfrak{A}, i, j$, and g—that is, $\alpha^{\mathfrak{A}.g}(\langle i,j\rangle)$ or the function value of the intension of α when applied to the point of reference $\langle i,j\rangle$. These notions may be introduced by the following recursive definition.

1 If α is a constant, then $\alpha^{\mathfrak{A},g}$ is $F(\alpha)$.
2 If α is a variable, then $\alpha^{\mathfrak{A},i,j,g}$ is $g(\alpha)$.
3 If $\alpha \in \mathrm{ME}_a$ and u is a variable of type b, then $[\lambda u\alpha]^{\mathfrak{A},i,j,g}$ is that function h with domain $D_{b,A,I,J}$ such that whenever x is in that domain, $h(x)$ is $\alpha^{\mathfrak{A},i,j,g'}$, where g' is the $\mathfrak{A}$-assignment like g except for the possible difference that $g'(u)$ is x.
4 If $\alpha \in \mathrm{ME}_{\langle a,b\rangle}$ and $\beta \in \mathrm{ME}_a$, then $[\alpha(\beta)]^{\mathfrak{A},i,j,g}$ is $\alpha^{\mathfrak{A},i,j,g}(\beta^{\mathfrak{A},i,j,g})$ (that is, the value of the function $\alpha^{\mathfrak{A},i,j,g}$ for the argument $\beta^{\mathfrak{A},i,j,g}$).
5 If $\alpha,\beta \in \mathrm{ME}_a$, then $[\alpha = \beta]^{\mathfrak{A},i,j,g}$ is 1 if and only if $\alpha^{\mathfrak{A},i,j,g}$ is $\beta^{\mathfrak{A},i,j,g}$.
6 If $\phi \in \mathrm{ME}_t$, then $[\neg\phi]^{\mathfrak{A},i,j,g}$ is 1 if and only if $\phi^{\mathfrak{A},i,j,g}$ is 0; and similarly for $\wedge, \vee, \rightarrow, \leftrightarrow$.

7 If $\phi \in ME_t$ and u is a variable of type a, then $[\vee u\phi]^{\mathfrak{A},i,j,g}$ is 1, if and only if there exists $x \in D_{a,A,I,\mathcal{J}}$ such that $\phi^{\mathfrak{A},i,j,g'}$ is 1, where g' is as in 3; and similarly for $\wedge u\phi$.
8 If $\phi \in ME_t$, then $[\Box\phi]^{\mathfrak{A},i,j,g}$ is 1 if and only if $\phi^{\mathfrak{A},i',j',g}$ is 1 for all $i' \in I$ and $j' \in \mathcal{J}$;[9] $[W\phi]^{\mathfrak{A},i,j,g}$ is 1 if and only if $\phi^{\mathfrak{A},i,j',g}$ is 1 for some j' such that $j \leq j'$; and $j \neq j'$; and $[H\phi]^{\mathfrak{A},i,j,g}$ is 1 if and only if $\phi^{\mathfrak{A},i,j',g}$ is 1 for some j' such that $j' \leq j$ and $j' \neq j$.
9 If $\alpha \in ME_a$, then $[\hat{}\alpha]^{\mathfrak{A},i,j,g}$ is $\alpha^{\mathfrak{A},g}$.[10]
10 If $\alpha \in ME_{\langle s,a\rangle}$, then $[\check{}\alpha]^{\mathfrak{A},i,j,g}$ is $\alpha^{\mathfrak{A},i,j,g}(\langle i,j\rangle)$.

If ϕ is a *formula* (that is, member of ME_t), then ϕ is *true* with respect to $\mathfrak{A}, i, j$ if and only if $\phi^{\mathfrak{A},i,j,g}$ is 1 for every $\mathfrak{A}$-assignment g.

It will be useful to call attention to some particular meaningful expressions of intensional logic. If $\gamma \in ME_{\langle a,t\rangle}$ and $\alpha \in ME_a$, then γ denotes (that is, has as its extension) a set (or really the characteristic function of a set) of objects of type a, and we may regard the formula $\gamma(\alpha)$, which denotes truth exactly in case the object denoted by α is a member of that set, as *asserting* that the object denoted by α is a member of the set denoted by γ. If $\gamma \in ME_{\langle a,\langle b,t\rangle\rangle}$, $\alpha \in ME_a$, and $\beta \in ME_b$, then γ may be regarded as denoting a (two-place) *relation*, and $\gamma(\beta,\alpha)$ is to be the expression $\gamma(\alpha)(\beta)$, which asserts that the objects denoted by β and α stand in that relation. If $\gamma \in ME_{\langle s,\langle a,t\rangle\rangle}$ and $\alpha \in ME_a$; then γ denotes a *property*, and $\gamma\{\alpha\}$ is to be the expression $[\check{}\gamma](\alpha)$, which asserts that the object denoted by α has that property. If $\gamma \in ME_{\langle s,\langle a,\langle b,t\rangle\rangle\rangle}$, $\alpha \in ME_a$ and $\beta \in ME_b$, then γ may be regarded as denoting a *relation-in-intension*, and $\gamma\{\beta,\alpha\}$ is to be the expression $[\check{}\gamma](\beta,\alpha)$, which asserts that the objects denoted by β and α stand in that relation in intension. If u is a variable of type a and ϕ a formula, then $\hat{u}\phi$ is to be $\lambda u\phi$, which denotes the set of all objects of type a that satisfy ϕ (with respect to the place marked by u), and $\hat{u}\phi$ is to be $[\hat{}\hat{u}\phi]$, which denotes the property of objects of type a expressed by ϕ. If $\alpha \in ME_e$, then α^* is to be $\hat{P}[P\{\hat{}\alpha\}]$, where P is $v_{0,\langle s,\langle\langle s,e\rangle,t\rangle\rangle}$.

3 Translating English into Intensional Logic

We first introduce a mapping f from the categories of English to the types of intensional logic. Accordingly, f is to be a function having *Cat* as its domain and such that

$f(e) = e$,
$f(t) = t$,
$f(A/B) = f(A//B) = \langle\langle s, f(B)\rangle, f(A)\rangle$ whenever $A, B \in Cat$.

The intention is that English expressions of any category A are to translate into expressions of type $f(A)$.[11]

In all that follows let g be a fixed biunique function such that (1) the domain of g is the set of basic expressions of our fragment of English other than **be, necessarily**, and the members of B_T, and (2) whenever $A \in Cat$, $\alpha \in B_A$, and α is in the domain of g, $g(\alpha) \in Con_{f(A)}$. Let j, m, b, n be particular distinct members of Con_e. (If we had introduced a definite well-ordering of the constants of intensional logic, we could at this point have explicitly defined g, j, m, b, and n. Such details would, however, be irrelevant to our present concerns.) Let u, v be the particular individual variables $v_{0,e}, v_{1,e}$ respectively; x, y, x_n be the particular individual-concept variables $v_{1,\langle s,e\rangle}$, $v_{3,\langle s,e\rangle}, v_{2n,\langle s,e\rangle}$ respectively (for any natural number n); p be the proposition variable $v_{0,\langle s,t\rangle}$; P, Q be the variables $v_{0,\langle s,\langle\langle s,e\rangle,t\rangle\rangle}, v_{1,\langle s,\langle\langle s,e\rangle,t\rangle\rangle}$, which range over properties of individual concepts; $\mathscr{P}$ be the variable $v_{0,\langle s,\langle\langle s,\langle\langle s,e\rangle,t\rangle\rangle,t\rangle\rangle}$, which ranges over properties of properties of individual concepts; M be the variable $v_{0,\langle s,\langle e,t\rangle\rangle}$, which ranges over properties of individuals; S be the variable $v_{0,\langle s,\langle e,\langle e,t\rangle\rangle\rangle}$, which ranges over two-place relations-in-intension between individuals; and G be the variable $v_{0,\langle s,\langle e,f(IAV)\rangle\rangle}$.

We shall now consider some rules of translation, T1–T17, which will be seen to correspond to the syntactic rules S1–S17 respectively and to constitute, in a sense to be made precise below, a definition of the translation relation.

Translation rules

Basic rules

T1. (a) If α is in the domain of g, then α translates into $g(\alpha)$.
(b) **be** translates into $\lambda\mathscr{P}\lambda x\mathscr{P}\{\hat{y}[{}^\vee x = {}^\vee y]\}$.
(c) **necessarily** translates into $\hat{p}[\Box{}^\vee p]$.
(d) **John**, **Mary**, **Bill**, **ninety** translate into j^*, m^*, b^*, n^* respectively.
(e) **he**$_n$ translates into $\hat{\mathrm{P}}\ \mathrm{P}\{x_n\}$.

T2. If $\zeta \in \mathrm{P}_{\mathrm{CN}}$ and ζ translates into ζ', then **every** ζ translates into $\hat{\mathrm{P}}\wedge x[\zeta'(x) \rightarrow \mathrm{P}\{x\}]$, **the** ζ translates into $\hat{\mathrm{P}}\vee y[\wedge x[\zeta'(x) \leftrightarrow x = y] \wedge \mathrm{P}\{y\}]$, $F_2(\zeta)$ translates into $\hat{\mathrm{P}} \vee x[\zeta'(x) \wedge \mathrm{P}\{x\}]$.

T3. If $\zeta \in \mathrm{P}_{\mathrm{CN}}$, $\phi \in \mathrm{P}_{\mathrm{t}}$, and ζ, ϕ translate into ζ', ϕ' respectively, then $F_{3,n}(\zeta, \phi)$ translates into $\hat{x}_n[\zeta'(x_n) \wedge \phi']$.[12]

Rules of functional application

T4. If $\delta \in \mathrm{P}_{\mathrm{t/IV}}, \beta \in \mathrm{P}_{\mathrm{IV}}$, and δ, β translate into δ', β' respectively, then $F_4(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T5. If $\delta \in \mathrm{P}_{\mathrm{IV/T}}$, $\beta \in \mathrm{P}_{\mathrm{T}}$, and δ, β translate into δ', β' respectively, then $F_5(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T6. If $\delta \in \mathrm{P}_{\mathrm{IAV/T}}$, $\beta \in \mathrm{P}_{\mathrm{T}}$, and δ, β translate into δ', β' respectively, then $F_5(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T7. If $\delta \in \mathrm{P}_{\mathrm{IV/t}}$, $\beta \in \mathrm{P}_{\mathrm{t}}$, and δ, β translate into δ', β' respectively, then $F_6(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T8. If $\delta \in \mathrm{P}_{\mathrm{IV//IV}}$, $\beta \in \mathrm{P}_{\mathrm{IV}}$, and δ, β translate into δ', β' respectively, then $F_6(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T9. If $\delta \in \mathrm{P}_{\mathrm{t/t}}, \beta \in \mathrm{P}_{\mathrm{t}}$, and δ, β translate into δ', β' respectively, then $F_6(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

T10. If $\delta \in \mathrm{P}_{\mathrm{IV/IV}}$, $\beta \in \mathrm{P}_{\mathrm{IV}}$, and δ, β translate into δ', β' respectively, then $F_7(\delta, \beta)$ translates into $\delta'({}^\wedge\beta')$.

Rules of conjunction and disjunction

T11. If $\phi, \psi \in P_t$ and ϕ, ψ translate into ϕ', ψ' respectively, then ϕ **and** ψ translates into $[\phi \wedge \psi]$, ϕ **or** ψ translates into $[\phi \vee \psi]$.

T12. If $\gamma, \delta \in P_{IV}$ and γ, δ translate into γ', δ' respectively, then γ **and** δ translates into $\hat{x}[\gamma'(x) \wedge \delta'(x)]$, γ **or** δ translates into $\hat{x}[\gamma'(x) \vee \delta'(x)]$.

T13. If $\alpha, \beta \in P_T$ and α, β translate into α', β' respectively, then α **or** β translates into $\hat{P}[\alpha'(P) \vee \beta'(P)]$.

Rules of quantification

T14. If $\alpha \in P_T, \phi \in P_t$, and α, ϕ translate into α', ϕ' respectively, then $F_{10,n}(\alpha, \phi)$ translates into $\alpha'(\hat{x}_n \phi')$.

T15. If $\alpha \in P_T, \zeta \in P_{CN}$, and α, ζ translate into α', ζ' respectively, then $F_{10,n}(\alpha, \zeta)$ translates into $\lambda y \alpha'(\hat{x}_n[\zeta'(y)])$.

T16. If $\alpha \in P_T$, $\delta \in P_{IV}$, and α, δ translate into α', δ' respectively, then $F_{10,n}(\alpha, \delta)$ translates into $\lambda y \alpha'(\hat{x}_n[\delta'(y)])$.

Rules of tense and sign

T17. If $\alpha \in P_T, \delta \in P_{IV}$, and α, δ translate into α', δ' respectively, then
$F_{11}(\alpha, \delta)$ translates into $\neg\alpha'(\hat{}\delta')$,
$F_{12}(\alpha, \delta)$ translates into $W\alpha'(\hat{}\delta')$,
$F_{13}(\alpha, \delta)$ translates into $\neg W\alpha'(\hat{}\delta')$,
$F_{14}(\alpha, \delta)$ translates into $H\alpha'(\hat{}\delta')$,
$F_{15}(\alpha, \delta)$ translates into $\neg H\alpha'(\hat{}\delta')$.

The precise import of the rules T1–T17 is that the translation relation may be defined as the smallest binary relation satisfying them; that is to say, an expression ϕ is characterized as *translating into* an expression ϕ' if the pair $\langle \phi, \phi' \rangle$ is a member of every binary relation R such that T1–T17 hold (with the condition that one expression translates into another replaced by the condition that the relation R holds between the two expressions).

The translation relation is of course not a function; a meaningful expression of English may translate into several different expressions of intensional logic. We could, however, speak of *the translation* of a given meaningful expression of English corresponding to any given analysis tree for that expression; the rather obvious definition of this notion will be omitted here. The interpretations of intensional logic may, by way of the translation relation, be made to play a second role as interpretations of English.[13] Not all interpretations of intensional logic, however, would be reasonable candidates for interpretations of English. In particular, it would be reasonable in this context to restrict attention to those interpretations of intensional logic in which the following formulas are true (with respect to all, or equivalently some, worlds and moments of time):

1 $\bigvee u \square [u = \alpha]$, where α is j, m, b, or n,
2 $\square [\delta(x) \rightarrow \bigvee u\ x = \hat{}u]$, where δ translates any member of B_{CN} other than **price** or **temperature,**
3 $\bigvee M \bigwedge x \square [\delta(x) \leftrightarrow M\{\check{}x\}]$, where δ translates any member of B_{IV} other than **rise** or **change**,
4 $\bigvee S \bigwedge x \bigwedge \mathcal{P} \square [\delta(x, \mathcal{P}) \leftrightarrow \mathcal{P}\{\hat{y} S\{\check{}x, \check{}y\}\}]$, where δ translates **find, lose, eat, love,** or **date,**
5 $\bigwedge \mathcal{P} \bigvee M \bigwedge x \square [\delta(x, \mathcal{P}) \leftrightarrow M\{\check{}x\}]$, where δ translates **seek** or **conceive,**
6 $\bigwedge p \bigvee M \bigwedge x \square [\delta(x, p) \leftrightarrow M\{\check{}x\}]$, where δ translates **believe that** or **assert that,**
7 $\bigwedge \mathrm{P} \bigvee M \bigwedge x \square [\delta(x, P) \leftrightarrow M\{\check{}x\}]$, where δ translates **try to** or **wish to,**
8 $\bigvee G \bigwedge \mathcal{P} \bigwedge Q \bigwedge x \square [\delta(\mathcal{P})(Q)(x) \leftrightarrow \mathcal{P}\{\hat{y}[[\check{}G](\check{}y)(Q)(x)]\}]$, where δ translates **in,**
9 $\square$ [**seek**$'(x, \mathcal{P}) \leftrightarrow$ **try-to**$'(x, \hat{}[$**find**$'\ (\mathcal{P})])]$, where **seek**$'$, **try-to**$'$, **find**$'$ translate **seek, try to, find** respectively.

The truth of (1) guarantees that proper nouns will be "logically determinate" according to the interpretations under consideration, that is, will have extensions invariant with respect to possible worlds and moments of time. In view of (2), "ordinary" common nouns (for example, **horse**) will denote sets of *constant* individual concepts (for example, the set of constant functions on worlds and moments having horses as their values; from an intuitive viewpoint, this is no different from the set of horses). It would be unacceptable to impose this condition on such "extraordinary" common nouns as **price** or **temperature**; the individual concepts in their extensions would in the most natural cases be functions whose values vary with their temporal arguments. The truth of (3) is the natural requirement of *extensionality* for intransitive verbs, that of (4) the condition of extensionality (or extensional first-order reducibility) for transitive verbs, and that of (8) the condition of extensionality (or extensional first-order reducibility) for prepositions. The *intensional* (or nonextensional) transitive verbs **seek** and **conceive,** as well as the verbs **believe that, assert that, try to, wish to** of other categories, are nevertheless *extensional with respect to subject position*, and this is expressed by imposing conditions (5)–(7). Condition (9) is the natural definition of **seek** as **try to find**.

Several notions of a logically possible interpretation may reasonably come into consideration, depending on whether, and if so how many, conditions analogous to (1)–(9), stemming from our intended system of translation, are to be imposed. For present purposes we may perhaps resolve the matter as follows: by a *logically possible interpretation* understand an interpretation of intensional logic in which formulas (1)–(9) are true (with respect to all worlds and moments of time). Logical truth, logical consequence, and logical equivalence, for formulas of intensional logic, are to be characterized accordingly. For instance, a formula ϕ of intensional logic is construed as *logically true* if it is true in every logically possible interpretation, with respect to all worlds and moments of time of that interpretation; and two formulas ϕ and ψ of intensional logic are *logically equivalent* if and only if the biconditional $[\phi \leftrightarrow \psi]$ is logically true.

If δ is an expression of intensional logic of such type as to translate a transitive or intransitive verb, then δ_* is to be an expression designating the set of individuals or relation between individuals that naturally corresponds to the set or relation designated by δ. In particular, if $\delta \in \mathrm{ME}_{f(\mathrm{IV})}$, then δ_* is to be the expression $\hat{u}\delta([\hat{\ }u])$; and if $\delta \in \mathrm{ME}_{f(\mathrm{TV})}$, then δ_* is to be $\lambda v\hat{u}\delta([\hat{\ }u], [\hat{\ }v^*])$. Notice that since $f(\mathrm{CN}) = f(\mathrm{IV})$, this characterization is also applicable in the case in which δ translates a common noun.

It is a consequence of principles (2), (3), (4) that if δ is among the constants involved in those principles (that is, constants translating "ordinary" common nouns or "extensional" transitive or intransitive verbs), then δ is definable in terms of δ_*. More exactly, the following formulas are logically true (*Editors' note*: The first formula below actually holds only for B_{IV}, not for B_{CN}):

$\square[\delta(x) \leftrightarrow \delta_*(\check{\ }x)]$, if δ translates any member of B_{CN} or B_{IV} other than **price, temperature, rise,** or **change;**
$\square[\delta(x, \mathscr{P}) \leftrightarrow \mathscr{P}\{\hat{y}\delta_*(\check{\ }x, \check{\ }y)\}]$, if δ translates any member of B_{TV} other than **seek** or **conceive**.

Notice that although the verb **be** (or its translation) is not covered by principle (4), it is by the last principle above. The reason why the extensionality of **be** was not explicitly assumed is that it can be proved. (More precisely, the analogue of (4) in which δ is the expression translating **be** is true in all interpretations (with respect to all worlds and moments).)

4 Examples

The virtues of the present treatment can perhaps best be appreciated by considering particular English sentences and the precisely interpreted sentences of intensional logic that translate them. I shall give a list of such examples. It is understood that each English sentence listed below translates into some formula logically equivalent to each of the one or more formulas of intensional logic listed with it, and that every formula into which the English sentence translates is logically equivalent to one of those formulas. It should be emphasized that this is not a matter of vague intuition, as in elementary logic courses, but an assertion to which we have assigned exact significance in preceding sections and which can be rigorously proved. (The constants of intensional logic that translate various basic expressions of English are designated below by primed variants of those expressions.)

The first five examples indicate that in simple extensional cases symbolizations of the expected forms are obtained.

Bill walks : $\mathbf{walk}'_*(b)$
a man walks : $\vee u[\mathbf{man}'_*(u) \wedge \mathbf{walk}'_*(u)]$
every man walks : $\wedge u[\mathbf{man}'_*(u) \rightarrow \mathbf{walk}'_*(u)]$
the man walks : $\vee v \wedge u\,[[\mathbf{man}'_*(u) \leftrightarrow u = v] \wedge \mathbf{walk}'_*(v)]$
John finds a unicorn : $\vee u\,[\mathbf{unicorn}'_*(u) \wedge \mathbf{find}'_*(j, u)]$

The next sentence, though superficially like the last, is ambiguous and has two essentially different symbolizations corresponding to the two analysis trees presented above; the first gives the *de dicto* reading; and the second the *de re*.

$$\textbf{John seeks a unicorn}: \begin{cases} \mathbf{seek}'({}^{\wedge}j, \hat{P}\bigvee u[\mathbf{unicorn}'_*(u) \wedge P\{{}^{\wedge}u\}]) \\ \bigvee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{seek}'_*(j, u)] \end{cases}$$

The source of the ambiguity of **John seeks a unicorn** will perhaps be clarified if we compare that sentence with the intuitively synonymous **John tries to find a unicorn,** which contains no intensional verbs but only the extensional verb **find** and the "higher-order" verb **try to**. Here, though perhaps not in **John seeks a unicorn,** the ambiguity is clearly a matter of scope, and indeed depends on the possibility of regarding either the component **find a unicorn** or the whole sentence as the scope of the existential quantification indicated by **a unicorn**.

$$\textbf{John tries to find a unicorn}: \begin{cases} \mathbf{try\text{-}to}'({}^{\wedge}j, \hat{y}\bigvee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{find}'_*({}^{\vee}y, u)]) \\ \bigvee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{try\text{-}to}'({}^{\wedge}j, \hat{y}\, \mathbf{find}'_*({}^{\vee}y, u))] \end{cases}$$

It might be suggested, as in Quine (1960) or Montague (1969), that intensional verbs be allowed only as paraphrases of more tractable locutions (such as **try to find**).[14] Such a proposal, however, would not be naturally applicable, for want of a paraphrase, to such intensional verbs as **conceive** and such intensional prepositions as **about**; and I regard it as one of the principal virtues of the present treatment, as well as the one in Montague (1970b), that it enables us to deal directly with intensional locutions. The next example accordingly concerns **about** and gives us, as intuition demands, one reading of **John talks about a unicorn** that does not entail that there are unicorns.

$$\textbf{John talks about a unicorn}: \begin{cases} \mathbf{about}'(\hat{P}\bigvee u[\mathbf{unicorn}'_*(u) \wedge P\{{}^{\wedge}u\}])({}^{\wedge}\mathbf{talk}')({}^{\wedge}j) \\ \bigvee u[\mathbf{unicorn}'_*(u) \wedge \mathbf{about}'({}^{\wedge}u^*)({}^{\wedge}\mathbf{talk}')({}^{\wedge}j)] \end{cases}$$

The next two examples indicate that our uniform symbolization of **be** will adequately cover both the **is** of identity and the **is** of predication; views along this line, though not the rather complicated analysis of **be** given here, may be found in Quine (1960).

Bill is Mary : $b = m$
Bill is a man : $\mathbf{man}'_*(b)$

The next few examples concern an interesting puzzle due to Barbara Hall Partee involving a kind of intensionality not previously observed by philosophers. From the premises **the temperature is ninety** and **the temperature rises,** the conclusion **ninety rises** would appear to follow by normal principles of logic; yet there are occasions on which both premises are true, but none on which the conclusion is. According to the following symbolizations, however, the argument in question turns out not to be valid. (The reason, speaking very loosely, is this. **The temperature** "denotes" an individual concept, not an individual; and **rise,** unlike most verbs, depends for its applicability on the full behavior of individual concepts, not just on their extensions with respect to the actual world and (what is more relevant here) moment of time. Yet the sentence **the temperature is ninety** asserts the identity not of two individual concepts but only of their extensions.)

the temperature is ninety : $\bigvee y[\bigwedge x[\textbf{temperature}'(x) \leftrightarrow x = y] \wedge [{}^{\vee}y] = n]$
the temperature rises : $\bigvee y[\bigwedge x[\textbf{temperature}'(x) \leftrightarrow x = y] \wedge \textbf{rise}'(y)]$
ninety rises : $\textbf{rise}'({}^{\wedge}n)$

We thus see the virtue of having intransitive verbs and common nouns denote sets of individual concepts rather than sets of individuals – a consequence of our general development that might at first appear awkward and unnatural. It would be possible to treat the Partee argument itself without introducing this feature, but not certain analogous arguments involving indefinite rather than definite terms. Notice, for instance, that **a price rises** and **every price is a number** must not be allowed to entail **a number rises.** Indeed they do not according to our treatment; to see this, perhaps it is enough to consider the first premise, which, unlike **a man walks,** requires individual-concept variables (and not simply individual variables) for its symbolization.

a price rises : $\bigvee x[\textbf{price}'(x) \wedge \textbf{rise}'(x)]$

The next example shows that ambiguity can arise even when there is no element of intensionality, simply because quantifying terms may be introduced in more than one order.

a woman loves every man : $\begin{cases} \bigvee u[\textbf{woman}'_*(u) \wedge \bigwedge v[\textbf{man}'_*(v) \rightarrow \textbf{love}'_*(u, v)]] \\ \bigwedge v[\textbf{man}'_*(v) \rightarrow \bigvee u[\textbf{woman}'_*(u) \wedge \textbf{love}'_*(u, v)]] \end{cases}$

The next example indicates the necessity of allowing verb phrases as well as sentences to be conjoined and quantified. Without such provisions the sentence **John wishes to find a unicorn and eat it** would (unacceptably, as several linguists have pointed out in connection with parallel examples) have only a "referential" reading, that is, one that entails that there are unicorns.

John wishes to find a unicorn and eat it :

$$\begin{cases} \bigvee u[\textbf{unicorn}'_*(u) \wedge \textbf{wish-to}'({}^{\wedge}j, \hat{y}[\textbf{find}'_*({}^{\vee}y, u) \wedge \textbf{eat}'_*({}^{\vee}y, u)])] \\ \textbf{wish-to}'({}^{\wedge}j, \hat{y}\bigvee u[\textbf{unicorn}'_*(u) \wedge \textbf{find}'_*({}^{\vee}y, u) \wedge \textbf{eat}'_*({}^{\vee}y, u)]) \end{cases}$$

The next example is somewhat simpler, in that it does not involve conjoining or quantifying verb phrases; but it also illustrates the possibility of a nonreferential reading in the presence of a pronoun.

Mary believes that John finds a unicorn and he eats it :

$$\begin{cases} \bigvee u[\textbf{unicorn}'_*(u) \wedge \textbf{believe-that}'({}^{\wedge}m, {}^{\wedge}[\textbf{find}'_*(j, u) \wedge \textbf{eat}'_*(j, u)])] \\ \textbf{believe-that}'({}^{\wedge}m, {}^{\wedge}\bigvee u[\textbf{unicorn}'_*(u) \wedge \textbf{find}'_*(j, u) \wedge \textbf{eat}'_*(j, u)]) \end{cases}$$

On the other hand, in each of the following examples only one reading is possible, and that the referential:

(1) **John seeks a unicorn and Mary seeks it,**
(2) **John tries to find a unicorn and wishes to eat it,**
$\vee u[\mathbf{unicorn}'_{*}(u) \wedge \mathbf{try\text{-}to}'(\hat{}j, \hat{y}\ [\mathbf{find}'_{*}(\check{}y, u)]) \wedge \mathbf{wish\text{-}to}'(\hat{}j, \hat{y}\,[\mathbf{eat}_{*}\,(\check{}y, u)])]$

This is, according to my intuitions (and, if I guess correctly from remarks in Partee (1970), those of Barbara Partee as well), as it should be; but David Kaplan would differ, at least as to (2). Let him, however, and those who might sympathize with him consider the following variant of (2) and attempt to make nonreferential sense of it:

(2′) **John wishes to find a unicorn and tries to eat it.**

Of course there are other uses of pronouns than the ones treated in this paper – for instance, their use as what have been called in Geach (1962, 1967) and Partee (1970) *pronouns of laziness*, that is, as "standing for" longer terms bearing a somewhat indefinite relation to other expressions in the sentence in question (or preceding sentences within the discourse in question). For instance, it is not impossible to construe **it** in (2) as standing for **the unicorn he finds** (that is, **the unicorn such that he finds it**), **a unicorn he finds**, or **every unicorn he finds,** and in this way to obtain a nonreferential reading of that sentence; but this is not a reading with which David Kaplan would be content.

Notes

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1 The medieval and twentieth-century philosophical literature has pointed out a number of such difficulties, most of them involving so-called intensional contexts. I am indebted to Barbara Hall Partee for pointing out others, both in conversation and in her provocative paper Partee (1970). (This remark should not, however, be taken as implying agreement with any of Professor Partee's conclusions.)
2 With the exception that in Montague (1970b) a number of intuitively plausible ambiguities were for simplicity ruled out.
3 In connection with imperatives and interrogatives truth and entailment conditions are of course inappropriate, and would be replaced by fulfilment conditions and a characterization of the semantic content of a correct answer.
4 It was perhaps the failure to pursue the possibility of syntactically splitting categories originally conceived in semantic terms that accounts for the fact that Ajdukiewicz's proposals have not previously led to a successful syntax. They have, however, been employed semantically in Montague (1970a) and, in a modified version, in Lewis (1970).
5 This way of constructing an underlying unambiguous language, though convenient here, would be unsuitable in connection with fragments of natural language exhibiting greater syntactical complexities of certain sorts.

6 In particular, in talks before the Southern California Logic Colloquium and the Association for Symbolic Logic in April and May of 1969, and in the paper Montague (1970b). The addition of tenses is rather routine in the light of the discussion in Montague (1968); and it would be possible to replace the tense operators by predicates, thus preserving exactly the language in Montague (1970b), in the manner indicated in Montague (1970c).

7 Clause (8) is of course vague but can be eliminated in a familiar way. To be exact, the recursive definition given above can be replaced by the following explicit definition: ME_a is the set of all objects α such that αRa, where R is the smallest relation such that clauses (1)–(7) hold (with all parts of the form "$\beta \in ME_a$" replaced by "βRa").

8 Or possible individuals. If there are individuals that are only possible but not actual, A is to contain them; but this is an issue on which it would be unethical for me as a logician (or linguist or grammarian or semanticist, for that matter) to take a stand.

9 [Richmond H. Thomason's note: Here, $\square$ is interpreted in the sense of "necessarily always."]

10 [Richmond H. Thomason's note: The form of this definition is not quite correct, since $\alpha^{\mathfrak{A},g}$ is undefined when α is not a constant. But the intention is clear; what is to be defined recursively is $\alpha^{\mathfrak{A},i,j,g}$. Clauses (1) and (9) should be revised to read as follows.

1 If α is a constant then $\alpha^{\mathfrak{A},i,j,g}$ is $F(\alpha)(\langle i,j\rangle)$.

9 If $\alpha \in ME_a$ then $[\hat{\ }\alpha]^{\mathfrak{A},i,j,g}$ is that function h with domain $I \times \mathcal{J}$ such that whenever $\langle i,j\rangle \in I \times \mathcal{J}$, $h(\langle i,j\rangle) = \alpha^{\mathfrak{A},i,j,g}$.

The *intension* $\alpha^{\mathfrak{A},g}$ of α relative to $\mathfrak{A}$ and g is then defined explicitly:

$\alpha^{\mathfrak{A},g}$ is that function h with domain $I \times \mathcal{J}$ such that whenever $\langle i,j\rangle \in I \times \mathcal{J}, h(\langle i,j\rangle) = \alpha^{\mathfrak{A},i,j,g}$. It then follows as a corollary that $[\hat{\ }\alpha]^{\mathfrak{A},i,j,g} = \alpha^{\mathfrak{A},g}$ for all $\langle i,j\rangle \in I \times \mathcal{J}$.]

11 The simplicity and uniformity of the present correspondence stands in remarkable contrast to the *ad hoc* character of the type assignment in Montague (1970b).

12 [Richmond H. Thomason's note: To avoid collision of variables, the translation must be $\hat{x}_m[\zeta'(x_m) \wedge \psi]$, where ψ is the result of replacing all occurrences of x_n in φ' by occurrences of x_m, where m is the least even number such that x_m has no occurrences in either ζ' or φ'.]

13 Alternatives are possible. For instance, we could instead consider *direct* interpretations of English induced by interpretations of intensional logic in conjunction with our translation procedure; the precise general construction is given in Montague (1970b). Though this would probably be the best approach from a general viewpoint, it would introduce slight complications that need not be considered in the present paper.

14 Strictly speaking, this would mean, within the framework of the present paper, introducing a syntactic operation F such that, for example, F **(John tries to find a unicorn)** = **John seeks a unicorn,** a syntactic rule to the effect that $F(\phi) \in P_t$ whenever $\phi \in P_t$, and a corresponding translation rule that whenever $\phi \in P_t$ and ϕ translates into ϕ', $F(\phi)$ translates into ϕ'.

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