

8.3.

28. While writing down the notes yesterday, and this morning still while pondering a little more, there has been the ever increasing feeling that I "was burning", namely turning around something very close, very simple-minded too surely, without quite getting hold of it yet. In such a situation, it is next to impossible just to leave it at that and come to the "ordre du jour" (namely stacks) - and even ^{"little"} the reflection I was about to write down last night (but it was really too late then to go on) will have to wait I guess, about the "geometric realization functors", as I feel it is getting me off rather, maybe just a little, from where it is "burning" !

There was one question flaring up yesterday (p.27) which I nearly dismissed as kind of silly, namely whether two localization functors

$$(*) \quad M \rightarrow (\text{Hot})$$

obtained in such and such a way were isomorphic (maybe even canonically so ??) provided they defined the same notion of "weak equivalences", namely arrows transformed into isomorphisms by the localization functors. Now this maybe isn't so silly after all, in view of the following

Assumption: The category of equivalences of (Hot) with itself, and ^{of} natural isomorphisms (possibly even any morphisms) between such, is equivalent to the one point category.

This means 1) any equivalence $(\text{Hot}) \xrightarrow{\sim} (\text{Hot})$ is isomorphic to the identity functor, and 2) any automorphism of the identity functor (possibly even any endomorphism?) is the identity.

Maybe these are facts well-known to the experts, maybe not - it is not my business here anyhow to set out to prove such kind of things. It looks pretty plausible, because if there was any non trivial autoequivalence of (Hot) , or automorphism of its identity functor, I guess I would have heard about it, or something of the sort would flip to my mind. It would not be so if we abelianized (Hot) some way or other, as there would be the loop and suspension functors, and homotheties by -1 of $\text{id}_{(\text{Hot})}$.

This assumption now can be rephrased, by stating that a localization functor $(*)$ $(\#)$ from any category M into (Hot) is well determined, up to a unique isomorphism, when the corresponding class $W \subset \text{FX}(M)$ of weak equivalences is known, in positive response to yesterday's silly question!

Such situation $(\#)$ seems to me to merit a name. As the word "model category" has already been used in a somewhat different and more sophisticated sense by Quillen, in the context of homotopy, I rather use another