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Outline: (1) Language for some elementary algebraic topology. (2) Application to generalizations of Hochschild complexes. (3) Examples for (a) Invariants on mapping spaces (generalization of Chern character) (b) Known constructions related to deformations of Laplacians.

Definition/Lemma: A commutative associative unital differential graded algebra is a strict monoidal functor from the category of finite sets, maps of sets, and disjoint unions of sets to the category of chain complexes, chain maps, and tensor products.

Why? We have $\emptyset \mapsto 1$, $1 \mapsto A$, $k \mapsto A^{\otimes k}$, $(2 \to 1) \mapsto (A \otimes A \to A)$.

Generalization: A partial DGA is a lax monoidal functor from the category of finite sets to the category of chain complexes, i.e., there is a natural equivalence $T\colon A(j \sqcup k) \to A(j) \otimes A(k)$ that is a quasi-isomorphism.

Generalize the category of finite sets: (1) Coalgebras. (2) Any operad. (3) Based finite sets can be generalized to modules, comodules etc. Then [Wilson] partial algebras can be functorially replaced by E_∞-algebras.

Example: If X is a space, then a map $f\colon j \to k$ induces a map $X^k \to X^j$. Take chains or cochains: $C_*(X^k) \to C_*(X^j)$ and $C^*(X^j) \to C^*(X^k)$. This construction works for PL chains and cochains. Künneth gives us a map $C_*(X^j) \otimes C_*(X^k) \to C_*(X^{j \sqcup k})$. Gives a partial coalgebra on $C_*(X)$ and a partial algebra on $C^*(X)$. The latter algebra by a theorem of Mandell determines the integral homotopy type.

Let Y be any finite simplicial space, A a partial algebra. We can compose them as functors and obtain a simplicial object in chain complexes, hence we obtain a total complex $\mathrm{CH}^Y(A)$, which is a generalization of Hochschild complex.

For $A = \Omega(X)$, $\mathrm{CH}^Y(A)$ computes cohomology of X^Y if X is sufficiently connected.