Non-perturbative effects and D-brane instanton resummation in string theory

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Motivation & introduction

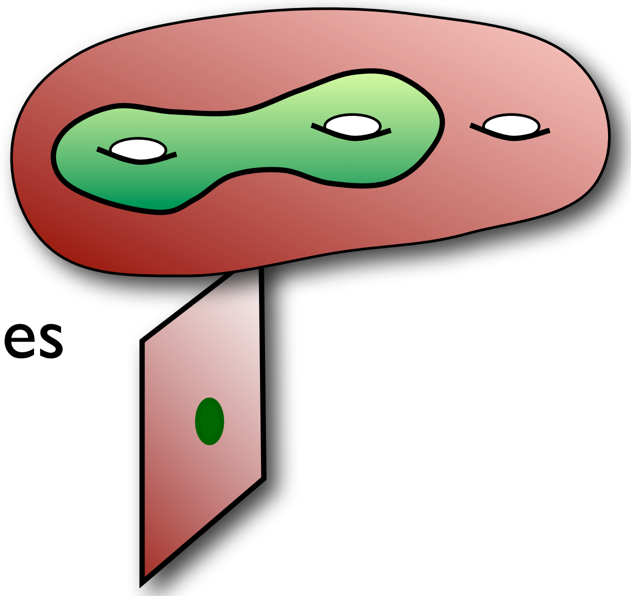
Non-perturbative effects in string theory are relevant in a number of phenomenological model building issues

- Kahler moduli stabilization in KKLT models
- Generation of perturbatively forbidden couplings

Neutrino masses, Yukawas, mu term, ...

Focus on type II (or orientifolds):
D-brane instantons

Euclidean D-branes wrapped on CY cycles and localized on 4d Minkowski



D-brane instanton effects

[Becker's, Strominger; Witten; Harvey, Moore; ...]

Violate certain perturbatively exact U(1) global symmetries

- Ex: Take one complex structure modulus in IIA CY orientifold

$$T = t + i\,a = \int_C \mathrm{Re}\,\Omega + i \int_C C_3$$

PQ symmetry $a \to a + \lambda$ violated by D2-brane instanton $\simeq e^{-T}$

⇒ Stabilization of moduli perturbatively protected by PQ

[Kachru, Kallosh, Linde, Trivedi]

Need to sum over all such instantons!

Only very partial results...

e.g. classify contributing instantons [Denef, Douglas, Florea+Grassi, Kachru]

Field theory operators

- With 4d gauge D6-branes, gauging of PQ by U(1) in U(N)
  From D6-branes on C'

$$\int_{C'\times M_4} C_5 \wedge \mathrm{tr}\, F \;\rightarrow\; \int_{M_4} B_2 \wedge \mathrm{tr}\, F \;\rightarrow\; \int d^4x\, (\, \partial_\mu a \,+\, A_\mu\,)^2$$

⇒ Instanton on C violates $U(1)_{C'}$ by $n=I_{CC'}$ units

⇒ Fermion zero modes at $I_{CC'}$ intersections

$$\int d\lambda\, d\tilde{\lambda}\, e^{-T+\lambda\Phi\tilde{\lambda}} \;=\; e^{-T}\,\Phi$$

⇒ Role in generating perturbatively forbidden couplings

$$e^{-T}\,\Phi_1\ldots\Phi_n$$

[Blumenhagen, Cvetic, Weigand; Ibanez, AU; Florea, Kachru, McGreevy, Saulina]

Need to sum over all such instantons!

Essentially no results...

Particles and instantons

A whole lot is known about the spectrum of BPS particles in CY compactifications [Denef, Moore; ...]

Sometimes even full list at any point in moduli space

e.g. conifold [Jafferis, Moore]

toric singus with no 4-cycles [Aganagic, Ooguri, Vafa, Yamazaki]

Relate BPS D-brane particles to BPS D-brane instantons via circle compactification to 3d and T-duality

[Ooguri, Vafa '96]

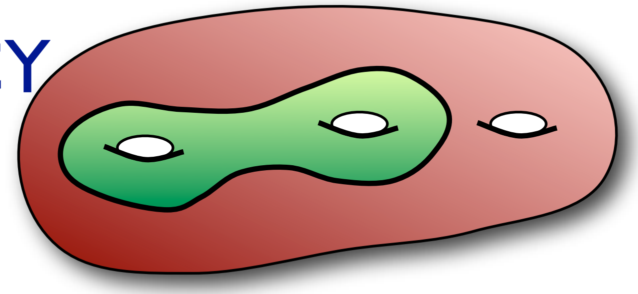
One-loop diagram of BPS particles ⇔ BPS D-brane instanton

Momentum of BPS particles ⇔ Multiwrapping of instanton

Bound states of BPS particles ⇔ Multi-instanton processes

D1/D(-1) instantons in type IIB on CY [Collinucci, Soler, A.U]

- Consider 4d D2/D0 particles in IIA on CY



Compactification to 3d + Tduality gives D1/D(-1)-instantons on IIB on same CY e.g. corrections to kinetic term of Kahler moduli

- Multiplicities of D2/D0 can be counted by lift to M-theory in terms of Gopakumar-Vafa invariants $n_{g,[C]}$ count 5d BPS particles from M2-branes on 2-cycles C

All at once: One-loop of 5d particles on T^2

$$Z_{\rm M2} = \frac{1}{4\pi}\sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+\tau n|^{3/2}} \left(1+2\pi\,|m+\tau n|\,k_a\,t^a\right) e^{-S_{k_1,k_2}}$$

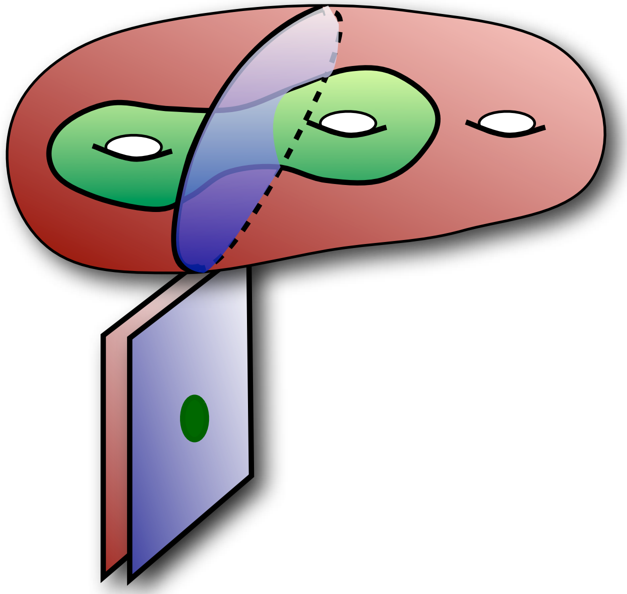
$$Z_{\rm D-inst} = \frac{1}{4\pi}\sum_{k_a} n_{k_a} \sum_{m\neq 0,n\in\mathbf{Z}} \frac{|z+q|^{1/2}}{|p|^{3/2}} \left[1+\sum_{k=1}^{\infty}\frac{\Gamma(3/2+k)}{k!\,\Gamma(3/2-k)}\left(4\pi|p\tau_2||z+q|\right)^{-k}\right] e^{-S_{(p,q)}}$$

Reproduces [Robles-Llana, Rocek, Saueressig, Theis, Vandoren, '06]

D(-1) instantons in type IIB with O7/D7

Understand better the role of O-planes and gauge D-branes

- Instantons localized on top of O-plane
- Instantons leading to gauge operators



Consider a simple 8d example
type IIB on T^2 with O7-planes and D7's
D(-1)-instantons yield tr F^4, $(\mathrm{tr}F^2)^2$, R^4
etc corrections

Become 4d N=2 gauge functions upon compactification on K3

Using heterotic/type IIB orientifold duality, can be computed from heterotic as threshold corrections

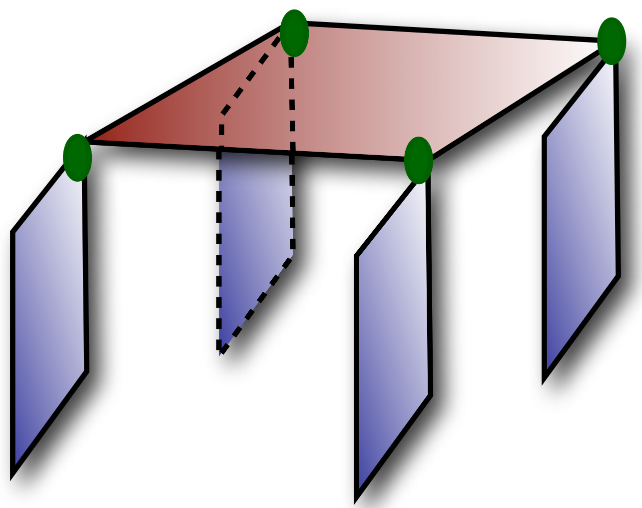
$$\frac{t_8\,\mathrm{Tr}\,F^4}{4}\log\left|\frac{\eta(4T_{\rm h})}{\eta(2T_{\rm h})}\right|^4+\frac{t_8\,(\mathrm{Tr}\,F^2)^2}{16}\log\left(\mathrm{Im}\,T_{\rm h}\,\mathrm{Im}\,U_{\rm h}\frac{|\eta(2T_{\rm h})|^8\,|\eta(U_{\rm h})|^4}{|\eta(4T_{\rm h})|^4}\right)$$

$$+\,2\,t_8\,\mathrm{Pf}\,F\,\log\left|\frac{\eta(T_{\rm h}+1/2)}{\eta(T_{\rm h})}\right|^4$$

[Lerche, Stieberger; Gutperle; Kiritsis, Obers, Pioline; Gava, Narain, Sarmadi]

The instanton computation

[Billo, Ferro, Frau, Gallot, Lerda, Pesando]



- In type IIB can be directly computed in instanton language
- Complicated integration over instanton moduli space using "localization"
- Heavy computation, explicit only order by order in instanton number; extrapolation
- Result is

$$t_8 \,\mathrm{Tr}\, F^4 \left\{ \left(\frac{\pi \mathrm{i} T_{\mathrm{h}}}{12} - \frac{1}{2} \sum_{k=1}^{\infty} \left(d_k q_{\mathrm{h}}^{4k} - d_k q_{\mathrm{h}}^{2k} \right) \right) + \mathrm{c.c.} \right\}$$

$$+ \, t_8 \, (\mathrm{Tr}\, F^2)^2 \left\{ \frac{1}{16} \log \left(\mathrm{Im}\, T_{\mathrm{h}} \,\mathrm{Im}\, U_{\mathrm{h}} \, |\eta(U_{\mathrm{h}})|^4 \right) + \frac{1}{8} \left(\sum_{k=1}^{\infty} \left(d_k q_{\mathrm{h}}^{4k} - 2 d_k q_{\mathrm{h}}^{2k} \right) + \mathrm{c.c.} \right) \right\}$$

$$+ \, 8 \, t_8 \,\mathrm{Pf}\, F \left(\sum_{k=1}^{\infty} d_{2k-1} q_{\mathrm{h}}^{2k-1} + \mathrm{c.c} \right)$$

with q=exp (2πiT) the instanton factor

- Full agreement with heterotic result

One-loop of D0's

[Gutperle; Petersson, Soler, AU]

The whole D(-1) instanton sum can be obtained from a one-loop diagram of BPS particles in T-dual type I'
9d IIA on S^1 and two O8 with 16 D8's each i.e. $SO(16)^2$

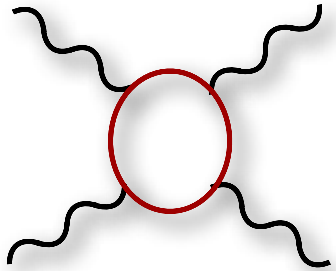
with Wilson line breaking SO(16) to the 8d $SO(8)^2$

- Bound states of BPS particles stuck on top of O-plane
- BPS particles charged under 4d group

The F^4 term from a one-loop of particle in representation R:

$$\Delta = \sum_m \int d^8p \int \frac{dt}{t} e^{-t(\mathbf{p}^2 + \frac{m}{R}^2 + n^2)} \, t^4 \, \mathrm{tr}_R F^4 \; = \sum_w \frac{1}{w} q^{nw} \, e^{2\pi i w A} \mathrm{tr}_R F^4$$

BPS particles:
- Bound state of 2n D0's, in 120 of SO(16)
- Bound state of 2n+1 D0's, in 128 of SO(16)



Full result computable, agreement with IIB and heterotic results

Interpretation in Horava-Witten theory

[Petersson, Soler, AU]

Lift IIA with two O8's to Horava-Witten

Each 10d boundary on S^1 with WL breaking E8 to SO(16)

- Momentum 2n is WL invariant and gives 120
- Momenbum 2n+1 is WL odd and gives 128

Each 10d boundary on T^2 with WLs breaking E8 to $SO(8)^2$

One-loop diagram of E8 gauge bosons with WL effects

- Clearly, same computation & result
- Manifest interpretation of modular functions of $\Gamma(2)$
- Modular transformations of T2 preserving the WLs

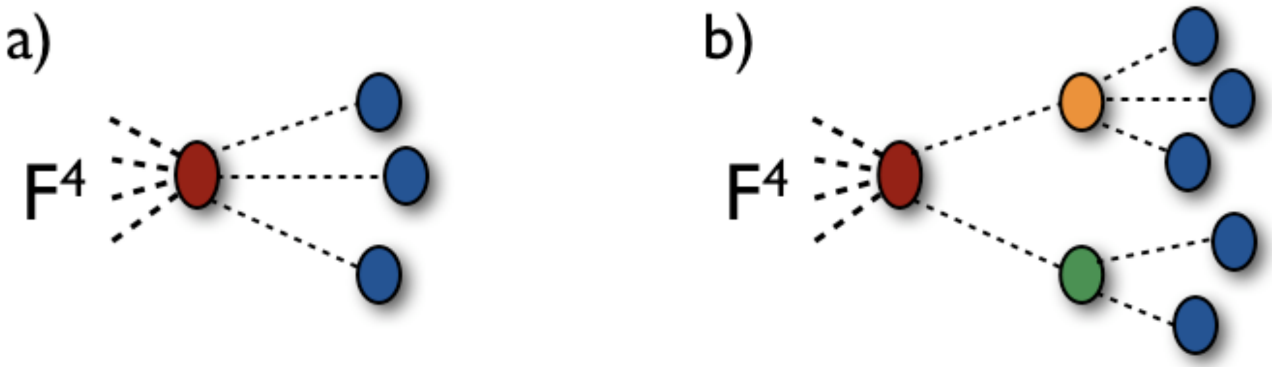
Multi-instantons vs. Polyinstantons

[Petersson, Soler, AU]

Mapping to BPS particle loops allows a nice interpretations of contributions involving multiple instantons

- **Multi-instantons:** [Garcia-Etxebarria, AU]
  Correspond to diagrams with one BPS bound states of particles

- **Poly-instantons:** [Blumenhagen, Schmidt-Sommerfeld]
  Correspond to diagrams with multiparticle states
  Higher loop diagram



Superpotentials?

Compactification on K3 leads to gauge kinetic functions in 4d N=2

Similar techniques for gauge functions in 4d N=1 using orbifolds
(corrections lie in N=2 subsectors, as in heterotic)

- What about genuine computations of superpotentials?
Not there yet...

But tantalizing hints:

Open and unoriented versions of Gopakumar-Vafa invariants

Expected to describe BPS multiplicities of particles in theories with 4 susys

Looking for a clever duality to 4d instantons...

Final comments

Non-perturbative effects in string theory are in rich interface

- Duality
- Moduli stabilization and susy breaking
- Phenomenological applications

Basic physics understood, new physics in resummation

- Duality
- Modular properties

Started exploiting interplay of particles and instantons

- Successful with 8 susys, or subsectors of 4d N=1
- Hoping to find lessons for 4d N=1 superpotentials