Physics Letters B 277 (1992) 306-312
North-Holland

PHYSICS LETTERS B

Some remarks on the supersymmetrization of the Lorentz Chern–Simons form in $D=10$ $N=1$ supergravity theories

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Received 10 December 1991

We show that all criticisms raised by J.S. Gates and collaborators to our solution of the Lorentz Chern–Simons supersymmetrization problem are groundless.

1. Introduction

The discovery in 1984 of the Green–Schwarz mechanism [1] and the advent of heterotic string theory [2] posed a conceptually simple but technically difficult problem. When the axion field strength is modified according to the rule

$$H_{\mu\nu\lambda}=\partial_{[\mu}B_{\nu\lambda]}+\beta\Omega^{(\mathrm{YM})}_{\mu\nu\lambda}(A)$$
$$\to \hat{H}_{\mu\nu\lambda}=H_{\mu\nu\lambda}-\gamma\Omega^{(\mathrm{L})}_{\mu\nu\lambda}(\omega)\ ,$$

where $\Omega^{\mathrm{L}}(\omega)$ is the Lorentz Chern–Simons three-form, the $N=1$, $D=10$ SUGRA+SYM theory is no longer supersymmetric, since the new interaction violates SUSY as given by the old transformation rules [#1]. The problem is that of restoring supersymmetry by deriving new SUSY rules for all the fields of the theory and, as a bonus of supersymmetry, also the new field equations following from the closure of the algebra. In a second run one might also obtain the lagrangian from which such field equations follow.

The first part of this program has been achieved and corresponds to a solved, fully understood problem. Indeed in a series of papers [3–13] due to the authors of the present letter (and in one instance [5] also to Ferrara and Porrati) the explicit derivation of the SUSY rules and of the field equations has been obtained in closed form. The main bulk of these results dates back to the years 1987–1988.

We go back to such an old topic in order to react to a campaign of criticisms raised by J.S. Gates and collaborators against our work. These criticisms are specifically contained in refs. [14,15] which have been published in Physics Letters B as well as in other preprints or drafts [16].

We will proceed as follows: first we summarize our results and the logic underlying their derivation. Then we analyze in detail the criticisms of refs. [14–16]. We will show not only that the criticisms of refs. [14–16] are groundless, but also that some alternative procedures in refs. [14–17] are wrong.

[#1] Here γ is a constant whose value is fixed by anomaly cancellation while β is fixed by supersymmetry. In the conventions of refs. [3,4] their value is $\gamma=-\frac{1}{32}$ and $\beta=-4$.

2. Anomaly free supergravity in the component formalism

The field content of $N=1$, $D=10$ SUGRA+SYM theory is the following: (1) the vielbein $V^a_\mu(x)$; (2) the gravitino $\psi_\mu(x)$; (3) the axion $B_{\mu\nu}(x)$; (4) the dilatino $\chi(x)$; (5) the dilaton $\sigma(x)$; (6) the gauge boson $A_\mu(x)$; (7) the gaugino $\lambda(x)$. In addition, if one uses the first order formalism, as we do, one has the spin connection $\omega^{ab}_\mu(x)$. It is determined from the torsion equation

$$\mathscr{D}_{[\mu}(\omega)V^a_{\nu]}-\tfrac{1}{2}\mathrm{i}\,\bar\psi_\mu\Gamma^a\psi_\nu=T^{abc}V_{b\mu}V_{c\nu}\,, \tag{1}$$

once the torsion tensor T^{abc} (fully antisymmetric) is expressed as a function of the seven physical fields.

One of the main results of our construction is provided by the differential equation that determines T_{abc} in terms of the physical fields. Let

$$H_{\mu\nu\rho}=\partial_{[\mu}B_{\nu\rho]}-\tfrac{1}{2}\mathrm{i}\,\mathrm{e}^{4/3\sigma}\,\bar\psi_{[\mu}\Gamma^d\psi_\rho V_{\nu]d}+\beta\Omega^{(\mathrm{YM})}_{\mu\nu\rho}(A)-\gamma\Omega^{(\mathrm{L})}_{\mu\nu\rho}(\omega) \tag{2}$$

be the axion field strength. Its supercovariant form is

$$H_{abc}=V^\mu_aV^\nu_bV^\rho_c(H_{\mu\nu\rho}-\gamma X^{(2,1)}_{\mu\nu\rho}-\gamma X^{(1,2)}_{\mu\nu\rho})\,, \tag{3}$$

where $X^{(2,1)}_{\mu\nu\rho}$ and $X^{(1,2)}_{\mu\nu\rho}$ are three-forms constructed out of T_{abc} itself, ψ_μ and the supercovariant gravitino field strength

$$\rho_{ab}=V^\mu_aV^\nu_b(\mathscr{D}_{[\mu}(\omega)\psi_{\nu]}-\tfrac{1}{36}\Gamma_m\Gamma_{ijk}\psi_{[\mu}V^m_{\nu]}T^{ijk})\,. \tag{4}$$

The explicit expressions for $X^{(2,1)}_{\mu\nu\rho}$ or $X^{(1,2)}_{\mu\nu\rho}$ can be found in the original papers (e.g. in eq. (5.1a) and table 10 of ref. [4] or, more explicitly, in eqs. (16), (17) of ref. [11]). The differential equation for T_{abc} is

$$T_{abc}=[-3H_{abc}+4\mathrm{i}\,\mathrm{Tr}(\bar\lambda\gamma_{abc}\lambda)-2\gamma W_{abc}]\,\mathrm{e}^{-4/3\sigma}, \tag{5}$$

where W_{abc} is a shorthand for the following expression:

$$\begin{aligned}W_{abc}={}&\tfrac{1}{2}\Box T_{abc}+3T_{ij[a}R^{ij}{}_{bc]}+3T_{i[ab}R^i{}_{c]}\\&+4T_{lm[a}T^m{}_b{}^nT_{c]n}{}^l-\tfrac{2}{27}T_{abc}T^2-\tfrac{1}{2}\bar\rho_{ij}\Gamma^j{}_{abck}\rho^{ki}\\&-6\bar\rho_{i[a}\Gamma^{ij}{}_b\rho_{c]j}-3\bar\rho_{ij}\Gamma^j{}_{[ab}\rho_{c]}{}^i-3\bar\rho_{i[a}\Gamma_b\rho_{c]}{}^i\\&-9\bar\rho_{i[a}\Gamma^i\rho_{bc]}\,.\end{aligned} \tag{6}$$

In eq. (6) $R^{ab}{}_{cd}$ is the supercovariant Riemann tensor given by

$$\begin{aligned}&R^{ab}{}_{cd}+V^{[\mu}_cV^{\nu]}_d[R^{ab}_{\mu\nu}(\omega)-\bar\psi_\mu\theta^{ab}_mV^m_\nu\\&\quad-\tfrac{5}{6}\mathrm{i}T^{abc}\bar\psi_\mu\Gamma_e\psi_\nu-\tfrac{1}{36}T_{ijk}\bar\psi_\mu\Gamma^{abijk}\psi_\nu]\,,\end{aligned} \tag{7}$$

with θ^{ab}_c defined as

$$\theta_{ab|c}=2\mathrm{i}\Gamma_c\rho_{ab}-3\mathrm{i}\Gamma^m{}_{[ab}\rho_{c]m}\,. \tag{8}$$

As one can see, eq. (5) is a highly complicated non linear equation for T_{abc}. When $\gamma=0$ everything simplifies and one gets the algebraic identification

$$T_{abc}=-3V^\mu_aV^\nu_bV^\rho_cH_{\mu\nu\rho}\,\mathrm{e}^{4/3\sigma}+4\mathrm{i}\,\mathrm{e}^{-4/3\sigma}\,\mathrm{Tr}(\bar\lambda\Gamma_{abc}\lambda)\,. \tag{9}$$

When $\gamma\neq0$ eq. (5) can be solved perturbatively in γ yielding, at each order γ^n, T_{abc} as a functional of H_{abc}.

The key equation (5) comes from the closure of the supersymmetry algebra that is represented by the following transformation rules:

$$\delta V^a_\mu=\mathrm{i}\bar\epsilon\Gamma^a\psi_\mu\,, \tag{10a}$$

$$\delta\psi_\mu=\mathscr{D}_\mu\epsilon+\tfrac{1}{36}\Gamma_a\Gamma_{ijk}\epsilon V^a_\mu T^{ijk}\,, \tag{10b}$$

$$\delta B_{\mu\nu}=\mathrm{i}\,\mathrm{e}^{4/3\sigma}\,\bar\epsilon\Gamma^a\psi_{[\mu}V^a_{\nu]}+\gamma(\epsilon\rfloor X^{(1,2)})_{\mu\nu}+\gamma(\epsilon\rfloor X^{(2,1)})_{\mu\nu}\,, \tag{10c}$$

$$\delta\chi=-2\mathrm{i}\,\Gamma^a\epsilon V^\mu_a(\partial_\mu\sigma-\tfrac{1}{4}\bar\psi_\mu\chi)+\mathrm{i}\Gamma_{abc}\epsilon Z^{abc}\,, \tag{10d}$$

$$\delta\sigma=\tfrac{1}{4}\bar\epsilon\chi\,, \tag{10e}$$

$$\delta A_\mu=-2\mathrm{i}\bar\lambda\Gamma_a\epsilon V^a_\mu\,, \tag{10f}$$

$$\delta\lambda=-\tfrac{1}{4}\Gamma_{ab}\epsilon F^{ab}\,, \tag{10g}$$

where Z^{abc} is a short hand for a certain expression in T_{abc} and the other fields given for instance in table 10 of ref. [4] or in eqs. (18a),(18b) of ref. [11], and where F^{ab} is the supercovariant gauge boson field strength:

$$F_{ab}=V^\mu_aV^\nu_b(\partial_{[\mu}A_{\nu]}-\tfrac{1}{2}[A_\mu,A_\nu]+2\mathrm{i}\,\bar\lambda\Gamma_a\psi_{[\mu}V^a_{\nu]})\,. \tag{11}$$

Furthermore $(\epsilon\rfloor X^{(1,2)})_{\mu\nu}$ and $(\epsilon\rfloor X^{(2,1)})_{\mu\nu}$ are the expressions obtained from the corresponding three-forms $X_{\rho\mu\nu}$ applying the formal operation $\epsilon_\rho\,\delta/\delta\psi_\rho$.

From the closure of the commutator algebra associated with the transformations (10), one obtains not only eq. (5) for the torsion T_{abc} but also the other field equations for all the seven physical fields. Their explicit form has been recently obtained by one of us with the aid of a dedicated symbolic manipulation program [11,12,18].

As the reader can see, all our results have been summarized in a component form without ever mentioning superspace, Bianchi identities and superfields. This is not the most economical way of presenting our formulae, which are equivalently, but more elegantly, written as rheonomic parametrizations (for a review see ref. [19]) of the superspace curvatures fulfilling Bianchi identities [= closure of the SUSY algebra]. We want to emphasize that in this approach going from superspace to components simply amounts to take the $\theta\to 0$ limit of all the relevant superfield equations. No choice of Wess–Zumino gauge is ever made; indeed one never considers any explicit θ-expansion of superfields. Moreover the SUSY transformations in eq. (10) which correspond to superspace diffeomorphisms are uniquely determined once the rheonomic parametrization of the superform curvatures is given, i.e. certain constraints are imposed on the curvatures themselves.

In order to discuss further the criticisms contained in refs. [14–16] it is convenient to go back to the more elegant superform language. For an easier comparison with refs. [14–17], we use the notation of refs. [2,5,9,10] which is completely equivalent to that of refs. [1,3,4,8,11,12] utilized so far.

In the notation of refs. [2,5,9,10] we have the supervielbein one-form $E^A=(V^a,\ \psi^\alpha)$, the supertorsion two-form $T^A=(T^a+\frac{1}{2}\mathrm{i}\,\bar\psi\Gamma^a\psi,\ \rho^\alpha)$, the Lorentz and Yang–Mills two-form supercurvatures $R_A{}^B=\frac{1}{2}E^CE^DR_{DCA}{}^B$, $F=\frac{1}{2}E^CE^DF_{DC}$. Furthermore one has the three-form

$$H=\mathrm{d}B+c_1\,\mathrm{Tr}(A\mathrm{d}A+\tfrac{2}{3}A^3)$$
$$-c_2\,\mathrm{Tr}(\Omega\,\mathrm{d}\Omega+\tfrac{2}{3}\Omega^3)\ , \qquad (12)$$

where c_1, c_2 correspond to the β, γ constants previously introduced. The Bianchi identities are

$$\Delta T^A=E^BR_B{}^A\ , \qquad (13a)$$

$$\Delta R_A{}^B=0\ , \qquad (13b)$$

$$\mathrm{d}H=c_1\,\mathrm{Tr}(FF)-c_2\,\mathrm{Tr}(RR)\ , \qquad (13c)$$

$$\Delta F=0\ . \qquad (13d)$$

Let us come now to the discussion of the constraints. The torsion constraints which are in one-to-one correspondence with the SUSY transformations (10a,10b) are

$$T_{\alpha\beta}{}^c=2(\Gamma^c)_{\alpha\beta}\ , \qquad (14a)$$

$$T_{\alpha\beta}{}^\gamma=0=T_{\alpha b}{}^\gamma,\quad T^\gamma_{a\beta}=(\Gamma_a)_{\beta\epsilon}(\Gamma_{ijk})^{\epsilon\gamma}T^{ijk}\ , \qquad (14b)$$

The constraints (14a) and (14b) are a possible choice but one can prove that once (14a) is fixed, (14b) as well as all other choices found in the literature can be obtained by field redefinitions [9] (the same result has been proven by Shapiro and Taylor [20] and adopted recently by Howe in ref. [21]).

We pause a moment on this subject because here the criticisms of refs. [14–16] fall more copiously. In particular in ref. [15] we meet the assertion that the result in ref. [9] is wrong and our field redefinitions are added assumptions, since the superconformal group is the largest superspace symmetry in this context (and does not include the field redefinitions of ref. [9]). Moreover in ref. [16] it is claimed that, with the above field redefinitions, qualified as singular gauge transformations, we are just gauging away the gravitino; hence, it is concluded that our results are inconsistent.

Let us straighten this matter and disentangle the issues involved. First of all, one can take the attitude that (14a), (14b) are just one's choice. We fix the vielbein and the gravitino SUSY transformation to be of the form (10a), (10b) and proceed. All the rest follows: we have a closed SUSY algebra and a set of field equations, namely a consistent supersymmetric theory. In particular there has been no loss of the gravitino, whose presence is in front of everyone's eyes. The next question is whether by choosing (14a), (14b) we are losing generality. The answer contained in ref. [9] and ref. [20] is clearly no: we are dealing with linear and invertible fields redefinitions involving tensor superfields, scalar under superdiffeomorphisms. Only (14a) is the significant choice, in other words if the SUSY transformation for the vielbein is the canonical one (10a), then the gravitino transformation can always be brought to the canonical form (10b) by a suitable field redefinition. This can be proven, if one wishes, in component language without even mentioning superspace, Wess–Zumino gauge and the like, or else one can work with superfields. No ad hoc assumptions are made or needed: the constraints in (14b) lead to a solution of the

Bianchi identities which is completely equivalent to the solution one would obtain with other consistent choices. Here by equivalence we mean that the two solutions can be obtained from each other through a field redefinition: *this field redefinition is not to be confused with a symmetry transformation.*

The very nice feature of the constraints (14a), (14b) is that the Bianchi identities (13a), (13b) are satisfied, independently of the presence or absence of CS-terms in the H-field strength, provided T_{abc} obeys the equations $\mathscr{D}^a T_{abc}=0$ and $R^{ab}_{ab}=\frac{2}{3}T_{ijk}T^{ijk}$.

Let us now discuss the constraints on H and F corresponding to the SUSY rules (10c) and (10f) respectively. For F one writes

$$F_{\alpha\beta}=0\,. \tag{15}$$

This constraint is well known, it concerns only the YM sector and will not be commented upon here. On the other hand the constraints for the three-form H are obtained in the following way: first one proves that

$$\mathrm{Tr}(RR)+\mathrm{d}X+K\,, \tag{16}$$

with X and K suitable Lorentz and gauge invariant superforms and that K has the same structure as $\mathrm{Tr}(FF)$ [2]. Then one defines $\hat{H}=H+c_2X$ so that (13c) becomes

$$\mathrm{d}\hat{H}=c_1\,\mathrm{Tr}(FF)-c_2K\,. \tag{17}$$

Now the problem is conceptually the same as in the case $c_2=0$ [22]. So one can impose on the components of $\hat{H}$ the conditions

$$\hat{H}_{\alpha\beta\gamma}=0\,, \tag{18a}$$

$$\hat{H}_{\alpha\beta\gamma}=\Gamma_{\alpha\beta\gamma}\phi\,, \tag{18b}$$

where $\phi\equiv \mathrm{e}^{4/3\sigma}$. The constraints (18) are consistent with the Bianchi identities and yield a unique set of equations of motion, derived in explicit form by Pesando in refs. [11,12].

As discussed in ref. [10], in the splitting between $\mathrm{d}X$ and K in (16) there is a possible freedom which may lead to physically different theories (non-minimal anomaly free supergravities [10]) (see also ref. [23]). Of course this freedom must be compatible with the Bianchi identities, i.e. the new K must have the same structure as $\mathrm{Tr}(FF)$. In particular in ref. [23] strong arguments are given that the $\zeta(3)$ stringy correction (the Grisaru, Zanon and Van de Ven [24] term) can be accounted for in this way. This is to point out that, contrary to the claim of refs. [14–16], our approach is capable of reproducing stringy corrections.

3. Browsing through the work of Gates and collaborators

We finally address few more specific criticisms contained in refs. [14–16]. Whenever possible we will use the notation of refs. [14–17], except for the convention that $\psi_{p,q}$ will denote the part of a superform ψ homogeneous of degree p in the vector-like supervielbein $E^a\equiv V^a$ and homogeneous of degree q in the spinor-like supervielbein E^α. A translation dictionary between refs. [6,7,9,10] and refs. [14–17] is shown in table 1. The essence of the would be criticisms of ref. [14] is contained in the claim that in refs. [3–13] a constraint is imposed on $G_{3,0}$ (equation (4.1) of ref. [14]). (This would imply that the construction of refs. [3–13] is not a correct Wess–Zumino construction and would lead to inconsistencies at the component level.) This is not the case: in refs. [3–13] the only constraints are on $G_{(0,3)}$, $G_{(1,2)}$ and $G_{(2,1)}$. Using the notation of ref. [14], we have

$$G_{(0,3)}=\gamma' X_{(0,3)}\,,$$

$$G_{(1,2)}=\tfrac{1}{2}\mathrm{i}\,E^cE^\alpha E^\beta(\sigma_c)_{\alpha\beta}+\gamma' X_{(1,2)}+\gamma' Q_{(1,2)}\,,$$

$$G_{(2,1)}=\gamma' X_{(2,1)}+\gamma' Q_{(2,1)}\,, \tag{19}$$

The resulting equation at level (3,0) (i.e. eq. (4.1) of ref. [14]) reveals that no constraint is imposed on $G_{(3,0)}$: indeed eq. (4.1) of ref. [14] comes from the consistent solution of the Bianchi identity $\mathrm{d}G=0$ in

[#2] In the notation of refs. [3–5,8,11–13] the three **120** supertensors related by the aforementioned equations are named H_{abc}, T_{abc} and Z_{abc} and correspond in the given order to G_{abc}, T_{abc} and ϕ_{abc}.

Table 1

Refs. [6,7,9,10]	Refs. [14–17]
$\mathrm{d}B$	G
X	Q
combination of YM and Lorentz CS	X
Γ_c	σ_c

the sector (2,2) once it has been solved in the sectors (0.4) and (1,3). More precisely $[\mathrm{d}G]_{(2,2)}=0$ gives two relations involving three supertensors: G_{abc}, T_{abc} and ϕ_{abc} (the **120** of $\mathrm{D}_\alpha \mathrm{D}_\beta\phi$) [#2]. Eq. (4.1) of ref. [14] is obtained by eliminating ϕ_{abc} from these two relations and obtaining T_{abc} in terms of something else, *which is not a constraint* since T_{abc} is an auxiliary superfield (this is eq. (4.8) of ref. [7] or, equivalently eq. (5.11) of ref. [4] or eq. (5) of the present paper: it is a very complicated equation since the supertensor T_{abc}, which is the unknown, appears in a highly non-linear way). Alternatively one can eliminate T_{abc}, in which case eq. (4.1) of ref. [14] gives the (complicated) expression of the **120** in the sector $\mathrm{D}_\alpha \mathrm{D}_\beta\phi$, *which is not a constraint either*: indeed this **120** supertensor just defines the SUSY transformation rule of the dilatino (see eq. (5.11b) of ref. [4] or equivalently eq. (10d) of the present paper). Since eq. (4.1) of ref. [14] is not a constraint, the construction of refs. [3–13] is perfectly Wess–Zumino and consistent with the component approach.

Another statement contained in ref. [14] is the claim that in refs. [3–13] the roles of G_{abc} and T_{abc} are confused. It is really hard to understand where this claim comes from. Eq. (4.8) of ref. [7] (or equivalently eq. (5) of the present paper) is

$$T_{abc}=-\frac{2}{3\phi}(G_{abc}+\gamma' X_{abc})-\frac{2}{3\phi}\gamma' Q_{abc}(T,\dots)$$
$$+\frac{16}{\phi}\gamma' b(\bar\lambda\sigma_{abc}\lambda)+\gamma' L_{abc}(T,\dots)\,, \tag{20}$$

where $b=\beta'/\gamma'$ is a finite number and the notation of ref. [14] has been used (except for L_{abc} which is the one as in ref. [7], not the one appearing in ref. [14]). Q_{abc} and L_{abc} are complicated functions of T_{abc}. Solving (20) iteratively (as one must do if one wants to compare the solution of refs. [3–13] with string theory) one just gets the expression of T_{abc} (*not* of G_{abc}) in terms of γ'. All this has already been discussed in the component formulation.

Finally we notice that the particular form of the constraints we use, including those on the three-form H, have been retrieved by one of us [25] in a totally different framework, namely from the cancellation of the κ-anomaly in a Green–Schwarz formulation of the 2D σ-model.

Now, as mentioned in the introduction, we want to show that refs. [14–17] contain an error. Since $\mathrm{d}G=0$ and $\mathrm{d}X=b\,\mathrm{tr}(FF)+\mathrm{tr}(RR)$, applying to eq. (19) the exterior differential d, one obtains

$$\mathrm{tr}(RR)_{0,4}+(\mathrm{d}Q)_{0,4}=0\,, \tag{21}$$

$$\mathrm{tr}(RR)_{1,3}+(\mathrm{d}Q)_{1,3}=0\,. \tag{22}$$

The difference between refs. [3–13] and refs. [14–17] is in the choice one makes for $Q_{1,2}$ and $Q_{2,1}$ in order to define the constraints (19). Both refs. [3–13] and refs. [14–17] choose a $Q_{1,2}$ compatible with eq. (21). However it is important to remark that (21) *leaves an arbitrariness* in the choice of $Q_{1,2}$, say

$$Q_{1,2}=\mathring{Q}_{1,2}+Y_{1,2}\,, \tag{23}$$

where $\mathring{Q}$ is what one gets from $\mathrm{tr}(RR)_{0,4}$ using the identity (2.4) of ref. [14], while

$$Y_{1,2}=E^a E^\beta E^\gamma(\sigma_a)_{\beta\delta}Y^\delta_\gamma\,. \tag{24}$$

Indeed from eq. (24) it follows that

$$(\mathrm{d}Y_{1,2})_{0,4}=0\,, \tag{25a}$$

since

$$Y^\delta_{(\gamma}\sigma^a_{\alpha\epsilon}\sigma^a_{\beta)\delta}=0\,. \tag{25b}$$

In refs. [14,17] the choice $Y_{1,2}=0$ is made, but this choice is incompatible with eq. (22).

The main result of ref. [6] is that one can choose $Y_{1,2}$ (and $Q_{2,1}$) in such a way that eq. (22) be satisfied. It turns out that Y^γ_δ is just given by $(\sigma_{abcd})^\gamma_\delta \mathrm{D}^a T^{bcd}$.

It is *not correct* to claim, as in ref. [14], that once $\mathring{Q}_{1,2}$ satisfies the Bianchi identity $\mathrm{d}G=0$ in the sector (0,4), the solution of the Bianchi identities in the other sectors is a mere exercise. One must consistently solve the Bianchi identity also in the sector (1,3) and only then the game becomes a (complicated) exercise.

In ref. [14] the term $Y_{1,2}$ was neglected. That this is not allowed can be further exposed by the following considerations:

(i) In the component approach the choice of $Q_{1,2}$ gives rise to a contribution in the SUSY transformation of the axion field $B_{\mu\nu}$,

$$\delta_\epsilon B_{\mu\nu}=\dots+c_2\bar\epsilon^\alpha M^a_{\alpha\beta}\psi^\beta_{[\mu}V^a_{\nu]}\,.$$

Obviously the closure of the SUSY algebra determines the form of the coefficient $M^a_{\alpha\beta}$ in terms of physical fields. Forgetting $Y_{1,2}$ means to forget an essential contribution to $M^a_{\alpha\beta}$ proportional to the derivative $D_{[a}T_{bcd]}$ of the torsion.

(ii) Since the authors of ref. [14] use exactly the same constraints as in ref. [8], a direct comparison of formulae and results is easily established. Ref. [14] merely presents a subset of the results in ref. [8] as we now show: eqs. (2.1), (2.2) and (2.3) in ref. [14] are simply

$$X \equiv \Omega^{(L)} = \mathrm{Tr}(\omega d\omega + \tfrac{2}{3}\omega^2)\,,$$

$$dX = \mathrm{tr}(RR)\,,$$

$$dG = 0\,,$$

where $G = dB = H + c_2\Omega^{(L)}$. Moreover eq. (2.5) in ref. [14] can be read directly from eqs. (7), (10), (11), (14) and (17) in ref. [8], once these last equations are evaluated at $Y_{1,2}=0$, i.e. with the choice (20) of ref. [8] which implies (21), as explained in ref. [8]. Finally eqs. (3.1) in ref. [14] are linear combinations of eqs. (22) in ref. [8]. As it has been clearly stressed in ref. [8] these equations do not provide the correct linear order solution of the Bianchi identities and the choice $Y_{1,2}=0$ is *definitely wrong*: it solves the linearized Bianchi identity but it does not provide the first order term of the unique all order solution. In a perturbative approach this can be checked by a second order calculation, which has never been attempted in refs. [14–16].

4. Conclusions

All criticisms raised by Gates et al. are groundless. The correctness of our approach can be summarized in the following results:

(i) Closed SUSY algebra with explicit SUSY transformation rules for every physical field in terms of physical fields.

(ii) Complete understanding of the generality of the constraints and their derivation even from the Green–Schwarz κ-anomaly cancellation requirement [25].

(iii) Explicit equations of motion for all the physical fields, admitting in particular Calabi–Yau spaces as exact solutions.

(iv) Clear separation between the supersymmetrization of the Lorentz Chern–Simons and the discussion of the higher order string corrections with an explicit localization of where they sit (the example of the $\zeta(3)$ correction explicitly worked out [#3].

(v) Full-fledged comparison with the dual formulation in terms of the seven-form.

(vi) Comparison of the $D=10$ case with the $D=4$ one. Extension of the Bonora–Pasti–Tonin theorem to $D=4$, with the proof that in the lower dimensional case it reproduces the results obtained by Ferrara, Girardello, Cecotti and Villasante in different ways. Understanding of why in $D=4$ one introduces no ghosts, while in $D=10$ one does introduce ghosts.

Finally we emphasize that contrary to the statement in ref. [14] it is not true that the (1,3) sector of the H Bianchi identities is automatically solved once the (0,4) sector is solved. Explicit calculations show that, keeping the constraints (14) fixed once and for all, the H Bianchi identities in the (1,3) sector imply the term $D_a T_{bcd}\Gamma^{abcde}_{\alpha\beta}$ Gates et al. are missing. It is a crucial term responsible in particular for the appearance of ghosts. Possibly, if one relaxes the constraints (14) at order γ then the crucial cocycle mentioned above might be shifted from the (1,2) sector of the H curvature to another place. Apart from the fact that this is not obvious, it is also irrelevant. Indeed since we can go back to the constraints (14) by means of a field redefinition, the same field redefinition will place back the cocycle to its canonical position in H.

[#3] We should stress that in refs. [14–17] there is often a confusion between *minimal* supersymmetrization of the Lorentz Chern–Simons term and *complete effective theory* for the superstring massless sector. In this paper we have been discussing the first issue. Ghosts appear necessarily in *minimal* anomaly free supergravity. We interpret this as an incompatibility between the simultaneous requirements of (a) locality, (b) unitarity, (c) absence of anomalies, (d) supersymmetry. It is not clear to us whether there exists a ghost free complete non-minimal effective theory (see the discussion in refs. [7, 9]).

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