Dan Freed.

Orientifolds.

Joint with J. Distler and G. Moore.

Σ is a compact 2-manifold (worldsheet) and X is a smooth 10-manifold (spacetime). Actually X is an orbifold.

Orientifold: we also have a double cover of X called X_w. X_w has an involution σ. Case I: σ is trivial. This is type I string. Case II: $X_w \to X$ has a section. This is type II string.

Differential cohomology. Suppose h is a cohomology theory. We have $h \otimes \mathbf{Q} = H(h(\bullet, \mathbf{Q}))$. Let $h_{\mathbf{R}} = h(\bullet, \mathbf{R})$. Now we let $\check{h}$ be a homotopy fibered product of h and $\Omega(h_{\mathbf{R}})$.

Exact sequences: $0 \to h(M, h_{\mathbf{R}} \otimes \mathbf{R}/\mathbf{Z})^{q-1} \to \check{h}^q(M) \to \Omega(M, h_{\mathbf{R}})^q \to 0$. The second map can be thought of as the curvature map. Another one: $0 \to \Omega \to \check{h}^q(M) \to h^q(M) \to 0$.

Examples: $\check{H}^1(M) = \mathrm{Map}(M, S^1)$. $H^1(M) = \pi_0(\mathrm{Map}(M, S^1))$. $\check{H}^2(M)$ consists of vector bundles over M with connection. $\check{H}^3(M)$ consists of gerbes.

Twistings of $KR(X_w)$. Pass to a locally equivalent groupoid $Y_w \to Y$. The usual construction gives us a simplicial manifold Y.

Definition: A twisting of $KR(X_w)$ is a locally equivalent $Y_w \to Y$ and $\tau = (d, L, \theta)$. such that $d: Y_0 \to \mathbf{Z}$ is continuous, $L \to Y_1$ is a $\mathbf{Z}/2$-graded hermitian line bundle, $\theta: {}^{\phi(f)}L_g \otimes L_f \to L_{gf}$, where $f: a \to b$ and $g: b \to c$.

Cohomology group: For $K(X)$ the first four homotopy groups are $\mathbf{Z}$, $\mathbf{Z}/2$, 0, $\mathbf{Z}$. For $KO(X_w)$: $\mathbf{Z}$, $\mathbf{Z}/2$, $\mathbf{Z}/2$. For $KR(X_w)$: $H^0(X, \mathbf{Z}) \times H^1(X, \mathbf{Z}/2) \times H^{w+3}(X, \mathbf{Z})$ as a *set*.

String theory. An NSNS superstring background is: X a smooth 10-orbifold with a metric and a real function; $\pi: X_w \to X$ an orientifold double cover; $\check{\beta}$ a differential twisting of $KR(X_w)$ (B-field); $K: R(\beta) \to T^{KO}(TX - 2)$ an isomorphism of twisting of $KO(X)$.

2d Theory: A *worldsheet* is a 2-manifold Σ with a metric and a spin structure α on the orientation double cover $\hat{\Sigma} \to \Sigma$.