Adjusted Higher Gauge Theory: Connections and Parallel Transport

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Topological Quantum Field Theory Seminar
Lisbon, 7.5.2021

Based on:

- arXivs: 1705.02353,1908.08086 with Lennart Schmidt
- arXiv:1911.06390 with Hyungrok Kim
- arXiv:2105.????? with Leron Borsten and Hyungrok Kim

Motivation

Representation theory suggests and string theory predicts a mysterious superconformal field theory in six dimensions

People call this Theory X or The (2,0)-Theory.

Little is known. No Lagrangian exists.

We know:

- It describes stacks of M5-branes with gravity turned off (just as Yang–Mills theory describes stack of D-branes)
- It has Wilson surfaces as observables (just as Yang–Mills has Wilson lines)
  It is a theory of ("self-dual") strings

Conjecture

The (2,0)-theory is classically a higher gauge theory.

"But Witten has said there is no Lagrangian!"

"... by hunting for unicorns we may find other creatures that are useful in understanding the theory more generally."

Neil Lambert

Wish:

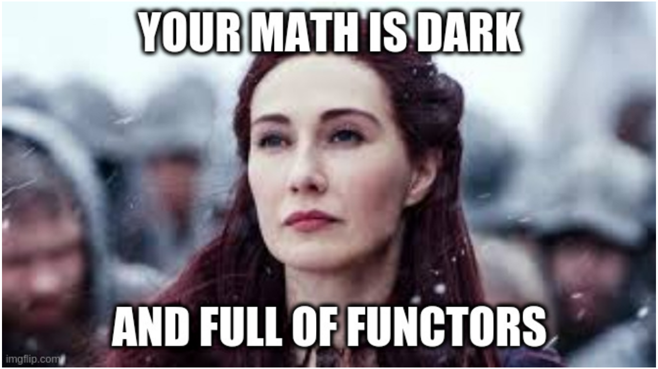
Reality:

or

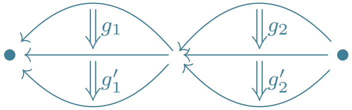
Actual outline

- Sketch: Higher Principal Bundles with Connections
- Vanishing of Fake Curvature and Implications
- Adjusted Higher Gauge Theory
- Adjusted Higher Parallel Transport
- Origin of Adjustment: EL_∞-algebras

Sketch: Higher Principal Bundles with Connections



Non-abelian parallel transport of strings problematic:



Consistency of parallel transport requires:

$$(g_1' g_2')(g_1 g_2) = (g_1' g_1)(g_2' g_2)$$

This renders group G abelian.

Eckmann and Hilton, 1962
Physicists 80'ies and 90'ies

Way out: 2-categories, Higher Gauge Theory.

Two operations $\circ$ and $\otimes$ satisfying Interchange Law:

$$(g_1' \otimes g_2') \circ (g_1 \otimes g_2) = (g_1' \circ g_1) \otimes (g_2' \circ g_2) \ .$$

Lie 2-group

A Lie 2-group is a

- monoidal category, morph. invertible, obj. weakly invertible.
- Lie groupoid + product $\otimes$ obeying weakly the group axioms.

Simplification: strict Lie 2-groups $\stackrel{1:1}{\longleftrightarrow}$ x-modules(Lie groups)

Crossed modules of Lie groups

Pair of Lie groups (G, H), written as $(\mathsf{H} \xrightarrow{\mathsf{t}} \mathsf{G})$ with:

- left automorphism action $\rhd : \mathsf{G} \times \mathsf{H} \to \mathsf{H}$
- group homomorphism $\mathsf{t} : \mathsf{H} \to \mathsf{G}$ such that

$$\mathsf{t}(g \rhd h) = g\mathsf{t}(h)g^{-1} \quad \text{and} \quad \mathsf{t}(h_1) \rhd h_2 = h_1 h_2 h_1^{-1}$$

Also: strict Lie 2-algebras $\stackrel{1:1}{\longleftrightarrow}$ crossed modules of Lie algebras

Cocycles of Principal Bundles from Functors

The cover $\bigsqcup_a U_a$ of a manifold M encoded in the Čech groupoid:

$$\check{\mathscr{C}}(U)\ :\quad \bigsqcup_{a,b} U_{ab} \rightrightarrows \bigsqcup_a U_a\ ,\qquad U_{ab}\circ U_{bc} = U_{ac}\ .$$

Principal G-bundle

Transition functions are nothing but a functor $g : \check{\mathscr{C}}(U) \to (\mathsf{G} \rightrightarrows *)$

$$\begin{array}{ccc} \bigsqcup_{a,b} U_{ab} & \xrightarrow{\ g_{ab}\ } & \mathsf{G} \\ \downdownarrows & & \downdownarrows \\ \bigsqcup_a U_a & \xrightarrow{\ *\ } & * \end{array} \qquad\qquad g_{ab}g_{bc} = g_{ac}$$

Equivalence relations: natural isomorphisms.

Use higher categories: Higher bundles including gerbes

Semistrict Categorified Lie Algebras $\leftrightarrow$ L_∞-algebras

Recall: Chevalley–Eilenberg algebra of a Lie algebra $\mathfrak{g}$:

- Graded vector space $V = \mathfrak{g}[1]^*$, coords. ξ^α, $|\xi^\alpha| = 1$.
- Vector field $Q = -\frac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma\frac{\partial}{\partial\xi^\alpha}$, $\quad Q^2 = 0$ and $|Q| = 1$.
- Lie bracket $[\tau_\alpha, \tau_\beta] = f^\gamma_{\alpha\beta}\tau_\gamma$, $\quad Q^2 = 0 \;\Leftrightarrow$ Jacobi identity

Generalize to Chevalley–Eilenberg algebra of L_∞-algebra:

- $\mathfrak{g} = \oplus_{i\leq 0}\mathfrak{g}_i$, $\qquad Q$ most general with $Q^2 = 0$ and $|Q| = 1$
- Structure constants in Q: $\mu_i : \mathfrak{g}^{\wedge i} \to \mathfrak{g}$, $\;|\mu_i| = 2 - i$.
- $Q^2 = 0 \;\Leftrightarrow$ homotopy Jacobi identities

Example:

- $\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0$, Q quadratic: differential crossed module.

Local connection data

- Ideas: Atiyah, Strobl et al., Sati, Schreiber, Stasheff
- Recall: Chevalley-Eilenberg algebra of Lie algebra $\mathfrak{g}$:

$$\mathrm{CE}(\mathfrak{g}) = C^\infty(\mathfrak{g}[1])\ , \quad Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma$$

- Double to Weil algebra $\quad(\mathrm{CE}(\mathfrak{inn}(\mathfrak{g})))$

$$\mathrm{W}(\mathfrak{g}) := C^\infty(\overset{\sigma\xi^\alpha}{T[1]}\overset{\xi^\alpha}{\mathfrak{g}[1]})\ , \quad Q = Q_{\mathrm{CE}} + \sigma\ , \quad \sigma Q_{\mathrm{CE}} = -Q_{\mathrm{CE}}\sigma$$

- Potentials/curvatures/Bianchi identities from dga-morphisms

$$(A, F) : \mathrm{W}(\mathfrak{g}) \to \Omega^\bullet(M) = W(M)$$
$$\xi^\alpha \mapsto A^\alpha$$
$$(\sigma\xi^\alpha) = Q\xi^\alpha + \tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \mapsto F^\alpha = (\mathrm{d}A + \tfrac{1}{2}[A, A])^\alpha$$
$$Q(\sigma\xi^\alpha) = -f^\alpha_{\beta\gamma}(\sigma\xi^\alpha)\xi^\beta \mapsto (\nabla F)^\alpha = 0$$

- Gauge transformations: homotopies between dga-morphisms
- Topological invariants: invariant polynomials in $\mathrm{W}(\mathfrak{g})$

Gluing things together

Notice:

- Local connections can be glued together to global object
- Best: analogous construction to Atiyah algebroid.
- Everything clear in principle.

> *"Category theory is the subject where you can leave the definitions as exercises."*
>
> *John Baez*

Cocycles for Principal 2-Bundles

Consider a manifold M with cover (U_a)

Object	Principal G-bundle	Principal $(\mathsf{H} \xrightarrow{\mathsf{t}} \mathsf{G})$-bundle
Cochains	(g_{ab}) valued in G	(g_{ab}) valued in G, (h_{abc}) valued in H
Cocycle	$g_{ab}g_{bc} = g_{ac}$	$\mathsf{t}(h_{abc})g_{ab}g_{bc} = g_{ac}$ $h_{acd}h_{abc} = h_{abd}(g_{ab} \rhd h_{bcd})$
Coboundary	$g_a g'_{ab} = g_{ab}g_b$	$g_a g'_{ab} = \mathsf{t}(h_{ab})g_{ab}g_b$ $h_{ac}h_{abc} = (g_a \rhd h'_{abc})h_{ab}(g_{ab} \rhd h_{bc})$
gauge pot.	$A_a \in \Omega^1(U_a) \otimes \mathfrak{g}$	$A_a \in \Omega^1(U_a) \otimes \mathfrak{g}$, $B_a \in \Omega^2(U_a) \otimes \mathfrak{h}$
Curvature	$F_a = \mathrm{d}A_a + A_a \wedge A_a -$	$\mathcal{F}_a = \mathrm{d}A_a + \frac{1}{2}[A_a, A_a] - \mathsf{t}(B_a)$ $H_a = \mathrm{d}B_a + A_a \rhd B_a$
Gauge trafos	$\tilde{A}_a := g_a^{-1}A_a g_a + g_a^{-1}\mathrm{d}g_a$	$\tilde{A}_a := g_a^{-1}A_a g_a + g_a^{-1}\mathrm{d}g_a + \mathsf{t}(\Lambda_a)$ $\tilde{B}_a := g_a^{-1} \rhd B_a + \tilde{A}_a \rhd \Lambda_a + \mathrm{d}\Lambda_a - \Lambda_a \wedge \Lambda_a$

Remarks:

- A principal $(1 \xrightarrow{\mathsf{t}} \mathsf{G})$-bundle is a principal G-bundle.
- A principal $(\mathsf{U}(1) \xrightarrow{\mathsf{t}} 1) = \mathsf{BU}(1)$-bundle is an abelian gerbe.

Vanishing of Fake Curvature and Implications

Why should the fake curvature(s) vanish?

$$\mathcal{F} := \mathrm{d}A + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B) = 0$$

Without this condition:

- For $\mu_3 \neq 0$: infinitesimal gauge transformations do not close:

$$[\delta_{c_0}, \delta_{c_1}]A = \delta_{[c_0,c_1]}A + \tfrac{1}{2}\mu_3(\mathcal{F}, A, A)$$

- For $\mu_3 = 0$, higher gauge transformations do not close
- Higher parallel transport is not reparametrization invariant
- Self-duality equation $H = \star H$ is not gauge-covariant:

$$H \to \tilde{H} = g \rhd H - \mathcal{F} \rhd \Lambda$$

With this condition:

- Principal $(1 \xrightarrow{\mathsf{t}} \mathsf{G})$-bundle is flat principal G-bundle.
- Higher connections are locally abelian!

Argument

- Lie 2-group (crossed module) $(\mathsf{H} \xrightarrow{\mathsf{t}} \mathsf{G}, \triangleright)$, $(\mathfrak{h} \xrightarrow{\mathsf{t}} \mathfrak{g}, \triangleright)$
- Potential forms: $A \in \Omega^1(\mathbb{R}^d, \mathfrak{g})$, $B \in \Omega^2(\mathbb{R}^d, \mathfrak{h})$
- Fake flatness: $\mathcal{F} := \mathrm{d}A + \frac{1}{2}[A, A] + \mathsf{t}(B) = 0$
- Gauge transformations: $g \in \Omega^0(\mathbb{R}^d, \mathsf{G})$, $\Lambda \in \Omega^1(\mathbb{R}^d, \mathfrak{h})$

$$A \mapsto \tilde{A} = g^{-1}Ag + g^{-1}\mathrm{d}g + \mathsf{t}(\Lambda_1)$$
$$B \mapsto \tilde{B} = g^{-1} \triangleright B + \mathrm{d}\Lambda_1 + \tilde{A} \triangleright \Lambda_1 + \tfrac{1}{2}[\Lambda_1, \Lambda_1]$$

- A and gauge transformations restrict to $\mathsf{G}^\circ = \mathsf{G}/\mathrm{im}(\mathsf{t})$
- $F^\circ = 0$ and non-abelian Poincaré lemma: gauge with $\tilde{A}^\circ = 0$
- $\tilde{A} \in \mathrm{im}(\mathsf{t})$, gauge away with Λ-transformation: $\tilde{\tilde{A}} = 0$
- connection is abelian with $\tilde{\tilde{B}} \in \ker(\mathsf{t})$!

1908.08086, see also Gastel (2018)

Solution: Adjusted Higher Gauge Theory

Adjustment for skeletal string algebra

Example: Skeletal string Lie 2-algebra: $\mathfrak{string}(\mathfrak{g}) = (\mathbb{R} \to \mathfrak{g})$

- Unadjusted action of differential of Weil algebra: Q_{W}:

$$t^\alpha \mapsto -\tfrac{1}{2} f^\alpha_{\beta\gamma} t^\beta t^\gamma + \hat{t}^\alpha \qquad r \mapsto \tfrac{1}{3!} f_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma$$
$$\hat{t}^\alpha \mapsto -f^\alpha_{\beta\gamma} t^\beta \hat{t}^\gamma \qquad \hat{r} \mapsto -\tfrac{1}{2} f_{\alpha\beta\gamma} t^\alpha t^\beta \hat{t}^\gamma$$

- Adjusted action of Q_{W}

$$t^\alpha \mapsto -\tfrac{1}{2} f^\alpha_{\beta\gamma} t^\beta t^\gamma + \hat{t}^\alpha \qquad r \mapsto \tfrac{1}{3!} f_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma - \kappa_{\alpha\beta} t^\alpha \hat{t}^\beta + \hat{r}$$
$$\hat{t}^\alpha \mapsto -f^\alpha_{\beta\gamma} t^\beta \hat{t}^\gamma \qquad \hat{r} \mapsto \kappa_{\alpha\beta} \hat{t}^\alpha \hat{t}^\beta$$

- Adjustment governed by Killing form $\kappa_{\alpha\beta}$.
- Projection $\mathsf{W}(\mathfrak{string}(\mathfrak{g})) \to \mathsf{CE}(\mathfrak{string}(\mathfrak{g}))$ unmodified
- Redefinition of curvature: $\hat{r} \mapsto \hat{r} - \kappa_{\alpha\beta} t^\alpha \hat{t}^\beta$.
- Simply: coordinate change on Weil algebra

Adjusted kinematical data

Gauge potentials:

$$(A, B) \in \Omega^1(U) \otimes \mathfrak{g} \oplus \Omega^2(U)$$

Curvatures:

$$F := \mathrm{d}A + \tfrac{1}{2}[A, A]$$

$$H := \mathrm{d}B - \tfrac{1}{3!}\mu_3(A, A, A) + \chi_{\mathrm{sk}}(A, F)$$

$$= \mathrm{d}B + \underbrace{(A, \mathrm{d}A) + \tfrac{1}{3}(A, [A, A])}_{\mathrm{cs}(A)}$$

Bianchi identities:

$$\mathrm{d}F + [A, F] = 0$$

$$\mathrm{d}H - \chi_{\mathrm{sk}}(F, F) = \mathrm{d}H - (F, F) = 0$$

Gauge transformations:

$$\delta A = \mathrm{d}\Lambda_0 + \mu_2(A, \Lambda_0) \qquad \delta F = -\mu_2(F, \Lambda_0)$$

$$\delta B = \mathrm{d}\Lambda_1 + (\Lambda_0, F) - \tfrac{1}{2}\mu_3(A, A, \Lambda_0) \qquad \delta H = 0$$

Remarks

- Above example: Sati/Schreiber/Stasheff (2009)
- Physicists studying supergravity were there first:
  - Nucl. Phys. B 195 (1982) 97
  - Phys. Lett. B 120 (1983) 105
- Many more examples: tensor hierarchies
- Without adjustment: BRST algebra "open"
- With adjustment: BRST algebra closes
- With adjustment:

$$\begin{array}{ccccccccc} 0 & \longleftarrow & \mathsf{CE}(\mathfrak{g}) & \longleftarrow & \mathsf{W}(\mathfrak{g}) & \longleftarrow & \mathsf{inv}(\mathfrak{g}) & \longleftarrow & 0 \\ & & \updownarrow\cong & & \updownarrow\cong & & \updownarrow\cong & & \\ 0 & \longleftarrow & \mathsf{CE}(\tilde{\mathfrak{g}}) & \longleftarrow & \mathsf{W}(\tilde{\mathfrak{g}}) & \longleftarrow & \mathsf{inv}(\tilde{\mathfrak{g}}) & \longleftarrow & 0 \end{array}$$

Adjusted Higher Parallel Transport

Ordinary parallel transport

Usual functorial perspective on parallel transport (locally!):

$$\Phi\colon \mathcal{P}U \longrightarrow \mathsf{BG} \qquad \begin{array}{ccc} \mathsf{paths} & \xrightarrow{\Phi_1} & \mathsf{G} \\ \downdownarrows & & \downdownarrows \\ U & \xrightarrow{\Phi_0} & * \end{array}$$

- Modulo technicalities (thin homotopy, sitting instances)
- Composition of paths $\Rightarrow$ multiplication of group elements
- Connection: $g = \mathbb{1} + \iota_X A$ for inf. paths in direction X
- Conversely: $g(\gamma) = \mathrm{P}\exp\int_\gamma A$
- Readily extends to higher gauge theory:
  - Higher path groupoid
  - Higher gauge group, as one-object higher groupoid
  - But: requires fake curvatures to vanish!

Baez, Schreiber, Waldorf, ...

Towards adjusted parallel transport

- Ordinary parallel transport: $\Phi\colon \mathcal{P}U \longrightarrow \mathsf{BG}$
- This "sees" connections, but we adjust only curvatures!
- Short exact sequence of groupoids:

$$* \longrightarrow \begin{array}{c}\mathsf{G}\\ \downdownarrows\\ \mathsf{G}\end{array} \hookrightarrow \mathsf{Inn(G)} \longrightarrow \begin{array}{c}\mathsf{G}\\ \downdownarrows\\ *\end{array} \longrightarrow *$$

- $\mathsf{Inn(G)}$ is inner derivation Lie 2-group of G
- Derived parallel transport functor:

$$\begin{array}{ccc}\mathcal{P}U & \hookrightarrow & \mathcal{P}_{(2)}U\\ \Phi\downarrow & & \downarrow\mathit{\Phi}\\ \mathsf{BG} & \hookrightarrow & \mathsf{BInn(G)}\end{array}$$

- $\mathit{\Phi}$ fully determined/equivalent to Φ

Adjusted higher parallel transport

A bit technical, so here are the steps: Hyungrok Kim+CS

- Unadjusted higher parallel transport requires fake curvature
- Can construct adjusted derived parallel transport functor

$$\begin{array}{ccccc} \mathcal{P}U & \hookrightarrow & \mathcal{P}_{(2)}U & & \mathcal{P}_{(3)}U \\ \downarrow{\scriptstyle \Phi} & & \downarrow{\scriptstyle \Phi} & \to & \downarrow{\scriptstyle \Phi} \\ \mathsf{BG} & \hookrightarrow & \mathsf{BInn(G)} & & \mathsf{BInn}_{\rm adj}(\mathscr{G}) \end{array}$$

such that for every pair of endpoints $x_0, x_1 \in U$,

$$\begin{array}{ccccc} & & \mathcal{P}_{(3)}U(x_0,x_1) & & \\ & & \downarrow{\scriptstyle \Phi^{\rm adj}(x_0,x_1)} & \searrow{\scriptstyle \Phi^{\rm adj}_{\rm curv}(x_0,x_1)} & \\ \mathscr{G} & \hookrightarrow & \mathsf{Inn}_{\rm adj}(\mathscr{G}) & \xrightarrow{\Pi} & \mathsf{B}\mathscr{G} \end{array}$$

commutes.

Remarks

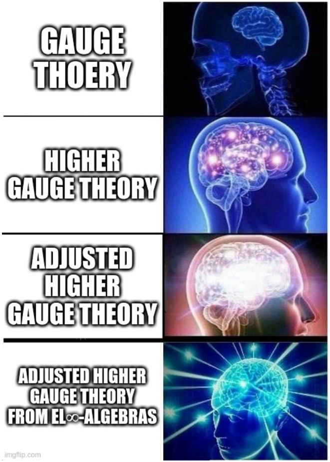
Adjusted Higher Parallel Transport Functor:

$$
\begin{array}{ccccc}
\mathcal{P}U & \hookrightarrow & \mathcal{P}_{(2)}U & & \mathcal{P}_{(3)}U \\
\downarrow{\scriptstyle \Phi} & & \downarrow{\scriptstyle \mathit{\Phi}} & \rightarrow & \downarrow{\scriptstyle \mathit{\Phi}} \\
\mathsf{BG} & \hookrightarrow & \mathsf{BInn(G)} & & \mathsf{BInn}_{\mathrm{adj}}(\mathscr{G})
\end{array}
$$

$\mathsf{BInn}_{\mathrm{adj}}(\mathsf{String}(n))$ is a bit hard to construct:

- Use quasi-isomorphic ("equivalent") strict version of $\mathfrak{string}(n)$
- $\mathfrak{inn}(\mathfrak{string}(n))$ is then a 2-crossed module of Lie algebras
- Readily integrates to 2-crossed module of Lie groups
- Adjustment rotates potentials/curvatures in functor $\mathit{\Phi}$

Origin of adjustment: EL_∞-algebras



Evident question:

Where do the structure constants for adjustment come from?

Observation:

There is a family of quasi-isomorphic weak Lie 2-algebras

$$\mathfrak{string}^{\mathrm{wk},\alpha}_{\mathrm{sk}}(\mathfrak{g}) \;:=\; (\mathbb{R} \xrightarrow{0} \mathfrak{g})\;,$$

$$\varepsilon_1(r) = 0\;,$$

$$\varepsilon_2(x_1, x_2) = [x_1, x_2]\;, \quad \varepsilon_2(x_1, r) = 0\;,$$

$$\varepsilon_3(x_1, x_2, x_3) = (1-\alpha)(x_1, [x_2, x_3])\;,$$

$$\mathsf{alt}(x_1, x_2) = -2\alpha(x_1, x_2)$$

Conjecture:

Adjustment data from alternators in weak Lie n-algebras

Lie 2-algebras: equivalent to differential graded vector space $\mathfrak{L}$ with

$$\varepsilon_2 : \mathfrak{L} \otimes \mathfrak{L} \to \mathfrak{L}~, \quad |\varepsilon_2| = 0~, \quad \mathsf{alt} : \mathfrak{L} \otimes \mathfrak{L} \to \mathfrak{L}~, \quad |\mathsf{alt}| = -1$$

Roytenberg (2007)

Generalize, extending differential ideal: $h\mathcal{L}ie$-algebras

$h\mathcal{L}ie$-algebras

Graded vector space $\mathfrak{L}$ with

$$\varepsilon_1 \;:\; \mathfrak{L} \to \mathfrak{L}~, \quad |\varepsilon_1| = 1~, \quad \varepsilon_2^i \;:\; \mathfrak{L} \otimes \mathfrak{L} \to \mathfrak{L}~, \quad |\varepsilon_2^i| = -i$$

such that

$$\begin{aligned}
\varepsilon_1(\varepsilon_1(x_1)) &= 0~,\\
\varepsilon_1(\varepsilon_2^i(x_1,x_2)) &= \pm\varepsilon_2^i(\varepsilon_1(x_1),x_2) \pm \varepsilon_2^i(x_1,\varepsilon_1(x_2)) + \varepsilon_2^{i-1}(x_1,x_2) \mp \varepsilon_2^{i-1}(x_2,x_1)\\
\varepsilon_2^i(\varepsilon_2^i(x_1,x_2),x_3) &= \pm\varepsilon_2^i(x_1,\varepsilon_2^i(x_2,x_3)) \mp \varepsilon_2^i(x_2,\varepsilon_2^i(x_1,x_3)) \mp \varepsilon_2^{i+1}(x_2,\varepsilon_2^{i-1}(x_3,x_1))\\
\varepsilon_2^j(\varepsilon_2^i(x_1,x_2),x_3) &= \pm\varepsilon_2^{i+1}(x_2,\varepsilon_2^{j-1}(x_3,x_1))\\
\varepsilon_2^i(\varepsilon_2^j(x_1,x_2),x_3) &= \pm\varepsilon_2^j(x_1,\varepsilon_2^i(x_2,x_3)) \mp \varepsilon_2^i(x_2,\varepsilon_2^j(x_1,x_3)) \pm \varepsilon_2^{i+1}(x_3,\varepsilon_2^{j-1}(x_1,x_2))
\end{aligned}$$

Generalizes hemistrict Lie 2-algs and specializes dg-Leibniz algs.

(Rough) picture

- Adjustments in tensor hierarchy: $\varepsilon_2^i(-,-)$ of $h\mathcal{L}ie$-algebras
- Homotopy $h\mathcal{L}ie$-algebras: EL_∞-algebras
- Each EL_∞-algebra is quasi-isomorphic to
  - L_∞-algebras (antisymmetrization)
  - $h\mathcal{L}ie$-algebras (strictification)
  - minimal models
- Non-trivial family of EL_∞-algebras over each (?) L_∞-algebra
- Usual definition of Weil algebra too naive
- Should be defined with respect to the EL_∞-Family.
- This then yields adjusted Weil algebras

Other applications

- They underlie generalized/exceptional/extended geometry
- They suggest an integration of Leibniz algebras
- Small cofibrant replacement of $\mathcal{L}ie$ over finite characteristic

- Usual connections on non-abelian gerbes are not suitable for non-flat higher gauge theories.
- There is, however, a generalized notion of higher gauge theory, correcting this: adjusted higher gauge theory.
- The adjustment happens at the level of the Weil algebra of the higher gauge algebra.
- This leads to adjusted curvatures, adjusted higher parallel transport, etc.
- The data needed for adjusting the Weil algebra originate in the higher products of EL_∞-algebras.

Thank You!