What Every Physicist Should Know About String Theory

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I thought that explaining these matters is possibly suitable for a session devoted to the centennial of General Relativity.

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Anyone who has studied physics is familiar with the fact that while physics – like history – does not precisely repeat itself, it does rhyme, with similar structures at different scales of lengths and energies. We will begin today with one of those rhymes – an analogy between the problem of quantum gravity and the theory of a single particle.

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$$I = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{g}(R+\Lambda),$$

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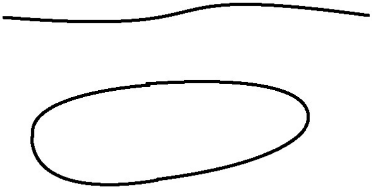

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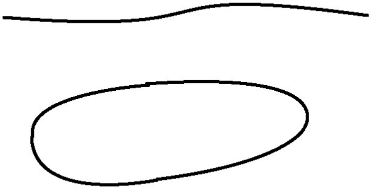

In contrast to the 4d case, there is no Riemann curvature tensor in 1 dimension so there is no close analog of the Einstein-Hilbert action.

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$$I = \int \mathrm{d}t \sqrt{g} \left(\frac{1}{2} \sum_{i=1}^{D} g^{tt} \left(\frac{dX_i}{\mathrm{d}t} \right)^2 - \frac{1}{2} m^2 \right)$$

where $g = (g_{tt})$ is a 1×1 metric tensor and I have written $m^2/2$ instead of Λ.

If we introduce the "canonical momentum"

$$P_i = \frac{\mathrm{d}X_i}{\mathrm{d}t}$$

then the "Einstein field equation" is just

$$\sum_i P_i^2 + m^2 = 0.$$

In other words, the wavefunction $\Psi(X)$ should obey the corresponding differential equation

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$$I = \int \mathrm{d}t \sqrt{g} \left(\frac{1}{2} g^{tt} \left(-\left(\frac{\mathrm{d}X_0}{\mathrm{d}t} \right)^2 + \sum_{i=1}^{D-1} \left(\frac{\mathrm{d}X_i}{\mathrm{d}t} \right)^2 \right) - m^2 \right).$$

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Now the equation obeyed by the wavefunction is a Klein-Gordon equation in *Lorentz* signature:

$$\left(\frac{\partial^2}{\partial X_0^2} - \sum_{i=1}^{D-1} \frac{\partial^2}{\partial X_i^2} + m^2 \right) \Psi(X) = 0.$$

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$$\left(-G^{IJ} \frac{D}{DX^I} \frac{D}{DX^J} + m^2 \right) \Psi(X) = 0.$$

This is the massive Klein-Gordon equation in curved spacetime.

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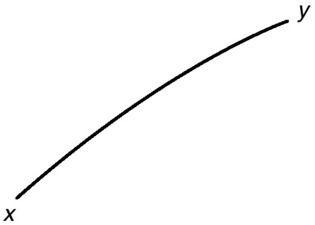

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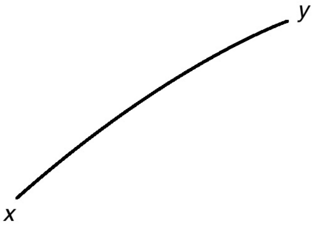

Part of the process of evaluating the path integral in a quantum gravity theory is to integrate over the metric on the one-manifold, modulo diffeomorphisms. But up to diffeomorphism, this one-manifold has only one invariant, the total length τ, which we will interpret as the elapsed proper time.

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$$G(x, y; \tau) = \int \frac{\mathrm{d}^D p}{(2\pi)^D} e^{iP\cdot(y-x)} \exp(-\tau(P^2 + m^2)).$$

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But we have to remember to do the "gravitational" part of the path integral, which in the present context means to integrate over τ.

Thus the complete path integral for our problem – integrating over all metrics $g_{tt}(t)$ and all paths $X(t)$ with the given endpoints, modulo diffeomorphisms – gives

$$G(x, y) = \int_0^\infty \mathrm{d}\tau \, G(x, y; \tau) = \int \frac{\mathrm{d}^D p}{(2\pi)^D} e^{ip\cdot(y-x)} \frac{1}{p^2 + m^2}.$$

This is the standard Feynman propagator in Euclidean signature, and an analogous derivation in Lorentz signature (for both the spacetime M and the particle worldline) gives the correct Lorentz signature Feynman propagator, with the $i\varepsilon$.

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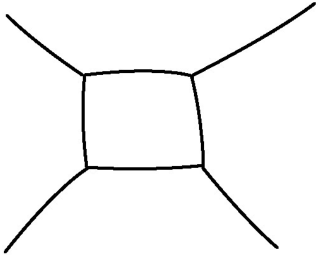

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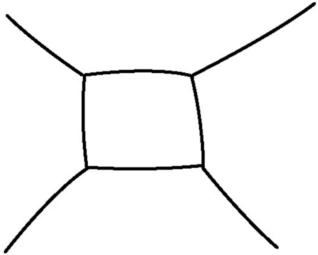

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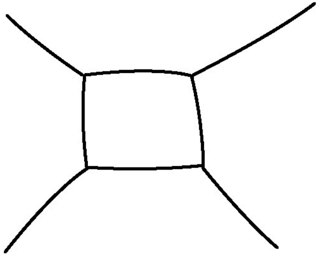

Our "quantum gravity" action makes sense on such a graph. We just take the same action that we used before, summed over all of the line segments that make up the graph.

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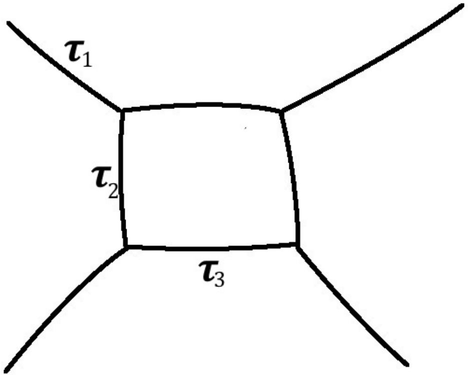
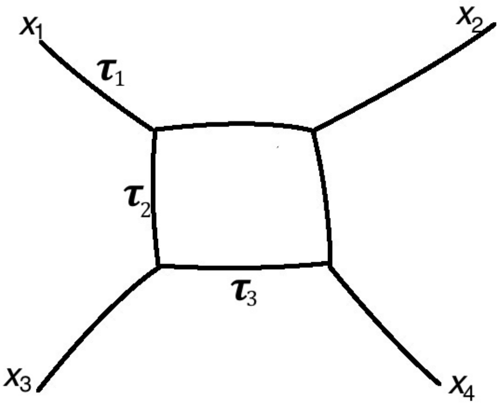
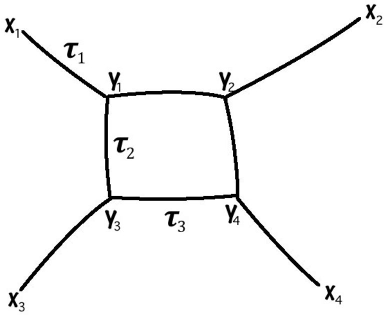


(I did not label all of them.)

The natural amplitude to compute is one in which we hold fixed the positions $x_1, \ldots, x_4$ of the external particles and integrate over all the τ's and over the paths the particle follow on the line segments.



To integrate over the paths, we just observe that if we specify the positions $y_1, \ldots, y_4$ at all the vertices (and therefore on each end of each line segment)



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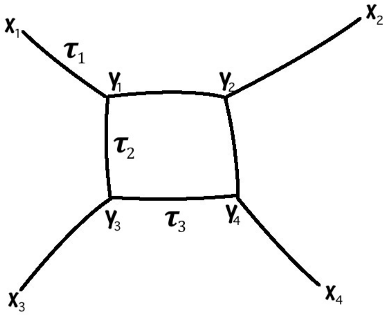

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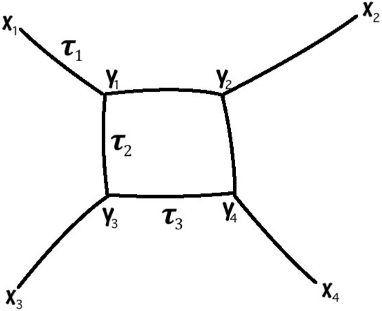

then the computation we have to do on each line segment is the same as before and gives the Feynman propagator. Integrating over the y_i will just impose momentum conservation at vertices, and we arrive at Feynman's recipe to compute the amplitude attached to a graph: a Feynman propagator for each line, and an integration over all momenta subject to momentum conservation.

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So the integral over 2d metrics promises to not be trivial at all.

This is related to the fact that a 2d metric in general is a 2×2 symmetric matrix constructed from 3 functions

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but a diffeomorphism

$$\sigma^i \to \sigma^i + h^i(\sigma), \quad i = 1, 2$$

(where σ^i are coordinates on the "worldsheet") can only remove two functions.

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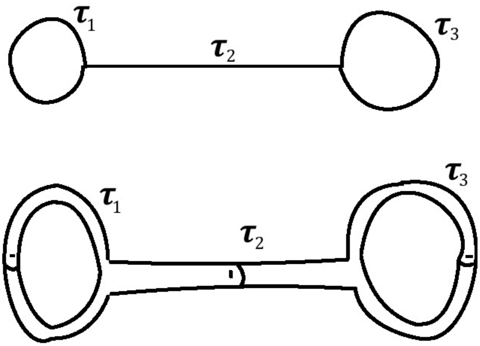
is *conformally-invariant*, that is, it is invariant under

$$g_{ij} \to e^\phi g_{ij}$$

for any real function ϕ on Σ. If we require conformal invariance as well as diffeomorphism invariance, then this is enough to make any metric g_{ij} on Σ locally trivial (locally equivalent to δ_{ij}), as we had in the quantum-mechanical case.

Some very pretty 19th century mathematics now comes into play



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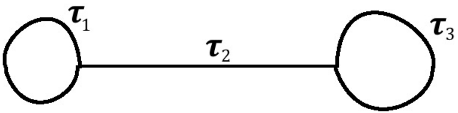

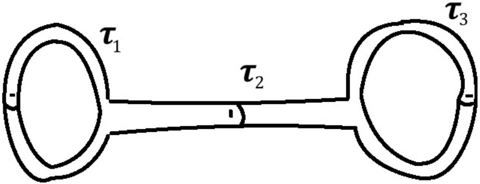

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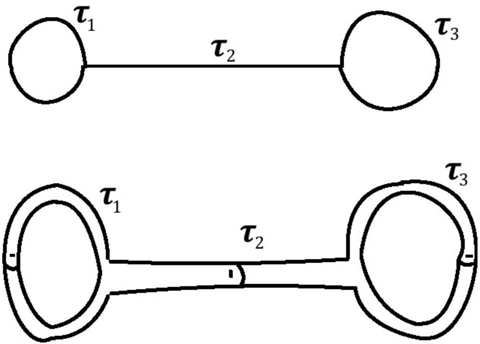
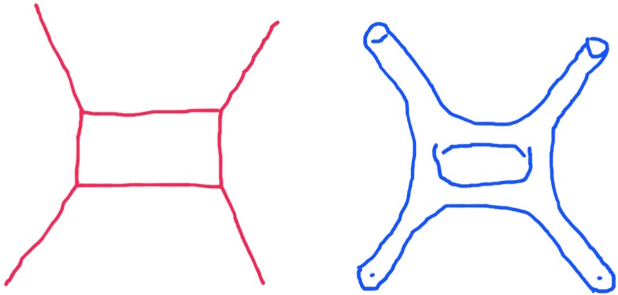


It turns out that, just as in the 1d case, the metric g_{ij} can be parametrized by finitely many parameters. Two big differences: The parameters are now complex rather than real, and their range is restricted in a way that allows no possibility for an ultraviolet divergence.

To underscore how a two-manifold is understood as a generalization of a Feynman graph, I've drawn alongside each other the one-loop diagrams for $2 \to 2$ scattering in the 1d or 2d case:



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The external states in a Feynman diagram were just the states in this quantum mechanics.

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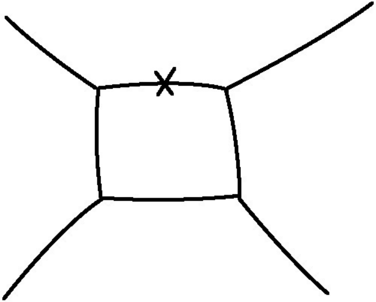

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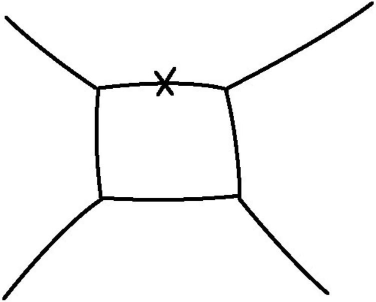

(To calculate the effects of a perturbation, we insert $\int \mathrm{d}t \sqrt{g}\, \mathcal{O}$, integrating over the position on the graph where the operator $\mathcal{O}$ is inserted. I just drew one possible insertion point.)

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that represents a fluctuation in the spacetime metric automatically represents a state in the quantum mechanics. Therefore the theory describes quantum gravity in spacetime.

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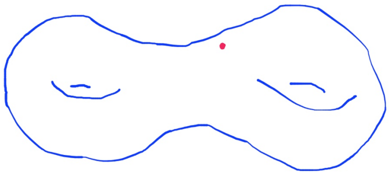

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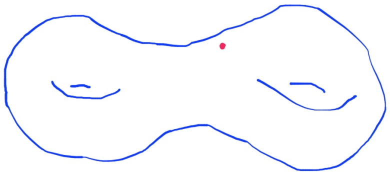

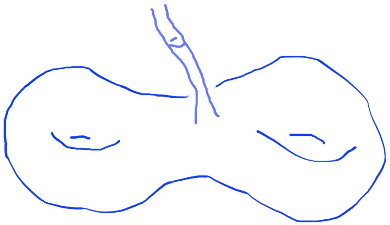

The basic idea can be seen if we write the metric of a plane in polar coordinates:

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In terms of $u = \log r$, $-\infty < u < \infty$, this is now

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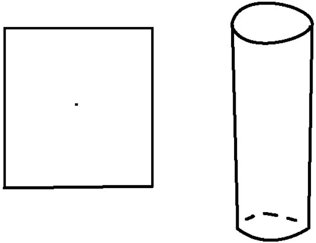
which describes a cylinder.



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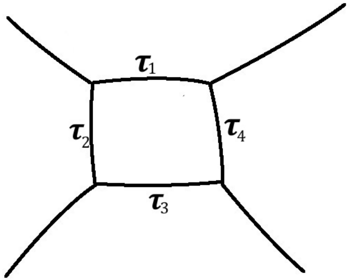

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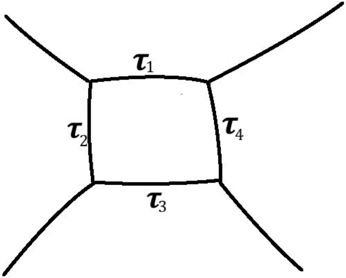

So in the example shown, ultraviolet divergences can occur for $\tau_1, \tau_2, \tau_3, \tau_4$ going simultaneously to 0.

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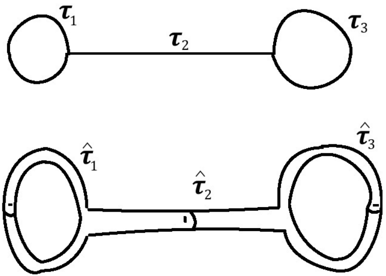

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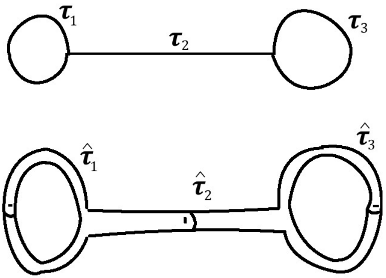

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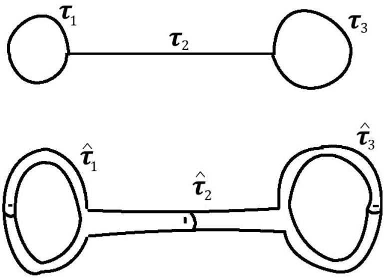

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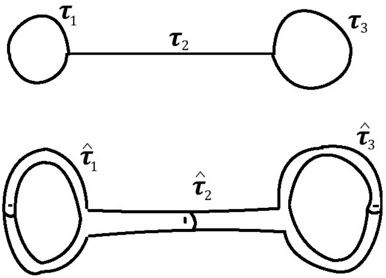

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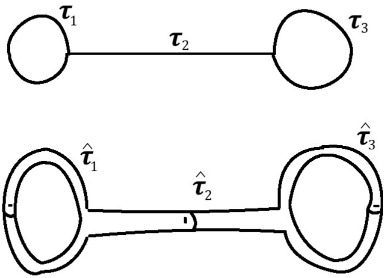

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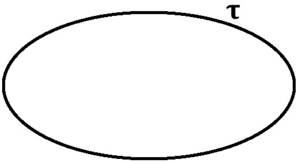

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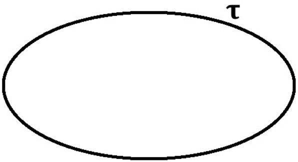

The resulting expression for the 1-loop cosmological constant is

$$\Gamma_1 = \frac{1}{2}\int_0^\infty \frac{\mathrm{d}\tau}{\tau}\mathrm{Tr}\,\exp(-\tau H)$$

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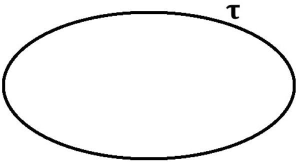

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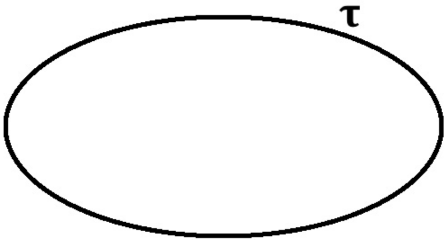
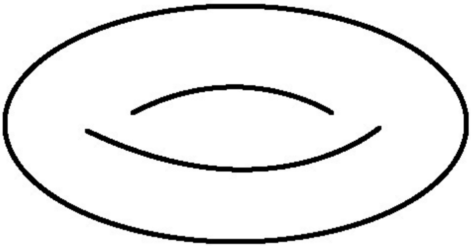
Going to string theory means replacing the classical one-loop diagram



Going to string theory means replacing the classical one-loop diagram with its stringy counterpart, which is a torus



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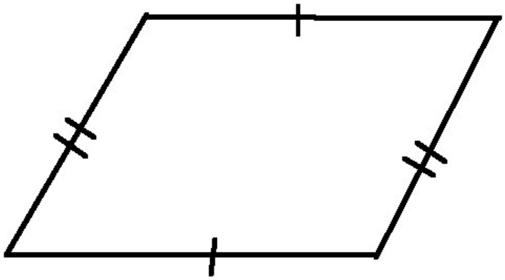
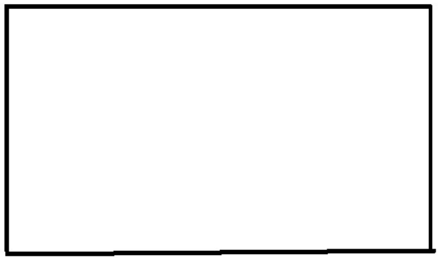


But to explain the idea without extraneous details, I will consider only rectangles instead of parallelograms:



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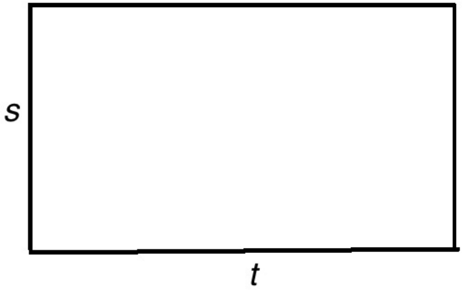

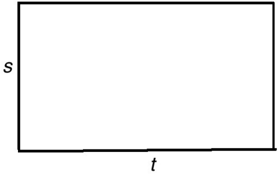

Now only the ratio

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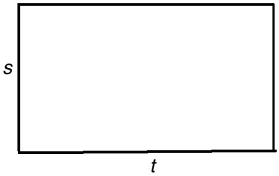

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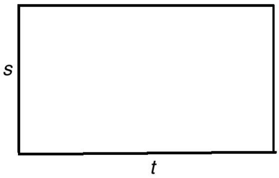

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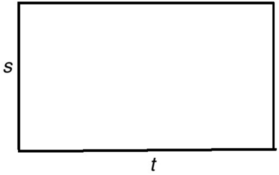

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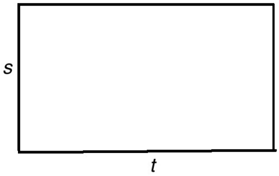

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So we can restrict to $t \geq s$, so that the range of u is

$$1 \leq u < \infty.$$

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There is no ultraviolet divergence, because the lower limit on the integral is 1 instead of 0.

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I have explained a special case, but this is a general story. The stringy formulas generalize the field theory formulas, but without the region that can give ultraviolet divergences in field theory. The infrared region ($\tau \to \infty$ or $u \to \infty$) lines up properly between field theory and string theory and this is why a string theory can imitate field theory in its predictions for the behavior at low energies or long times and distances.

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Let us focus on the following fact: The spacetime M with its metric tensor $G_{IJ}(X)$ was encoded as the data that enabled us to define a 2d conformal field theory that we used in this construction. Moreover, that is the only way that spacetime entered the story.

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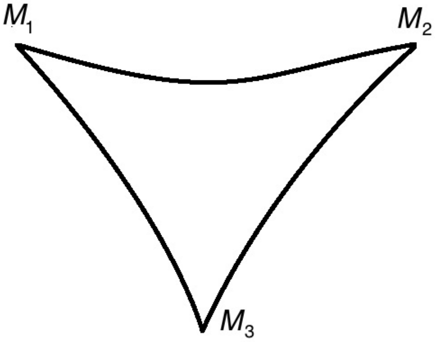

Many other nonclassical things can happen.

Many other nonclassical things can happen. It can happen that from a classical point of view, the spacetime develops a singularity, but actually the 2d conformal field theory remains perfectly good, meaning that the physical situation in string theory is perfectly sensible.

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This is not a complete explanation of the sense in which, in the context of string theory, spacetime emerges from something deeper. A completely different side of the story involves quantum mechanics and the duality between gauge theory and gravity. However, what I have described is certainly one important piece of the puzzle, and one piece that is relatively well understood. It is at least a partial insight about how spacetime as conceived by Einstein can emerge from something deeper, This is not a complete explanation of the sense in which, in the context of string theory, spacetime emerges from something deeper. A completely different side of the story involves quantum mechanics and the duality between gauge theory and gravity. However, what I have described is certainly one important piece of the puzzle, and one piece that is relatively well understood. It is at least a partial insight about how spacetime as conceived by Einstein can emerge from something deeper, and thus I hope this topic has been suitable as part of a session devoted to the centennial of General Relativity.