Topics in D4-D8 Holographic QCD

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with

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R.C. Brower & S. Mathur & C.I. Tan
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contents

- pure QCD
- bi-quark mesons
- solitonic baryons
- nucleon-meson dynamics
- NN-potential, deuteron, nuclei, and ……

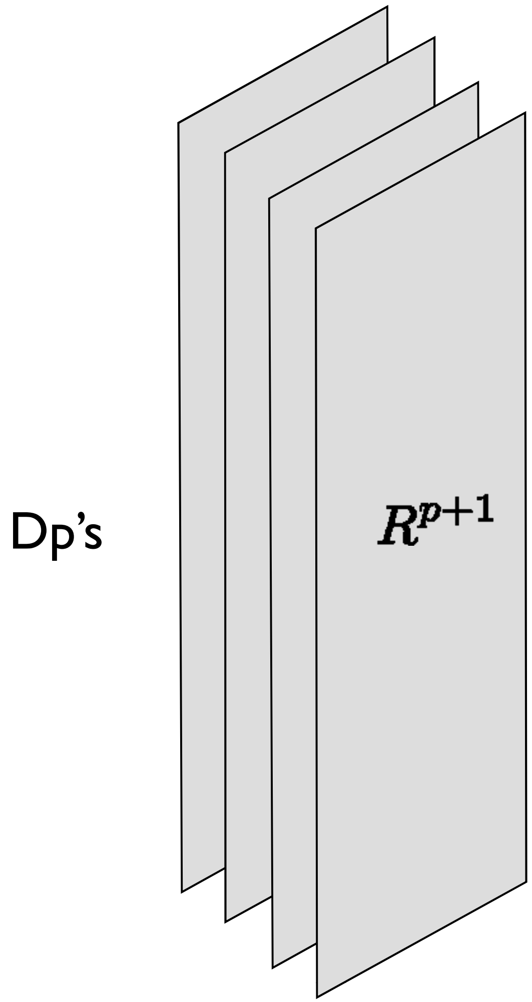
holography elevates global symmetries to local (gauge) symmetries:

Poincare symmetry → gravity

holography elevates global symmetries to local (gauge) symmetries:

Poincare symmetry → gravity

→ string theory



(p+1) - dim maximally SUSY U(n) Yang-Mills
+ Chern-Simons couplings to
background RR-fields

$$\mu_{\rm p} \int dx^{p+1} \frac{1}{4e^{\Phi}} \sqrt{-h}\, \mathrm{tr}\, |2\pi\alpha' F|^2$$

$$+ \quad \mu_{\rm p} \int \sum_{k=0}^{p} C_{k+1} \wedge \mathrm{tr}\, e^{2\pi\alpha' F}$$

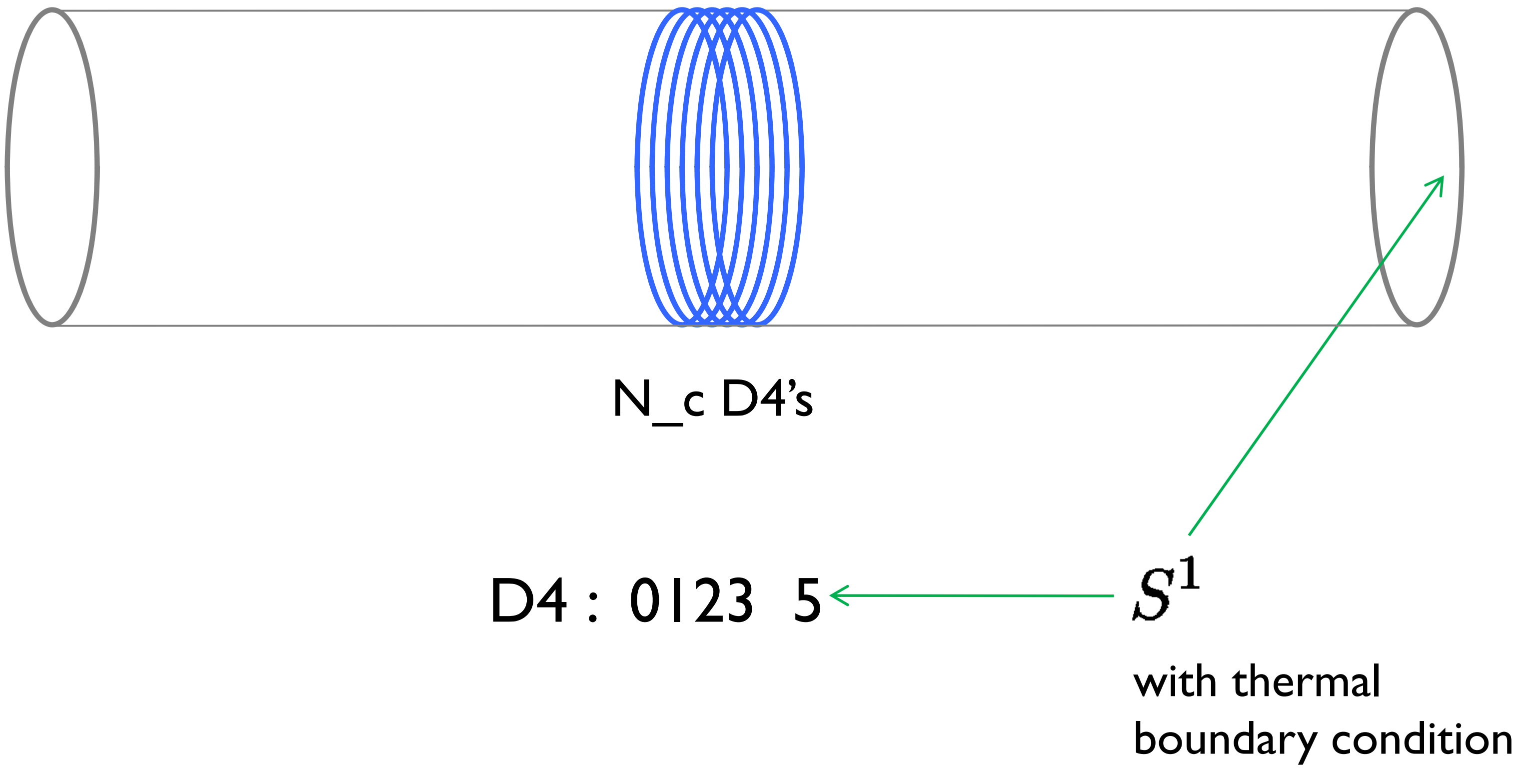
$+$ susy completion

$+$ **higher dimensional corrections**

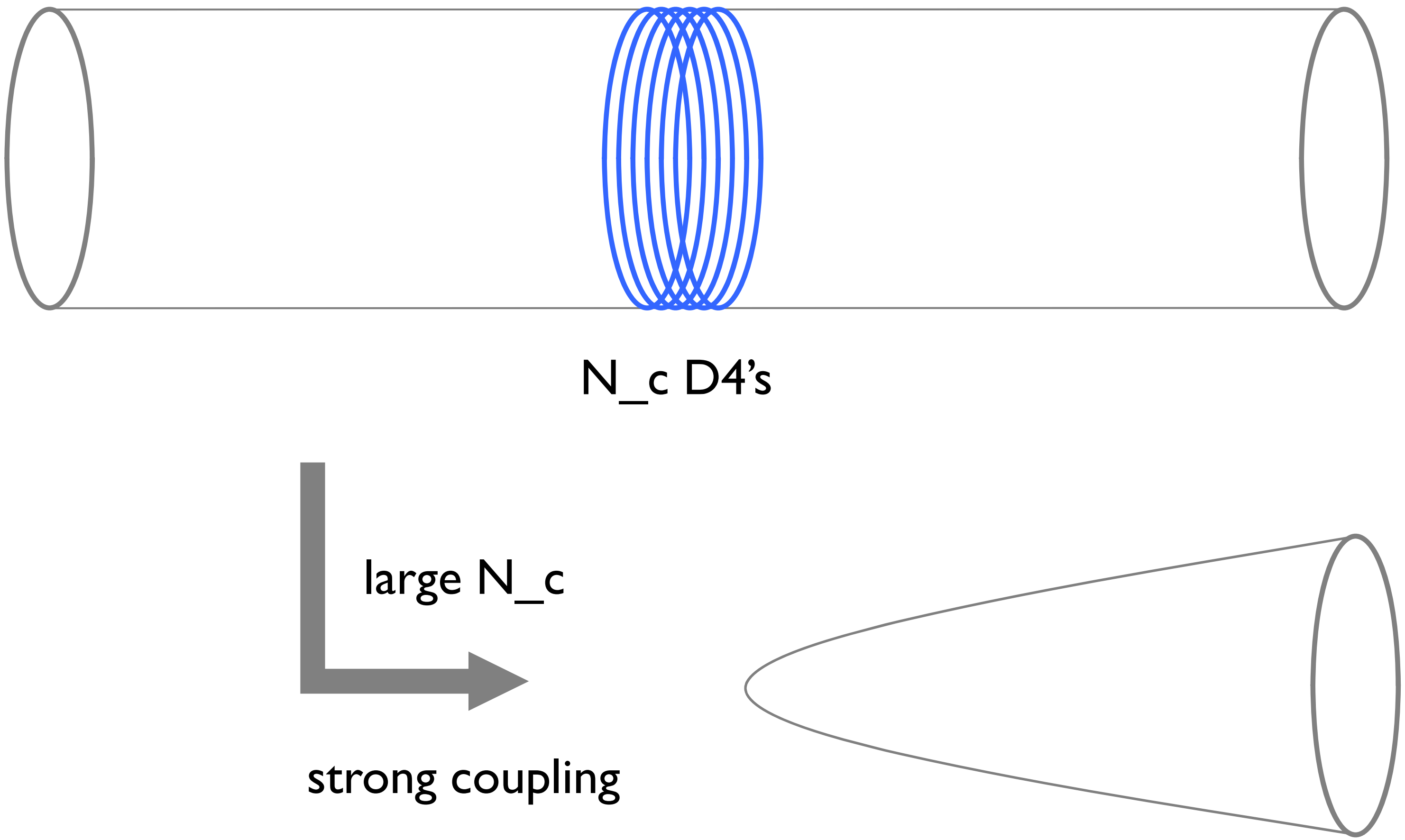
$$\mu_{\rm p} = \frac{2\pi}{(4\pi^2\alpha')^{(p+1)/2}}$$

Holographic QCD without flavor

Witten 1998



Holographic QCD without flavor



the dual geometry: explicit metric

$$N_c \gg 1, \;\; g_{YM}^2 N_c \gg 1$$

Gibbons and Maeda 1988

$$G_{9+1} = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{3+1} + f(u) d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

R^{3+1} $\quad$ S^1

occupied by N_c D4's

$$u_0 \leq u < \infty$$

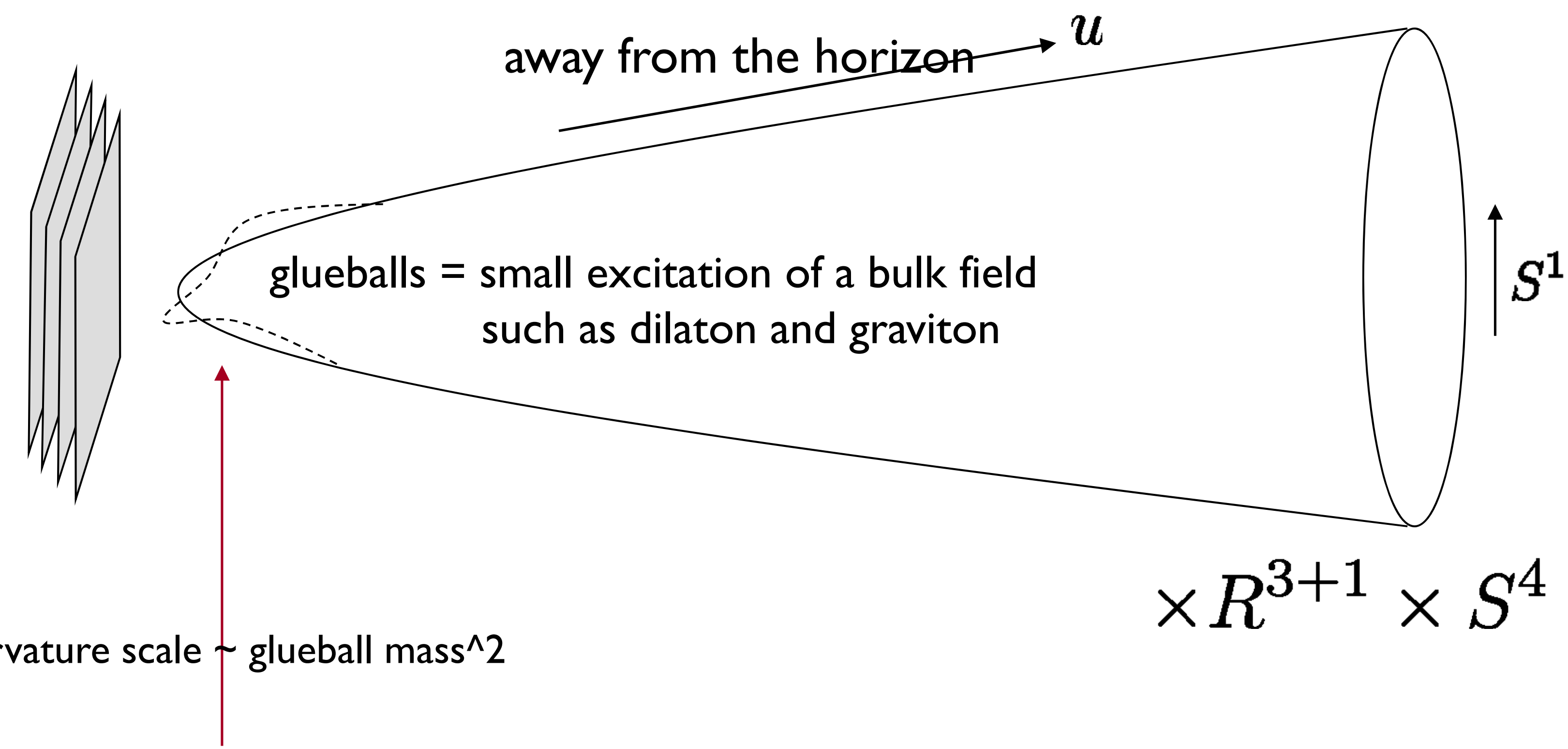
$$f(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

$$e^{\Phi} = g_s \left(\frac{u}{R}\right)^{3/4}$$

$$\int_{S^4} dC_3^{\mathrm{RR}} = 2\pi N_c$$

Holographic QCD without flavor

$$N_c \gg 1,\ \ g_{YM}^2 N_c \gg 1$$

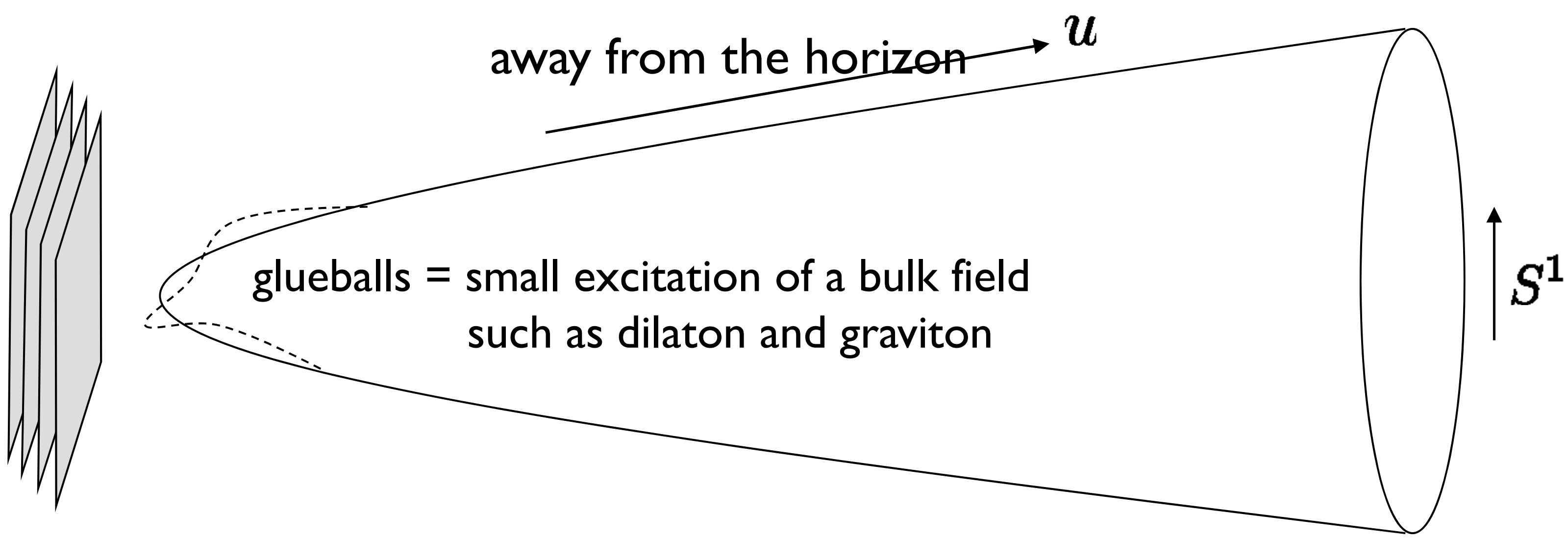


$$M_{KK}^2 \equiv 9u_0/4R^3$$

$$\int_{S^4} dC_3^{\rm RR} = 2\pi N_c$$

Holographic QCD without flavor

$N_c \gg 1, \;\; g_{YM}^2 N_c \gg 1$



For glueballs with spin no larger than 2

10% match with lattice results on mass ratios had been reported.

C. Csaki, H. Ooguri, Y. Oz, J. Terning 1998
C. Brower, S.D. Mathur, C.I. Tan 2000

hQCD theta-angle by Witten (1998)

QCD has one more parameter, θ, an angle which is unavoidable due to instanton processes and multiplies the Pontryagin density.

$$L = \frac{1}{4g^2}\mathrm{tr}F^2 + \frac{\theta}{8\pi^2}\mathrm{tr}F\tilde{F}$$

when nontrivial, it breaks CP explicitly.

dilute instanton gas approximation generate a vacuum energy of type

$$\left[\int_{moduli} e^{-S_{instanton}} Det'_{1-loop}\right] \times \cos\theta$$

suggesting a nontrivial periodic vacuum energy in full QCD

hQCD theta-angle by Witten (1998)

this type of operator is already present in D4-brane action

$$\mu_4 \int dx^{4+1} \frac{1}{4e^{\Phi}} \sqrt{-h}\, \mathrm{tr}\, |2\pi\alpha' F|^2 + \mu_4 \int \sum_{k=0}^{p} C_{k+1} \wedge \mathrm{tr}\, e^{2\pi\alpha' F} + \cdots$$

$$2\pi^2 \alpha'^2 \mu_4 \int C_1 \wedge \mathrm{tr} F \wedge F + \cdots$$

$$2\pi^2 \alpha'^2 \mu_4 \int_{S^1} C_1 \int_{R^{3+1}} \mathrm{tr} F \wedge F + \cdots$$

hQCD theta-angle by Witten (1998)

$$\theta \sim \langle C_\tau \rangle$$

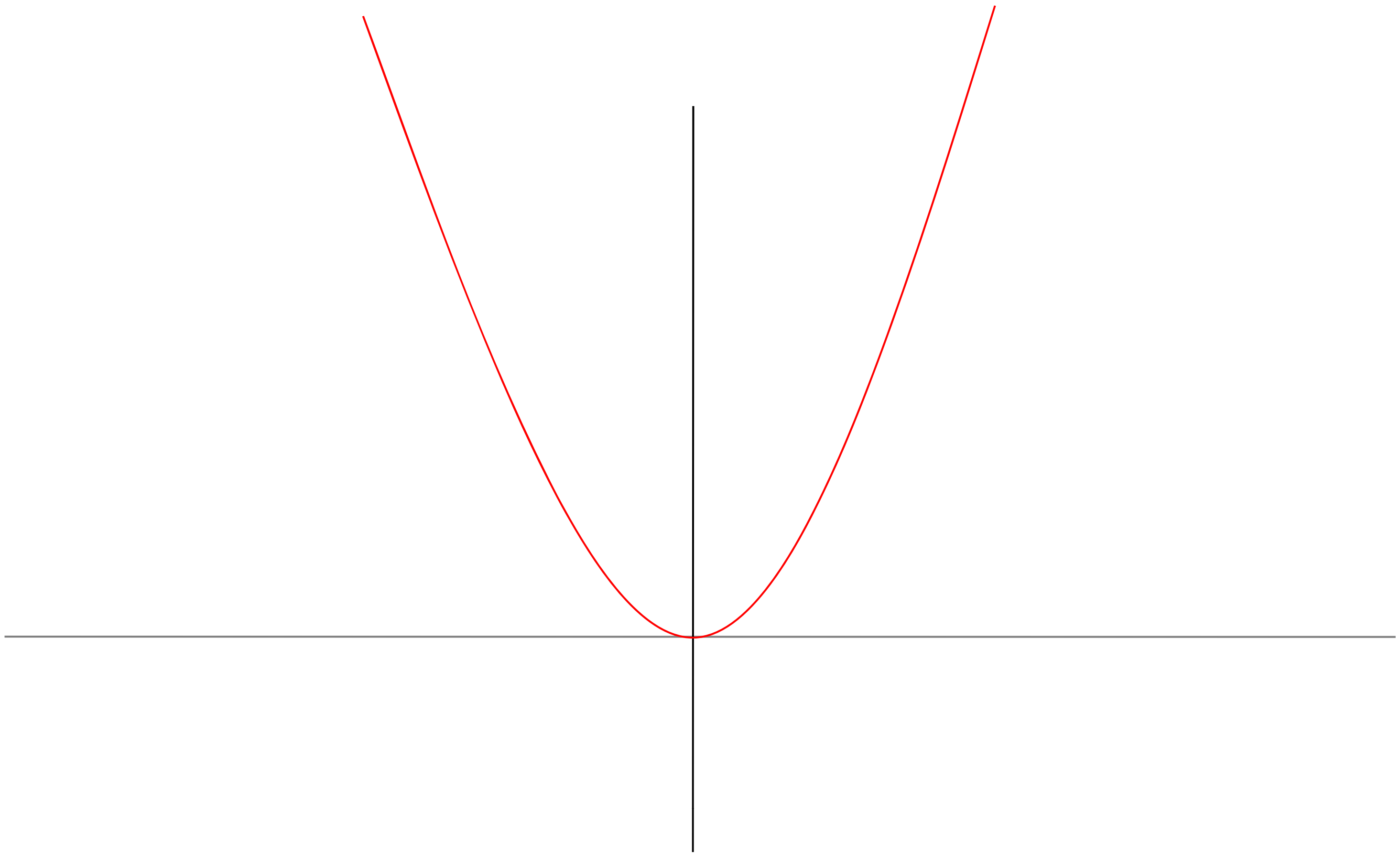
$$S_{IIA} = \cdots + \int |G_2 \equiv dC_1|^2 + \cdots = \cdots + \int_{S^4 \times R^{3+1}} \int dU d\tau U^4 |(G_2)_{U\tau}|^2 + \cdots$$

$$G_2 \simeq \frac{\theta}{U^4} dU \wedge d\tau$$ → magnetic flux

$$\mathcal{E}(\theta) = \int dU d\tau U^4 |(G_2)_{U\tau}|^2 \sim \theta^2$$

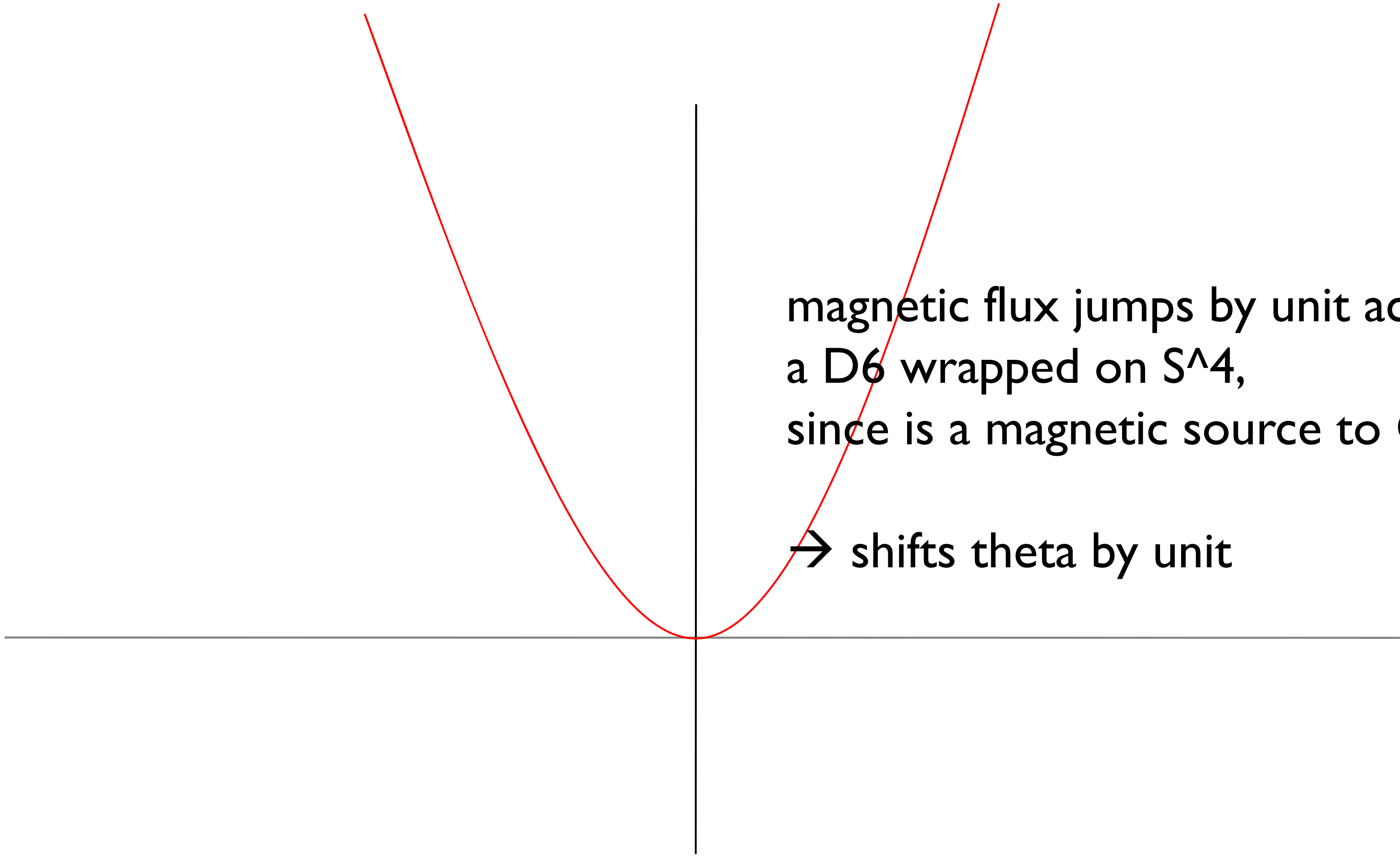
hQCD theta-angle by Witten (1998)

periodic ?



hQCD theta-angle by Witten (1998)

periodic ?

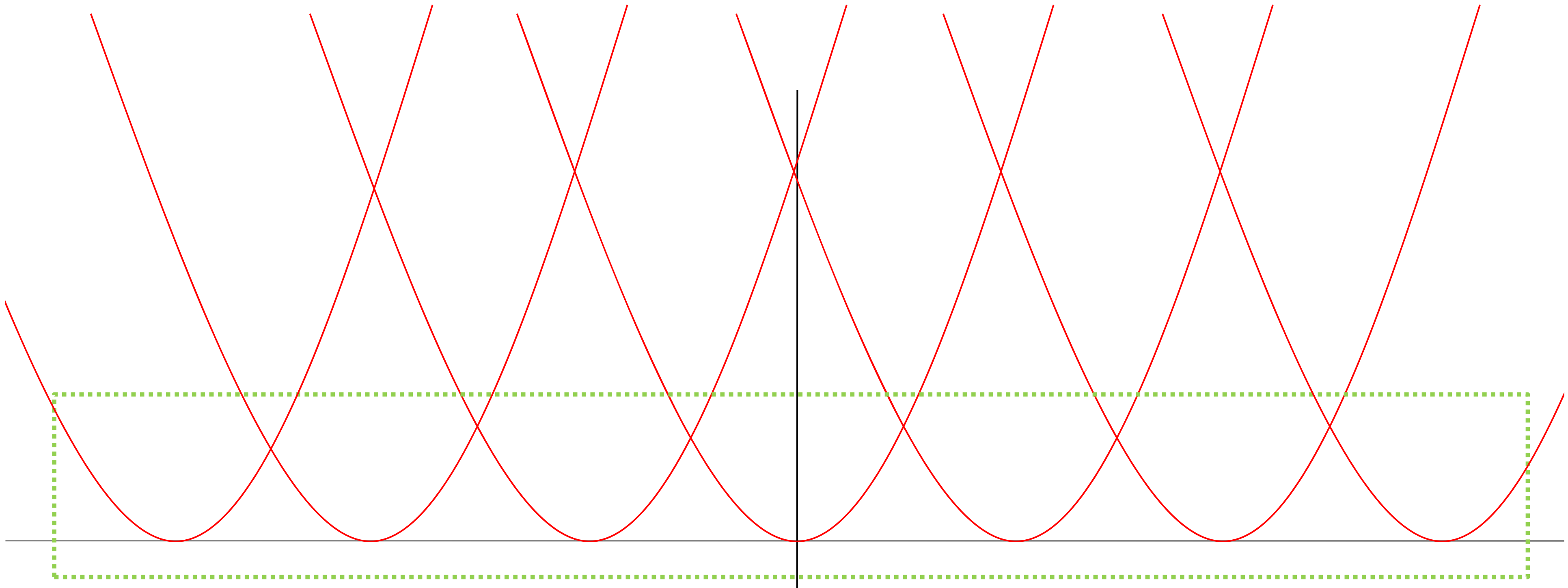


magnetic flux jumps by unit across
a D6 wrapped on S^4,
since is a magnetic source to C

→ shifts theta by unit

hQCD theta-angle by Witten (1998)

periodic ?



periodic but nonanalytic

$$\mathcal{E}(\theta) \sim \min_k (\theta - 2\pi k)^2$$

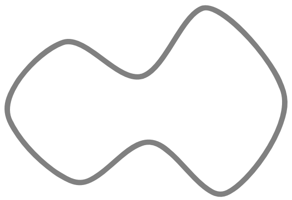
although somewhat strange, the behavior is not unfamiliar.

similar vacuum energy is known for 1+1 dimensional QED with theta angle

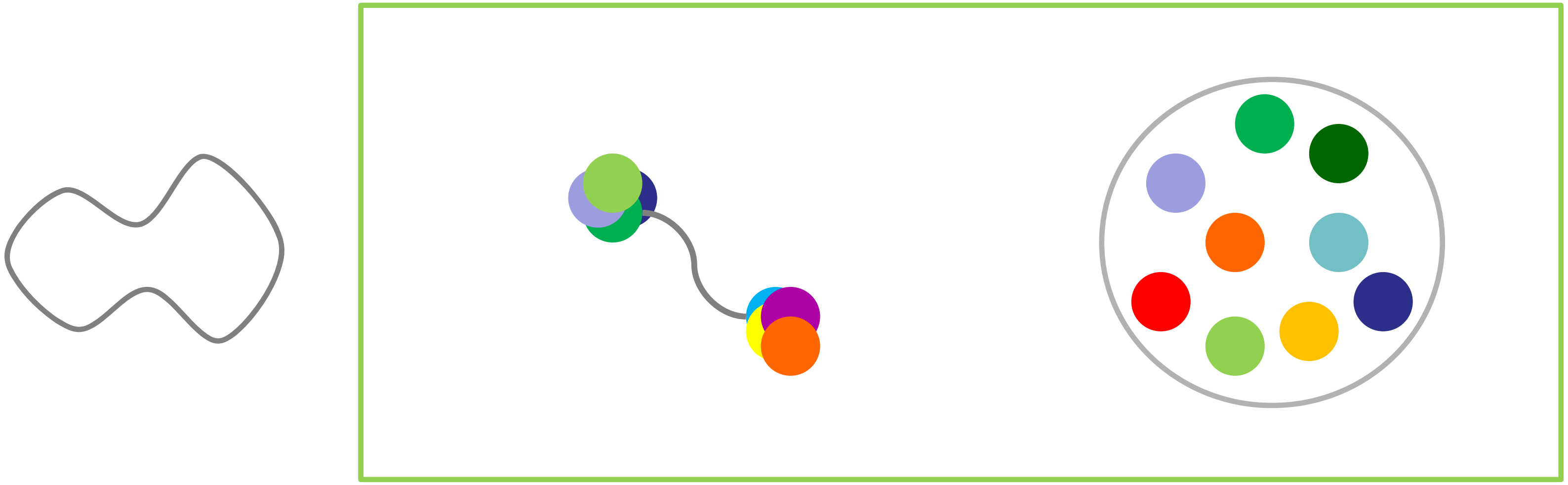
$$L = \frac{1}{2e^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} + i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi$$

$$\mathcal{E}(\theta) = \min_k \frac{e^2}{4\pi}\left(\frac{\theta}{2\pi} - k\right)^2$$

where the "analog" of D6-domain wall is
the Schwinger process of electron-positron pair creation

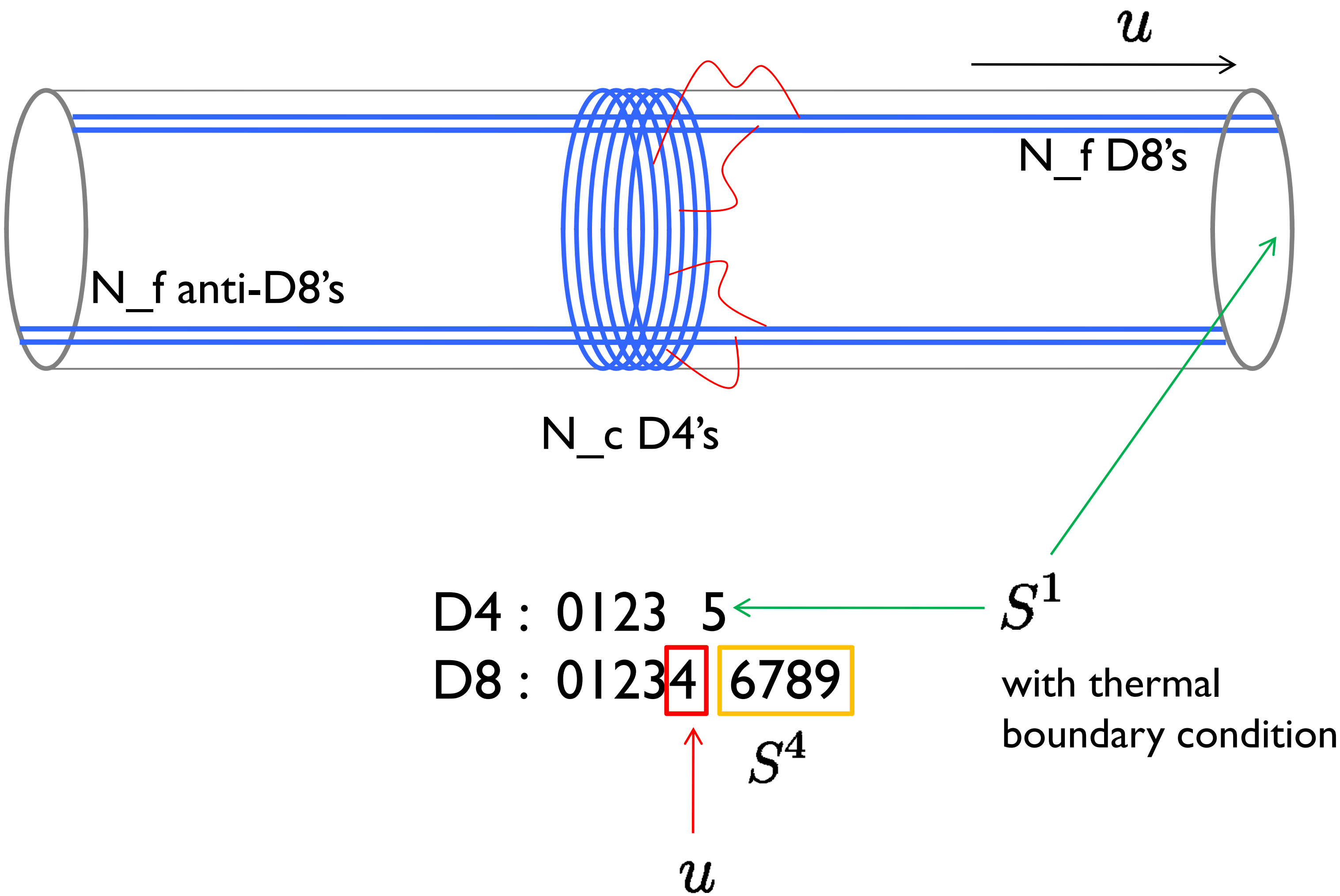


even the lightest glueballs are yet to be identified experimentally,
with the most likely candidates at 1.5 GeV and 1.7 GeV

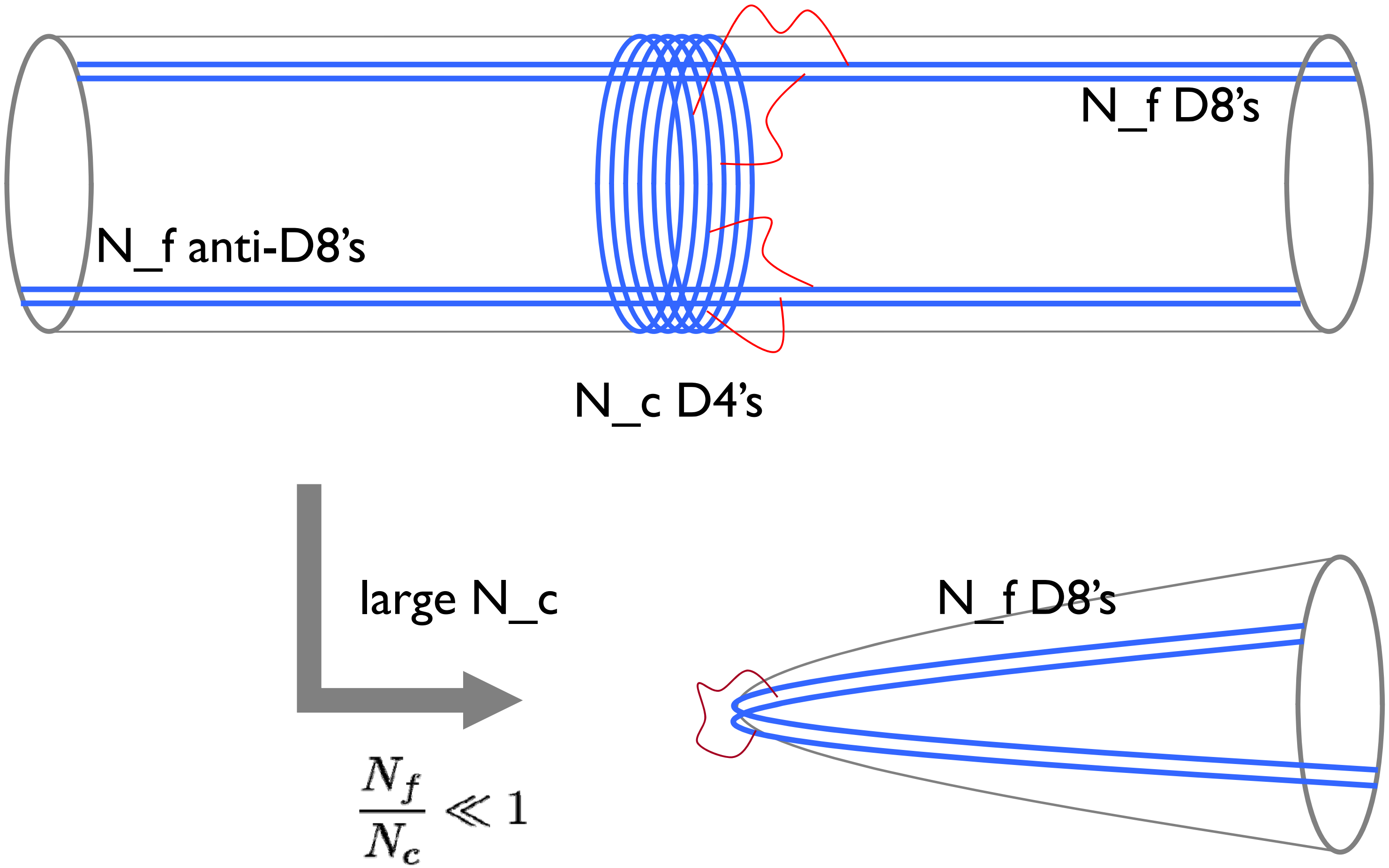


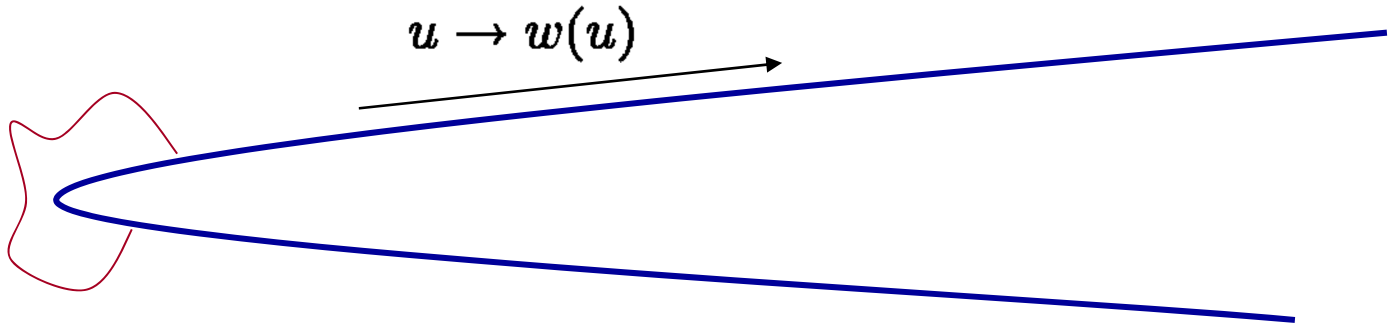
mesons and baryons would be more relevant for QCD

Adding Massless Quarks: Sakai-Sugimoto



Massless Quarks → Bi-quark Meson





Only 5D gauge field vector remain massless
due to compactification on S^4 and also due to broken SUSY.

holographic QCD mesons in a nutshell

$$\int dx^{3+1} \int dw \frac{1}{4e(w)^2} \operatorname{tr} F^{mn} F_{mn} + \frac{N_c}{24\pi^2} \int \omega_5(A)$$

↑ holographic direction

$$\frac{1}{e(w)^2} = \frac{(g_{YM}^2 N_c) N_c}{108\pi^3} \boxed{M_{KK}} \frac{u(w)}{u_0}$$

M_{KK} ~ 0.94 GeV

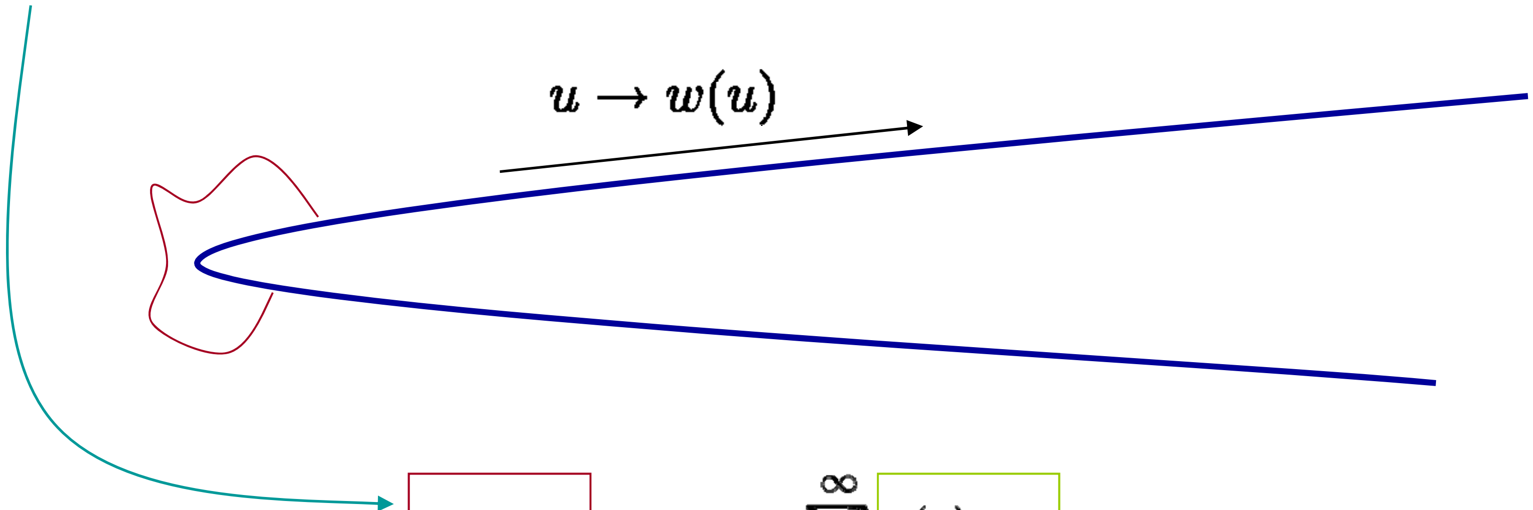
Sakai+Sugimoto, 2004

holography elevates global symmetries to local (gauge) symmetries:

4D flavor symmetry → 5D flavor gauge theory

Mesons

Pions and (pseudo-)vector mesons are contained in U(N_f) gauge field on N_f D8's



$$
\begin{aligned}
A_5(x^\mu, w) &= \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w) \\
A_\mu(x^\mu, w) &= \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)
\end{aligned}
$$

Vector and Axial Vector Mesons

Pions and (pseudo-)vector mesons are contained in U(N_f) gauge field on N_f D8's

$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

eaten up → massive vector mesons

$$(F_{\mu 5})^2 = (\sum_n (\partial_w \psi_n)(a_\mu^{(n)} - \partial_\mu \phi^{(n)}))^2 + \cdots$$

$$\int dw \ (F_{\mu 5})^2 = \sum_n m_n^2 (a_\mu^{(n)} - \partial_\mu \phi^{(n)})^2 + \cdots$$

Pions

Pions = Wilson line of U(N_f) gauge field on N_f D8's

$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

$$e^{2i\pi(x^\mu)/f_\pi} = U(x^\mu) \equiv P e^{i\int A_5(x^\mu, w)dw}$$

Pions

$$e^{2i\pi(x^\mu)/f_\pi} = U(x^\mu) \equiv P e^{i\int A_5(x^\mu,w)dw}$$

$$\int dx^{3+1} \left\{ \frac{f_\pi^2}{4} \mathrm{tr} \left(U^{-1}\partial U\right)^2 + \frac{1}{32 e_{Sk}^2} \mathrm{tr} \left[U^{-1}\partial U, U^{-1}\partial U\right]^2 \right\} + \cdots$$

$$f_\pi^2 = \frac{(g_{YM}^2 N_c) N_c}{54\pi^2} M_{KK}^2$$

$$\frac{1}{e_{Sk}^2} \simeq \frac{61 (g_{YM}^2 N_c) N_c}{54\pi^2}$$

D4-D8 Mesons

Chiral Lagrangian of Pions + Infinite Towers of Vector and Axial Vector Mesons

$$\int dx^{3+1} \left\{ \frac{f_\pi^2}{4} \mathrm{tr}\left(U^{-1}\partial U\right)^2 + \frac{1}{32 e_{Sk}^2} \mathrm{tr}\left[U^{-1}\partial U, U^{-1}\partial U\right]^2 \right\}$$

$$+ \int dx^{3+1} \left\{ \sum_n \frac{1}{4} (\partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)})^2 - \sum_n m_n^2 (a_\mu^{(n)} - \partial_\mu \phi^{(n)})^2 \right\} + \cdots$$

$$+ \int dx^{3+1} \mathcal{L}_{interactions}$$

$$+ \int dx^{3+1} \mathcal{L}_{WZW}$$

$$f_\pi \sim 93 MeV, \quad m_\rho \sim 770 MeV$$

$$\downarrow$$

$$M_{KK} \sim 0.94 GeV, \quad \lambda = g_{YM}^2 N_c \sim 17$$

D4-D8 Mesons

how about the U(1)Goldstone boson which should get massive via anomaly ?

$$\int_{D8} \sum_{k=0}^{p} C_{k+1} \wedge \mathrm{tr}\, e^{2\pi\alpha' F} = \int_{D8} \cdots + C_7 \wedge F + \cdots$$

$$d(dC_1) = d * dC_7 = \delta_{D8} \wedge \mathrm{tr} F$$

$$dC_1 \to dC_1 - \delta_{D8} \wedge \mathrm{tr} A$$

D4-D8 Mesons

how about the U(1)Goldstone boson which should get massive via anomaly ?

$$dC_1 \to dC_1 - \delta_{D8} \wedge \mathrm{tr} A$$

$$\theta \to \theta + \frac{\sqrt{2N_f}}{f_\pi}\eta'$$

$$\mathcal{E}(\theta) \sim \theta^2 \to V(\eta') \simeq m_{\eta'}^2 \eta'^2$$

$$m_\eta'^2 = \frac{1}{27\pi^2}\frac{N_f}{N_c}\lambda^2 M_{KK}^2$$

Sakai+Sugimoto, 2004

E. Witten, NPB156, 269-283 (1979)

$$m_{\eta}'^{2} \sim \frac{N_f}{N_c}$$

Baryons

=

quantized instanton solitons of the 5D flavor gauge theory

$$\int dx^{3+1} \int dw \frac{1}{4e(w)^2} \operatorname{tr} F^{mn} F_{mn} + \frac{N_c}{24\pi^2} \int \omega_5(A)$$

$$\frac{1}{e(w)^2} = \mu_8 (2\pi\alpha')^2 e^{-\Phi} V_{S^4} \left(\frac{u(w)}{R}\right)^{3/4}$$

$$\frac{1}{e(w)^2} = \frac{(g_{YM}^2 N_c) N_c}{108\pi^3} M_{KK} \frac{u(w)}{u_0}$$

$$\frac{u(w)}{u_0} \simeq 1 + \frac{1}{3} M_{KK}^2 w^2 + \cdots$$

holography elevates global symmetries to local (gauge) symmetries:

4D flavor symmetry → 5D flavor gauge theory

4D baryon number → 5D flavor U(1) charge

$$\int dx^{3+1} \int dw \frac{1}{4e(w)^2} \operatorname{tr} F^{mn} F_{mn} + \frac{N_c}{24\pi^2} \boxed{\int \omega_5(A)}$$

Chern-Simons terms

$$\sim \mathbf{3} \, \mathbf{tr} \, \boldsymbol{A} \wedge \boldsymbol{F} \wedge \boldsymbol{F} + \cdots$$

$$\sim 3A^{U(1)} \wedge \operatorname{tr} F \wedge F + \cdots$$

$$\int dx^{3+1} \int dw \frac{1}{4e(w)^2} \, \mathrm{tr}\, F^{mn} F_{mn} + \frac{N_c}{24\pi^2} \boxed{\int \omega_5(A)}$$

Chern-Simons terms

$$\sim 3\, \mathrm{tr}\, A \wedge F \wedge F + \cdots$$

$$\sim 3 A^{U(1)} \wedge \mathrm{tr}\, F \wedge F + \cdots$$

$$\int_{I \times R^3} tr(F \wedge F)/8\pi^2 = 1$$ ← → **unit baryon number**

↓

baryon = instanton soliton

+ U(1) charge N_c

Baryons: Classical Size

Using the ordinary instanton as trial configurations

$$A_m^a = \bar{\eta}_{mk}^a \partial_k \log\left(1 + \rho^2/(\vec{x}^2 + w^2)\right)$$

Energy can be estimated for small size limit as

$$E(\rho) = \frac{(g_{YM}^2 N_c) N_c}{27\pi} M_{KK} \times \left(1 + \frac{1}{6} M_{KK}^2 \rho^2 + \cdots\right) + \frac{e(0)^2 N_c^2}{20\pi^2 \rho^2} + \cdots$$

Extra F^2 energy due to increasing 1/e(w)^2

Coulomb energy due to the baryon #
which is now a gauge charge

Baryons: Properties

Minimization gives a definite soliton size of the holographic baryon

$$\rho_{\rm baryon} \simeq \frac{(2\cdot 3^7 \cdot \pi^2/5)^{1/4}}{M_{KK}\sqrt{g_{YM}^2 N_c}} \simeq \frac{9.6}{M_{KK}\sqrt{g_{YM}^2 N_c}}$$

Hong, Rho, Yee, Yi, hep-th/0701276
Hata, Sakai, Sugimoto, Yamato, hep-th/0701280

Baryons: Properties

quantization of such a soliton,
which is a long story of its own dating back to
Finkelstein et.al. in the 60's,
generates quantum particles of the following representations under
$SU(N\_f=2)_{\text{flavor}} \times SO(4)_{\text{little group}} = SU(N\_f=2) \times SU(2)_+ \times SU(2)_-$

$$[s] \otimes [s] \otimes [0]$$

$$[s] \otimes [0] \otimes [s]$$

with half-integral spin = isospin s.

the lowest lying states with s equal to ½,
5D analog of protons and neutrons,
can be packaged into a single Isospin ½ Dirac field.

Baryons: Properties

with small soliton size, we may introduce an effective (Dirac) field for the (isospin ½) baryon and try to incorporate the property of the latter into an effective action.

$$N_c \gg 1, \;\; g_{YM}^2 N_c \gg 1$$

Compton Size of Baryons << Soliton Size << Compton Size of Mesons

→ The baryon can be treated point-like for interaction with mesons, yet, the classical properties of the soliton make sense.

how do these baryons interact with rest of QCD ?

after a long song and dance…….

theory of Isospin/Spin ½ 5D Baryons interacting with D4-D8 mesons

Hong, Rho, Yee, P.Y., 2007

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(2_f)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(2_f)}\mathcal{B} \right]$$

kinetic term — mass term — a magnetic coupling term

$$g_5(0) = \frac{2\pi^2}{3}$$

cf) Hashimoto, Sakai, Sugimoto 2008

theory of Isospin/Spin ½ 5D Baryons interacting with D4-D8 mesons

Hong, Rho, Yee, P.Y., 2007

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(2_f)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(2_f)}\mathcal{B} \right]$$

$U(1)$ part

leading in $1/\lambda$

leading in $1/N_c$

For an illustrative purpose, consider a magnetic monopole soliton which by definition carries a magnetic charge, meaning that the Soliton has a long range tail of appropriate magnetic Coulomb field. If we want to introduce a local field theory of the monopole and the dual gauge field, we would have the kinetic terms like

$$\int \tilde{F}^2 - \int M^*(\partial - \tilde{A})^2 M \qquad\qquad d\tilde{A} = *\tilde{F}$$

with the minimal coupling to the dual photon gauge field.

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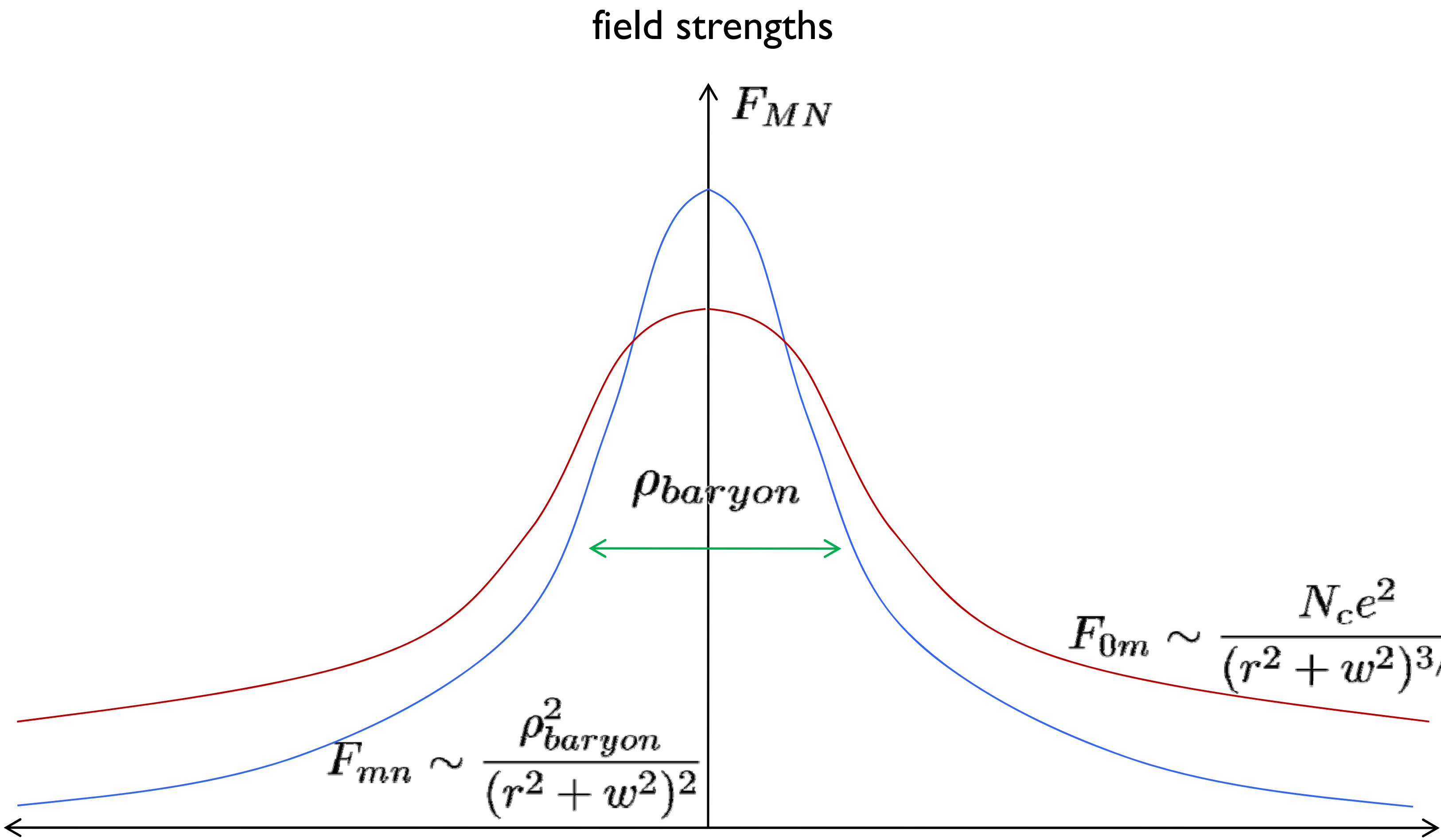
with the minimal coupling to the dual photon gauge field.

This obvious fact can be understood from that each quanta of M must be accompanied by a long range $\tilde{A}$ Coulomb field. In the effective field theory, the latter follows from EOM, whose right hand side is generated by the minimal coupling,

$$\nabla \tilde{F} = (M^*\partial M - \partial M^* M)$$

Recall the Shape of the Solitonic Baryon



The our soliton actually carries two types of long range gauge fields:
Electric Coulomb Field and Magnetic Self-Dual Instanton Field

$$F_{om} \sim \frac{e(0)^2}{(r^2 + w^2)^{3/2}}$$

$$F_{mk} \sim \frac{\rho^2}{(r^2 + w^2)^2}$$

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$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N_f)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \cdots \right]$$

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Adkins-Nappi-Witten Procedure for Instantons

how was the magnetic coupling to the nucleon derived ?
= how to adapt Adkins-Nappi-Witten (1982) for instanton soliton ?

$$\langle A_m \rangle = \langle\langle S^\dagger A_m S \rangle\rangle = \sum_a A_m^a \langle\langle S^\dagger \frac{\tau^a}{2} S \rangle\rangle = \sum_{a,b} A_m^a \langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle \frac{\tau^b}{2}$$

$$\bar{\eta}_{mn}^a \partial_n (\cdots)$$

quantum expectation value
of the instanton gauge field

$$\sum_{a,b} \partial_n (\cdots) \eta_{mn}^a \langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle \frac{\tau^b}{2}$$

can an excitation of a dirac field
emulate such a source ?

$$\langle A_m \rangle = \langle\langle S^\dagger A_m S \rangle\rangle = \sum_a A_m^a \langle\langle S^\dagger \frac{\tau^a}{2} S \rangle\rangle = \sum_{a,b} A_m^a \langle\langle \mathrm{tr} \left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle \frac{\tau^b}{2}$$

quantum expectation value
of the instanton gauge field

$$\langle\langle \mathrm{tr} \left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle = -\frac{1}{3} U_\alpha^{*p} \sigma_{pq}^b \tau_a^{\alpha\beta} \boxed{U_\beta^q}$$

spin-isospin ½ fermion field
on-shell two-component part of $\mathcal{B}$

$$\langle A_m \rangle = \langle\langle S^\dagger A_m S \rangle\rangle = \sum_a A_m^a \langle\langle S^\dagger \frac{\tau^a}{2} S \rangle\rangle = \sum_{a,b} A_m^a \langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle \frac{\tau^b}{2}$$

$$\langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b \right] \rangle\rangle = -\frac{1}{3} U_\alpha^{*p} \sigma_{pq}^b \tau_a^{\alpha\beta} U_\beta^q$$

shape of source term that can generate this type of solution

$$\partial_m A_n^a \bar{\eta}_{mn}^a U_\alpha^{*p} \sigma_{pq}^b \tau_a^{\alpha\beta} U_\beta^q$$

relativistic, gauge-invariant completion thereof

$$\bar{\mathcal{B}} \gamma^{mn} F_{mn} \mathcal{B}$$

Excited Baryons

what is the analog of this for baryons of arbitrary isospin ?

answer:

Excited Baryons

what is the analog of this for baryons of arbitrary isospin ?

answer:

$$ {}_s\langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b\right]\rangle\rangle_s = -\frac{1}{s(s+1)} U(s)^\dagger J_b I_a U(s) $$

$$ {}_s\langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S \tau_b\right]\rangle\rangle_{s+1} = -\frac{1}{2}\sqrt{\frac{2s+1}{2s+3}}\, U(s)^\dagger U(s+1)_{ba} $$

Excited Baryons

what is the analog of this for baryons of arbitrary isospin ?

answer:

$$_s\langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau_b\right]\rangle\rangle_s = -\frac{1}{s(s+1)} U(s)^\dagger J_b I_a U(s)$$

$$_s\langle\langle \mathrm{tr}\left[S^\dagger \frac{\tau^a}{2} S\tau_b\right]\rangle\rangle_{s+1} = -\frac{1}{2}\sqrt{\frac{2s+1}{2s+3}}\, U(s)^\dagger U(s+1)_{ba}$$

$\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}$ replaced by "$\bar{\mathcal{B}}_s\gamma\cdot F\mathcal{B}_s$" and "$\bar{\mathcal{B}}_s\langle\gamma\cdot F,\mathcal{B}_{s+1}\rangle$"

J. Park and P.Y. , 2008

Grygoryan+Lee+Yee 2009
for relativistic form for I=3/2

Nucleon-Meson Dynamics

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(2_f)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(2_f)}\mathcal{B} \right]$$

kinetic term mass term a magnetic coupling term

$$g_5(0) = \frac{2\pi^2}{3}$$

The above 5D effective action with **only one non-canonical term** is capable of reproducing all the interaction between Nucleons and the entire tower of pions and (pseudo-)vector mesons, including some subleading corrections in 1/N_c expansion. Furthermore, this effective action dictates all electromagnetic interaction, with a vector-dominance.

Nucleon-Meson Dynamics

KK reduction along the fifth direction

$$\mathcal{B}(x,w) = \begin{pmatrix} B_+(x) f_+(w) \\ B_-(x) f_-(w) \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\pm \partial_w f_\pm(w) + m_{\mathcal{B}}(w) f_\pm(w) = m_{\mathcal{N}} f_\mp(w)$$

take the smallest eigenvalue M_B ⟶ 4D nucleon mass

$$\mathcal{N}(x) \equiv \begin{pmatrix} B_+(x) \\ B_-(x) \end{pmatrix}$$ ⟶ 4D Dirac field for nucleons

Recall that pions and (axial-)vector mesons
are contained in U(N_f) gauge field on N_f D8's

$$
\begin{aligned}
A_5(x^\mu, w) &= \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w) \\
A_\mu(x^\mu, w) &= \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)
\end{aligned}
$$

Nucleon-Meson Dynamics

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B} \right]$$

$$\mathcal{B}(x,w) = \begin{pmatrix} B_+(x)f_+(w) \\ B_-(x)f_-(w) \end{pmatrix}$$

$$\mathcal{N}(x) \equiv \begin{pmatrix} B_+(x) \\ B_-(x) \end{pmatrix}$$

A_5

A_μ

$F_{5\mu}$

$F_{\mu\nu}$

$$\int dx^{3+1} \left[-i\bar{\mathcal{N}}\gamma^\mu\partial_\mu\mathcal{N} - im_{\mathcal{N}}\bar{\mathcal{N}}\mathcal{N} \right] + \cdots$$

Nucleon-Meson Dynamics

$$\int dx^{3+1}\ \left[-i\bar{\mathcal{N}}\gamma^{\mu}\partial_{\mu}\mathcal{N} - i m_{\mathcal{N}}\bar{\mathcal{N}}\mathcal{N}\right] + \cdots$$

meson-nucleon-nucleon or

$$\sim \int dw\ f_{+}(w)^{*}\psi_{n}(w)f_{\pm}(w)$$

meson-meson-nucleon-nucleon

$$\sim \int dw\ f_{+}(w)^{*}\psi_{n}(w)\psi_{m}(w)f_{\pm}(w)$$

for example, cubic terms:

$$\int dw\ \frac{g_5\rho^2}{e^2} f_{+}(w)^{*}\psi_{n}(w)f_{\pm}(w)$$

$$\int dw\ f_{+}(w)^{*}\psi_{n}(w)f_{\pm}(w)$$

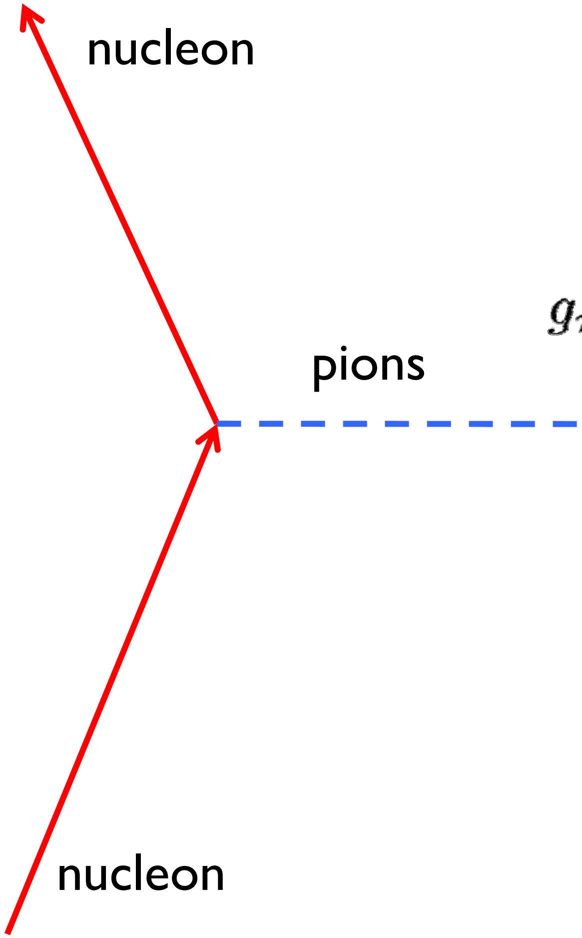
↔

$$g_{\pi\mathcal{N}\mathcal{N}}\ \bar{\mathcal{N}}\pi\gamma^5\mathcal{N}$$

$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}\rho^{(k)}_{\mu}\gamma^{\mu}\mathcal{N}$$

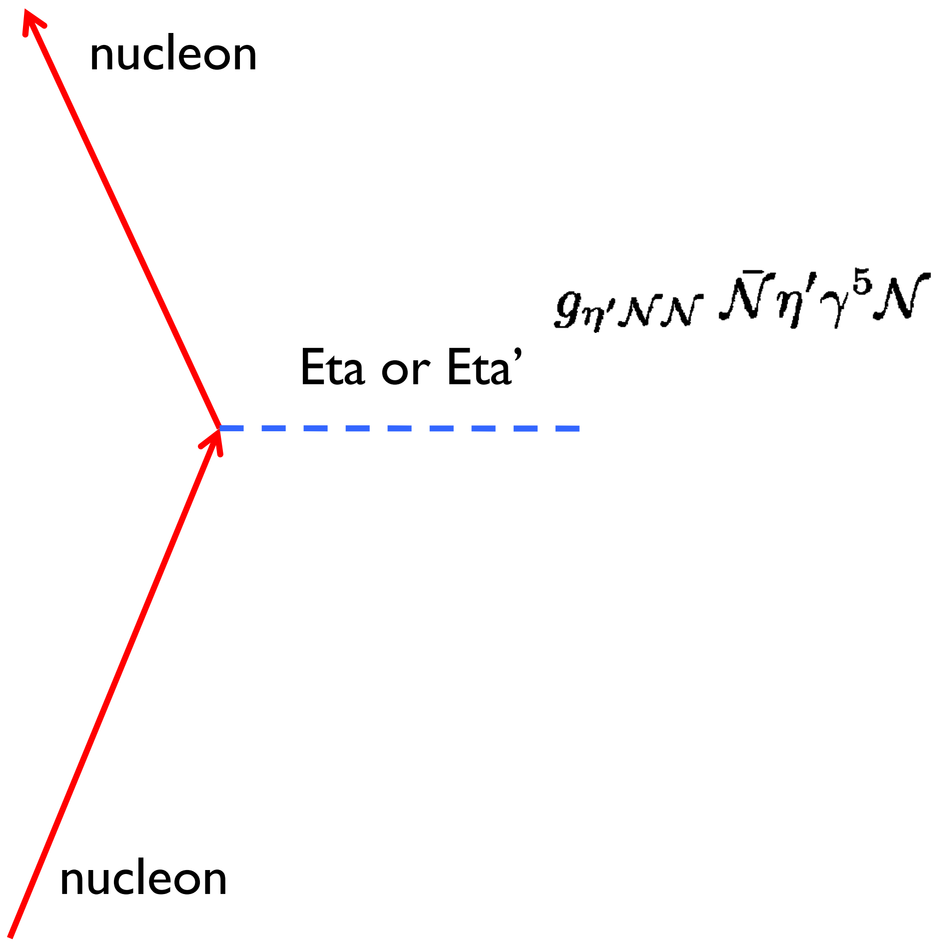
$$\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_{\nu}\rho^{(k)}_{\mu}\gamma^{\nu\mu}\mathcal{N}$$

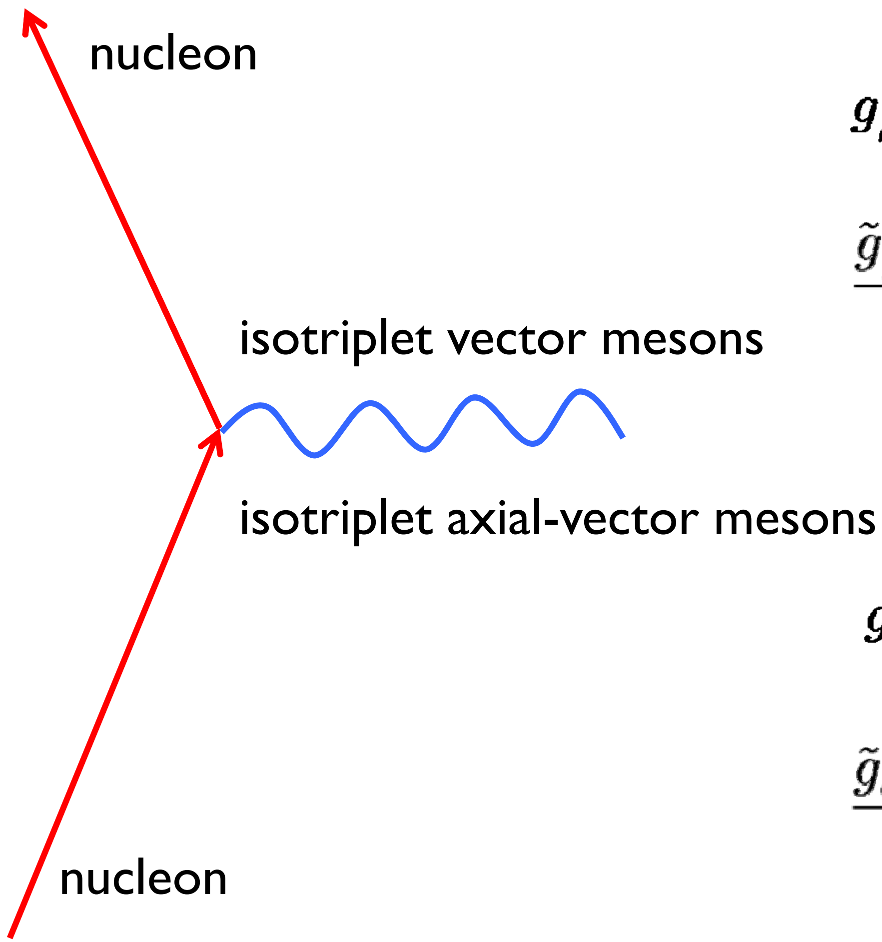
$$g_{a^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}a^{(k)}_{\mu}\gamma^{\mu}\gamma^5\mathcal{N}$$



$$g_{\pi\mathcal{N}\mathcal{N}}\,\bar{\mathcal{N}}\pi\gamma^5\mathcal{N} = \frac{g_A}{2f_\pi}\,\bar{\mathcal{N}}\partial_\mu\pi\gamma^\mu\gamma^5\pi$$

$$g_{\pi\mathcal{N}\mathcal{N}} = \frac{g_A m_{\mathcal{N}}}{f_\pi}$$


nucleon
$g_{\eta' \mathcal{N}\mathcal{N}} \, \bar{\mathcal{N}} \eta' \gamma^5 \mathcal{N}$
Eta or Eta'
nucleon

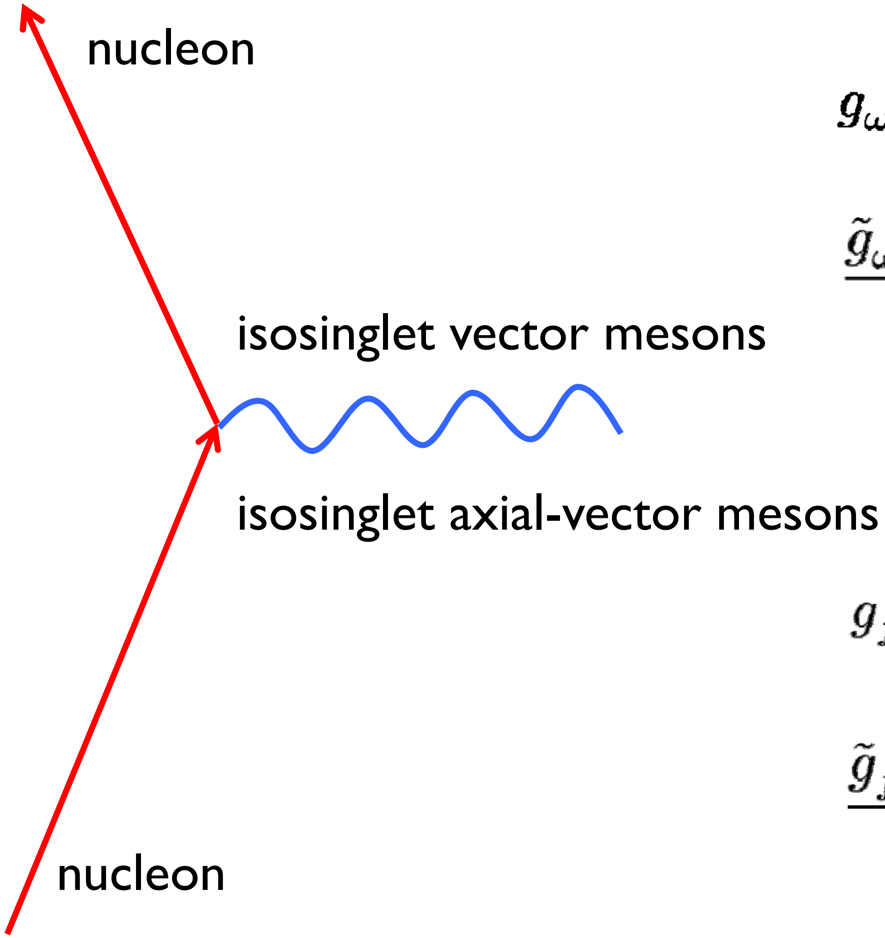


$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}\rho^{(k)}_{\mu}\gamma^{\mu}\mathcal{N}$$

$$\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_{\nu}\rho^{(k)}_{\mu}\gamma^{\nu\mu}\mathcal{N}$$

$$g_{a^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}a^{(k)}_{\mu}\gamma^{\mu}\gamma^{5}\mathcal{N}$$

$$\frac{\tilde{g}_{a^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_{\nu}a^{(k)}_{\mu}\gamma^{\nu\mu}\gamma^{5}\mathcal{N}$$



$$g_{\omega^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}\omega_\mu^{(k)}\gamma^\mu\mathcal{N}$$

$$\frac{\tilde{g}_{\omega^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_\nu\omega_\mu^{(k)}\gamma^{\nu\mu}\mathcal{N}$$

$$g_{f^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}f_\mu^{(k)}\gamma^\mu\gamma^5\mathcal{N}$$

$$\frac{\tilde{g}_{f^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_\nu f_\mu^{(k)}\gamma^{\nu\mu}\gamma^5\mathcal{N}$$

leading estimates of couplings, in the large N_c limit

Hong, Rho, Yee, P.Y., 2007
Kim, Lee, P.Y. 2009

$$\frac{g_{\pi NN}}{2m_N} M_{KK} \simeq \simeq \frac{2\cdot 3\cdot \pi}{\sqrt{5}} \times \sqrt{\frac{N_c}{\lambda}},$$

$$\frac{g_{\eta' NN}}{2m_N} M_{KK} \simeq \sqrt{\frac{3^9}{2}\pi^2} \times \frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}},$$

$$g_{\rho^{(k)} NN} \simeq \sqrt{2\cdot 3^3\cdot \pi^3}\, \hat{\psi}_{(2k-1)}(0) \times \frac{1}{N_c}\sqrt{\frac{N_c}{\lambda}},$$

$$g_{\omega^{(k)} NN} \simeq \sqrt{2\cdot 3^3\cdot \pi^3}\, \hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}},$$

$$\frac{\tilde{g}_{\rho^{(k)} NN}}{2m_N} M_{KK} \simeq \sqrt{\frac{2^2\cdot 3^2\cdot \pi^3}{5}}\, \hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}},$$

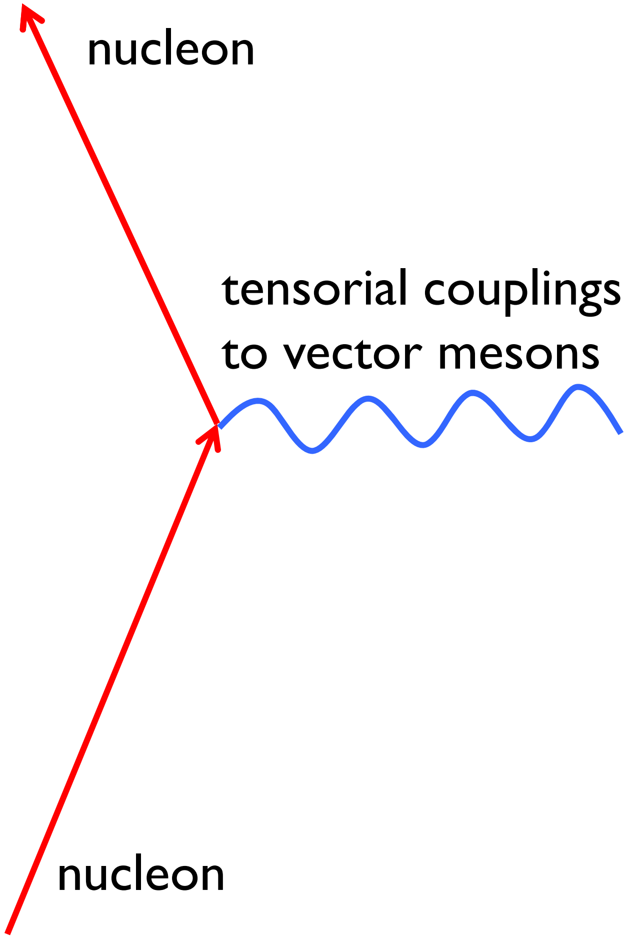
$$g_{a^{(k)} NN} \simeq \sqrt{\frac{2^2\cdot 3^2\cdot \pi^3}{5}}\, \hat{\psi}_{(2k)}{}'(0) \times \sqrt{\frac{N_c}{\lambda}} \times \epsilon(\lambda),$$

$$g_{f^{(k)} NN} \simeq \sqrt{\frac{3^9\cdot \pi^5}{2}}\, \hat{\psi}_{(2k)}{}'(0) \times \frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}} \times \epsilon(\lambda)$$

$$\epsilon(\lambda) \equiv 1 - \frac{\sqrt{2\cdot 3^5\cdot \pi^2/5}}{\lambda} + O(\lambda^{-2})$$

Predictions / Postdictions

Kim, Lee, P.Y. 2009



$$\frac{\tilde{g}_{\rho NN}}{g_{\rho NN}} \simeq 6$$

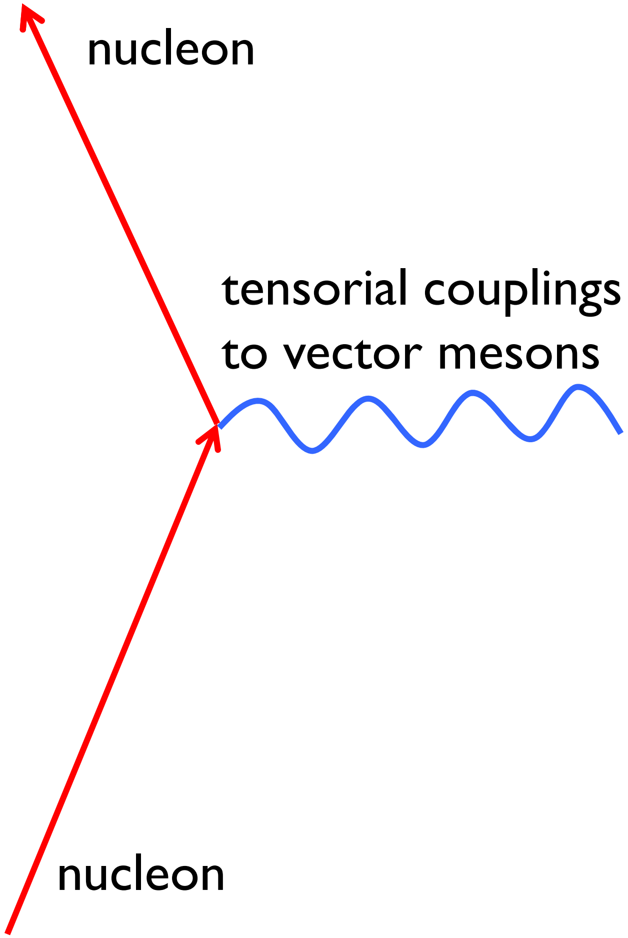
$$\tilde{g}_{\omega NN} = 0$$

$$\tilde{g}_{a NN} = 0$$

$$\tilde{g}_{f NN} = 0$$

Predictions / Postdictions

Kim, Lee, P.Y. 2009



$$\frac{\tilde{g}_{\rho NN}}{g_{\rho NN}} \simeq 6$$

$$\tilde{g}_{\omega NN} = 0$$

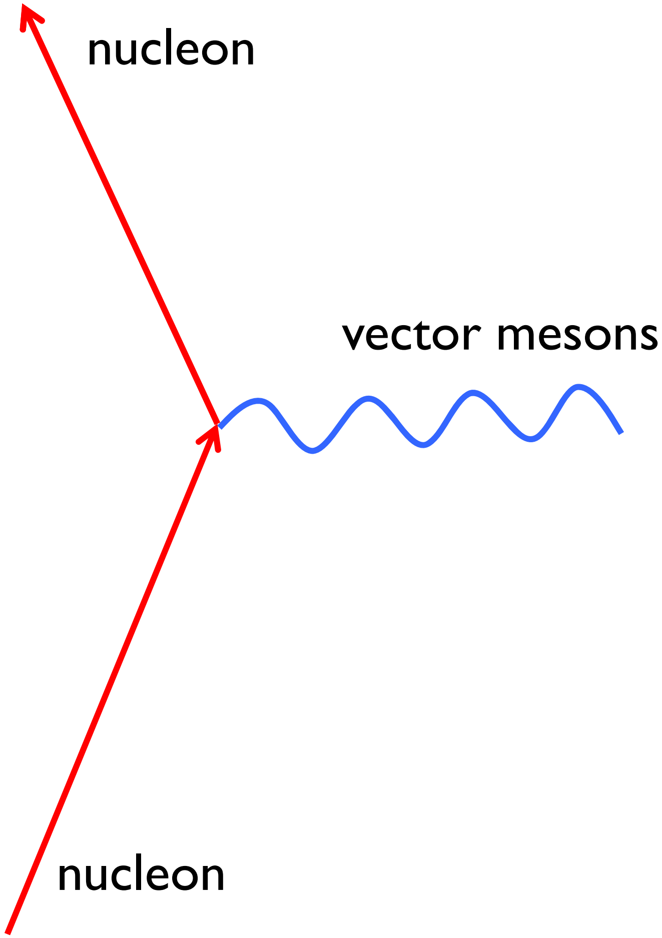
match NN-scattering experiments quite precisely !!

$$\tilde{g}_{a NN} = 0$$

$$\tilde{g}_{f NN} = 0$$

Predictions / Postdictions

Hong, Rho, Yee, P.Y., 2007



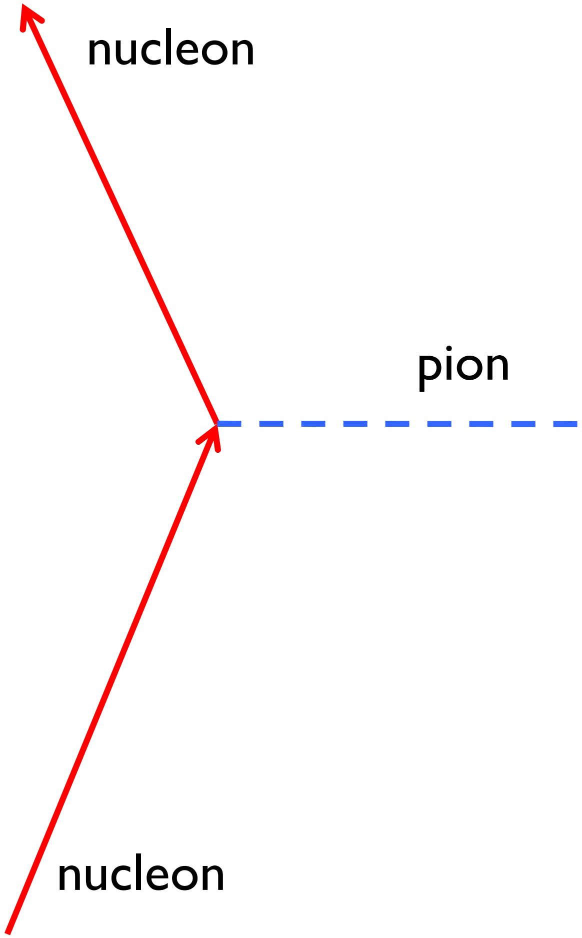
from the kinetic term

$$\frac{g_{\omega NN}}{g_{\rho NN}} \simeq N_c + \delta \sim \mathbf{3.6}$$

from the magnetic term

$$\left(\frac{g_{\omega NN}}{g_{\rho NN}}\right)_{exp} \sim 4$$

Predictions / Postdictions



$$g_A = \int dw \, \frac{2 g_5(w) \rho^2}{e^2(w)} f_+(w)^* \partial_w \psi_0(w) f_+(w) + \cdots$$

$$\simeq \frac{2 g_5(w) \rho^2}{e^2(w)} \partial_w \psi_0(w) \Big|_{w=0} + O(N_c^0)$$

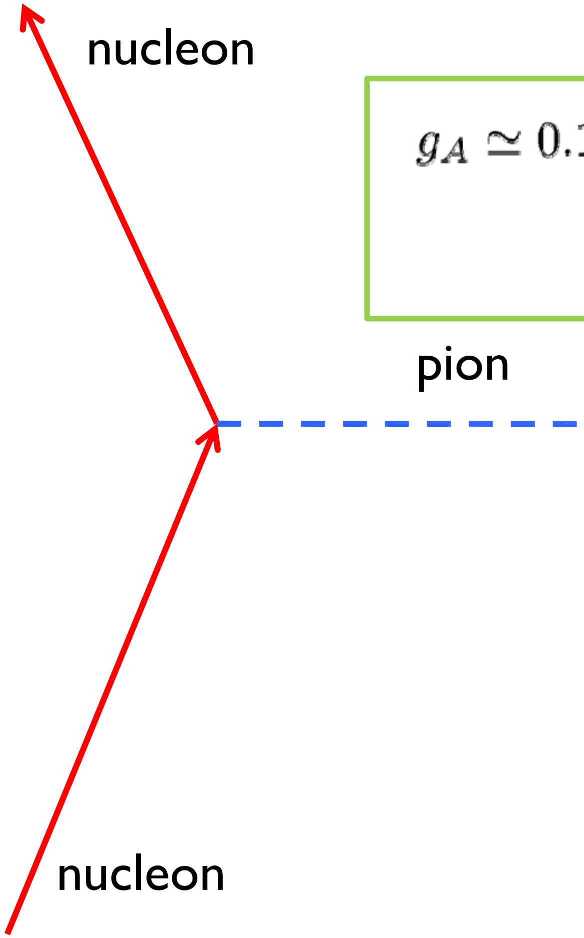
$$= 0.18 N_c \times (4/\pi) + O(N_c^0)$$

$$\rightarrow 0.18(N_c + 2) \times (4/\pi) + O(N_c^{-1})$$

suggested by constituent quark models and gives an excellent agreement with data $(g_A)_{exp} \simeq 1.27$ for N_c=3

Predictions / Postdictions

Hong, Rho, Yee, P.Y., 2007



$$g_A \simeq 0.18(N_c + 2) \times (4/\pi) + O(N_c^{-1}) \simeq 1.16 + 0.14$$

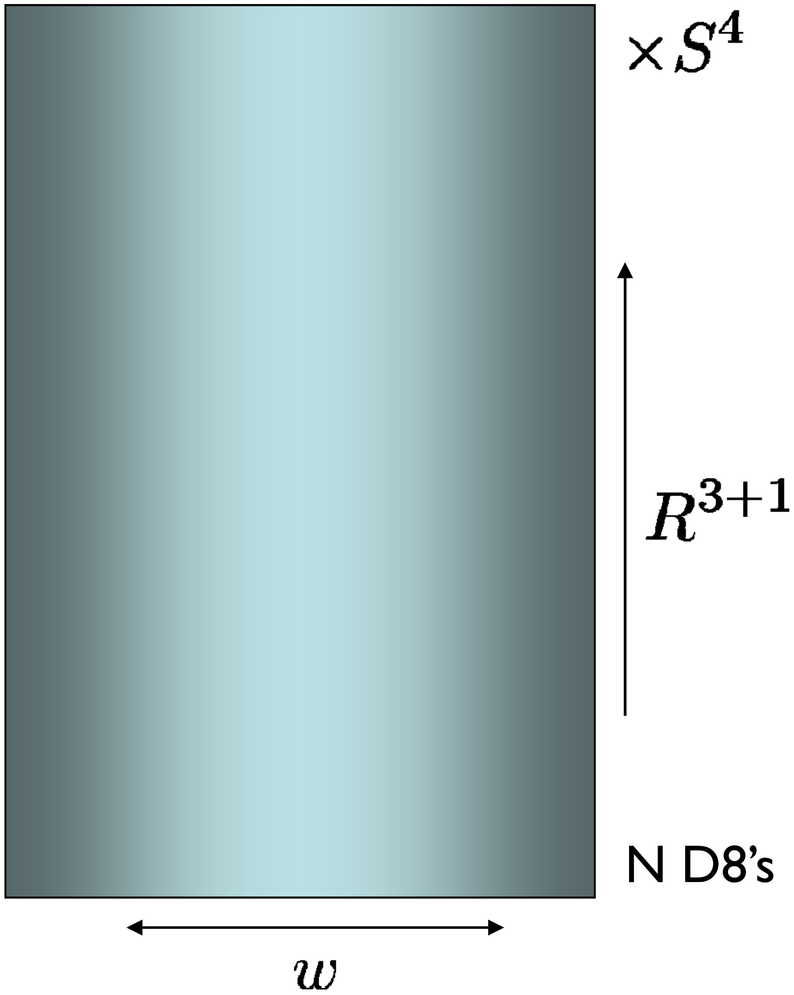
$$(g_A)_{exp} \simeq 1.27$$

N.B.
the leading N_c coefficient
$0.18 \times (4/\pi)$

is about 25% larger than
that of the Skyrmion picture
a la Adkins-Nappi-Witten

nuclei ?

a holographic deuteron = two instanton solitons ?



a holographic deuteron = two instanton solitons ?

$$S \simeq \int dx^{3+1} dw \frac{1}{4e(0)^2} \left(1 + \frac{1}{3} w^2 M_{KK}^2 + \cdots \right) \text{tr}\, F^{mn} F_{mn} + \frac{N_c}{24\pi^2} \int \omega_5(A)$$

$$\sim \frac{N_c}{8\pi^2} \text{tr}\, A \wedge F \wedge F + \cdots$$

zeroth order: ordinary R^4 instantons with aribtrary size and positions

first order: instantons of fixed size with N_c electric charges (=unit baryon#) each

a holographic deuteron = two instanton solitons ?

the leading interation between two such solitons with small distance
is the five-dimenional Coulomb repulsion between the two

(the 5D electric coupling constant)^2 is $e(0)^2 \simeq 1/\lambda N_c M_{KK}$

$$V(r) \sim e(0)^2 N_c^2 \frac{1}{r^2} \sim \frac{N_c}{\lambda} \frac{1}{M_{KK} r^2} > 0$$

see Hashimoto, Sakai, and Sugimoto, Jan. 2009
for complete detail on this short distance behavior

cf) Kim+Zahed 2009

a holographic deuteron = two instanton solitons ?

the leading interation between two such solitons with small distance is the five-dimenional coulomb repulsion between the two

(the 5D electric coupling constant)^2 is $e(0)^2 \simeq 1/\lambda N_c M_{KK}$

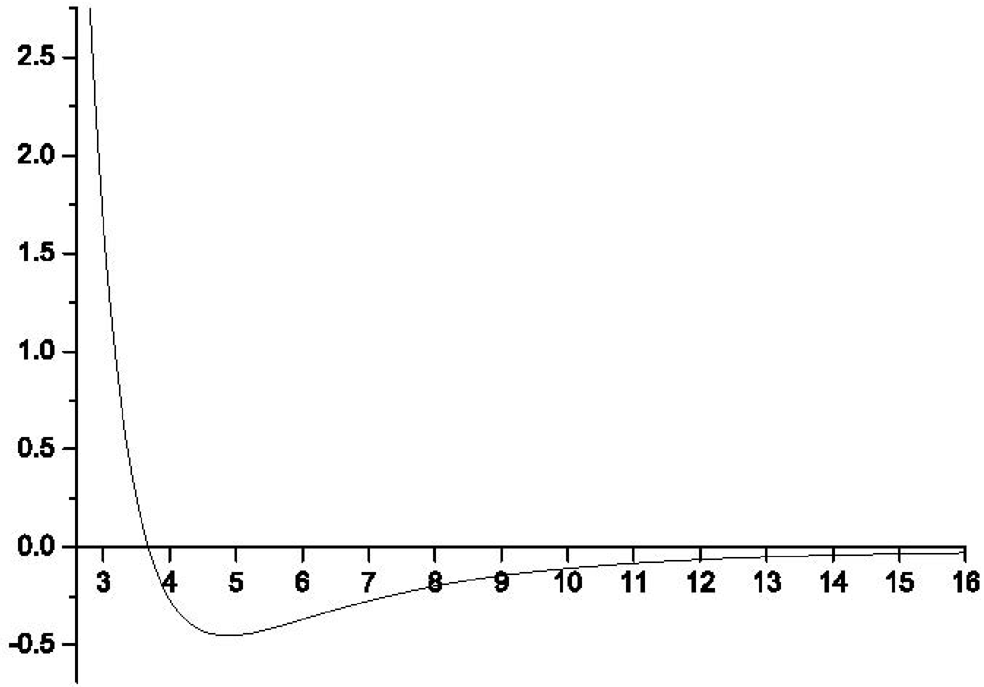
$$V(r) \sim e(0)^2 N_c^2 \frac{1}{r^2} \sim \frac{N_c}{\lambda} \frac{1}{M_{KK} r^2} > 0$$

which is unversally repulsive (with some isospin/spin-dependence)

→

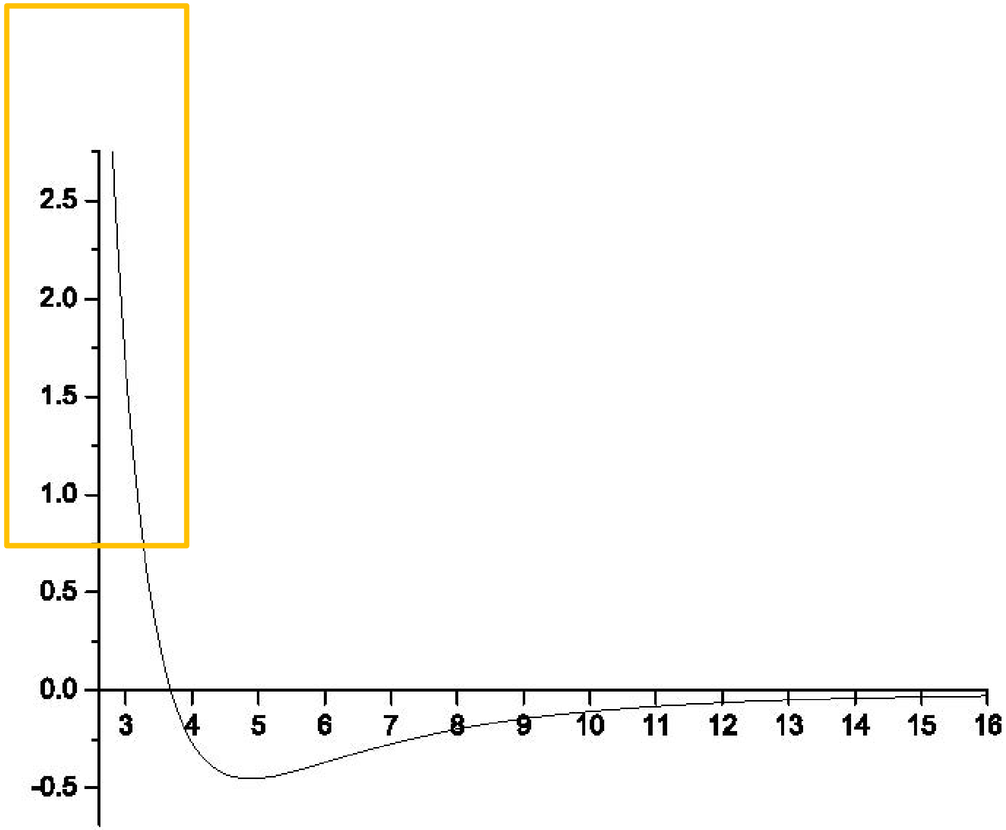
no deuteron where flat R^4 instanton picture is reliable

typical NN-potential

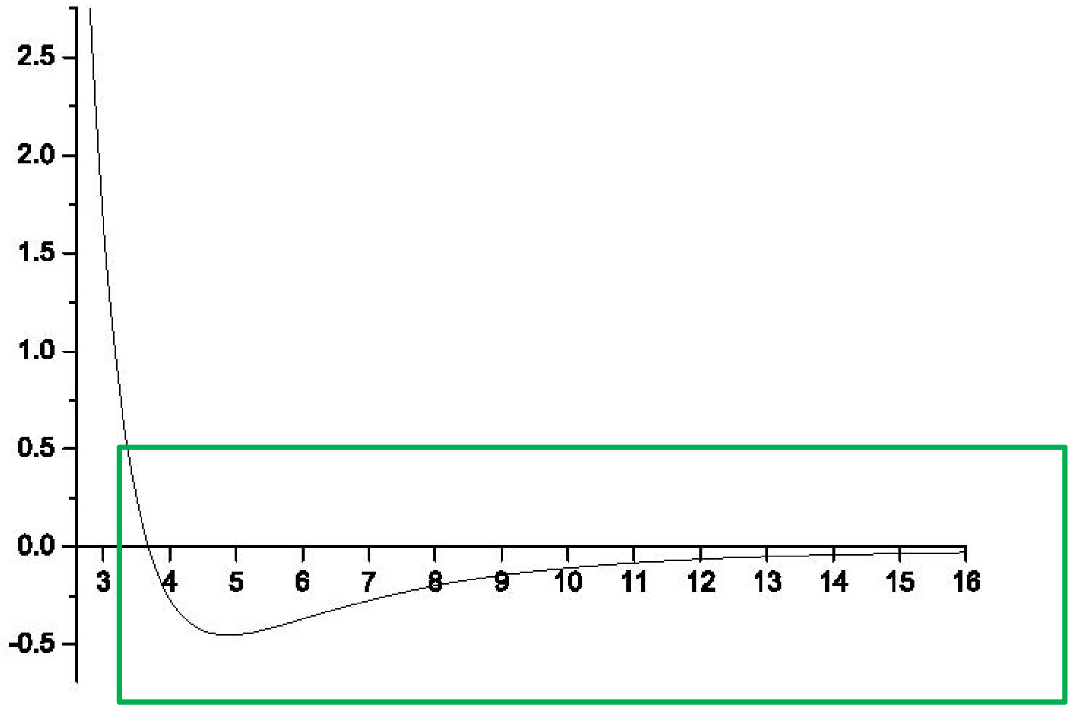


typical NN-potential

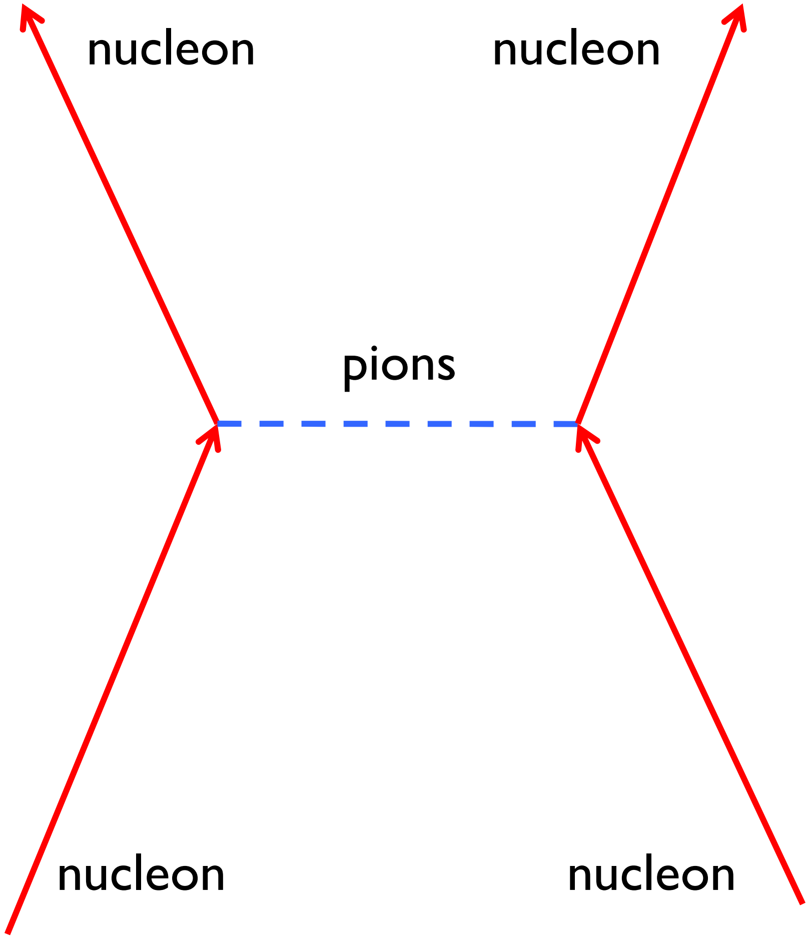
instantons +
electric charge
in nearly flat R^4



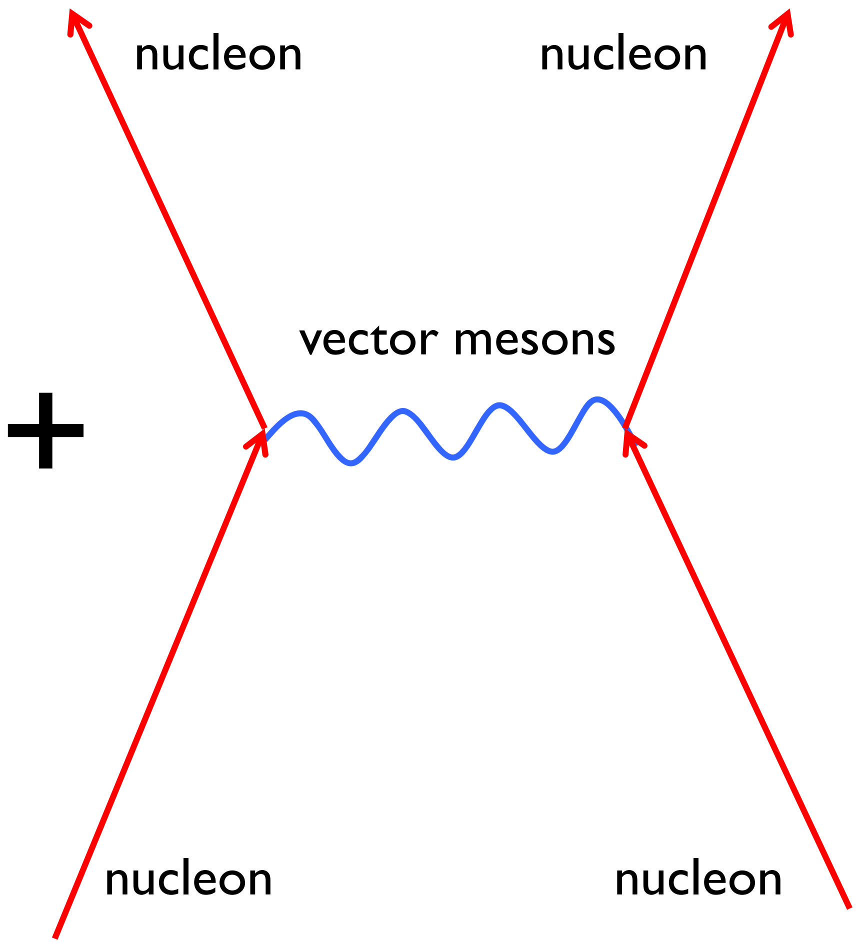
typical NN-potential



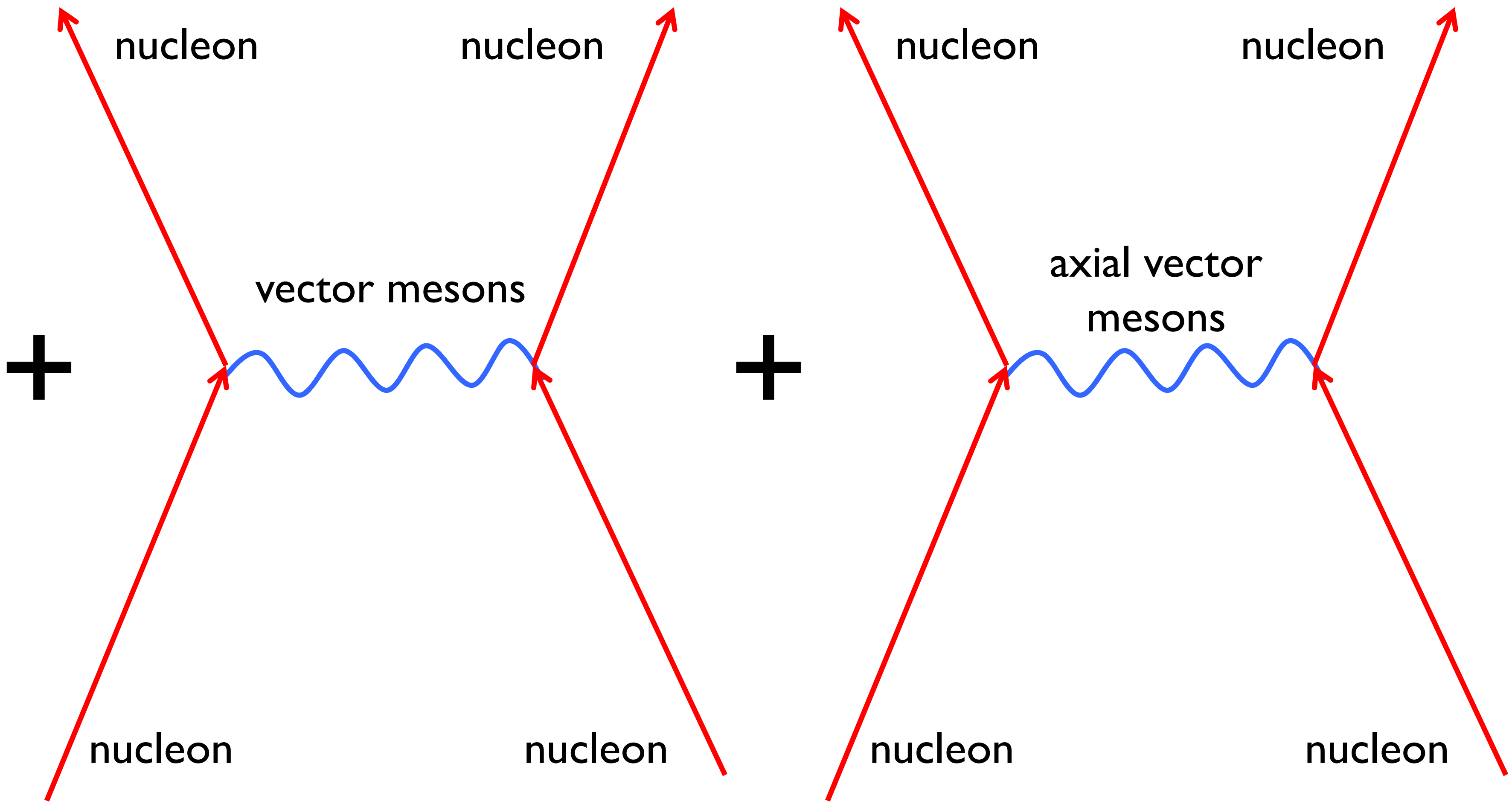
two-body nucleon potential from meson exchanges

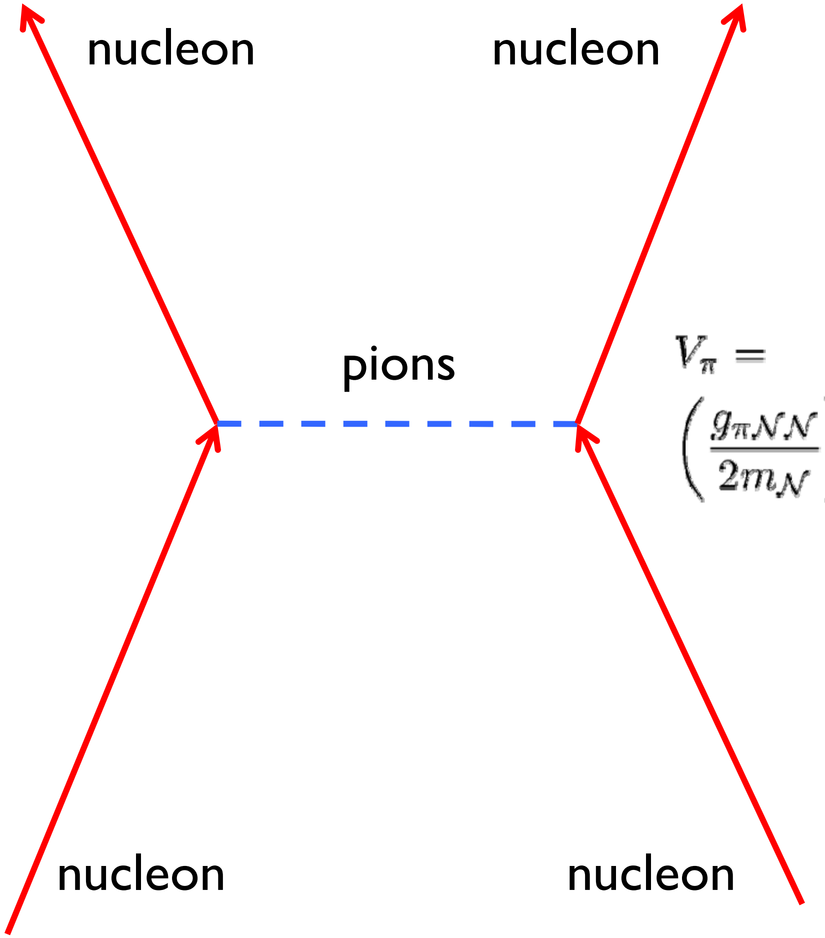


two-body nucleon potential from meson exchanges



two-body nucleon potential from meson exchanges





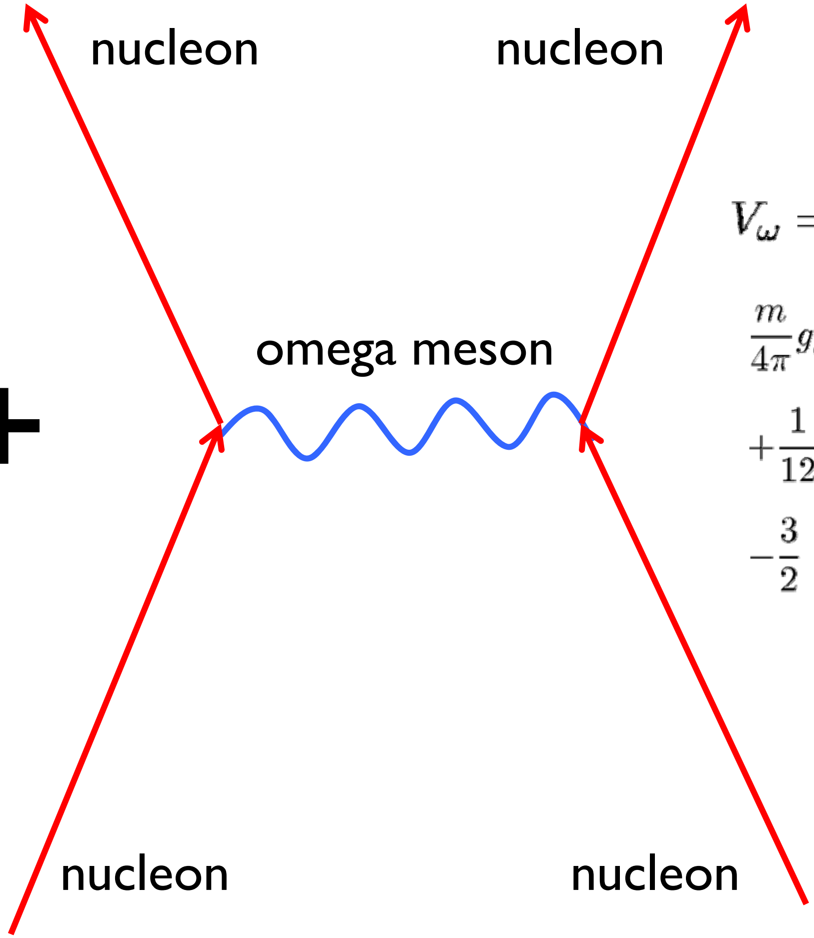
$$V_\pi = \left(\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_\mathcal{N}}\right)^2 \frac{m_\pi^3}{12\pi}[y_0(m_\pi r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + y_2(m_\pi r)S_{12}]\vec{\tau}_1\cdot\vec{\tau}_2 + \cdots$$

$$y_0(x) = \frac{e^{-x}}{x}$$

$$y_2(x) = \left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}$$

$$S_{12} = 3(\vec{\sigma}\cdot\hat{r})(\sigma_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

+

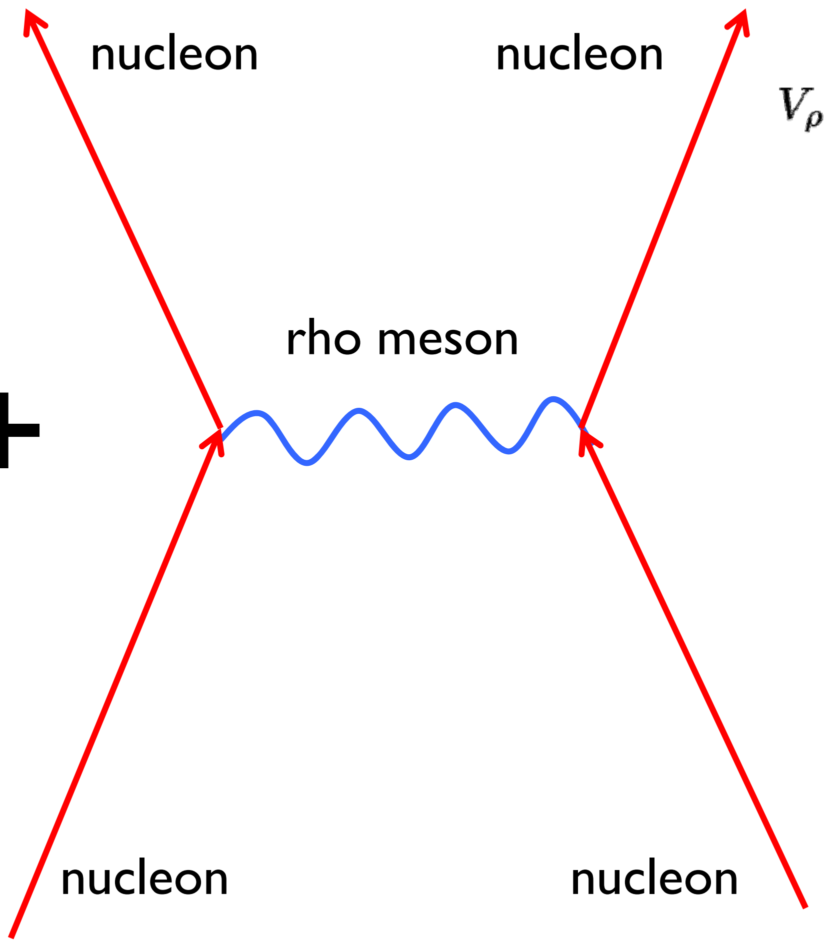


$$V_\omega =$$

$$\frac{m}{4\pi} g^2_{\omega\mathcal{N}\mathcal{N}} \left\{ \left(1 - \frac{1}{4}\frac{m^2}{m^2_{\mathcal{N}}}\right) y_0(mr) \right.$$
$$+ \frac{1}{12}\left(\frac{m}{m_{\mathcal{N}}}\right)^2 \left[2y_0(mr)\vec{\sigma}_1 \cdot \vec{\sigma}_2 - y_2(mr) S_{12}(\hat{r})\right]$$
$$\left. - \frac{3}{2}\left(\frac{m}{m_{\mathcal{N}}}\right)^2 \frac{y_1(mr)}{mr} \vec{L}\cdot\vec{S} + \frac{1}{16}\left(\frac{m}{m_{\mathcal{N}}}\right)^4 \frac{y_2(mr)}{m^2 r^2} Q_{12} \right\} + \cdots$$

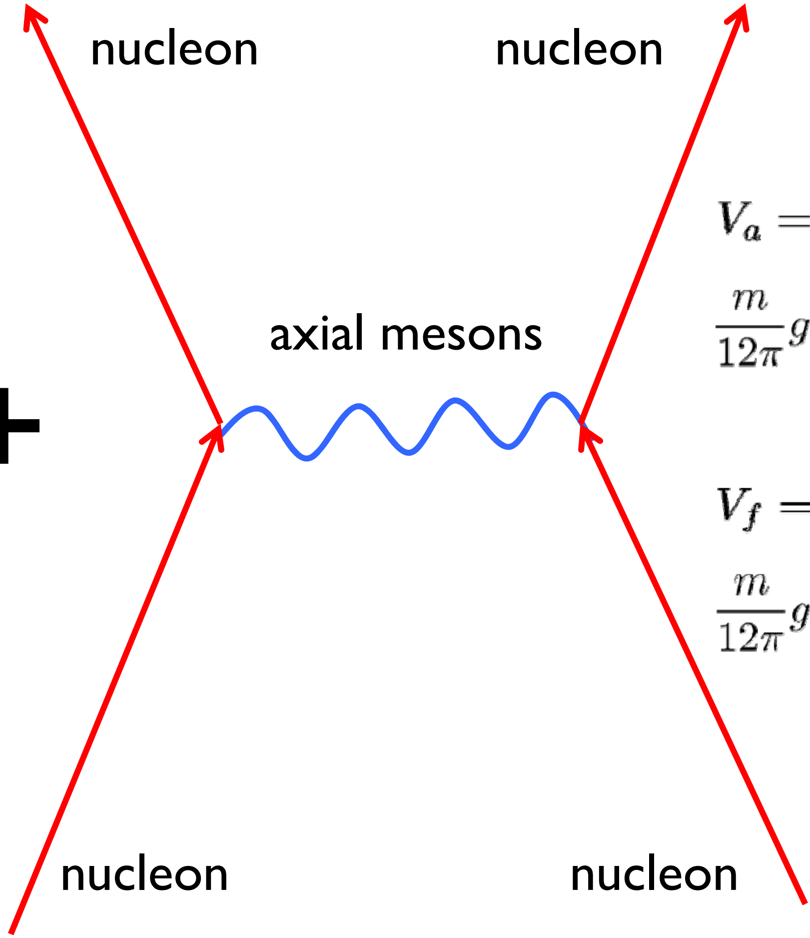
$$\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2),$$
$$Q_{12} = \frac{1}{2}\left((\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\right)$$

+



$$
\begin{aligned}
V_\rho = \\
&\frac{m}{4\pi}\Bigg\{\Bigg[g^2_{\rho\mathcal{N}\mathcal{N}}\left(1-\frac{1}{4}\frac{m^2}{m^2_{\mathcal{N}}}\right)+g_{\rho\mathcal{N}\mathcal{N}}\left(\frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}m\right)\frac{m}{m_{\mathcal{N}}} \\
&\qquad+\frac{1}{4}\left(\frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}m\right)^2\left(\frac{m}{m_{\mathcal{N}}}\right)^2\Bigg]y_0(mr) \\
&+\frac{1}{3}\Bigg[\left(\frac{m}{2m_{\mathcal{N}}}g_{\rho\mathcal{N}\mathcal{N}}+\frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}m\right)^2 \\
&\qquad+\frac{1}{8}\left(\frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}m\right)^2\left(\frac{m}{m_{\mathcal{N}}}\right)^2\Bigg]\Big[2y_0(mr)\vec{\sigma}_1\cdot\vec{\sigma}_2-y_2(mr)S_{12}(\hat{r})\Big] \\
&-\left(\frac{m}{m_{\mathcal{N}}}\right)^2\Bigg[\frac{3}{2}g^2_{\rho\mathcal{N}\mathcal{N}} \\
&\qquad+2g_{\rho\mathcal{N}\mathcal{N}}\tilde{g}_{\rho\mathcal{N}\mathcal{N}}+\frac{3}{2}\left(\frac{\tilde{g}_{\rho\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}m\right)^2\Bigg]\frac{y_1(mr)}{mr}\vec{L}\cdot\vec{S} \\
&+\left(\frac{m}{m_{\mathcal{N}}}\right)^4\Bigg[\frac{1}{16}g^2_{\rho\mathcal{N}\mathcal{N}} \\
&\qquad+\frac{1}{2}g_{\rho\mathcal{N}\mathcal{N}}\tilde{g}_{\rho\mathcal{N}\mathcal{N}}+\frac{1}{2}\tilde{g}^2_{\rho\mathcal{N}\mathcal{N}}\Bigg]\frac{y_2(mr)}{m^2r^2}Q_{12}\Bigg\}\times\vec{\tau}_1\cdot\vec{\tau}_2 \\
&+\cdots
\end{aligned}
$$

+



$$V_a =$$

$$\frac{m}{12\pi} g^2_{a\mathcal{N}\mathcal{N}} \left[-2 y_0(mr) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + y_2(mr) S_{12}(\hat{r}) \right] \vec{\tau}_1 \cdot \vec{\tau}_2 + \cdots$$

$$V_f =$$

$$\frac{m}{12\pi} g^2_{f\mathcal{N}\mathcal{N}} \left[-2 y_0(mr) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + y_2(mr) S_{12}(\hat{r}) \right] + \cdots$$

Recall

$$\int dx^{3+1}\left[-i\bar{\mathcal{N}}\gamma^{\mu}\partial_{\mu}\mathcal{N}-im_{\mathcal{N}}\bar{\mathcal{N}}\mathcal{N}\right]+\cdots$$

meson-nucleon-nucleon or

$$\sim\int dw\; f_+(w)^*\psi_n(w)f_\pm(w)$$

meson-meson-nucleon-nucleon

$$\sim\int dw\; f_+(w)^*\psi_n(w)\psi_m(w)f_\pm(w)$$

$$\int dw\;\frac{g_5\rho^2}{e^2}f_+(w)^*\psi_n(w)f_\pm(w)$$

$$\int dw\; f_+(w)^*\psi_n(w)f_\pm(w)$$

↔

$$g_{\pi\mathcal{N}\mathcal{N}}\;\bar{\mathcal{N}}\pi\gamma^5\mathcal{N}$$

$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}\rho_\mu^{(k)}\gamma^\mu\mathcal{N}$$

$$\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{m_{\mathcal{N}}}\bar{\mathcal{N}}\partial_\nu\rho_\mu^{(k)}\gamma^{\nu\mu}\mathcal{N}$$

$$g_{a^{(k)}\mathcal{N}\mathcal{N}}\bar{\mathcal{N}}a_\mu^{(k)}\gamma^\mu\gamma^5\mathcal{N}$$

Recall large N_c behaviors

$$\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}M_{KK} \simeq \simeq \frac{2\cdot 3\cdot\pi}{\sqrt{5}}\times\sqrt{\frac{N_c}{\lambda}},$$

$$\frac{g_{\eta'\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}M_{KK} \simeq \sqrt{\frac{3^9}{2}\pi^2}\times\frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}},$$

$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}} \simeq \sqrt{2\cdot 3^3\cdot\pi^3}\,\hat{\psi}_{(2k-1)}(0)\times\frac{1}{N_c}\sqrt{\frac{N_c}{\lambda}},$$

$$g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \simeq \sqrt{2\cdot 3^3\cdot\pi^3}\,\hat{\psi}_{(2k-1)}(0)\times\sqrt{\frac{N_c}{\lambda}},$$

$$\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}M_{KK} \simeq \sqrt{\frac{2^2\cdot 3^2\cdot\pi^3}{5}}\,\hat{\psi}_{(2k-1)}(0)\times\sqrt{\frac{N_c}{\lambda}},$$

$$g_{a^{(k)}\mathcal{N}\mathcal{N}} \simeq \sqrt{\frac{2^2\cdot 3^2\cdot\pi^3}{5}}\,\hat{\psi}_{(2k)}{}'(0)\times\sqrt{\frac{N_c}{\lambda}}\times\epsilon(\lambda),$$

$$g_{f^{(k)}\mathcal{N}\mathcal{N}} \simeq \sqrt{\frac{3^9\cdot\pi^5}{2}}\,\hat{\psi}_{(2k)}{}'(0)\times\frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}}\times\epsilon(\lambda)$$

$$\epsilon(\lambda)\equiv 1-\frac{\sqrt{2\cdot 3^5\cdot\pi^2/5}}{\lambda}+O(\lambda^{-2})$$

Recall large N_c behaviors

$$\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}M_{KK} \simeq \simeq \frac{2\cdot 3\cdot\pi}{\sqrt{5}}\times\sqrt{\frac{N_c}{\lambda}},$$

$$\frac{g_{\eta'\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}}M_{KK} \simeq \sqrt{\frac{3^9}{2}\pi^2}\times\frac{1}{\lambda N_c}\sqrt{\frac{N_c}{\lambda}},$$

$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}} \simeq \sqrt{2\cdot 3^3\cdot\pi^3}\,\hat{\psi}_{(2k-1)}(0)\times\frac{1}{N_c}\sqrt{\frac{N_c}{\lambda}},$$

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$$\lambda \equiv g_{YM}^2 N_c$$

$$\epsilon(\lambda) \equiv 1 - \frac{\sqrt{2\cdot 3^5\cdot\pi^2/5}}{\lambda} + O(\lambda^{-2})$$

$$\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} \simeq 8.43 \sqrt{\frac{N_c}{\lambda}},$$

$$g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \simeq \xi_k \sqrt{\frac{N_c}{\lambda}}, \qquad \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} \simeq \zeta_k \sqrt{\frac{N_c}{\lambda}}, \qquad g_{a^{(k)}\mathcal{N}\mathcal{N}} \simeq \chi_k \sqrt{\frac{N_c}{\lambda}}.$$

k	$m_{\omega^{(k)}} = m_{\rho^{(k)}}$	$\hat{\psi}_{(2k-1)}(0)$	ξ_k	ζ_k	$m_{a^{(k)}}$	$\hat{\psi}'_{(2k)}(0)$	χ_k
1	0.818	0.5973	24.44	8.925	1.25	0.629	9.40
2	1.69	0.5450	22.30	8.143	2.13	1.10	16.4
3	2.57	0.5328	21.81	7.961	3.00	1.56	23.3
4	3.44	0.5288	21.64	7.901	3.87	2.02	30.1
5	4.30	0.5270	21.57	7.874	4.73	2.47	36.9
6	5.17	0.5261	21.52	7.860	5.59	2.93	43.8
7	6.03	0.5255	21.50	7.852	6.46	3.38	50.5
8	6.89	0.5251	21.48	7.846	7.32	3.83	57.3
9	7.75	0.5249	21.48	7.843	8.19	4.29	64.1
10	8.62	0.5247	21.47	7.840	9.05	4.74	70.9

Large N_c Two-Body Nucleon Potential from Meson Exchanges

Kim, Lee, P.Y. 2009

$$V_{large\ N_c} = V_\pi^{hQCD} + \sum_{k=1}^{p} \left(V_{\rho^{(k)}}^{hQCD} + V_{\omega^{(k)}}^{hQCD} + V_{a^{(k)}}^{hQCD} \right)$$

$$V_\pi^{hQCD} = \frac{1}{4\pi} \left(\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \right)^2 \frac{1}{M_{KK}^2 r^3} S_{12}\, \vec{\tau}_1 \cdot \vec{\tau}_2 + \cdots$$

$$V_{\omega^{(k)}}^{hQCD} = \frac{1}{4\pi} \left(g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \right)^2 m_{\omega^{(k)}}\ y_0(m_{\omega^{(k)}} r) + \cdots$$

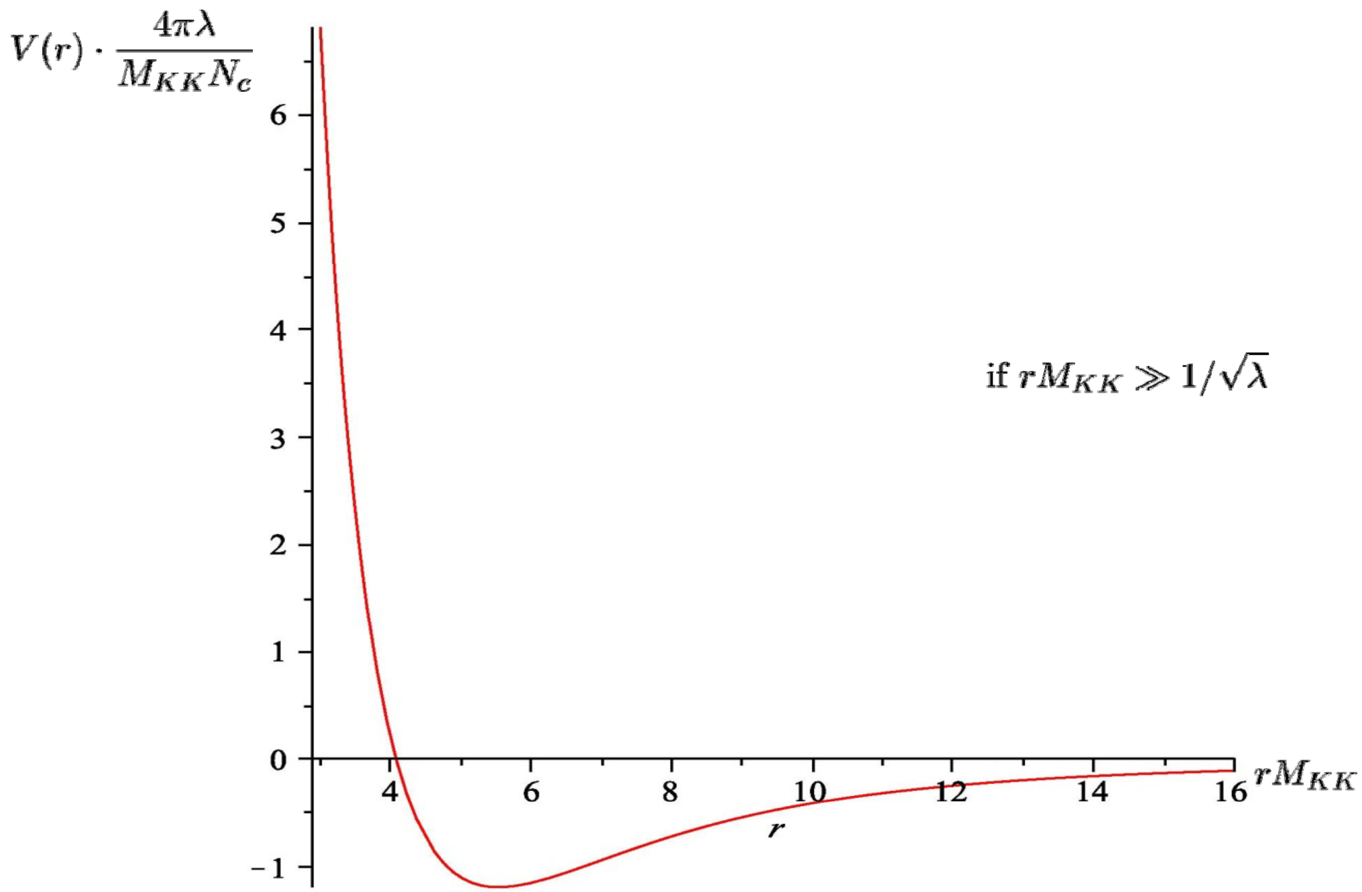
$$V_{\rho^{(k)}}^{hQCD} = \frac{1}{4\pi} \left(\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \right)^2 \frac{m_{\rho^{(k)}}^3}{3M_{KK}^2} [2y_0(m_{\rho^{(k)}} r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 - y_2(m_{\rho^{(k)}} r) S_{12}(\hat{r})]\, \vec{\tau}_1 \cdot \vec{\tau}_2 + \cdots$$

$$V_{a^{(k)}}^{hQCD} = \frac{1}{4\pi} \left(g_{a^{(k)}\mathcal{N}\mathcal{N}} \right)^2 \frac{m_{a^{(k)}}}{3} [-2y_0(m_{a^{(k)}} r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + y_2(m_{a^{(k)}} r) S_{12}(\hat{r})]\, \vec{\tau}_1 \cdot \vec{\tau}_2 + \cdots$$

$$y_0(x) = \frac{e^{-x}}{x}$$

$$y_2(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x}$$

$$S_{12} = 3(\vec{\sigma} \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



$$m_{\mathcal{N}} \sim \lambda N_c \quad \gg \quad E_{deformation} \sim N_c \quad \gg \quad E_{binding} \sim \frac{N_c}{\lambda}$$

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explanation of why holographic baryons make sense
despite its stringy size and huge mass ?

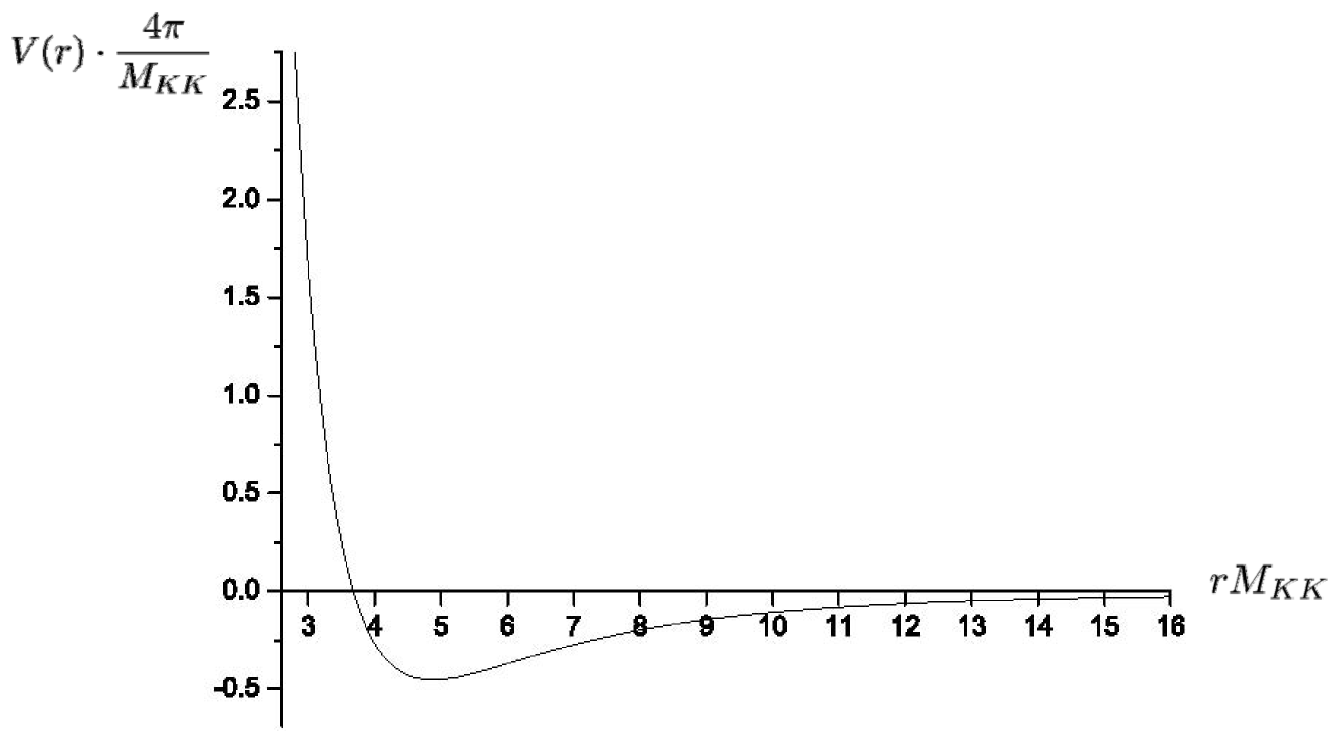
Two-Body Nucleon Potential from Meson Exchange in realistic QCD regimes

$$V_{N_c=3;\ \lambda=17} \simeq V_\pi + V_{\eta'} + V_{\rho^{(1)}} + V_{\omega^{(1)}}$$

$$\begin{aligned}
\frac{g_{\pi \mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &\simeq 4.27, \\
\frac{g_{\eta' \mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &\simeq 4.18, \\
g_{\rho^{(1)} \mathcal{N}\mathcal{N}} &\simeq 2.36, \\
g_{\omega^{(1)} \mathcal{N}\mathcal{N}} &\simeq 8.90, \\
\frac{\tilde{g}_{\rho^{(1)} \mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &\simeq 7.04.
\end{aligned}$$

$$\frac{\tilde{g}_{\rho^{(1)} \mathcal{N}\mathcal{N}}}{g_{\rho^{(1)} \mathcal{N}\mathcal{N}}} \sim 6$$ experimentally, also !!

$$V_{N_c=3;\,\lambda=17} \quad \leftarrow \quad V_\pi + V_{\eta'} + V_{\rho^{(1)}} + V_{\omega^{(1)}}$$



Kim, Lee, P.Y. 2009

is deuteron predictable?

(binding energy: 2.2Mev =0.12% of rest mass= a few % of NN-potential)

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Deuteron	hQCD (938)	hQCD (1130)	BonnB
Binding Energy (MeV)	2.3	2.2241	2.2246
D − state probability (%)	8.79	10	4.99
Quadrupole moment (fm^2)	0.44	0.474	0.278

courtesy of Youngman Kim

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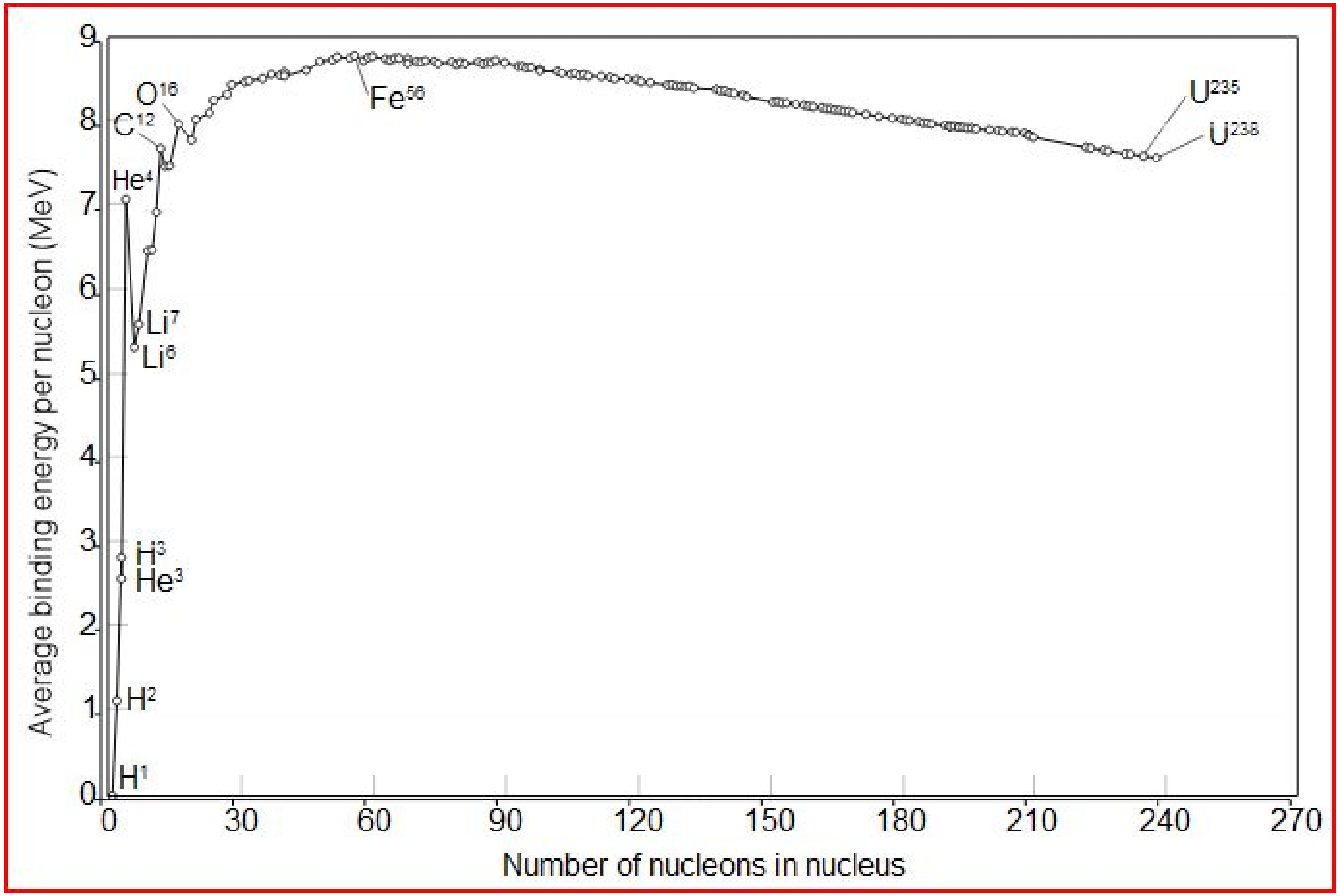
courtesy of Youngman Kim

however, the code (or the physics) turns out to be very sensitive to how we treat short-distance in the simulation, so this is NOT yet a prediction.

other nuclei ?



conventional wisdom calls for 3-body nucleon potential, not yet available

prospects

- **better tools for many baryons ?**
- **baryons as wrapped D-branes ?**
  (with K. Hashimoto, N. Iizuka)
- **neutron star: gravitating dense matter ?**
- **non-equilibrium dense matter in D4-D8: role of light mesons in RHIC/ALICE ?**
- **3^{rd} massive flavor ?**
  (see Hashimoto, Hirayama,Lin,Yee 2008)