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# Higher TQFT and categorical quantum mechanics

Recently new parallels between the foundations of quantum mechanics, quantum computation and topological quantum field theory (TQFT) have begun to take shape. Categorical quantum mechanics was conceived by Abramsky and Coecke about ten years ago in an attempt to clarify the conceptual foundations of the subject, while TQFT (pioneered by Atiyah, Segal and Witten) is one of the finest modern examples of the interaction between pure mathematics and theoretical physics. The two subjects are founded on a common mathematical language: symmetric monoidal categories (both ordinary categories and higher categories) with duals at various levels.

In recent years higher categories, under for example the headings of 'categorification' and 'defects', have generated a lot of activity in the field of TQFT. This has led to insights in algebra, geometry, topology, condensened matter physics, supersymmetric gauge theory, and string theory; the rich and growing literature on categorified knot invariants is a prime example here. These developments have many unexplored implications for categorical quantum mechanics, and for quantum computation more specifically. In the opposite direction, researchers in categorical quantum mechanics have made rapid progress on clarifying the conceptual problems of quantum information; this insight, as well as an emphasis on categorical structures not traditionally studied in TQFT, is an opportunity to examine the latter subject from a new angle.

The time is right for a workshop to intensify cross-fertilisation between TQFT and categorical quantum mechanics, by bringing together experts in higher category theory, TQFT, and quantum mechanics. The categorical language that these subjects share is sure to quickly catalyse new collaborations and research directions. ESI and Vienna are an ideal place to conduct such an activity: new interactions between different communities in pure mathematics and theoretical physics, with foundational quantum theory as the unifying framework.

The main advantage of categorical quantum mechanics over more standard presentations is that its diagrammatic language not only dramatically simplifies many proofs such as that of the no-cloning theorem, it also emphasises the formal similarities of quantum computation with logic and computer science. In this setting the connection between quantum and classical computation arises because both are embedded into the common framework of symmetric monoidal categories, with classical computation being modelled by the case where the tensor product is cartesian. The relation with logic is classical and is referred to as the Curry-Howard correspondence, which relates formulas and proofs of first-order propositional logic with objects and morphisms in cartesian closed categories, respectively.

The diagrammatic language goes beyond pure formalism: an important example in the symmetric monoidal category $\mathscr{C}$ of finite-dimensional Hilbert spaces is the observation that the data of an orthogonal basis of a Hilbert space $H$ may be equivalently expressed as the data of a commutative †-Frobenius monoid in $\mathscr{C}$ with underlying object $H$, that is, a collection of morphisms $H \otimes H \to H$, $\mathbb{C} \to H$, $H \to H \otimes H$, $H \to \mathbb{C}$ satisfying natural equations. In a similar way some of the basic concepts of quantum information may be internalised into the categorical language, and represented diagrammatically.

Another insight made possible by categorical quantum mechanics is a clear separation between syntax and semantics: the syntax of a quantum protocol may be represented abstractly by a string diagram, and then given semantic and operational meaning by realising that string diagram in the monoidal category of Hilbert spaces. From this point of view it is natural to expect that TQFT, which in its basic form is the study of monoidal functors from bordism categories of various kinds into finite-dimensional vector spaces, might also have an important role to play in models of computation.

Finally, one is naturally lead to study certain symmetric monoidal 2-categories: these arise when one allows quantum systems to store and extract information from classical environments, or more generally consider all interactions of open quantum systems.

The kinds of symmetric monoidal 2-categories that arise in connection with categorical quantum mechanics possess duality on many levels, and similar structures had previously been identified to be as fundamental to two-dimensional rational conformal field theories (CFTs) and TQFTs with defects. Frobenius monoids, which appear in categorical quantum mechanics as avatars of orthogonal bases, or 'witnesses of classical information', abound in the study of TQFT. In particular, special symmetric Frobenius monoids (on the representation category of vertex algebras) fully encode rational CFTs in the Fuchs-Runkel-Schweigert formalism, and they also feature prominently in a 2-categorical theory of generalised orbifolds.

Another, much more fundamental aspect of TQFT is the cobordism hypothesis, which guarantees that any symmetric monoidal (higher) category with sufficient duality gives rise to representations of some suitable bordism category. It continues to inspire new insights in its native territory, but as has already been mentioned such representations are clearly related to models of computation studied in categorical quantum mechanics, and it is reasonable to expect that the cobordism hypothesis may lead to a deeper understanding of 2-categorical quantum mechanics and higher categorical models of both quantum and classical computation.

The first intention of the proposed workshop is to review recent progress in these directions and discuss important open problems such as a unified 2-categorical treatment of open quantum systems including infinite-dimensional state spaces, or a 2-categorical version of homological mirror symmetry. The second and main purpose is to contribute to solutions of these challenges, in the spirit of earlier successes such as the idea of an anyonic topological quantum computer based on the understanding of three-dimensional TQFT. Accordingly, higher-dimensional TQFTs will be another focus. We believe that the 'quantum camp' and the 'TQFT camp' still have a lot to learn from one another.