Flux Quantization in F-theory and Freed-Witten anomaly

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MPI - Munich

String-Math, Bonn,
July 2012

Based on work with A. Collinucci,
arXiv: 1011.6388 , 1203.4542

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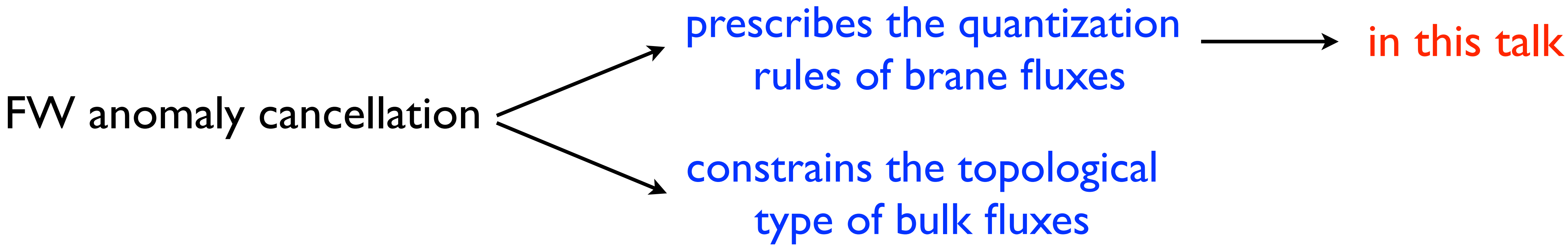

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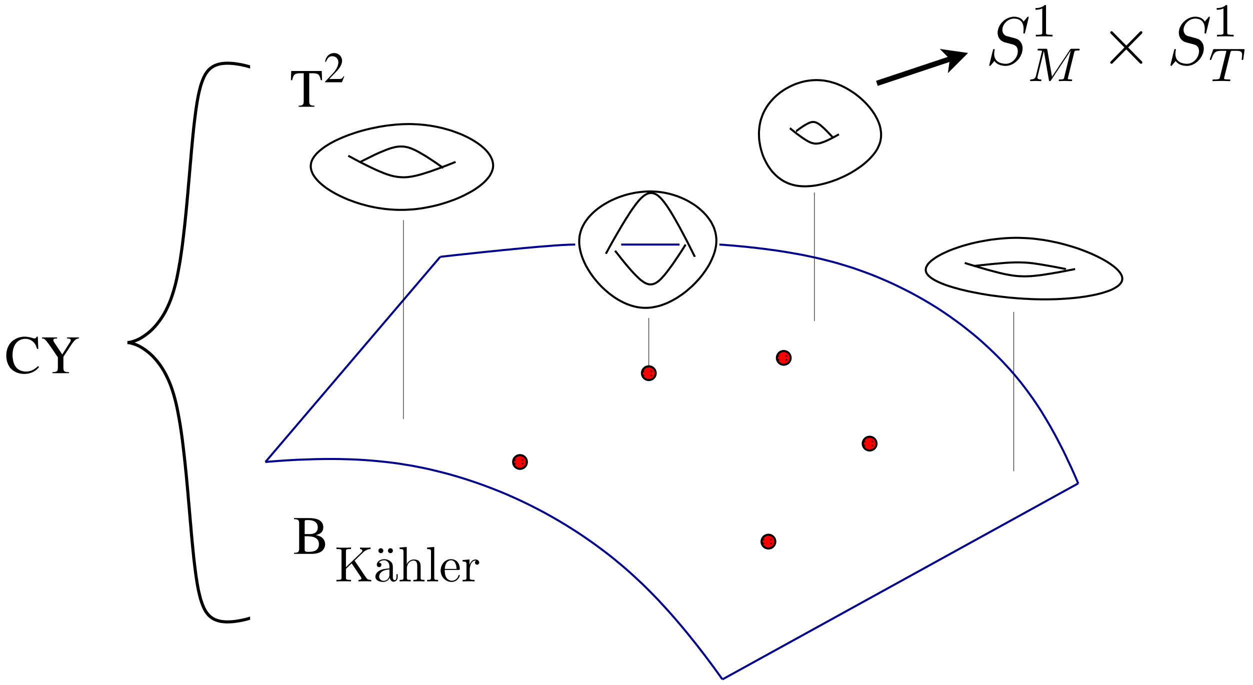

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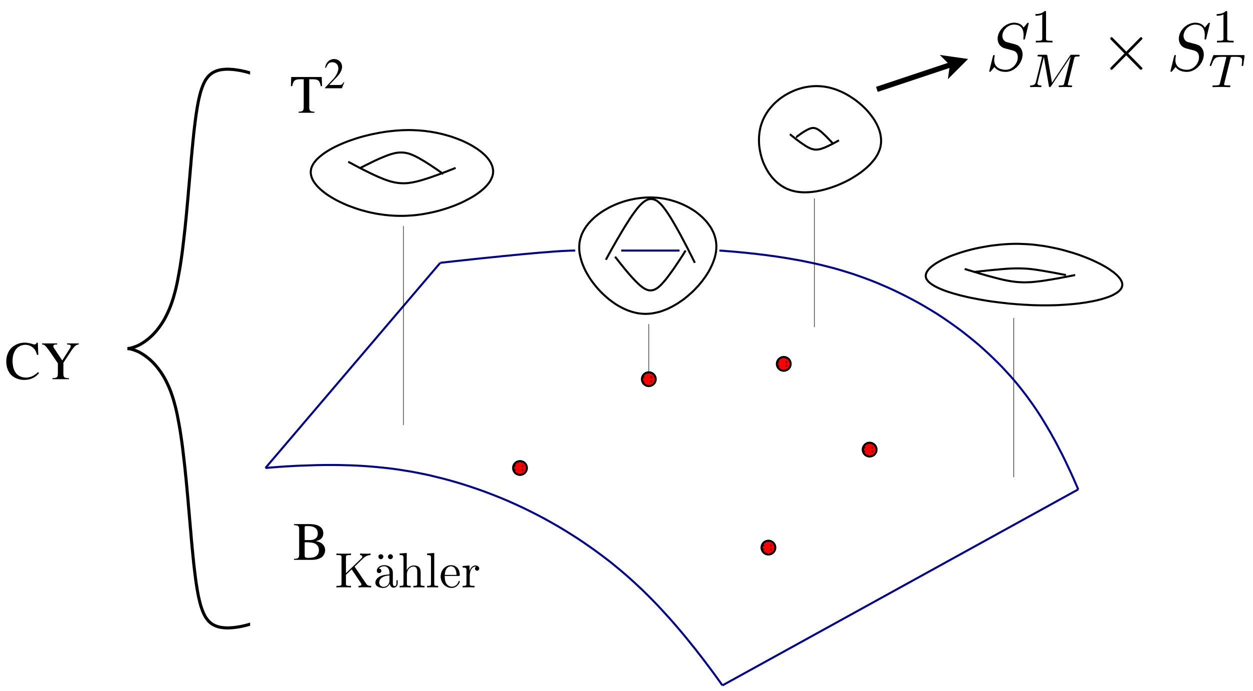

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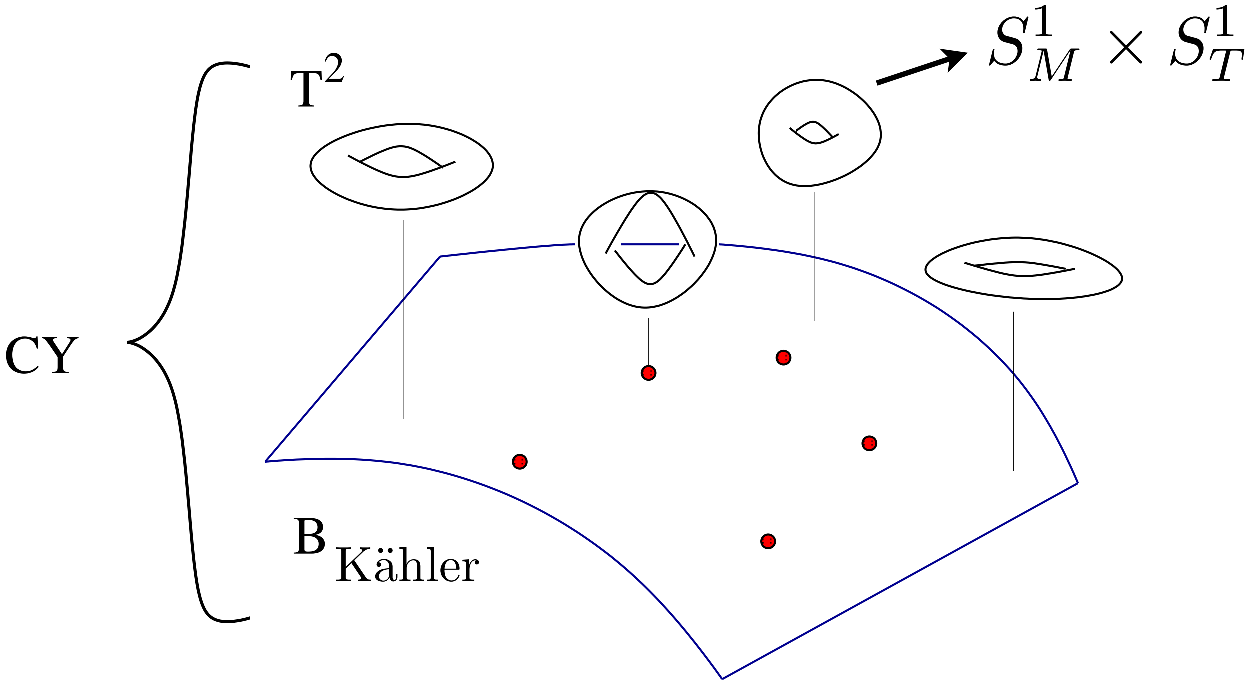

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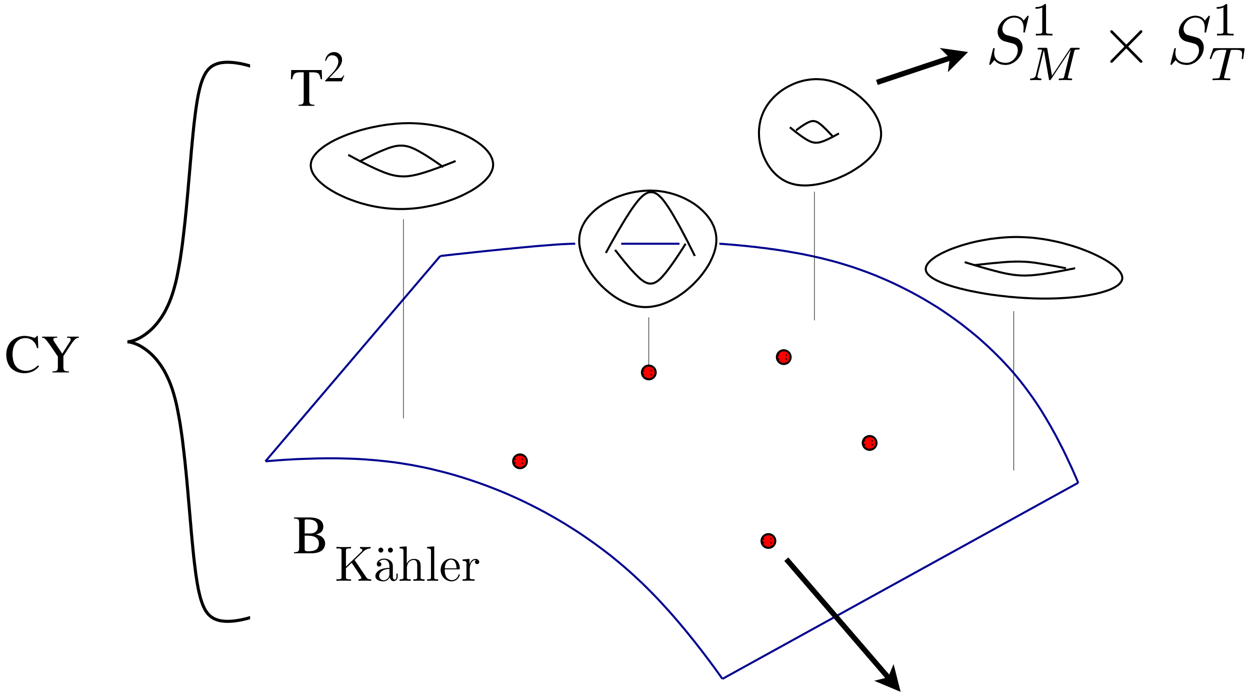

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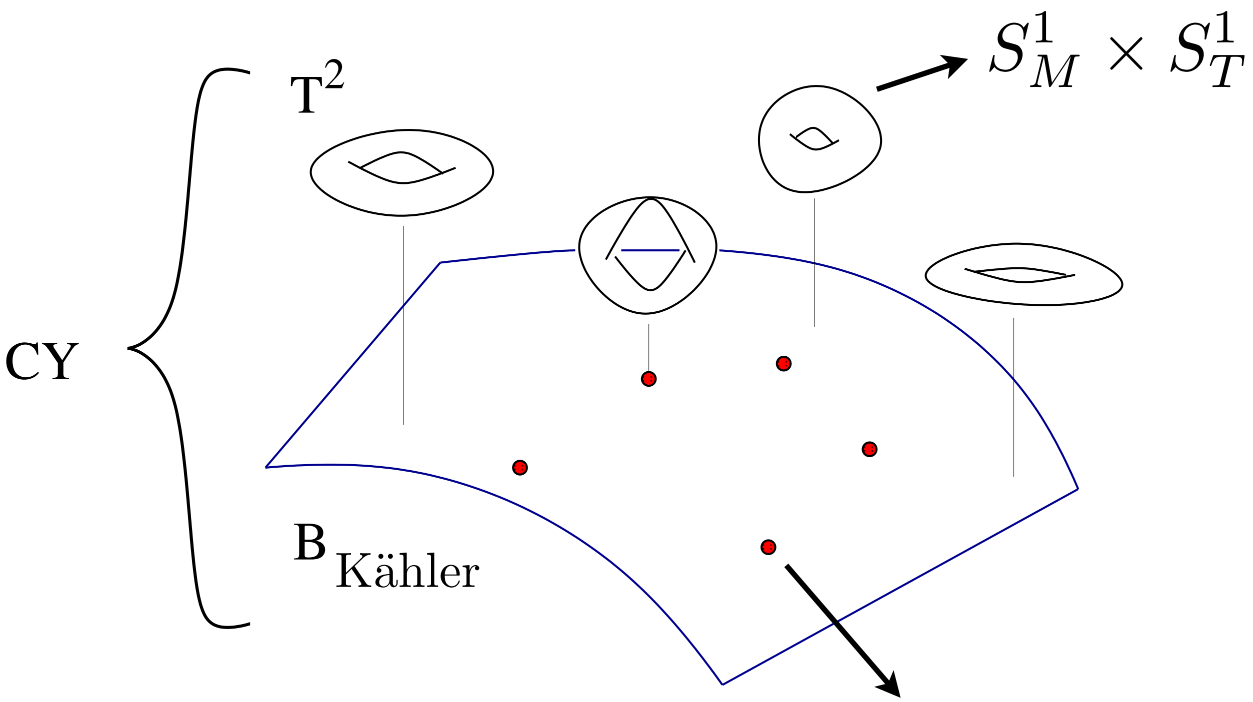

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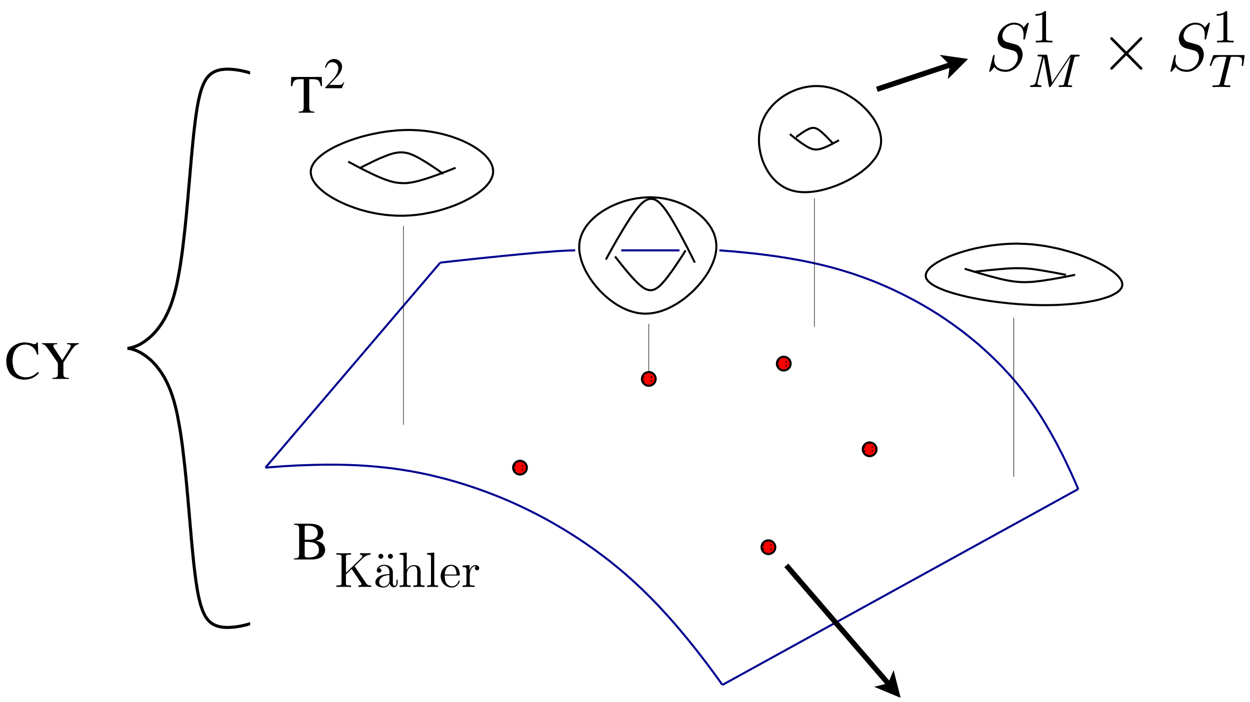

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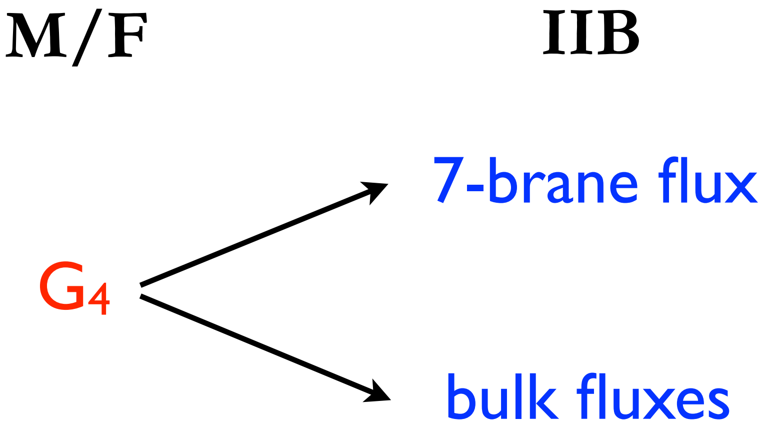

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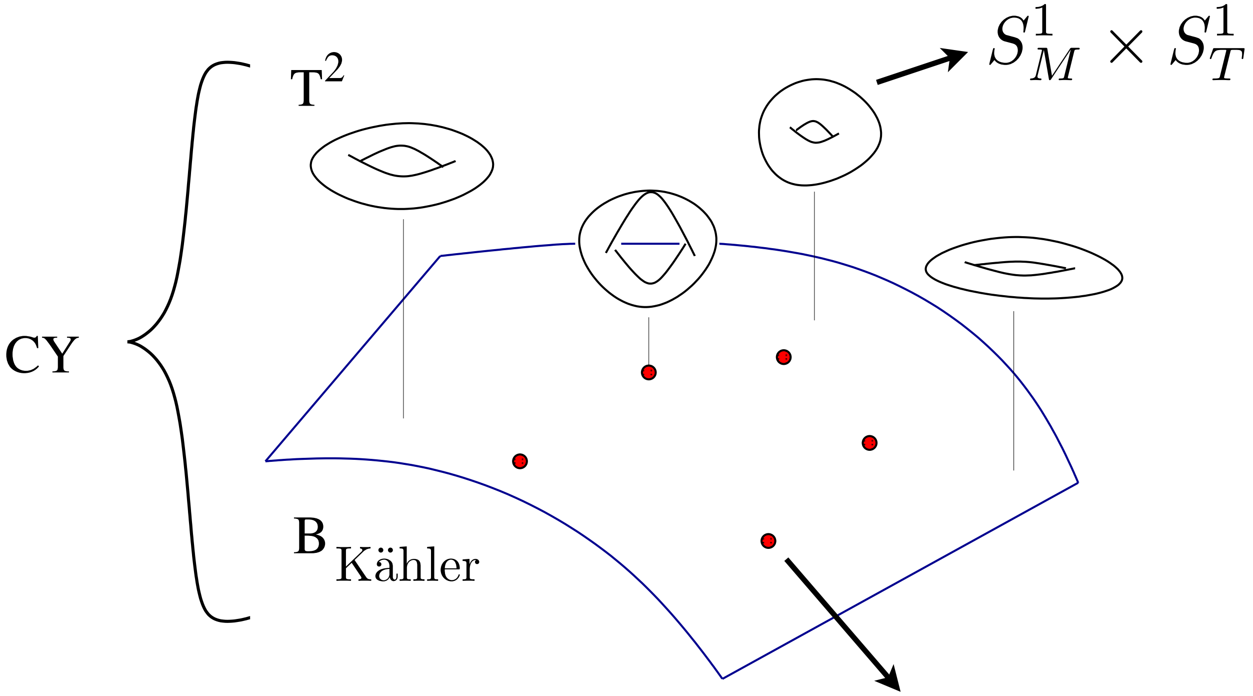

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M/F **IIB**

Fluxes: G₄ → 7-brane flux

G₄ → bulk fluxes

4D Lorentz invariance

⬇

G₄ must have one and only one leg along T^2

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detecting FW anomaly detecting M2 anomaly

The SU(2N) case

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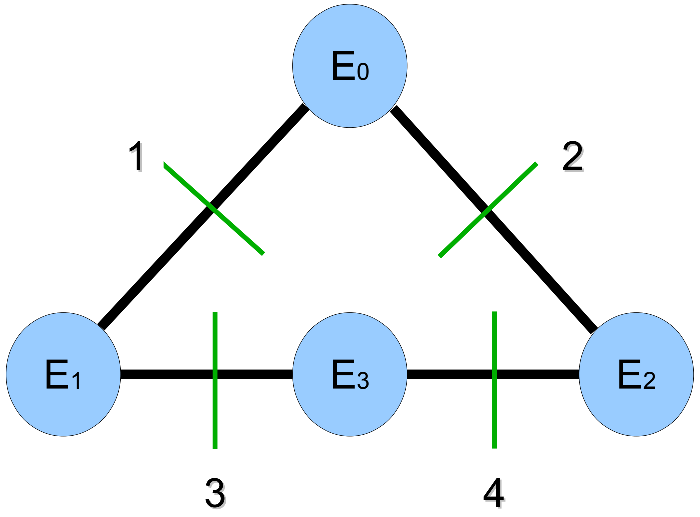
Resolved fiber over SU(4) locus ⟷ Affine Dynkin diagram of SU(4)



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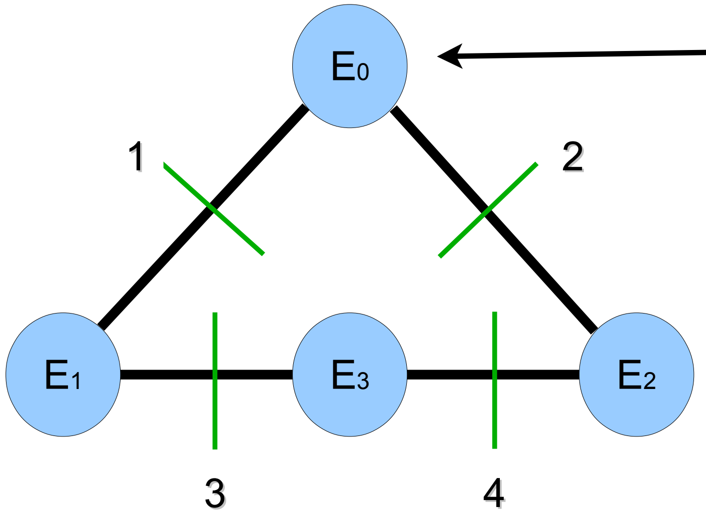

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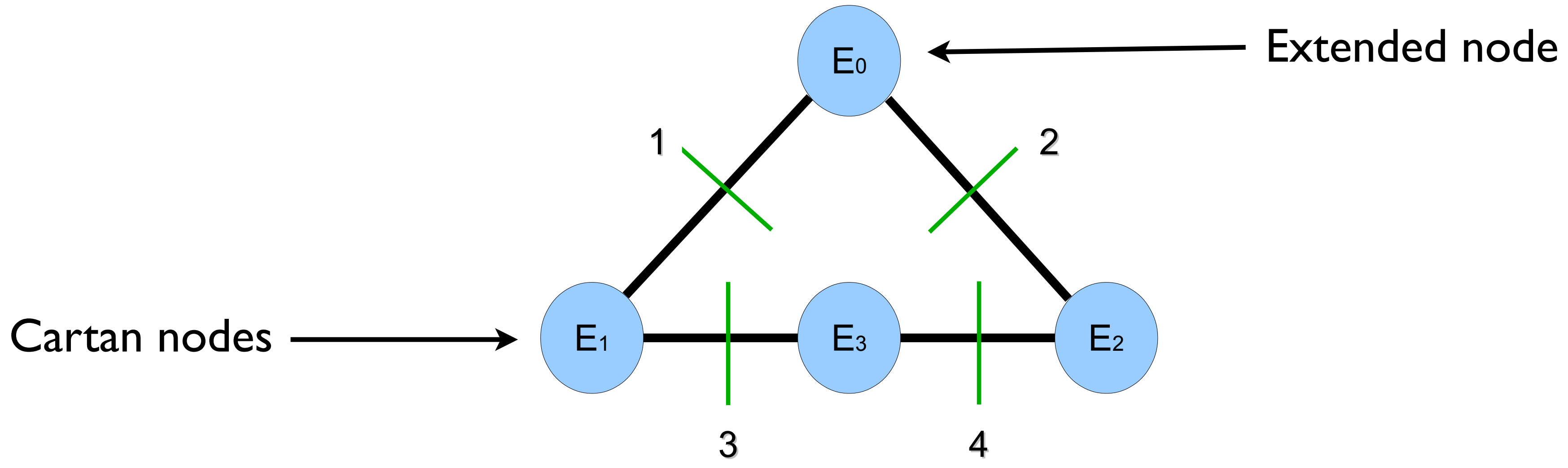

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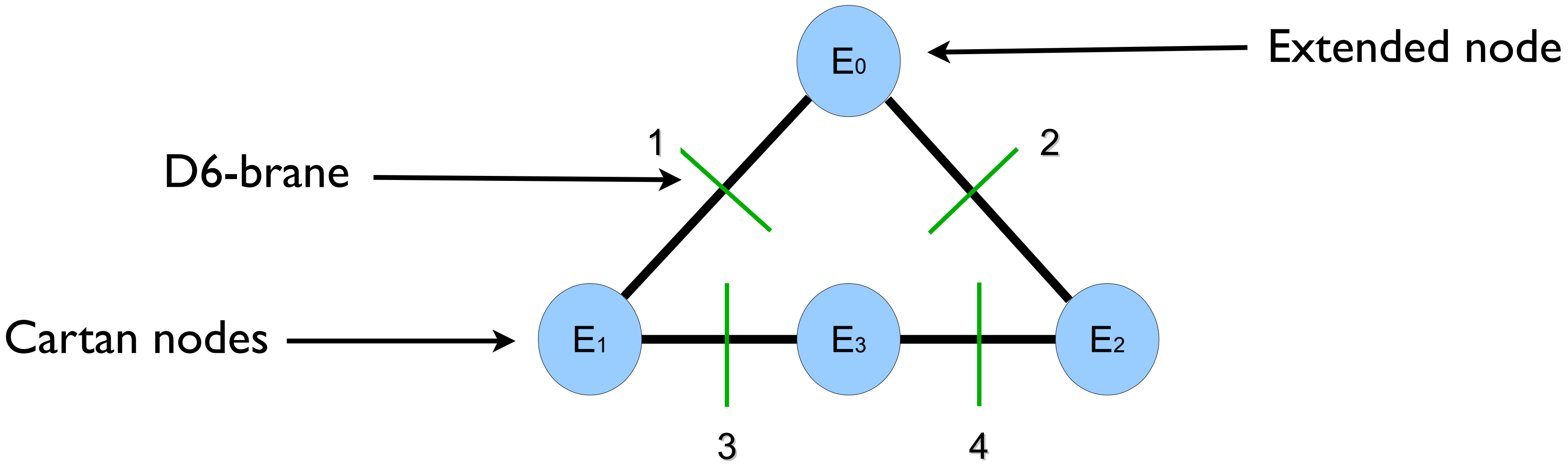

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D6-brane

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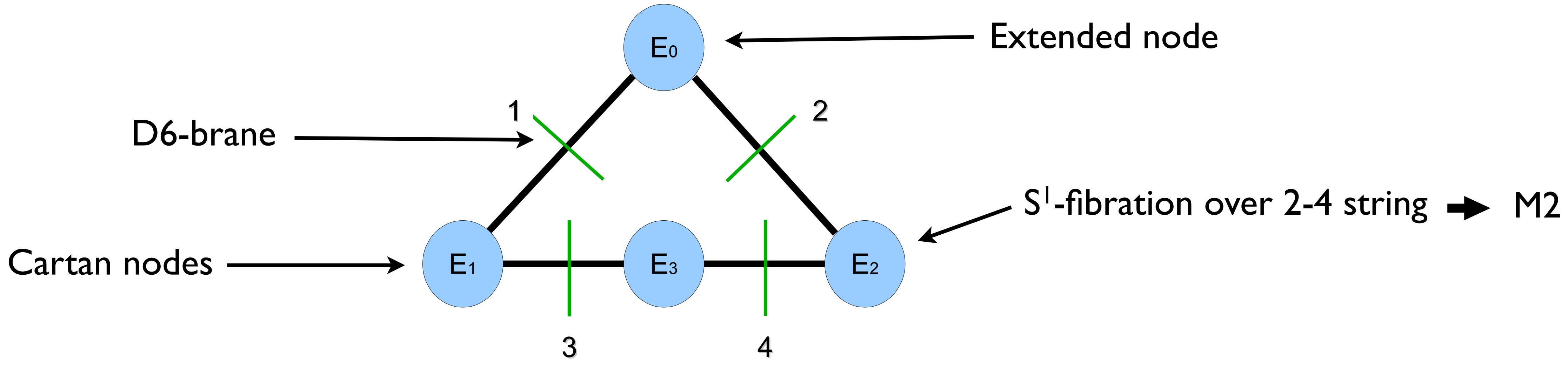

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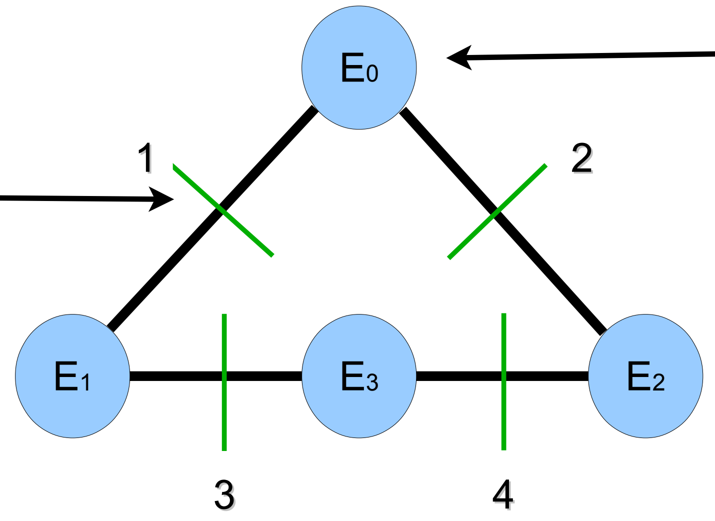

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Loops of i-j IIA open strings

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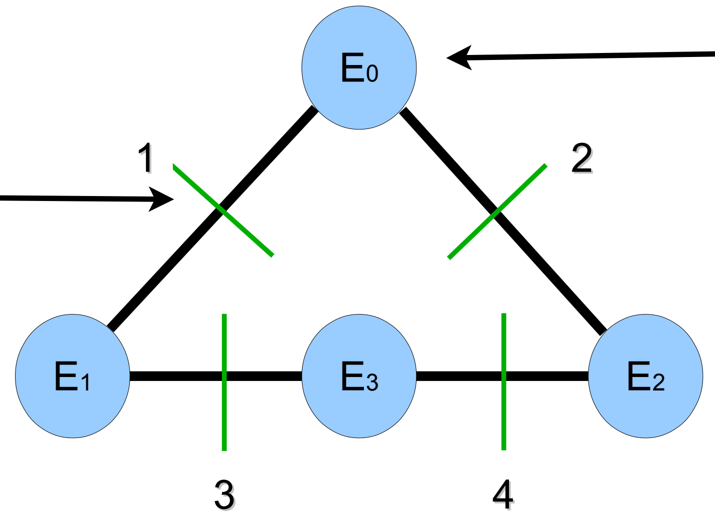

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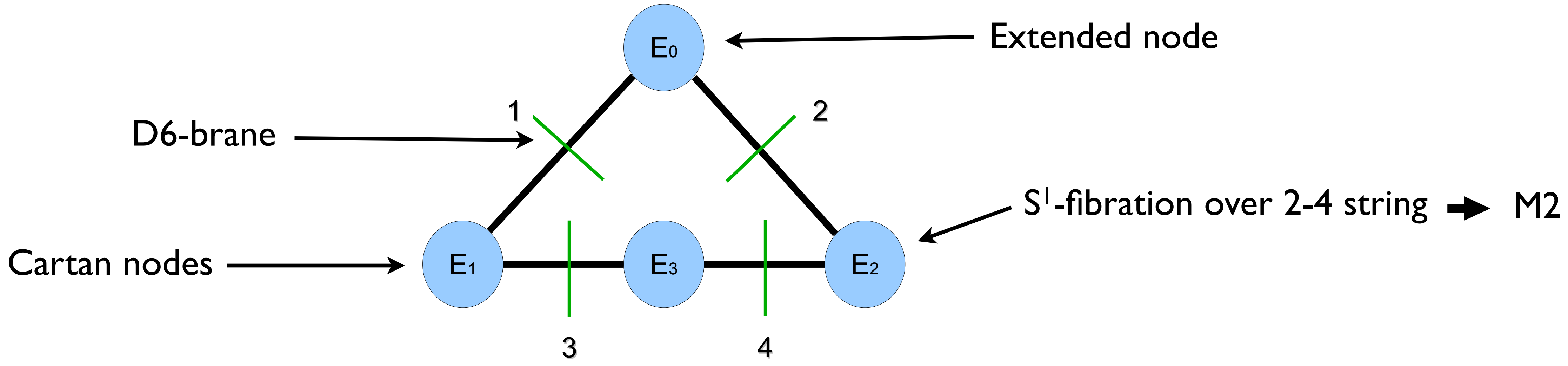

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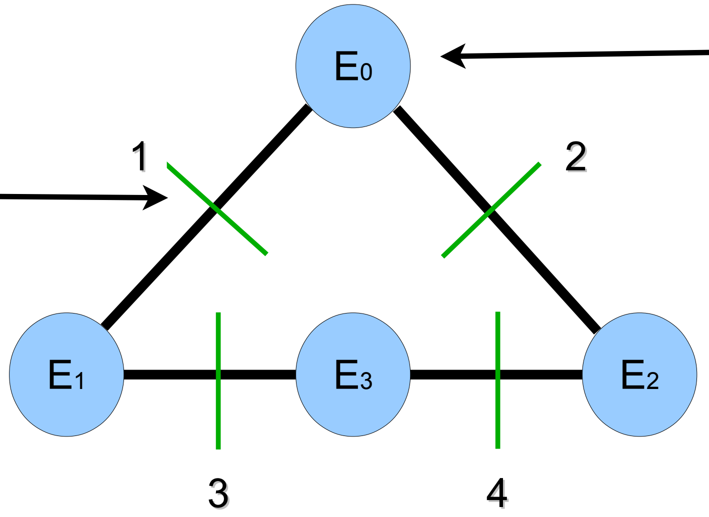

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↓ lift

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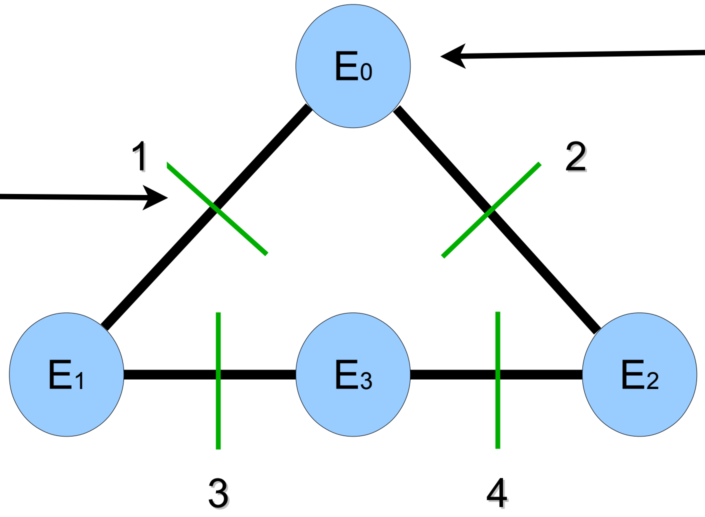

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⬇ lift

Loops of M2s = E_n fibered over $C^{(2)}$ ➡ $C^{(4)}$

However, these 4-cycles are NOT able to detect the M2 anomaly!

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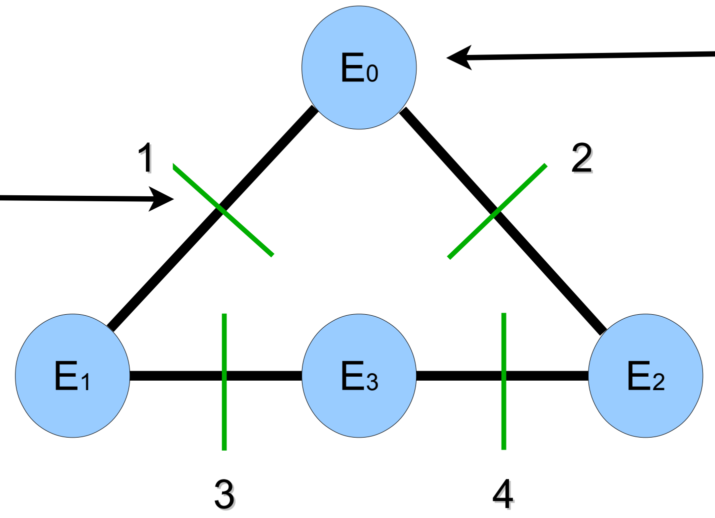

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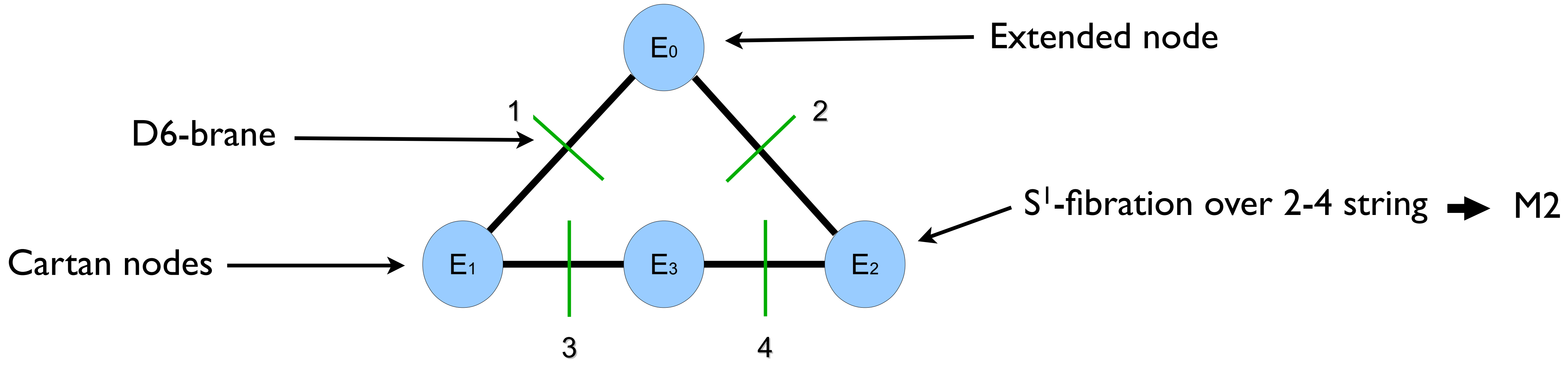

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$C^{(4)}$ complete-intersection ➡ $\int_{C^{(4)}} c_2$ is even

S.Krause, C.Mayrhofer, T.Weigand `12

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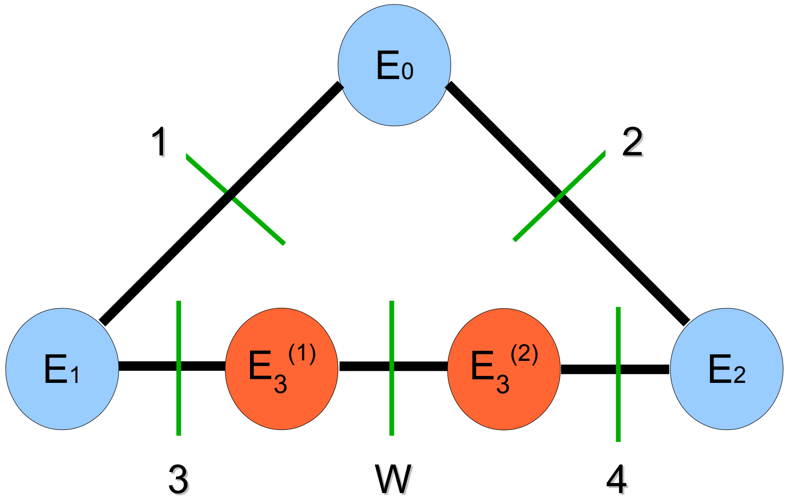

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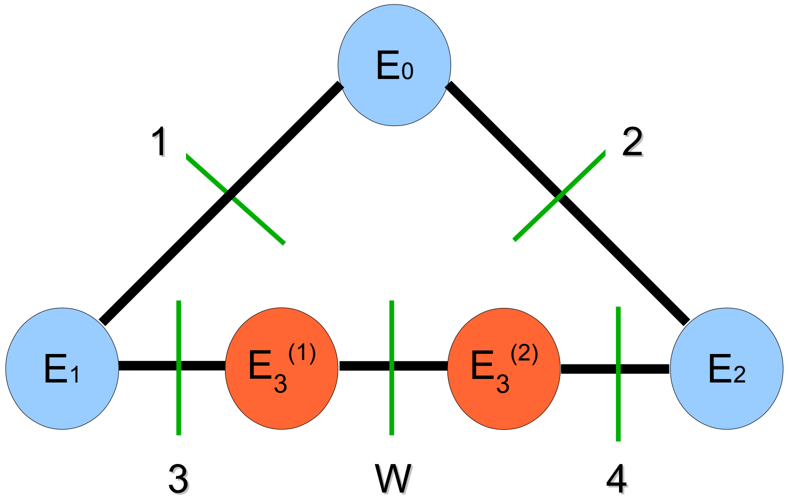

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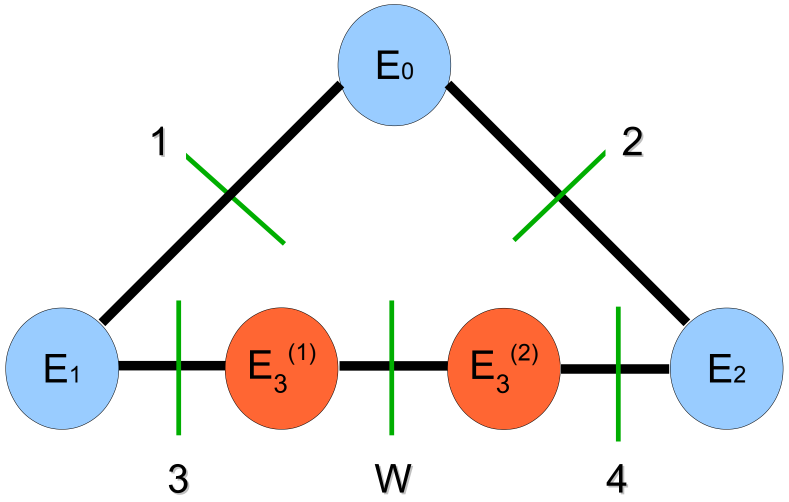

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➡ Constrain CY_4 complex structure
such that $S \cap W$ is reducible
and choose $C^{(2)}$ to be one component

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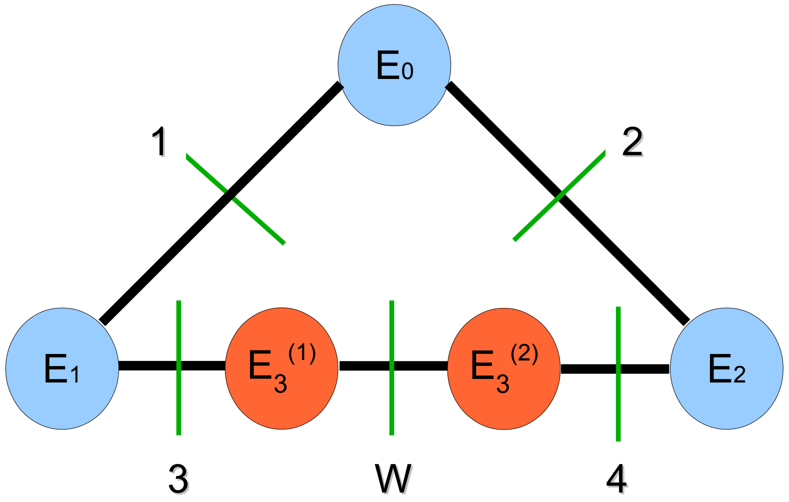

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➡ Constrain CY_4 complex structure such that $S \cap W$ is reducible and choose $C^{(2)}$ to be one component

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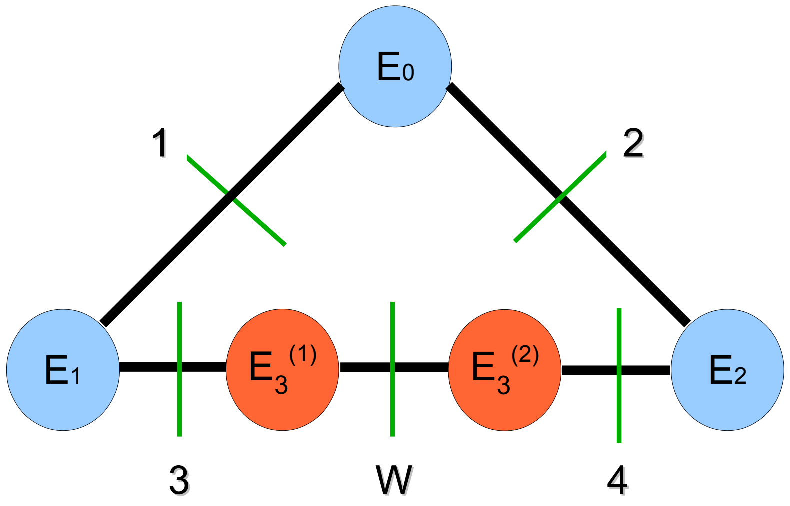

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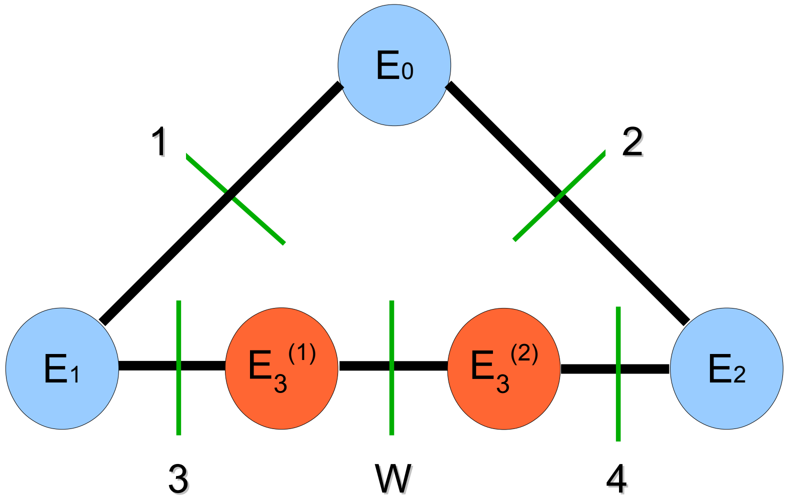

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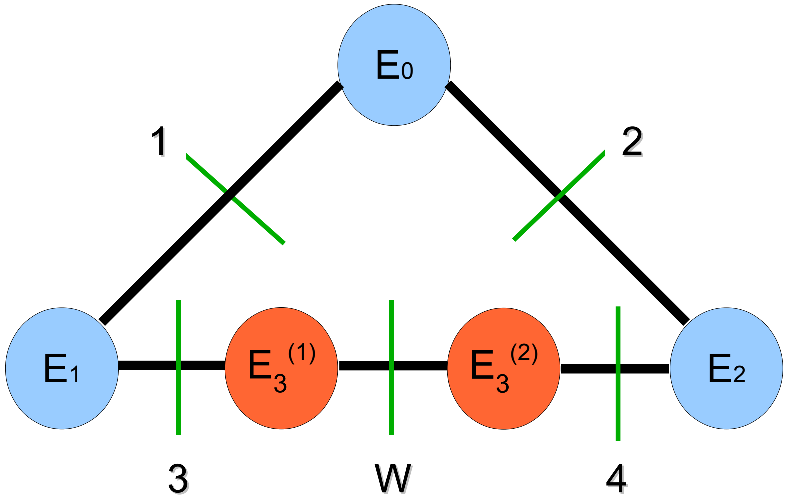

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The SU(2N+1) case

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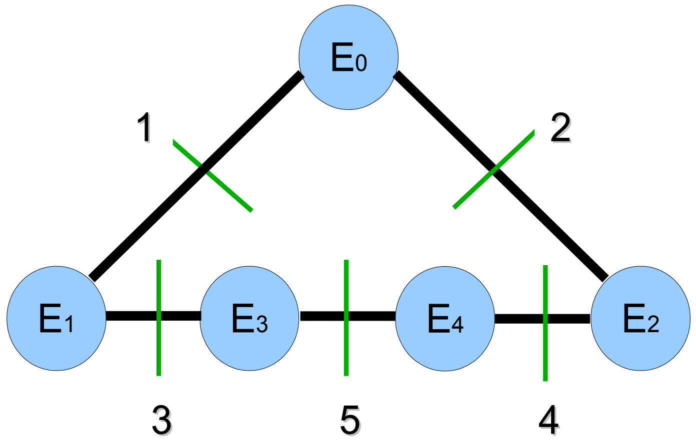

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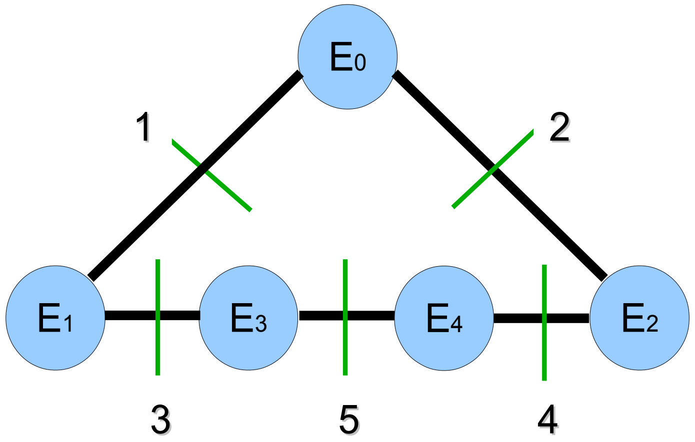

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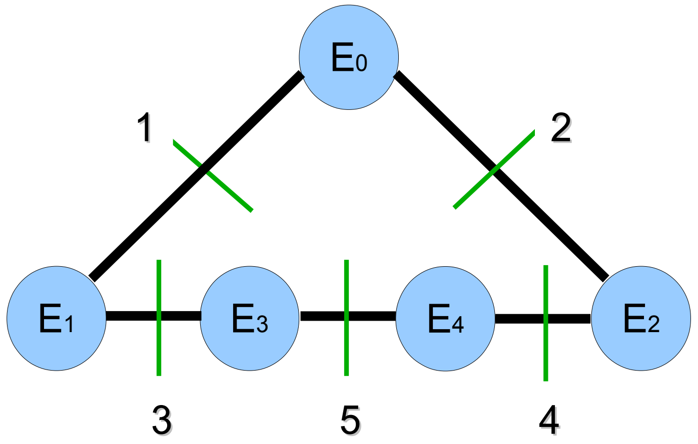

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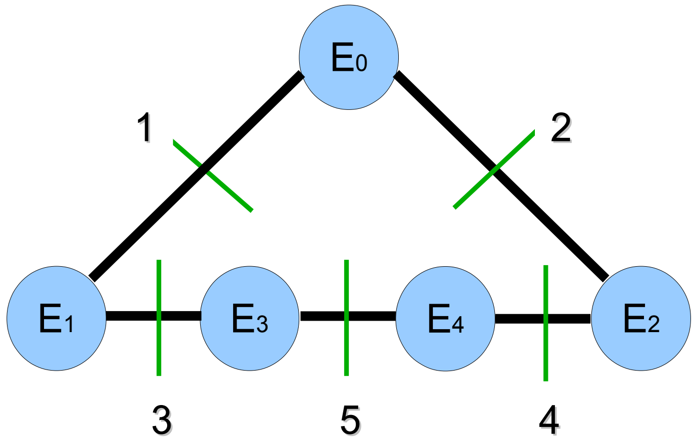

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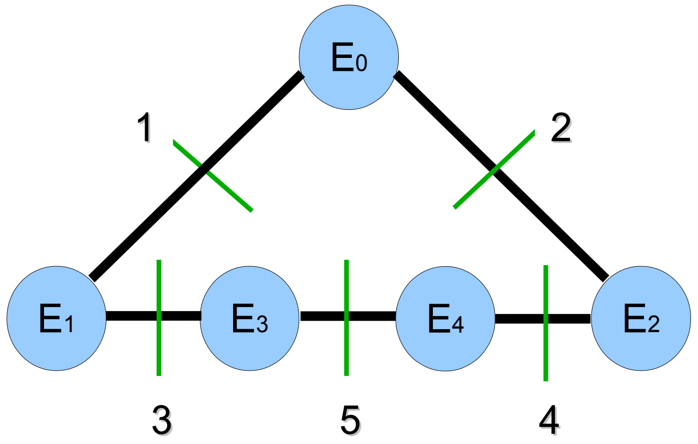

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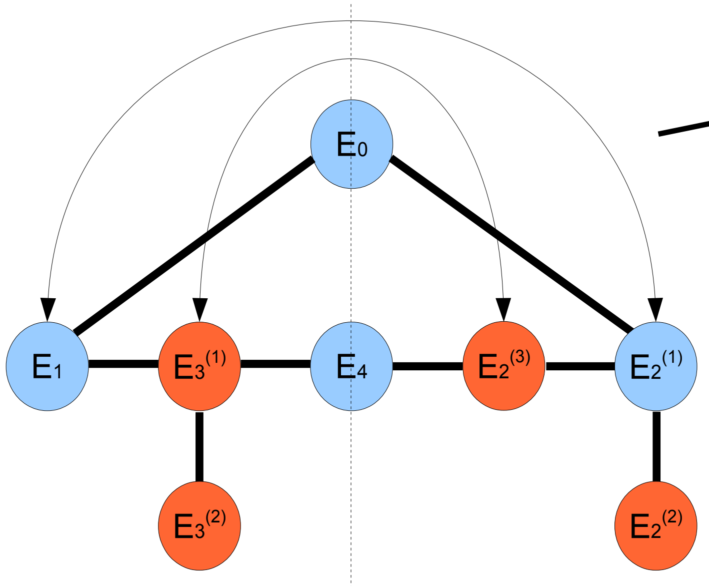

Affine Dynkin diagram of SO(10)

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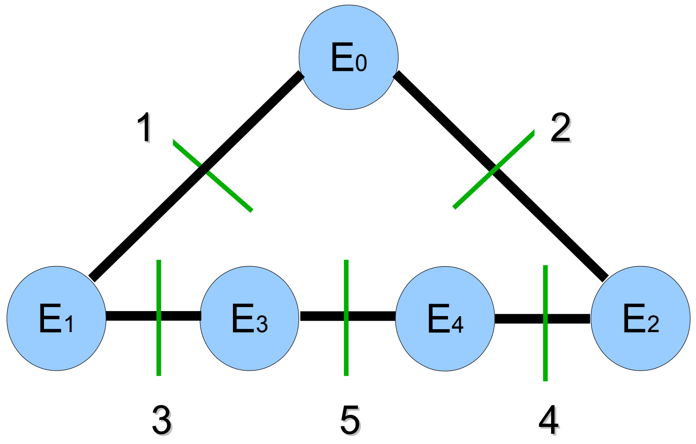

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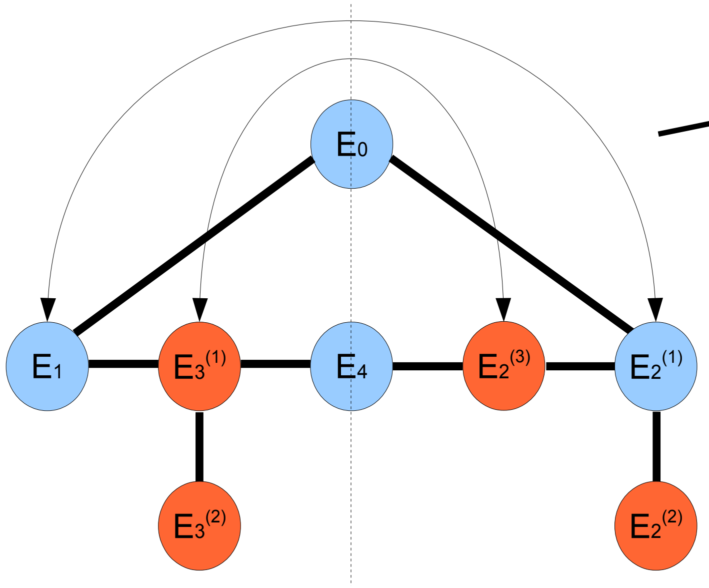

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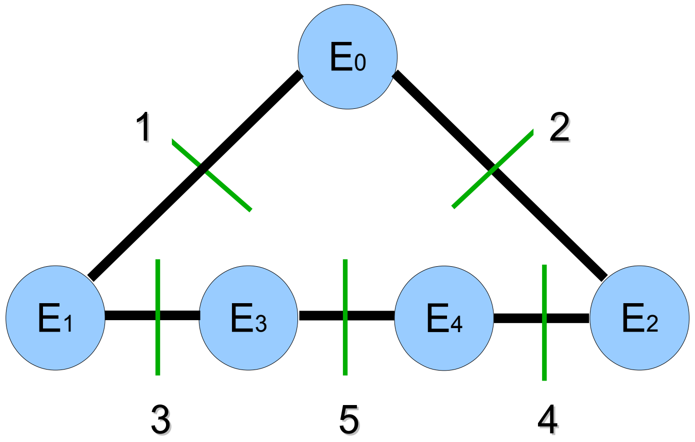

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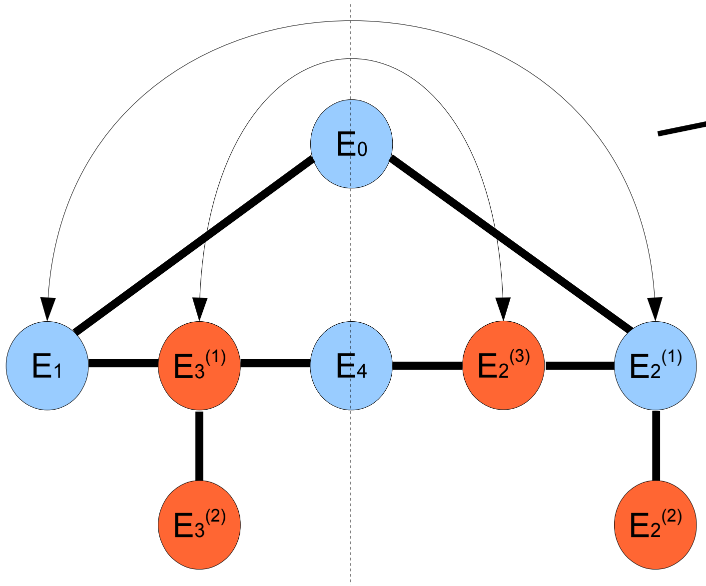

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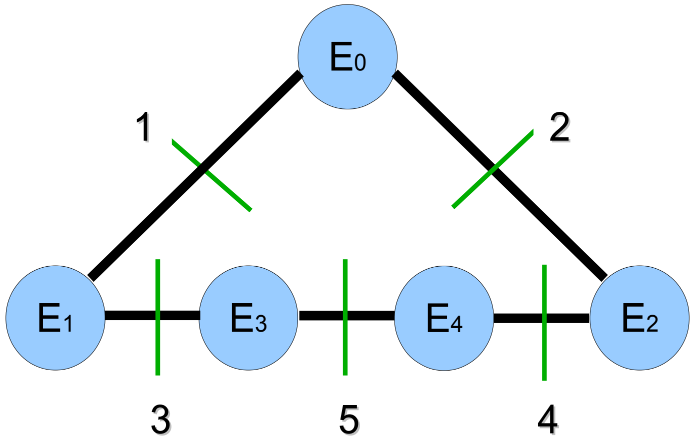

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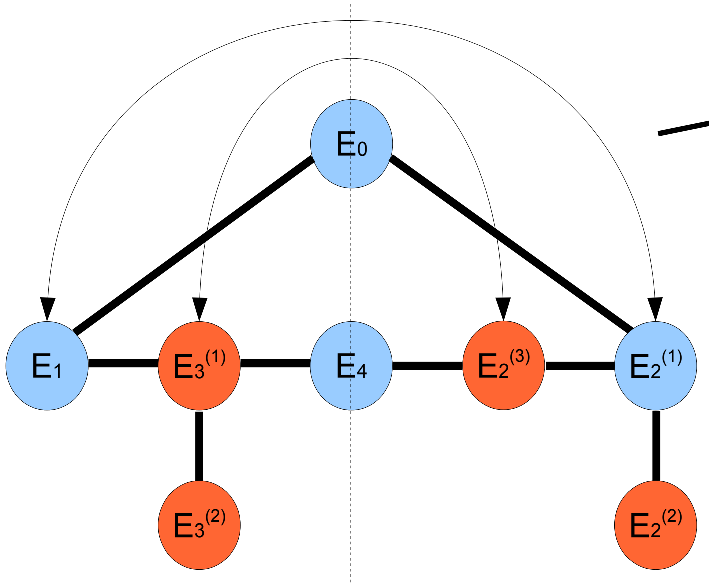

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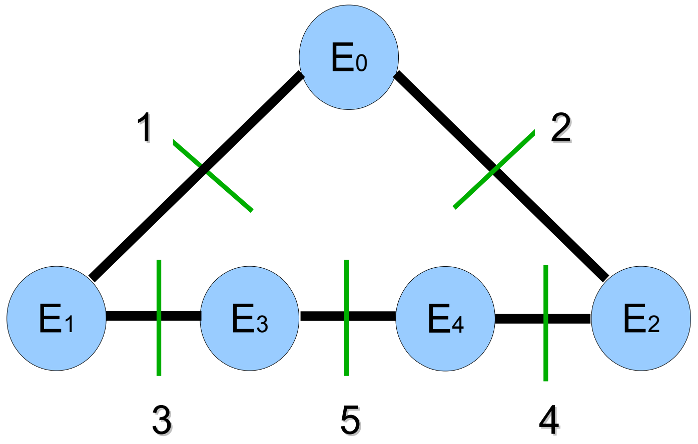

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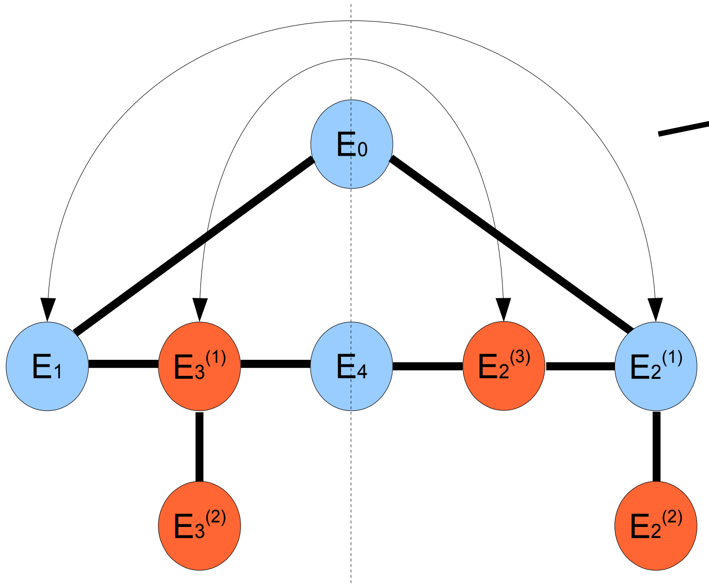

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This procedure works also for the SU(2N) series and lends better itself to treating the “U(1)-restricted” cases.

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- The class of $G_4|_{M5}$ must be pure torsion E.Witten `99

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