Research Statement *

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May 31, 2014

Algebraic topology/homotopy theory is a fundamental field of mathematics, dealing with the very nature of mathematical space and the concept of mathematical equivalence and invariance [AGP02]. In recent years this field is seeing an enormous boost, fueled to a considerable extent by the singular work of Jacob Lurie [L06, L09b, L1x]. This is motivated by geometry (in its refined version of "higher" and "derived" geometry [To14]) and cohomology theory (the theory of invariants) and solves deep problems that had originally been discussed by Alexander Grothendieck [Gr83]. At the same time, homotopy theory has in recent years been found to have a deep interpretation in formal logic itself, a connection that Vladimir Voevodsky has been proposing serves as the very foundations for modern mathematics [Voe10, UFP13].

Moreover, the most recent developments in algebraic and differential topology substantially owe to motivation coming from fundamental physics, notably from mathematical quantum field theory [SaSc11], hence the mathematical study of the basic theory that physicists use to understand phenomena seen in particle accelerators. This development revolves around and maybe culminates in the seminal mathematical classification of "local" quantum field theories via the cobordism hypothesis, conjectured by John Baez and James Dolan [BaDo95] and then formalized and proven by Jacob Lurie [L09a] (see [Ber10] for a review).

This algebraic classification result is supposed to be related to (higher, derived) geometry via a famous but notoriously mysterious process called *quantization* [Fre92, Sch14a], that turns "classical" (pre-quantum) geometry [FRS13] into the geometry of quantum field theory. Foundational work of Daniel Freed [Fr00, DFM11], and Michael Hopkins [HoSi05] and others, drawing on a long sequence of observations by Edward Witten (e.g. [Wi87, Wi96]), has shown that the subtle obstruction theory at the heart of pre-quantum field theory, – known by physicists as quantum anomaly cancellation [Fre86] – is mathematically captured by formulating quantum gauge fields as elements ("cocycles") in a "differential refinement" of generalized cohomology theory. However, despite the immense and deep contributions by these authors, it seemed fair to say that the development of this *differential cohomology theory* [Bu12] has been lacking behind its numerous intended applications.

More in detail, a long-standing open problem in algebraic topology and mathematical field theory was the definition of differential cohomology theories that are "twisted" [ABG10]. This plays a role notably in the quantum anomaly cancellation condition known as the Freed-Witten-Kapustin anomaly [Ka99, FSS13a] that appears in superstring theory, where it is twisted differential K-cohomology that is relevant. As this example shows, the question is of profound relevance for the foundations of the most advanced theories of fundamental physics. It was long known how to do the twist and the differential refinement separately, but their combination

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used to be elusive. The right framework was as much missing as it was known to be necessary [DFM11].

The various technical terms involved here witness that this is a question that involves dealing with the very foundations of modern geometry. Topology, differential structure, homotopy theory, generalized cohomology, fundamental physics (string physics, hence perturbative quantum gravity) all intimately interact in differential cohomology theory.

Motivated by this state of affairs, and based on work started in my PhD thesis [Sch05], I have, in recent years, re-developed differential cohomology and its application to mathematical physics as a theory that naturally has its place in a mathematical structure that deserves to be called a "higher cohesive topos" – whence the title of my Habilitation thesis [Sch13a]. This combines the key ingredient of Lurie's formulation of higher geometry [L06] (pairing traditional topos theory [MaMo92] with homotopy theory) with an old idea that had long been proposed by William Lawvere [Law86, Law94, Law97, Law07], but had by and large been ignored. Lawvere is famous for his groundbreaking contributions to the foundations of mathematics, and of abstract geometry and algebra, but what is less known is that to him all these developments were motivated as part of a mathematical foundation of physics. Indeed Lawvere introduced cohesive toposes (and synthetic differential geometry [MoR91]) as a foundation for continuum physics.[1] I found that combining this with the "higher" aspect of higher toposes, it yields a more powerful formalism that naturally encapsulates classical Hamilton-Jacobi-De Donder-Weyl field theory ([Sch13b], based on [FRS13]) and lends itself to a formalization of quantization via generalized cohomology theory [Sch14a].

This was, while natural, a new approach not followed by many, but now it is picking up momentum. In a talk at the Simons-Center in 2011 Michael Hopkins said [Ho11] that this is the context in which his seminal article [HoSi05] with Isadore Singer on the topic "should have been written". After a series of articles of mine with Domenico Fiorenza, Hisham Sati, and James Stasheff on this point, e.g. [FSS09, FSS10], Hopkins and Freed emphasized this again in [FrHo13].

Then recently Ulrich Bunke, Thomas Nikolaus and Michael Völkl found in [BNV13], in further development of [BuTa12, BuGe13], a considerable strengthening of the statement that higher cohesive topos theory is the right home for differential cohomology theory: they observe that *every* stable homotopy type in a higher cohesive topos automatically sits in a doubly interlocking exact sequence – the "differential cohomology diagram" – which identifies it with the representative of a generalized differential cohomology theory (the abstract mechanism is discussed also in section 4.1.2 of [Sch13a]). I have given a concise exposition of this state of affairs in [Sc14b].

Hence where I had originally thought of a higher cohesive topos as the natural home inside which one may find differential cohomology, this result shows that the situation is much better even: every stable object in a cohesive higher topos is a generalized differential cohomology theory and vice versa. Hence instead of titling my thesis "Differential cohomology in a cohesive ∞-topos" in hindsight the appropriate title is "Differential cohomology theory *is* cohesive ∞-topos theory". This is striking in that the axioms of cohesion are structurally very simple, much simpler than the common perception of the intricacies of differential cohomology theory would suggest. This unexpected structural simplicity allowed us, with Michael Shulman [ScSh12], to fully formally axiomatize differential cohomology in Vladimir Voevodsky's new foundations of mathematics already mentioned above [Voe10, UFP13], something that would be utterly

[1] See `http://ncatlab.org/nlab/show/William+Lawvere#MotivationFromFoundationsOfPhysics`.

impractical with the traditional definition of differential cohomology. It seems clear to me that this opens the door to a wealth of new constructions and results that may now be explored, an outlook on some of these I give in [Sch14a].

Notably, based on this result one finds a canonical solution to the old problem of formulating generalized cohomology that is both differential as well as twisted [BuNi14, Sc14b], thereby in particular putting the foundational work [DFM11] on solid ground.

These are exciting developments and it is clear from the available results that this is just the beginning of a rich theory that will further inform deep aspects both of algebraic/differential topology as well as mathematical quantum field theory.

Concretely, the central open mathematical problem that becomes tractable with this technology is a systematic mathematical formulation and study of *quantization* in (topological, local) field theory. The concept of quantization is central to fundamental physics since the beginning of the 20th century, and it has deeply inspired modern geometry and algebra. Notably Alain Connes' spectral (triple) formulation of non-commutative geometry [Co94] (and all that relates to it, e.g. quantum groups, quantum moduli spaces, etc.) is really the study of the deformation of classical geometry as seen by quantum mechanical particles. Generalizing this from quantum particles to quantum strings [So11] leads to quantum stringy geometry, the conceptual source for the mathematics of, for instance, homological mirror symmetry [ABDKMS09], topological T-duality [BuNi14] and geometric Langlands duality [KaWi06]. However, while punctual examples of quantization have been absorbed into mathematics this way, quantization as a whole proverbially remains a mystery. It is used as a powerful heuristics whose mathematical content however has to be guessed and proven anew in each new situation. It is not by chance that the quantization of Yang-Mills field theory is one of the Clay Millennium Problems [JWD].

A visionary proposal for which mathematical process might formalize quantization as such and hence eventually serve to explain and systematically derive its many heuristically found applications had been made in [Fre92] and further explored in [FHT07]. The idea is that the quantization process that takes geometric data to linear quantum data is given by fiber integration in generalized cohomology theories ("Umkehr maps", "Gysin maps"), indeed that the notoriously elusive formal definition of the path integral in quantum physics is given by fiber integration in a suitable generalized cohomology theory. For the highly interesting but still comparatively simple example of finite higher gauge theories (higher Dijkgraaf-Witten field theories) this is currently being worked out in great detail in [HL14], providing details for the sketch of a solution strategy in sections 3 and 8 of [FHLT09].

With my student Joost Nuiten we had begun to study how, more generally, quantization for physically relevant field theories may be formalized in this spirit using cohomology in higher cohesive toposes. This is the content of Nuiten's thesis [N13] and of a series of talks that I have recently given [Sch14a].

We find for each higher prequantum field theory in a cohesive ∞-topos (as formalized in [Sch13a]) that a suitable choice of generalized differential cohomology theory induces a topological quantum field theory in some dimension which carries a physical (non-topological) quantum field theory on its boundary and whose consistency (whose quantum anomaly cancellation) is controlled by another topological quantum field theory in one dimension higher – a hierarchical structure anticipated for instance in [Sa11, Fre12]. In particular ordinary quantum mechanics (quantum particles, hence 1-dimensional quantum field theory) we recover as the boundary field theory of a 2-dimensional Poisson-Chern-Simons theory, using the central result of another student of mine [Bon13], based on [FRS11]. Here it is noteworthy that the perturbative approximation to this 2d Poisson-Chern-Simons theory is the Poisson sigma-model; which is famously known [CaFe00] to have as boundary theory the perturbative approximation to the quantum mechanical system, its formal deformation quantization as famously found by Maxim Kontsevich. Hence what we find is a non-perturbative (full, exact) refinement of that result.

This pattern in the mathematical formalization, of physical field theories appearing on the boundary of topological field theory, is what physicists know as the *holographic principle.* In its incarnation as "AdS/CFT-duality" [Mal97, Wi98a] this has been the central topic in string theory since the turn of the millennium. While this is by and large far from being amenable to mathematical formalization at the moment, an often underappreciated fact (but see [GMMS04]) is that the famous and mathematically precise relation between 3-dimensional topological Chern-Simons field theory and the Wess-Zumino-Witten quantum string on its boundary [KaSa11] is an example of holography: states of Chern-Simons theory are the correlators of the theory on the boundary, this was Witten's seminal insight in [Wi89]. (Indeed, in our formalization via generalized cohomology, the quantum superstring appears as the boundary field theory of 3d Chern-Simons field theory.) In [Wi96] this principle was suggested to serve in higher dimensions as the very definition of the abelian sector for instance of the all-important 6-dimensional field theory on the worldvolume of the M5-brane. With coauthors we have discussed ingredients for how to generalize this from the abelian to the non-abelian case [FSS12a, FSS12b], more on that below.

I think that these results of [N13, Sch14a] already plausibly demonstrate that the problem of quantization finds a solution in terms of generalized differential cohomology formulated in higher cohesive topos theory. But this will be a vast topic, and plenty of further investigations are now waiting to be done. Some key points to be looked into would be the following.

1. One observation in [N13] is that defect structure [DKR11] in the 2d Poisson-Chern-Simons theory found in [Bon13] and cohomologically quantized in K-cohomology encodes the "universal orbit method" of geometric representation theory, that was found in [FHT05]. Using here the cohesive differential refinements of K-cohomology found in [BNV13] will refine this theory geometrically. This needs to be investigated.

2. In [N13, Sch14a] we observe that in direct analogy to how K-cohomology theory serves to quantize the boundaries of the 2d Poisson Chern-Simons theory [Bon13], so tmf-cohomology [AHR10] serves to quantize the boundaries of 3d Chern-Simons theory, this way recovering the Witten genus [Wi87] as the quantization of the superstring. This closely connects to other ongoing investigations into the deep role of tmf-cohomology in 3d/2d quantum field theory [StTe11, DoHe11]. In particular there is a direct connection to [Ni14] that we are looking into.

3. The discussion of the boundary quantization of the 2d Poisson-Chern-Simons theory depends crucially on the central result of one of my students [Bon13]. We expect that this is just the first instance of a whole tower of similar results on boundary conditions in higher dimensional non-perturbative AKSZ field theories as constructed in [FRS11], which will similarly govern higher field theories as indicated in the list of examples at the end of [FRS13]. This is ongoing joint work Domenico Fiorenza.

4. The seminal holographic relation between 3-dimensional Chern-Simons theory and the 2-dimensional worldsheet theory of the quantum string, as mentioned before, is expected to have a higher dimensional analog where a 7-dimensional Chern-Simons theory obtained from compactification of 11-dimensional supergravity carries on its boundary a

6-dimensional quantum field theory known as the *M5-brane worldvolume theory* or the $(2,0)$-*superconformal 6d QFT*. This is part of the $\mathrm{AdS}_7/\mathrm{CFT}_6$-duality conjecture in physics [Mal97, Wi96, Wi98b], but ambition to phrase this mathematically is being strongly felt [Fre12]. This 6-dimensional field theory is expected to be a source of profound structure, in particular its compactification on a torus is argued [Wi04] to be 4-dimensional (super-)Yang-Mills field theory, the field theory asked for in the Clay Millennium Problem [JWD].

Now in [FSS12a, FSS12b] we had described the 7-dimensional Chern-Simons theory appearing here in the refined higher cohesive prequantum geometry that serves as the input for the quantization procedure in [N13, Sch14a]. In [FSS13b] we discussed how in the prequantum geometry indeed the M5-brane arises as the boundary field theory of this 7d theory. Therefore one problem whose study naturally suggests itself here is to find a cohesive generalized differential cohomology theory which properly linearizes the coefficients of this 7d theory, so that our quantization procedure applies. While this is probably well out of reach with present insight (given that we just barely handle the tmf-coefficients relevant four dimensions further down) it is maybe remarkable as being at least a plausible and clearly describable proof strategy by which, via the above route, to eventually address the Clay Problem.

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