UNIVERSALITY OF FIBONACCI ANYONS IN TOPOLOGICAL QUANTUM COMPUTING

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Contents

1. Introduction and Overview

The concept of a computer whose calculations were governed by the laws of quantum mechanics is usually attributed first to Feynman [10]. Generally speaking, quantum computers are able to outcompete classical computers in certain classes of problems by drastically reducing the number of computations necessary to solve a given problem. This is often achieved by leveraging quantum entanglement between qubits in the physical system, allowing each computational operation in a quantum computer to perform the equivalent of multiple classical operations.

One of the main difficulties in building a quantum computer, however, is that it is notably more difficult to mitigate and handle errors. Quantum computers are often only advantageous over classical computers when they are able to leverage superposition of states of qubits. If no qubits in a quantum algorithm are put into a superposition of states by any operations or initialization, the algorithm can usually be equivalently executed classically. Therefore, a physical implementation of a quantum computer will need to handle decoherence, as this may collapse wavefunctions in unexpected ways, introducing unexpected errors into a calculation.

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So, a major problem that must be addressed in an implementation of a quantum computer is the effect of local perturbations. For example, consider a qubit corresponding to the z-component of an electron in a semiconductor quantum computer. At low temperatures, and in the presence of an external magnetic field, the randomly fluctuating magnetic field due to nuclear spins in the semiconductor can cause decoherence of a nearby electron , introducing an error in the computation [11].

The vulnerability in this last example is that the information in the qubit is encoded locally, as the z-component of the spin of a particle. This makes the information vulnerable to local perturbations, and therefore to errors. One possible solution to this issue is topological quantum computing. In a topological quantum computer, rather than encoding the information locally, information is encoded more globally, involving topological properties of a multi-quasiparticle system [4]. By topological properties, I mean those that are invariant under "bending" and "stretching", so that local perturbations do not change the encoded information. We will be more specific about this later.

The idea of a topological quantum computer is, in a crash-course format, as follows: in systems of two spatial dimensions, we can create particles, called anyons, that behave non-trivially under exchange. So if we exchange two particles, then exchange them back in the same (not the reverse!) direction, the quantum system picks up a phase, so we can tell that the particles were swapped and then put back. The particles' worldlines, or the paths they trace through time, then show a history of which particle was exchanged with which, and in what order, which are topological properties. We can, in theory, manipulate the system by braiding these worldlines as we please, which corresponds to what turns out to be a unitary operation on the system. Finally, we can measure the end state of the system by fusing together the particles, which determines a state.

The goal of this paper is to summarize how one might go about computing with a topological quantum computer, and focuses less on how a physical implementation may be achieved. In Section 2, we will introduce anyons, the particles that exchange non-trivially in the above paragraph. We will focus on Fibonacci anyons, the simplest known anyon model that can be used to build a universal quantum computer. We will also consider a Hilbert space of anyons and define a basis for the Hilbert space. In Section 3, we will discuss unitary operations on the space, which arise from braiding the anyons. We will also introduce a computational subspace and basis in this Hilbert space, allowing us to build qubits. In Section 4, we will discuss construction of a universal quantum computer with Fibonacci anyons. More specifically, we will talk about approximating quantum circuits with braids, showing that if we can control Fibonacci anyons, we can create a universal quantum computer.

2. Introducing Anyons

Topological quantum computation hinges on the use of anyons, a class of particles that emerges in quantum systems in two spatial dimensions. In this paper, we will be concerned with moving anyons, so it is most useful to think of these particles as existing in $2 + 1$ dimensions, where the last dimension is time. For our purposes, anyons become particularly interesting due to their exchange statistics. The central idea here is that swapping two anyons and then swapping them back introduces a phase into the system's wavefunction.

2.1. Anyon Exchange Statistics.

Consider two identical particles in 3+1 dimensions. If we take particle A and wind it around particle B, bringing particle A back to its original positions, particle A's worldline traces a path around particle B. But we are in 3 spatial

dimensions, so we can smoothly deform the worldline back to an increasingly tiny loop that does not encircle particle B, and in fact we can smoothly deform the worldline all the way down to one in which particle A never moves at all. Call the path in which particle A never moves the identity. Since these paths are topologically equivalent to one another (there exists a homotopy between them), the effect on the wavefunction must be the same in both cases [4]. So winding particle A around particle B must do nothing to the wavefunction in 3+1 dimensions.

Note that, topologically, winding particle A around particle B in the counter-clockwise direction is the same as exchanging particles A and B in a counter-clockwise direction, and then exchanging them again in the counter-clockwise direction to return the particles to their original positions. Let $|\psi_{AB}\rangle$ be the wavefunction of this two-particle system. Since particles A and B are identical, $||\psi_{AB}\rangle|^2$ must be invariant under exchange of the two particles. Suppose exchanging the two particles counter-clockwise introduces a phase $-\theta$:

$$|\psi_{AB}\rangle = e^{i\theta}\,|\psi_{BA}\rangle \tag{2.1}$$

where we write the relationship in terms of the "un-twisted" wavefunction $|\psi_{AB}\rangle$ for consistency with our later notation. Then exchanging the particles counter-clockwise twice, which we saw should be the same as doing nothing, gives

$$|\psi_{AB}\rangle = e^{i2\theta}\,|\psi_{AB}\rangle \tag{2.2}$$

Since this should be the same as doing nothing, we must have for some n

$$2\theta = 2\pi n \implies e^{i\theta} = \pm 1 \tag{2.3}$$

That is, exchanging two particles gives the wavefunction a phase of ± 1. These are the familiar exchange statistics for bosons and fermions! This holds for 3 or more spatial dimensions.

In 2+1 dimensions, the situation is a bit more unique. If we wind particle A around particle B, we can no longer smoothly deform particle A's worldline back to a tiny loop not encircling particle B - the loop will always get "caught" on particle B! Since this action is no longer equivalent to doing nothing, the above derivation changes. We now have no condition on what 2θ may be, and so exchanging particles A and B twice counter-clockwise can induce a non-trivial phase to the wavefunction. We now have

$$|\psi_{AB}\rangle = e^{-i2\theta}\,|\psi'_{AB}\rangle \tag{2.4}$$

where the prime is to denote that although the coordinate labels have been returned to the original setting, the wavefunction is no longer necessarily the same. Particles that have $e^{-i2\theta} \neq 1$ are called anyons. This is the magic that enables topological quantum computing to work.

2.2. **Physical Models of Anyons.** For a physical model of anyons, consider the Aharonov-Bohm effect, as presented in [7]. Consider two spatial dimensions, with a magnetic flux localized to some point. We can assume that since the flux is localized to a point, the magnetic field outside of this point is zero. However, in the Aharonov-Bohm effect, which can be experimentally observed in three dimensions, an electron moving in a loop around the flux gains a non-trivial phase in its wavefunction. The phase is directly related to the flux and charge of the particle encircling the flux. As you might expect in this context, the phase is a topological invariant of the electron's path, depending only on how many times the electron encircles the flux, and not depending on the exact geometry of the path.

Now, consider gluing a charge e around the circumference of the flux. This allows us to consider each flux-plus-charge as an anyon. Moving one flux-charge combination around another can be viewed as moving a charge around a flux, generating a phase by the Aharonov-Bohm effect. For a fuller description of this model, see [7], page 7. For more on how to build anyons with Majorana Zero Modes, see [9].

2.3. **Anyon Fusion and Fibonacci Anyons.** Rather than focus on a single physical model for anyons, we will focus on a more abstract model of anyons defined by what rules the anyons follow when they fuse.

Each anyon has an intrinsic quantum number that behaves like a charge or spin; here we will refer to it as a charge. Similarly, a group of anyons has a charge, and so long as that group does not interact with any other anyons, that charge will be conserved. The charge of a group of anyons is the charge of the particle that results from the fusion of all the anyons in the group. For example, if two pairs of anyon-anti-anyon pairs are created from vacuum (so that each pair has net charge zero), then fusing all the anyons together will result in a charge-zero anyon [1]. In this paper, the anyons can be labeled by this charge, so that an anyon of type a is an anyon with charge a.

For a group of anyons, one way to define a state of the system in its Hilbert space is by its ordered set of fusion outcomes. In the above example, if we label the 4 particles by $a, \bar{a}, b, \bar{b}$, we could fuse a and b to get c, c and $\bar{a}$ to get d, and d and $\bar{b}$ to get 0. We can write this state as $(c, d, 0)$, and this is a well-defined state of the system. Because our system is quantum, an arbitrary system is typically in a superposition of states, each with its own fusion outcomes. The set of possible fusion outcomes for a given initial set of anyons and group charge forms an orthonormal basis of a Hilbert space. For the Hilbert space of fusion outcomes that can come from the initial set of anyons $(a_1, \ldots, a_n)$ with group charge c, call this space $V^c_{a_1,\ldots,a_n}$. This can also be thought of as the space of states that start with anyons $(a_1, \ldots, a_n)$ and end with an anyon c.

There exists a class of anyons for which each pair of anyons has only one possible fusion outcome. Such anyons are called abelian anyons. For a given set of initial anyons and a set order of fusion, there is then only one possible fusion outcome, so the Hilbert space $V^c_{a_1,\ldots,a_n}$ has dimension 1 for abelian anyons. This makes them much less useful for quantum computation. The opposing class of anyons, non-abelian anyons, has more than one possible fusion outcome for a pair of anyons. This makes the Hilbert spaces $V^c_{a_1,\ldots,a_n}$ much richer, and more useful for computation.

Rather than pick a single physical model of anyons to focus on, we pick a mathematical model of non-abelian anyons defined by the rules governing their fusion. We focus on a model of anyons called Fibonacci anyons, which is the simplest model of anyons from which we can build a universal quantum computer [3]. We will explore how these anyons can work as the components of a universal quantum computer later; in this subsection we will just explore how the Hilbert spaces for these anyons look.

There are two possible charges for Fibonacci anyons: 0, which corresponds to the vacuum, or the absence of an anyon; and τ, which is the presence of an anyon. For Fibonacci anyons,

$$\bar{0} = 0 \tag{2.5}$$

$$\bar{\tau} = \tau \tag{2.6}$$

That is, the vacuum is its own antiparticle, and a Fibonacci anyon is its own antiparticle. With two particle types, there are 4 possible pairs of anyons. Let $\times$ denote fusion, and $+$ denote multiple possible outcomes, without any associated weighting. The fusion rules for Fibonacci anyons are then

$$0 \times 0 = 0 \tag{2.7}$$

$$0 \times \tau = \tau \tag{2.8}$$

$$\tau \times 0 = \tau \tag{2.9}$$

$$\tau \times \tau = 0 + \tau \tag{2.10}$$

The vacuum behaves as expected, and the only nontrivial fusion is then an anyon with itself.

Consider a line of n Fibonacci anyons, each with charge τ, but with group charge 0. This system exists in the Hilbert space $V^0_{\tau,\dots,\tau} := V^0_{\tau^n}$. We can write the basis as the ordered tuple of fusion outcomes from first to last. This tuple will be of length $n-1$, where the i^{th} entry is the result of the i^{th} fusion. If at any point a fusion results in a 0, the next fusion will be $0 \times \tau$. But by our fusion rules, this next fusion must result in a τ. So a tuple corresponding to a basis state cannot have two 0s in a row, since the first 0 necessitates a τ after it.

The first fusion can result in a τ, in which case the possible states are in the Hilbert space $V^0_{\tau^{n-1}}$. If the first fusion is a 0, then the next fusion must be a 1. So the possible states would then be in the HIlbert space $V^0_{\tau^{n-2}}$. Therefore, we get the relation

$$|V^0_{\tau^n}| = |V^0_{\tau^{n-1}}| + |V^0_{\tau^{n-2}}| \tag{2.11}$$

This is the Fibonacci recursion relation! There is only one way to get 0 from two τ anyons, and there is only one way to get 0 from three τ anyons (since the first fusion must be a τ), so

$$|V^0_{\tau,\tau}| = |V^0_{\tau,\tau,\tau}| = 1 \tag{2.12}$$

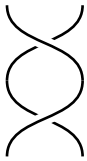
So the dimensions of the Hilbert spaces for 0-charge anyon configurations exactly follows the Fibonacci sequence: $|V_{\tau^n}|$ is the $n-1^{st}$ Fibonacci number. This is the reason for the name [7].

3. Unitary Operations on Anyons and Anyonic Qubits

Now that we have an idea of how to look at the state of a system of anyons, we can consider how to perform unitary operations on these states. As we mentioned in Section 2.1, the nontrivial exchange statistics of anyons are what make them interesting for quantum computation. Another way of thinking about exchanging particles is to think about braiding their worldlines. In a braid, there are n strands, fixed to n points on the bottom and n points on the top. The combination of the starting and endpoints of each strand with the intermediate interactions of the strands with one another between the ends determines the topological properties of the braid. As we will see in this section, a braid of anyons corresponds to a unitary operations on the system's wavefunction, so we will perform the unitary operations in our quantum computer with braids.

3.1. **Braiding Anyons.** Consider a straight line of anyons. The first elementary braid move is b_1, which passes the first anyon behind the second, leaving the first anyon in the second position, and vice versa.

We can also have the inverse operation b_1^{-1}, which passes the anyon in the first position in front of the anyon in the second position. The braid formed by performing b_1 and then b_1^{-1} is, as we expect, just the straight strands, or the identity I.



More generally, the braid group on n strands is generated by the $n-1$ elements b_i and the $n-1$ corresponding elements b_i^{-1}. These generators are subject to the relations

$$b_i b_i^{-1} = b_i^{-1} b_i = I \tag{3.1}$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1} \tag{3.2}$$

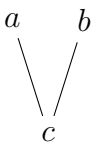
$$b_i b_j = b_j b_i \text{ if } |i-j| \geq 2 \tag{3.3}$$

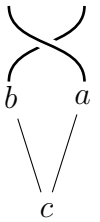
The first of these relations is true by definition of the inverse. The second relation is less obvious, but can be seen by taking any three adjacent strands and directly carrying out the braiding. The third relation is usually called far-commutativity, and just encodes the fact that it does not matter which of two braid moves we perform first if they involve entirely different strands in each order.

One of the reasons that topological quantum computation with non-abelian anyons is such an attractive idea is because almost all unitary operations on a system of anyons come from braiding [4]. Therefore, if we are being careful about our braiding, we are unlikely to accidentally perform an unintended unitary operation on our anyons.

3.2. **R-moves.** To this point, we have not been specific about how braiding anyons affects the state of the system. The relationship between a system of two anyons pre- and post-braiding is described by the R-move.

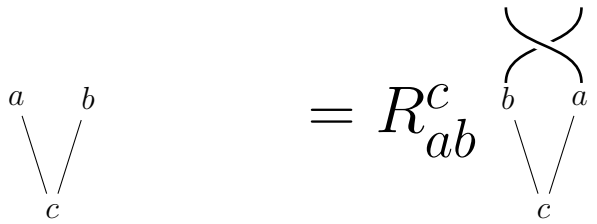
Recall that a state of a system of anyons is described by the fusion outcomes. For a system of two anyons, a and b, the system's state is described by the charge of the particle c to which they fuse. The fusion diagram for this scenario before braiding is shown on the left below. If we were to perform a braid on the anyons, say b_1, we get the fusion diagram on the right.





The first is a basis state of $V_{a,b}^c$, and the second is a basis state of $V_{b,a}^c$. Of course, these Hilbert spaces are one-dimensional, but when we extend this to more initial anyons, we get connections between basis states of $V_{a_1,a_2,\ldots,a_n}^c$ and $V_{a_2,a_1,\ldots,a_n}$ and so on [7].

This relationship between the basis states is the R-move, and we say that



where R_{ab}^c is the R coefficient associated with this set of initial and final anyons. The R coefficients for incompatible sets of anyons, such as R_{00}^τ, are zero. In addition, braiding with the vacuum is considered as not braiding at all, so coefficients like $R_{0\tau}^\tau$ are 1. Together, the R coefficients give rise to an isomorphism between bases of V_{ab}^c and V_{ba}^c. The map R extends to a unitary operation, so the inverse of an R coefficient R_{ab}^c, corresponding to the inverse braid b^{-1}, is the complex conjugate of R_{ab}^c [1].

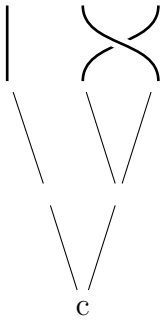
For Fibonacci anyons, the only non-trivial R coefficients are R_{11}^0 and R_{11}^1. Using quantum field theory and consistency relations [1], these coefficients turn out to be

$$R_{11}^0 = e^{-4\pi i/5} \tag{3.4}$$

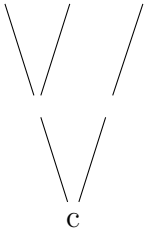
$$R_{11}^1 = e^{3\pi i/5} \tag{3.5}$$

See [4].

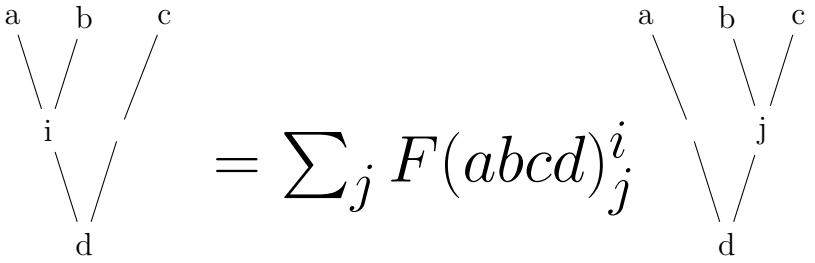
3.3. **F-moves.** The R moves give the relationship between the states of braided and unbraided sets of two anyons fusing to a given anyon. However, for more than two initial anyons, we need another move to relate a given basis state to an arbitrary braiding of that state's anyons. To see this, consider the braid b_2..



We can use an R move on the two rightmost strands to untwist them, but then we are still not left with the "standard" basis state



. What we need is the relationship shown below.



This is the F move, and it is both unitary and Hermitian. The F coefficients $F(abcd)^i_j$, as with the R moves, are zero for fusions incompatible with the fusion rules of the anyons. For Fibonacci anyons, the only non-trivial F coefficients are those for $abcd = 1111$. These coefficients are

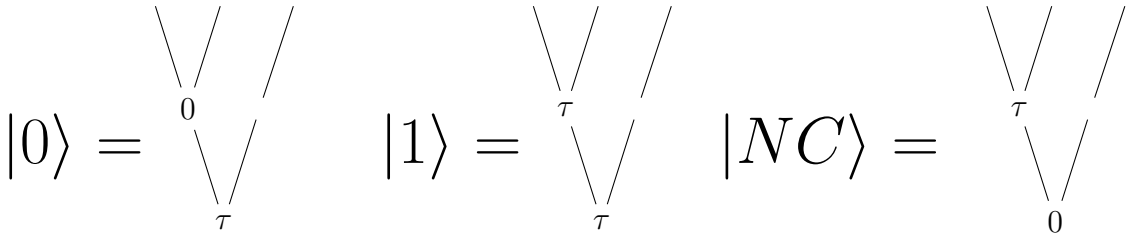
$$F(1111)^0_0 = \frac{1}{\phi} \tag{3.6}$$

$$F(1111)^0_1 = F(1111)^1_0 = \frac{1}{\sqrt{\phi}} \tag{3.7}$$

$$F(1111)^1_1 = -\frac{1}{\phi} \tag{3.8}$$

where ϕ is the golden ratio, $\phi = \frac{1+\sqrt{5}}{2}$. See [1], [7]. Together, the F and R moves are sufficient to determine the action of every braid on the basis states of a system of anyons [7]. We will see how these moves can be used to calculate the linear transformations corresponding to braids in Section 3.5.

3.4. **Building Qubits from Fibonacci Anyons.** Consider a line of 3 Fibonacci anyons. That is, the initial set of anyons is τ, τ, τ. Use the basis corresponding to the fusion outcomes when we fuse the two leftmost anyons at each stage. This Hilbert space is 3 dimensional. The basis we described has fusion outcomes, in order, $(0, 1)$, $(1, 1)$, and $(1, 0)$, since $(0, 0)$ is forbidden by the fusion rules for Fibonacci anyons. These fusion results are shown below, where for the rest of the paper, we assume that any unlabeled vertices on fusion diagrams are τ. We give these basis states the suggestive labels $|0\rangle$, $|1\rangle$, and $|NC\rangle$.

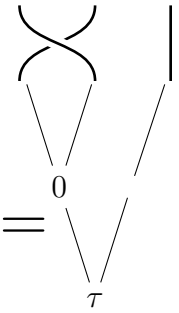


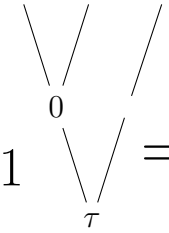
To build qubits, we will want a two-dimensional Hilbert space, so we can consider the span of only the first of these two options. In practice, it is popular to do this by instead considering a system of four anyons, consisting of two anyon-anti-anyon pairs created from the vacuum, so as to give the system a group charge of 0. Then, if we neglect the fourth anyon and do not braid it, the first three anyons must fuse to τ, as otherwise the final fusion will be $0 \times \tau$, which cannot give 0. Therefore, the only possible basis states of this system are the first

two, $|0\rangle$ and $|1\rangle$, as desired [5]. The states $|0\rangle$ and $|1\rangle$ are then the logical 0 and 1 for our computational basis, and we refer to $|NC\rangle$ as a non-computational state.

3.5. **Fibonacci Braid Matrices.** In the basis $\{|0\rangle, |1\rangle, |NC\rangle\}$, we can write the actions of the braid operators b_1 and b_2 as matrices. The matrices corresponding to b_1^{-1} and b_2^{-1} are then, naturally, the inverses of those matrices (equivalently, the conjugate transpose of those matrices, since they are unitary). Since the braid group on three strands is generated by b_1, b_2, b_1^{-1}, and b_2^{-1}, this set of matrices fully describes the action of braiding anyons on the system's wavefunction.

Following [1], we can write

$b_1 |0\rangle =$ 

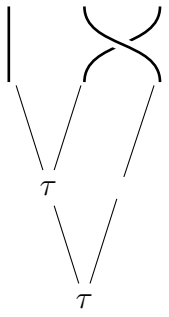
$= R_{11}^0$ 

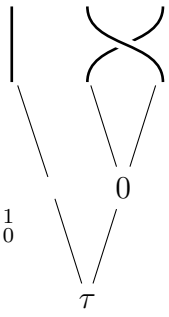
$= e^{-4\pi i/5} |0\rangle$

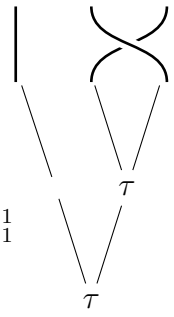
where we used the result for R_{11}^0 at (3.4). Similarly, we find that $b_1 |1\rangle = e^{3\pi i/5} |1\rangle$, and $b_1 |NC\rangle = e^{3\pi i/5} |NC\rangle$. In this basis, we can write

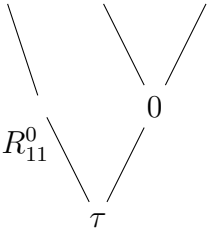
$$b_1 = \begin{pmatrix} e^{-4\pi i/5} & 0 & 0 \\ 0 & e^{3\pi i/5} & 0 \\ 0 & 0 & e^{3\pi i/5} \end{pmatrix} \tag{3.9}$$

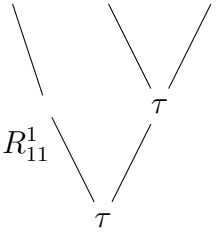
The derivation of the matrix for b_2 is a bit more involved, so we present the calculation of $b_2 |1\rangle$ here, and refer the reader to [1] for the action of b_2 on the other two basis elements.

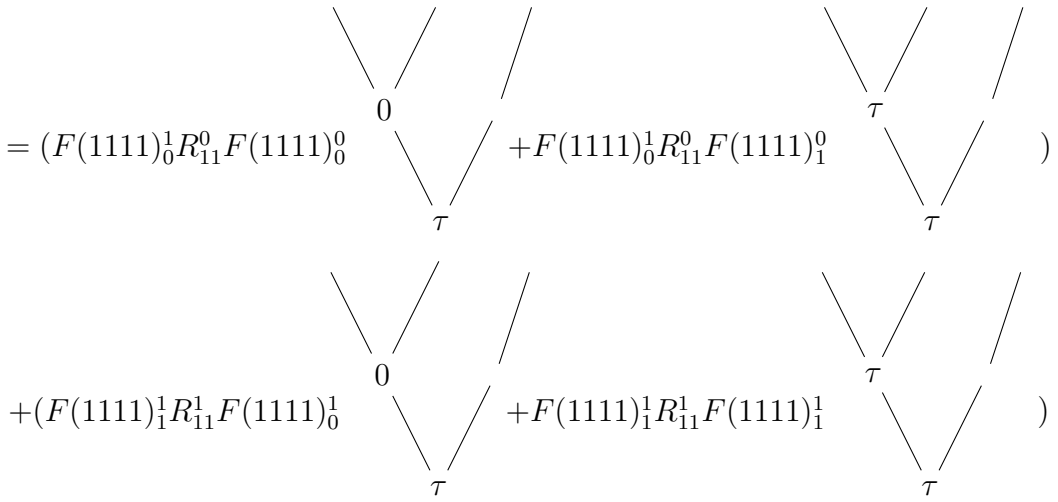
$b_2 |1\rangle =$ 

$= F(1111)_0^1$ 

$+F(1111)_1^1$ 

$= F(1111)_0^1 R_{11}^0$ 

$+F(1111)_1^1 R_{11}^1$ 



$$= (\phi^{-\frac{3}{2}} e^{-4\pi i/5} - \phi^{-\frac{3}{2}} e^{3\pi i/5}) |0\rangle + (\phi^{-1} e^{-4\pi i/5} + \phi^{-2} e^{3\pi i/5}) |1\rangle = \phi^{-\frac{1}{2}} e^{-3\pi i/5} |0\rangle - \phi^{-1} |1\rangle$$

As seen on page 17 of [1], we eventually get the matrix

$$b_2 = \begin{pmatrix} \phi^{-1} e^{4\pi i/5} & \phi^{-1/2} e^{-3\pi i/5} & 0 \\ \phi^{-1/2} e^{-3\pi i/5} & -\phi^{-1} & 0 \\ 0 & 0 & e^{3\pi i/5} \end{pmatrix} \tag{3.10}$$

Notice that each matrix is block diagonal, with the upper two rows forming the block in the matrix b_2. Therefore, any state that begins in the span of the computational basis stays in the span of the computational basis, and there is no "leakage" to $|NC\rangle$. Single qubit operations, then, are safe from leakage. However, this safety is not afforded to multiple-qubit operations, and so care must be taken to mitigate these errors [8].

4. Approximating Gates with Anyonic Qubits

Now that we have a way to build and act on qubits made of anyons, we turn to the problem of finding a set of braid operations that enact some unitary operation on the system. In particular, we want to create gates for a quantum computer. Unfortunately, braids that give exactly a particular unitary gate do not always exist [6]. Fortunately, the set of braiding operations on our 3 (or 4, with one anyon inert) anyon system is dense in $SU(2)$, meaning we can approximate a single qubit gate to arbitrary accuracy. In fact, using the Solovay-Kitaev algorithm, one can algorithmically compute a good approximation to any operation in $SU(2)$ in a manner that converges fairly quickly. In [2], this is presented as a classical algorithm. This reduces a problem in quantum computation to one in classical computation: given a unitary matrix, what product of braid operation matrices approximates the corresponding gate "well enough"?

This subject is very interesting in its own right. The most common method currently appears to be an exhaustive search method, where a maximum braidword length is defined, and all braidwords up to that length are considered. In [1], the authors mention that certain optimization involving eliminating redundancies due to the braid group's relations can significantly decrease the search space, and that braidwords converge to good accuracy fairly quickly, so the problem can be solved with limited efficiency.

Using this method, we can approximate the Hadamard gate. As given in [1], the braidword $b_2^{-4} b_1^{-4} b_2^2 b_1^4 b_2^2 b_1^{-2} b_2^{-2} b_1^{-2} b_2^{-2} b_1^2 b_2^{-2}$ is an approximation of the Hadamard gate, up to a global

phase that is does not affect any measurement of the system. The braidword gives exactly the matrix

$$e^{3.4558i}\begin{pmatrix}0.9997+0.0017i & 1.0003-0.0039i\\ 1.0003+0.0039i & -0.9997+0.0017i\end{pmatrix} \tag{4.1}$$

which is rather close to the Hadamard gate.

In [5], the authors found a braid that implements an approximation to a CNOT gate. The method takes the first two anyons in the control qubit together and braids them as a pair through the three anyons in the target qubit. If the two anyons in the control qubit fuse to the vacuum, it is as if nothing was ever braided through the second qubit. If the first two anyons fuse to a τ, then the gate approximates a NOT gate on the target qubit. By looking at the fusion diagrams for our computational basis, one can notice that the state is determined by the fusion of the first two anyons, so this corresponds to our CNOT gate.

This braid does introduce some leakage to the state $|NC\rangle$, and so it will need to be managed with the projective methods in [8]. However, any quantum circuit can be implemented using only a CNOT gate and single qubit operations [5], so this gives a universal quantum computer with qubits made from Fibonacci anyons.

Notes

Braids were drawn with Andrew Stacey's braids package, originating at the stackexchange post here: https://tex.stackexchange.com/questions/16897/how-to-make-nice-braids-diagrams. Fusion diagrams were drawn using tikzpicture trees.

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