The K-theory of the Category of $\mathcal{O}$-module objects over an Algebra Object over a Unital ∞-Operad $\mathcal{C}^{\otimes}$

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Fall Western Sectional Meeting, 2014

Outline

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Preliminary to the Motivation - the (well-known) Quillen Q-construction

Quillen's Q-construction - If $\mathcal{C}$ is an exact category, then define $\mathrm{Q}(\mathcal{C})$ as the category whose objects are the objects of $\mathcal{C}$ and morphisms $X \to Y$ are isomorphism classes of diagrams $Y \hookleftarrow P \rightsquigarrow X$, where $Y \hookleftarrow P$ is an admissible monomorphism and $P \rightsquigarrow X$ is an admissible epimorphism.
Composition in this category is given by pullbacks.
In this section, we will suppress Q; so by $\Omega\mathcal{C}$ we will mean $\Omega\mathrm{Q}(\mathcal{C})$.

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- It turns out that $\mathrm{Mod}_A(\Omega\mathcal{C}) \subset \Omega\mathrm{Mod}_A(\mathcal{C})$ is impossible.

Can the ∞-categorification hold?

- Can the impossibility $\mathrm{Mod}_A(\Omega\mathcal{C}) \not\subset \Omega\mathrm{Mod}_A(\mathcal{C})$ hold if we ∞-categorify it? I.e., can we obtain an impossibility:

$$\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) \not\supset \mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes}? \tag{1}$$

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- **Our long-term goal is to compute $\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})$, and perhaps generalize this to $\mathbf{K}(\mathcal{C}^{\otimes})$ if possible.** We won't do so here.

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- **Our long-term goal is to compute $\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})$, and perhaps generalize this to $\mathbf{K}(\mathcal{C}^{\otimes})$ if possible.** We won't do so here.
- In order to answer these, we will first introduce some preliminaries.

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1. Introduction
   - Motivation
   - Preliminaries
2. The Main Theorem
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3. Outlook

∞-categories

- Let us imagine a (simply-connected, for simplicity) topological space X with a points, P, Q, in it, which are connected by homotopic paths γ, Γ.

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- Let us imagine a (simply-connected, for simplicity) topological space X with a points, P, Q, in it, which are connected by homotopic paths γ, Γ.
- We can view X as an "∞-groupoid" as follows: the points of X can be viewed as objects, the paths as 1-morphisms, the homotopies as 2-morphisms, etc.

∞-categories, II

- This is called an ∞-groupoid, and the **homotopy hypothesis** states that the category $\infty\mathcal{G}\mathrm{rpd}$ of ∞-groupoids is is equivalent to the category Top of topological spaces.

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- We will define an ∞-category $\mathcal{O}$ to be a collection of objects along with an ∞-groupoid $\mathcal{H}\mathrm{om}_{\mathcal{O}}(X, Y)$ for any two objects $X, Y \in \mathrm{Obj}(\mathcal{O})$, with a composition law.

∞-Operads

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- Lurie has defined in his book, *Higher Algebra* ([[4]]) the concept of ∞-operads.
- I will try to offer a hand-wavy and informal definition of ∞-operads. Ordinary 1-categorical colored operads are multicategories, which allow for multiple inputs into the hom-set and one single output. Thus, if the 2-category of categories is denoted $\mathcal{C}\mathrm{at}$, and the 2-category of colored operads/multicategories is denoted $\mathcal{M}\mathrm{ult}$, then there is an inclusion $\mathcal{C}\mathrm{at} \hookrightarrow \mathcal{M}\mathrm{ult}$.

∞-Operads, II

- ∞-operads generalize this to the ∞-categorical setting by completing the diagram:

$$\begin{tikzcd} \mathcal{C}\mathrm{at} \arrow[r, hook] \arrow[d, hook] & \mathcal{M}\mathrm{ult} \arrow[d, hook] \\ \infty\mathcal{C}\mathrm{at} \arrow[r, hook, dashed] & \text{“}\infty\mathcal{O}\mathrm{perad}\text{”} \end{tikzcd} \tag{2}$$

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- We will use Lurie's abuse of terminology for ∞-operads; see [4, Remark 2.1.1.12]: if $\mathcal{O}^{\otimes} \to \mathrm{N}(\mathcal{F}\mathrm{in}_*)$ is an ∞-operad, we always will refer to $\mathcal{O}^{\otimes}$ as an ∞-operad.

Defining some important ∞-Operads

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Defining some important ∞-Operads

- Let $\mathcal{C}^{\otimes}$ be a unital ∞-operad.
- Let $\mathcal{O}^{\otimes}$ be a unital ∞-operad. Denote by $\mathbf{K}_{\mathcal{O}} \subseteq \mathcal{F}\mathrm{unct}(\Delta^1, \mathcal{O}^{\otimes})$ the full subcategory of $\mathcal{F}\mathrm{unct}(\Delta^1, \mathcal{O}^{\otimes})$ spanned by semi-inert morphisms in $\mathcal{O}^{\otimes}$. Let $\mathbf{K}^0_{\mathcal{O}}$ denote the full subcategory of $\mathbf{K}_{\mathcal{O}}$ spanned by the null morphisms in $\mathcal{O}^{\otimes}$. Let $\mathcal{P}\mathrm{reMod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$ be the simplicial set such that for every map of simplicial sets $X \to \mathcal{O}^{\otimes}$, there is a bijection $\mathcal{F}\mathrm{unct}_{\mathcal{O}^{\otimes}}(X, \mathcal{P}\mathrm{reMod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) \simeq$ $\mathcal{F}\mathrm{unct}_{\mathcal{F}\mathrm{unct}(\{1\}, \mathcal{O}^{\otimes})}(X \times_{\mathcal{F}\mathrm{unct}(\{0\}, \mathcal{O}^{\otimes})} \mathbf{K}^0_{\mathcal{O}}, \mathcal{C}^{\otimes})$. Let $\overline{\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}}$ be the full simplicial subset of $\mathcal{P}\mathrm{reMod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$ spanned by vertices $\overline{v}$ such that if $v = q(\overline{v}) \in \mathcal{O}^{\otimes}$, then $\overline{v}$ determines a functor $\{v\} \times_{\mathcal{O}^{\otimes}} \mathbf{K}_{\mathcal{O}} \to \mathcal{C}^{\otimes}$ which carries inert morphisms to inert morphisms, where $q : \mathcal{P}\mathrm{reMod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes} \to \mathcal{O}^{\otimes}$.

Defining some important ∞-Operads, Continued

- Construct a ∞-category $\mathcal{A}\mathrm{lg}_{\mathcal{O}'/\mathcal{O}}(\mathcal{C}^{\otimes})$ as follows: Let $p : \mathcal{C}^{\otimes} \to \mathcal{O}^{\otimes}$ be a fibration of ∞-operads and suppose there are ∞-operads $\alpha : \mathcal{O}'^{\otimes} \to \mathcal{O}'$. Let $\mathcal{A}\mathrm{lg}_{\mathcal{O}'/\mathcal{O}}$ be the full subcategory of $\mathcal{F}\mathrm{unct}_{\mathcal{O}^{\otimes}}(\mathcal{O}'^{\otimes}, \mathcal{C}^{\otimes})$ spanned by maps of ∞-operads. If $\mathcal{O}'^{\otimes} \cong \mathcal{O}^{\otimes}$ then $\mathcal{A}\mathrm{lg}_{\mathcal{O}'/\mathcal{O}}$ is denoted as $\mathcal{A}\mathrm{lg}_{/\mathcal{O}}$. If in $\mathcal{A}\mathrm{lg}_{/\mathcal{O}}$ we have that $\mathcal{O}^{\otimes} \cong \mathrm{N}(\mathcal{F}\mathrm{in}_*)$, then $\mathcal{A}\mathrm{lg}_{/\mathrm{N}(\mathcal{F}\mathrm{in}_*)}$ is written as $\mathcal{C}\mathcal{A}\mathrm{lg}(\mathcal{C}^{\otimes})$.

Defining some important ∞-Operads, Continued II

- In the following slide, we will make use of the ∞-category $\mathcal{A}\mathrm{lg}^{\Phi}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$, which we can define as follows. Define a set $\mathcal{P}\mathrm{re}\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$ with a map $\mathcal{P}\mathrm{re}\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes}) \to \mathcal{O}^{\otimes}$ so that the following is satisfied: for every simplicial set X with a map $X \to \mathcal{O}^{\otimes}$, there is a bijection $\mathcal{F}\mathrm{unct}_{\mathcal{O}^{\otimes}}(X, \mathcal{P}\mathrm{re}\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes})) \simeq \mathcal{F}\mathrm{unct}_{\mathcal{F}\mathrm{unct}(\{1\},\mathcal{O}^{\otimes})}(X \times_{\mathcal{F}\mathrm{unct}(\{0\},\mathcal{O}^{\otimes})} \mathbf{K}_{\mathcal{O}}, \mathcal{C}^{\otimes})$. Then we define $\mathcal{A}\mathrm{lg}^{\Phi}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$ as the full simplicial subset of $\mathcal{P}\mathrm{re}\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$ spanned by those vertices $F : \{Y\} \times_{\mathcal{F}\mathrm{unct}(\{0\},\mathcal{O}^{\otimes})} \mathbf{K}^{0}_{\mathcal{O}^{\otimes}} \to \mathcal{C}^{\otimes}$, where $Y \in \mathcal{O}^{\otimes}$, preserves inert morphisms. Here, $\mathbf{K}^{0}_{\mathcal{O}^{\otimes}}$ is the full subcategory of $\mathbf{K}_{\mathcal{O}^{\otimes}}$ spanned by the null morphisms in $\mathcal{O}^{\otimes}$.

Defining some important ∞-Operads, Continued III

- Let $\mathcal{O}^{\otimes}$ be a unital ∞-operad. Then let $\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$ denote the fiber product $\overline{\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}} \times_{\mathcal{A}\mathrm{lg}^{\Phi}_{/\mathcal{O}}(\mathcal{C}^{\otimes})} (\mathcal{O}^{\otimes} \times \mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes}))$. Let $A \in \mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$. Then let $\mathrm{Mod}^{\mathcal{O}}_{A}(\mathcal{C}^{\otimes})^{\otimes}$ denote the fiber product $\overline{\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}} \times_{\mathcal{A}\mathrm{lg}^{\Phi}_{/\mathcal{O}}(\mathcal{C}^{\otimes})} \{A\} \simeq \mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes} \times_{\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes})} \{A\}$, given by an equivalence $\mathcal{A}\mathrm{lg}_{/\mathcal{O}}(\mathcal{C}^{\otimes}) \simeq \mathcal{A}\mathrm{lg}^{\Phi}_{/\mathcal{O}}(\mathcal{C}^{\otimes})$, which in turn is induced by the equivalence $\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes} \simeq \overline{\mathrm{Mod}^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}}$. We refer to $\mathrm{Mod}^{\mathcal{O}}_{A}(\mathcal{C}^{\otimes})^{\otimes}$ as the ∞-operad of $\mathcal{O}$-module objects over A.

Higher Categorical $\mathbb{Q}$-Construction

- Let $\mathcal{O}(\Delta^n)$ denote $\mathcal{M}\mathrm{ap}(\Delta^{n,op} \star \Delta^n, \Delta^n)$, where $\star$ is the concatenation operation on Δ (see [1]).

Higher Categorical $\mathbb{Q}$-Construction

- Let $\mathcal{O}(\Delta^n)$ denote $\mathcal{M}\mathrm{ap}(\Delta^{n,op} \star \Delta^n, \Delta^n)$, where $\star$ is the concatenation operation on Δ (see [1]).
- Let $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})_{\dagger}^{\otimes}$ and $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes,\dagger}$ be subcategories of $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$ that contain all the equivalences. Call a pullback square

$$\begin{tikzcd} X \arrow[r] \arrow[d] & Y \arrow[d, two heads] \\ X' \arrow[r, tail] & Y' \end{tikzcd}$$

apprécié if $X' \to Y'$ and $Y \to Y'$ are morphisms of $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})_{\dagger}^{\otimes}$ and $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes,\dagger}$ respectively.

Higher Categorical $\mathbb{Q}$-Construction II

- A functor $X : \mathcal{O}(\Delta^n) \to \mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$ is apprécié if for integers $0 \leq i \leq k \leq \ell \leq j \leq n$, the square

$$\begin{tikzcd} X_{ij} \arrow[r, tail] \arrow[d, two heads] & X_{kj} \arrow[d, two heads] \\ X_{i\ell} \arrow[r, tail] & X_{k\ell} \end{tikzcd}$$

is an apprécié pullback.

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  is an apprécié pullback.
- **Defining the K-theory and the Q-construction:** Let $\mathrm{Q}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})$ denote the ∞-operad whose n-simplices are apprécié functors $\mathcal{O}(\Delta^n)^{\mathrm{op}} \to \mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$. Then define: $\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) := \Omega\mathrm{Q}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})$.

Outline

The Higher **K**-groups

- For the sake of posterity, I will define the higher **K**-groups. Define the homotopy groups $\pi_n(\mathcal{C}^\otimes)$ as $\mathcal{F}\mathrm{unct}_{\mathrm{s}\mathcal{S}\mathrm{et}}(\mathcal{C}^\otimes, \Delta[n]/\partial\Delta[n])$. Thus define $\mathbf{K}_n(\mathcal{C}^\otimes) := \pi_n(\mathbf{K}(\mathcal{C}^\otimes))$.

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- We will now proceed to our main theorem, which shows that $\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) \supset \mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes}$ is impossible.

Main Theorem

Theorem (D.)

Let $\mathcal{C}^{\otimes}$ be a unital ∞-operad. Then there is a stabilization from the first iteration of $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$:

$$\mathrm{Mod}_A^{\mathcal{O}}(\mathrm{Mod}_A^{\mathcal{O}}(\cdots(\mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes})\cdots)^{\otimes})^{\otimes} \simeq \mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes}$$

Additionally, it is impossible to have the inclusion

$$\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) \supset \mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes}.$$

There is a sequence of isomorphisms

$$\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}) \simeq \mathrm{Mod}_A^{\mathcal{O}}(\mathbf{K}(\mathcal{C}^{\otimes}))^{\otimes} \simeq \mathbf{K}(\mathcal{C}^{\otimes}).$$

If either $\mathcal{O}^{\otimes} = \mathbf{E}[0]^{\otimes} = \mathrm{N}(\mathcal{F}\mathrm{in}_*^{\mathrm{inj}})^{\otimes}$ or $\mathcal{O}^{\otimes}$ is a coherent ∞-operad and A is a trivial $\mathcal{O}$-algebra.

Outlook

- Can we compute $\mathbf{K}(\cdots\mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})\cdots)$ for any unital ∞-operad $\mathcal{C}^{\otimes}$?

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- Some ideas I've been looking into suggest that $\mathbf{K}(\cdots\mathbf{K}(\mathcal{C}^{\otimes})\cdots) = \mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$, for $\mathcal{C}^{\otimes}$ a unital ∞-operad.

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- Can we compute $\mathbf{K}(\cdots \mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})\cdots)$ for any unital ∞-operad $\mathcal{C}^{\otimes}$?
- Some ideas I've been looking into suggest that $\mathbf{K}(\cdots \mathbf{K}(\mathcal{C}^{\otimes})\cdots) = \mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$, for $\mathcal{C}^{\otimes}$ a unital ∞-operad.
- In fact, we can generalize and say that

$$\mathbf{K}(\cdots \mathbf{K}(\mathrm{Mod}_A^{\mathcal{O}}(\mathrm{Mod}_A^{\mathcal{O}}(\cdots(\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes})\cdots)^{\otimes})^{\otimes})\cdots)$$

is $\mathrm{Mod}_A^{\mathcal{O}}(\mathcal{C}^{\otimes})^{\otimes}$.

Thank You

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Thank you!

P.S.

These slides and notes can be obtained by emailing me at devalapurkarsanath@gmail.com.

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