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Addendum

1. RR fields

It is an abelian gauge field.

For string theory, maps

$$\Sigma^2 \longrightarrow \text{Minkowski}^{10}$$

quantize ...> $\mathbb{Z}/2$ - graded Hilbert space

$$\mathcal{H} = \mathcal{H}^0 \oplus \mathcal{H}^1$$

rep^n of Poincaré.

You only see the massless fields at long distance...

diff. forms of high degree

... RR fields.

So they live on Minkowski^{10}.

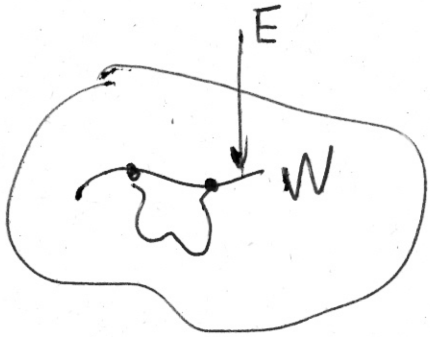
Put them on arbitrary X

$$
\begin{tikzcd}
 & \Omega^{\bullet}_{\text{closed}}(X) \arrow[d] \\
\boxed{\circ\circ\circ} \arrow[r] & H^{\bullet}_{dR}(X) \\
 & K^{\bullet}(X) \arrow[ul]
\end{tikzcd}
$$

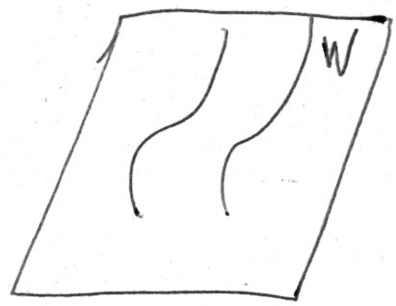
For RR fields, want this to be $K^{\bullet}(X)$

Why? • For anomaly cancellation reasons.

• Have to make 10d theory <u>and</u> 2d theory work.



$q : W \longrightarrow \mathbb{R}$



$$\int_W \sqrt{\frac{\hat{A}(W)}{\hat{A}(V)}}\, \mathrm{ch}\, E\; i^* C$$

plays the role of q

RR field

<u>Maxwell</u>

$$\int_W q\; i^* A$$

Notes for Urs

NS NS superstring background

1. X^{10} orbifold – metric, dilaton

2. $\begin{array}{c} X_w \circlearrowleft \sigma \\ \downarrow p \\ X \end{array}$ orientifold double cover

3. $\check{\beta}$ differential twisting of $KR(X_w)$

4. $\kappa : \mathcal{R}(\beta) \xrightarrow{\cong} \tau^{KO}(TX - 2)$ iso of twistings of $KO(X)$

What is this quadratic form?

the B-field is something which twists KR.

$$\begin{array}{ccc} \check{j} \in & KR^{\beta}(X_w) & \\ \downarrow & \downarrow & \\ \underbrace{\bar{\check{j}}\check{j}}_{\text{real structure}} \cdot \in & KR^{\bar{\beta}+\beta}(X_w) & \end{array}$$

real structure, so lifts to $KO^{\mathcal{R}(\beta)}(X)$.

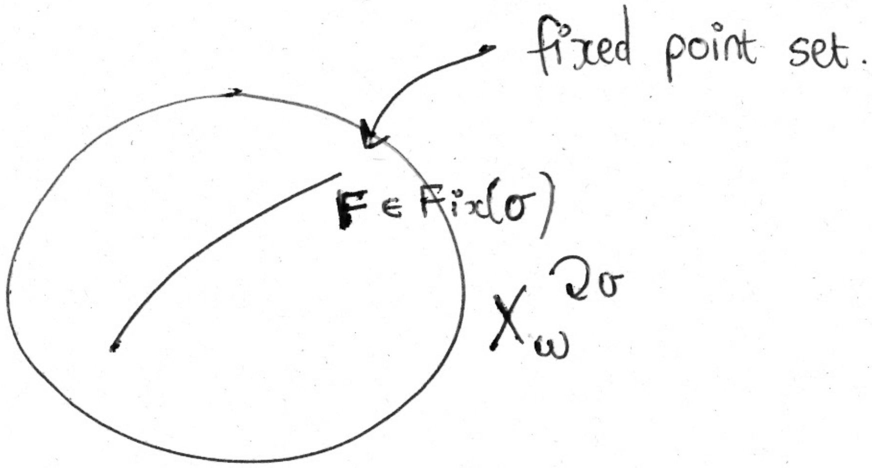
$$\int_{X/S} \kappa \bar{\check{j}}\check{j}$$

$$\begin{array}{ccc} KO^{\mathcal{R}(\beta)}(X) & & \\ \downarrow \kappa & & \\ \kappa \bar{\check{j}}\check{j} \in KO^{\tau^{KO}(TX-2)}(X) & \xrightarrow{\int_X} & KO^{-2}(S) \end{array}$$

$\xrightarrow[\text{of sign representation}]{\text{take coefficient}} \quad H^2(S)$

This is the input to the theory.

Now we can compute a formula for μ.



Use Atiyah-Singer localization theory in equivariant KO-theory.

We localize in this ring.

Wu class

$$\mu = i_* \left(\frac{\square(F)}{\psi^{-1}\left(\mathrm{Euler}^{KO}(\nu)\right)} \right) \in KR^{\beta}[\tfrac{1}{2}](X)$$

Adams square $\qquad$ localized

This is the story of the Ramond-Ramond field.

The cohomology theory needs to be <u>self-dual</u>

K theory ✓

bordism ✗

<u>Urs</u> Maxwell theory from 1850

is secretly about gerbes with connection.