SUPERGRAVITY AND COHOMOLOGY THEORY:

PROGRESS AND PROBLEMS IN D = 5

R. D'Auria, P. Fré and T. Regge

Istituto di Fisica Teorica, Università di Torino, Italy
and
Istituto di Fisica Nucleare, Sezione di Torino, Italy

In this contribution, emphasizing new developments, we plan to review the group manifold-rheonomic symmetry approach to supergravity[1,2] which has already been presented to other conferences[3]. In particular, we want to emphasize the central role of the mathematical concept of the graded Lie algebra cohomology class, which gives a constructive criterion for Lagrangians and which was not discussed in previous papers. The cohomological foundations of geometrical theories will be fully explained in a forthcoming paper[4].

The machinery of a geometrical field theory on a (super) group manifold is introduced as follows. Let G be a (super) group and $\mathbb{G}$ its (graded) Lie algebra. A basis of $\mathbb{G}$ is given by the generators T_A (A = 1, ..., n) which satisfy:

$$\left[T_A , T_B \right\} = C^{L}_{\ AB} T_L \tag{1}$$

On the manifold G, whose co-ordinates y^M are the group parameters themselves, we consider a $\mathbb{G}$ -valued 1-form (the <u>pseudo-connection</u>):

$$\mu = \mu^A T_A = dy^M \mu_M^{\ \cdot A} T_A \tag{2}$$

which will be the fundamental field of the theory.

In order to write down an action for μ^A we have to consider two kinds of objects which can be constructed out of it. One is the curvature 2-form R:

$$R = d\mu + \mu\wedge\mu \implies R^A \cdot T_A = R$$
$$R^A = d\mu^A + \frac{1}{2} C^A_{\cdot BC}\, \mu^B \wedge \mu^C \tag{3}$$

(when R = 0 we say that μ is a left-invariant 1-form); the other is the cochain ν^i. A cochain ν^i is a p-form with an index i in some finite-dimensional representation $D(T_A)$ of $\mathbb{G}$ such that it admits the following expansion

$$\nu^i = \nu^i_{A_1 \dots A_p}\, \mu^{A_1} \wedge \dots \wedge \mu^{A_p} \tag{4}$$

where $\nu^i_{A_1 \dots A_p}$ are constant numbers. With respect to this definition we remark that any p-form can be expanded in the basis of the μ^A (which is complete) but in general its components will be functions of the y^M co-ordinates and not constants, as we have assumed to be the case for the cochain. We can perform two operations on the cochains: one is the covariant derivative which maps a p cochain into a p + 1 cochain

$$\nabla : \nu^i \mapsto \nabla\nu^i = d\nu^i + \mu^A \wedge D(T_A)^i_{\cdot j}\, \nu^j \tag{5}$$

where $D(T_A)^i_{\cdot j}$ is the matrix representation of the generator T_A; the other operation is the contraction $\underline{A}|$ which maps p cochains into (p - 1) cochains. For every tangent vector $\vec{T}_A$ such that

$$\mu^A(\vec{T}_B) = \delta^A_B \tag{6}$$

and for every cochain (4) we define:

$$\underline{A}\rfloor \nu^{i} = p\, \nu^{i}{}_{A A_2 \ldots A_p}\, \mu^{A_2} \wedge \ldots \wedge \mu^{A_p} \tag{7}$$

Combining the contraction and the covariant derivative we also obtain a third operation which does not change the degree of the cochain and which is called the Lie derivative:

$$L_A \nu^{i} = \underline{A}\rfloor \nabla \nu^{i} + \nabla\left(\underline{A}\rfloor \nu^{i}\right) \tag{8}$$

These operations have several important formal properties and they all have a deep geometrical meaning which is basic to the discussion of Chevalley cohomology theory[5]; more will be said about them in a forthcoming paper[4]; here we just note that for any ν^i we get

$$\nabla\nabla\nu^{i} = R^{A} \wedge D(T_A)^{i}{}_{j}\, \nu^{j} \tag{9}$$

With these ingredients the action of our typical field theory will be the following:

$$\mathcal{A}[\mu] = \int_{M_{p+2}} R^{A}[\mu] \wedge \nu_A[\mu] \tag{10}$$

where the integration domain M_{p+2} is an arbitrary (p + 2) dimensional hypersurface of the manifold G, and the variational principle requires $\mathcal{A}[\mu]$ to be an extremum independently of the particular choice of M_{p+2}. From (10) we get the following equation of motion

$$\nabla \nu_A + (-)^{AB} R^{B} \wedge \underline{A}\rfloor \nu_B = 0 \tag{11}$$

which is an equation for (p + 1)-forms holding on the whole G-manifold. This latter statement means that the projection of (10) on any combination of (p + 1) tangent vectors $\vec{T}_{C_1}, \vec{T}_{C_2}, \ldots, \vec{T}_{p+1}$ is an equation of motion

$$\left(\nabla\nu_A + R^B{}_{\wedge\underline{A}}|\nu_B\,(-)^{AB}\right)\left(\vec{T}_{C_1},\ldots,\vec{T}_{C_{p+1}}\right) = 0 \tag{12}$$

Up to this point ν_A is fully arbitrary and therefore there is no criterion for selecting a particular Lagrangian. The criterion comes from the physical interpretation of the group G. In a theory which aims to be an extension of general relativity there must be a vacuum solution which corresponds to a flat space (= space without curvature) admitting symmetry under a group of motions. Excited states are the deformations of this flat space and are no longer symmetrical under the original group. The idea of geometrical theories on group manifolds is that G is indeed the group of motions of the vacuum which therefore corresponds to a left-invariant μ^A ($R^A = 0$). Such a physical requirement has the far-reaching consequence that $R^A = 0$ must be a solution of (11). This means

$$\nabla\nu_A = 0 \quad \text{if} \quad R^A = 0 \tag{13}$$

Now it is remarkable that (13) is preciyely the definition of a cocycle in Chevalley cohomology theory.

A cocycle is, in fact, a cochain which is covariantly closed, where closed means that its covariant derivative is zero modulo curvature. This immediately leads us into the realm of cohomology. In fact, calling coboundary an ω_A cochain which is covariantly exact, namely, is the covariant derivative of some other cochain:

$$\omega_A = \nabla\varphi_A \tag{14}$$

because of (9), we find that a coboundary is always a cocycle

$$\nabla\omega_A = \nabla\nabla\varphi_A = -C^B{}_{DA}R^B\wedge\varphi_B = 0 \text{ if } R^D = 0 \tag{15}$$

but the reverse is not always true. The equivalence classes of cocycles of degree p modulo the coboundaries of the same degree are called $H^p(\mathfrak{G},D)$, the p^{th} cohomology group of $\mathfrak{G}$ in the D representation (in our case D is the coadjoint representation). These cohomology classes are in a finite and small number, and depend entirely on the structure of the (super) group. For instance, a fundamental theorem[5] states that for any faithful representation of G there are no non-trivial cohomology classes if G is semi-simple. Now it is of the utmost importance that the action (10) depends only on the cohomology class and not on the particular cocycle representing it. In fact, if to ν_A we add a coboundary

$$\nu_A' = \nu_A + \nabla\varphi_A \tag{16}$$

the new action is

$$\mathcal{A}'[\mu] = \mathcal{A}[\mu] + \int_{M_{p+2}} R^A \wedge \nabla\varphi_A \tag{17}$$

where the second term on the right-hand side of (17) is, due to the Bianchi identity $\nabla R^A \equiv 0$, a pure divergence:

$$\int R^A \wedge \nabla\varphi_A = \int d\left(R^A \wedge \varphi_A\right) \tag{18}$$

and therefore does not contribute to the equations of motion. Hence we conclude that the possible geometrical theories which can be constructed with a supergroup G are in one-to-one correspondence with its cohomology classes. In particular, the already quoted theorem on semi-simple groups would rule out theories based on them. This difficulty can be overcome with the introduction of a larger cohomology theory which we shall discuss elsewhere[4]. For the purpose of this talk, we limit our discussion to the case of a non-semi-simple G which already includes the examples of gravity, ordinary D = 4 supergravity, and of the five-dimensional theory we shall discuss in the second part of this paper.

Once the correspondence between Lagrangians and cohomology has been established, a further restriction on the domain of possible theories comes from the observed fact that reasonable theories, although not invariant under the full G. are, however, exactly gauge invariant under some subgroup $H \subset G$. In most cases, H is the Lorentz group. Therefore, besides being a representative of a cohomology class, ν_A must be such that the action (10) is invariant under gauge transformations of H:

$$\mu^A \longmapsto \mu^A + \nabla \epsilon^A \qquad \begin{cases} \epsilon^K = 0 \\ \epsilon^H \neq 0 \end{cases} \tag{19}$$

(In (19) we have called H an index belonging to the subalgebra $\mathbb{H}$ of $\mathbb{G}$ and K an index belonging to the complement $\mathbb{K}$ of $\mathbb{H}$ in $\mathbb{G}$: $\mathbb{G} = \mathbb{H} \oplus \mathbb{K}$). Again the Chevalley theory comes to rescue us by supplying the concept of $\mathbb{H}$ - orthogonal $\mathbb{G}$ cohomology classes. By definition, a $\mathbb{G}$ cohomology class is orthogonal to the subalgebra $\mathbb{H}$ if, for all its representatives ν_A, we have

$$\left. \begin{aligned} {}_{\underline{H}}\rfloor \nu_A &= 0 \\ L_H \nu_A &= 0 \end{aligned} \right\} \text{ if } T_H \in \mathbb{H} \tag{20}$$

In a forthcoming paper it will be shown that the orthogonality of ν_A is sufficient to guarantee the gauge invariance of the action under H so that we can conclude by stating the following: "*For any pair* (G,H) *of a (super) group* G *and one of its sub-groups the possible geometrical theories are in one-to-one correspondence with the* $\mathbb{H}$*-orthogonal cohomology classes of* $\mathbb{G}$".

When ν_A is a cocycle its covariant derivative must be, by definition, proportional to the curvature and indeed we can show that :

$$\nabla \nu_A = R^B \wedge {}_{\underline{B}}\rfloor \nu_A \tag{21}$$

Therefore, Eq. (11) becomes

$$R^B \wedge \left(\underline{B} \rfloor \nu_A + (-)^{AB} \underline{A} \rfloor \nu_B \right) = 0 \tag{22}$$

which, when projected on all possible combinations of tangent vectors, becomes an algebraic equation for the intrinsic curvature components:

$$R^A_{\cdot BC} = R^A(\vec{T}_B, \vec{T}_C) \tag{23}$$

The essential features of the field theory described by our action (10) are determined by what sort of relations among $R^A_{\cdot BC}$ we get from (22). In order to discuss the various cases let us write the (graded) Lie algebra $\mathbb{G}$ in the following way:

$$\mathbb{G} = \mathbb{H} \oplus \mathbb{K} = \mathbb{H} \oplus \mathbb{I} \oplus \mathbb{O} \tag{24}$$

where $\mathbb{H}$ is the subalgebra and the complement $\mathbb{K}$ has been further decomposed into two subspaces which we shall call Inner and Outer, respectively.

<u>First possibility</u>

The only solution of (22) is $R^A_{\cdot BC} = 0$ for all values of A, B,C. In this case, the theory contains only the vacuum $R^A = 0$. It is a trivial theory.

<u>Second possibility</u>

Equation (22) admits solutions with some $R^A_{\cdot BC} \neq 0$. In all cases the theory is non-trivial but its properties are critically dependent on how many and which ones are the independent intrinsic component $R^A_{\cdot BC}$ parametrizing the most general solution of (22). The reason is that any theory on a group manifold, described by an action principle of type (10), being the action of a topological invariant, is symmetrical under an infinitesimal general co-ordinate transformation $y^M \mapsto y^M + \xi^M$. Such a transformation can be re-written as the following shift of the pseudo-connection $\mu^A \mapsto$ $\mapsto \mu^A + \delta\mu^A$, where (see (2)):

$$\delta\mu^A = \nabla\epsilon^A - 2\mu^F\epsilon^G R^A_{\cdot FG} \tag{25}$$

and

$$\epsilon^A = \xi^M \mu^{\cdot A}_M \tag{26}$$

It is apparent from (25) that, if some components $R^A_{\cdot F_1F_2}$ are determined to be zero, the co-ordinate transformation in the corresponding direction actually becomes a gauge transformation (19). Because of this, a theory based on an H-orthogonal cohomology class, which is automatically H-gauge invariant, must be H-factorized, namely (22) must imply:

$$H\text{-factorization} \iff {}_{H}\rfloor R^A = 0 \quad \text{if } T_H \in \mathbb{H} \tag{27}$$

Therefore, for a geometrical theory based on ν_A of type (20), the only non-vanishing components are those in the directions of $\mathbb{K}$. However, not all these components are independent; we call II (= inner), the space spanned by those directions of $\mathbb{K}$ such that the components of the curvature along them are independent. The complementary space ⊚ (= outer) is such that all the curvature components in such directions are just linear combinations of the inner components:

$$R^A_{\ OX} = \mathcal{C}^{A,I_1I_2}_{OX,B} R^B_{\ I_1I_2} \tag{28}$$

where χ is any index, 0 belongs to ⊚, I_1, I_2 belong to II and $\mathcal{C}^{A,I_1I_2}_{0,x,B}$ are some constant coefficients.

Rheonomic symmetry

We say that a theory is rheonomic symmetrical when the subspace II can be identified with space-time and $\circledcirc$ is non-empty. Identification with space-time means that II is spanned by the translation generators whose number d matches the number of dimensions of the Lorentz group SO(1,D - 1) contained in H. For example, in ordinary D = 4 supergravity $\mathbb{G}$ is the graded Poincaré algebra, $\mathbb{H}$ the SO(1,3) Lorentz algebra and $\mathbb{K} = \mathbb{P} \oplus \mathbb{Q}$ the direct sum of the translations $\mathbb{P}$ and of the supersymmetries $\mathbb{Q}$. The theory is rheonomic symmetrical because $\mathbb{II} = \mathbb{P}$ which contains exactly four translations, and $\circledcirc = \mathbb{Q}$.

When a theory is rheonomic symmetrical Eq. (25), supplemented with Eq. (28), tells us that the restriction of theory to space-time II contains as many extra symmetries besides $\mathbb{H}$ as there are generators in $\circledcirc$. In fact, every $\circledcirc$ general co-ordinate transformation is, due to (28), a transformation which involves only the space-time fields and their derivatives. In the case of supergravity, as discussed in Refs 2) and 3), the extra symmetry (rheonomic symmetry) is supersymmetry: however, the fermionic character of the transformation is accidental. In the D = 5 supergravity, which we shall presently discuss, we encounter an example of bosonic rheonomic symmetry.

D = 5 supergravity

It is well known that a supersymmetric theory of gravitation is 5-space-time dimensions besides the graviton must contain a complex spin 3/2 gravitino and also a spin-1 field. This is in order to match the number of physical degrees of freedom in the bosonic and in the fermionic sector[6]. In fact, in D = 5 the graviton has five and the vector field three degrees of freedom which together make eight; on the other hand, eight is precisely the number of polarizations of the complex gravitino. In view of this, the minimal supergroup G apt to describe D = 5 supergravity must have the following 24 generators:

$$\begin{array}{llcl} \text{Lorentz} & SO(1,4) : & J_{ab} & = 10 + \\ \text{Translations} & T_5 : & P_a & = 5 + \\ \text{Internal} & U(1) : & Z & = 1 + \\ \text{Supersymmetry} & \left\{ \begin{matrix} Q \\ \bar{Q} \end{matrix} \right. : & & = \underline{8} = \\ & & & \quad 24 \end{array} \tag{29}$$

It turns out that with these generators we can span the graded Lie algebra of $SU(2,2|1)$ [7,8] or of one of its contractions. If we call ω^{ab}, V^a, B, ξ, $\bar{\xi}$ the components of the pseudoconnection μ, respectively, along J_{ab}, P_a, Z, Q, and $\bar{Q}$ the structure of $SU(2,2|1)$ is given by the Maurer-Cartan equations which we obtain when the curvature is equal to zero in the following definitions:

$$
\begin{aligned}
R^{ab} &= d\omega^{ab} + \omega^{ac}\wedge\omega^{bd}\eta_{cd} + V^a\wedge V^b - i\,\bar{\xi}\wedge\Sigma^{ab}\xi \\
R^{a} &= dV^a - \omega^{ab}\wedge V^c\eta_{bc} - \frac{i}{2}\,\bar{\xi}\wedge\Gamma^a\xi \\
R^{\otimes} &= dB - i\,\bar{\xi}\wedge\xi \\
\rho &= d\xi + \frac{i}{2}\omega^{ab}\wedge\Sigma_{ab}\xi + \frac{i}{2}V^a\wedge\Gamma_a\xi - \frac{3i}{4}B\wedge\xi \\
\bar{\rho} &= \rho^{+}\Gamma_0
\end{aligned}
\tag{30}
$$

where η_{ab} is the flat metric of $D = 5$ Minkowski space and Γ_a the $D = 5$ gamma matrices ($\Sigma_{ab} = i/4\,[\Gamma_a, \Gamma_b]$. $SU(2,2\ 1)$ is a semi-simple supergroup and therefore we do not expect non-trivial cohomology classes. We can, however, obtain a non-semi-simple supergroup with the same number of generators performing the contraction. This is done by redefining

$$
\omega^{ab\,\prime} = \omega^{ab};\quad V^{a\,\prime} = e\,V^a;
$$

$$
B' = e\,B;\quad \xi' = \sqrt{e}\,\xi;\quad R^{ab\,\prime} = R^{ab};\quad R^{a\,\prime} = e\,R^a;
$$
$$
R^{\otimes\,\prime} = e\,R^{\otimes};\quad \rho' = \sqrt{e}\,\rho
$$

and performing the limit $e \to 0$ in Eq. (30). In this way we obtain the structural equations of the contracted non-semi-simple $\overline{SU(2,2|1)}$:

$$\begin{aligned} R^{ab} &= \mathcal{R}^{ab} \\ R^{a} &= \mathcal{D}V^{a} - \frac{i}{2}\,\bar{\xi}\wedge\Gamma^{a}\xi \\ R^{\otimes} &= dB - i\,\bar{\xi}\wedge\xi \\ \rho &= \mathcal{D}\xi \end{aligned} \tag{31}$$

where R^{ab} is the curvature of the SO(1,4) Lorentz subgroup $R^{ab} = d\omega^{ab} + \omega^{ac}\,\omega^{bd}\eta_{cd}$ and $\mathcal{D}V^{a}$ and $\mathcal{D}\xi$ are the SO(1,4) covariant derivatives of V^{a} and ξ, respectively:

$$\mathcal{D}V^{a} = dV^{a} - \omega^{ac}\wedge V^{d}\eta_{cd} \quad ; \quad \mathcal{D}\xi = d\xi + \frac{i}{2}\,\omega^{ab}\wedge\Sigma_{ab}\xi$$

Given the group and its curvature, in order to write an action we have to find the cohomology classes in the co-adjoint representation. To do this we have to know the form of the covariant derivative ∇ in such a representation. Let then, the action written as:

$$\mathcal{A} = \int\left\{-\frac{1}{2}\,R^{ab}\wedge\nu_{ab} - R^{a}\wedge\nu_{a} + \frac{3}{4}\,R^{\otimes}\wedge\nu_{\otimes} - \bar{m}\wedge\rho - \bar{\rho}\wedge m\right\} \tag{32}$$

It follows that the covariant derivative of the adjoint cochain $(\nu_{ab}, \nu_{a}, \nu_{\otimes}, m)$ of degree p is the following one:

$$\begin{aligned} \nabla\nu_{ab} &= \mathcal{D}\nu_{ab} + V_{a}\wedge\nu_{b} - V_{b}\wedge\nu_{a} - i\left(\bar{\xi}\Sigma_{ab}m - (-)^{p}\,\bar{m}\Sigma_{ab}\xi\right) \\ \nabla\nu_{a} &= \mathcal{D}\nu_{a} \\ \nabla\nu_{\otimes} &= d\nu_{\otimes} \\ \nabla m &= \mathcal{D}m - \frac{i}{2}\,\Gamma^{a}\xi\wedge\nu_{a} + \frac{3}{4}\,i\,\xi\wedge\nu_{\otimes} \end{aligned} \tag{33}$$

Now as the theory we want to construct has got to include five-dimensional gravitation, ν_{ab} must have, in addition, the Einstein term $\varepsilon_{abijk}V^i \wedge V^j \wedge V^k$ (in fact the component of the pseudoconnection along the translation generator P_a is to be identified with the fünfbein). This means that ν should be the most general cohomology class of order three containing the Einstein term. By explicit computations we have determined the complete cohomology group of order three of G = $\overline{SU(2,2|1)}$ orthogonal to the Lorentz group H = SO(1,4) [4]. It turns out to be composed of four elements so that the most general ν, which is a linear combination of these elements, contains three arbitrary parameters (in fact the over-all constant in front of the ν is irrelevant). Explicitly we find

$$\begin{aligned}
\nu_{ab} &= -\epsilon_{abijk} V^i \wedge V^j \wedge V^k + (\alpha_1 - 3) V_a \wedge V_b \wedge B + 2\alpha_2 \bar{\xi} \wedge \Sigma_{ab} \xi \wedge B \\
&\quad + 2\alpha_3 \bar{\xi} \wedge \Gamma_{[a} \xi \wedge V_{b]} \\
\nu_a &= \alpha_1 \left(\frac{i}{4} \bar{\xi} \wedge \Gamma_a \xi \wedge B - \frac{i}{2} \bar{\xi} \wedge \xi \wedge B \right) \\
\nu_\otimes &= i \bar{\xi} \wedge \xi \wedge B - 2i \bar{\xi} \wedge \Gamma_a \xi \wedge V^a \\
M &= -3 \Sigma_{ab} \xi \wedge V^a \wedge V^b + \\
&\quad + \frac{i}{2} \left(3 - \frac{\alpha_1}{2} \right) \Gamma_m \xi \wedge V^m \wedge B + \\
&\quad + i \alpha_2 \Sigma_{ab} \xi \wedge V^a \wedge V^b
\end{aligned} \tag{34}$$

In a recent paper[9], two of us have studied the bosonic limit of (34) as an independent theory. In this limit one sets $\xi = 0$ and disregards the corresponding Q generators. In this way G becomes $ISO(1,4) \otimes U(1)$, namely, the direct product of the Poincaré group in D = 5 times a U(1) internal group. The multiplet ν reduces to $\nu_{ab} = -\varepsilon_{abijk} V^i \wedge V^j \wedge V^k$ + cost $x V_a \wedge V_b \wedge B$ which is the most general SO(1,4) orthogonal cohomology class of order three for $ISO(1,4) \otimes U(1)$. The corresponding action:

$$\mathcal{A} = \int \left\{ R^{ab} \wedge V^i \wedge V^j \wedge V^k \epsilon_{abijk} + \text{cost}\, R^{ab} \wedge V_a \wedge V_b \wedge B \right\} \tag{35}$$

is, as it should be, gauge invariant under SO(1,4) but not under U(1). From the equations of motion, however, it follows that the theory is factorized and rheonomic symmetrical. In fact, the components of the curvature along SO(1,4) are all zero while on the other hand, we have:

$$\begin{aligned}
R^{a}_{\cdot bc} &= \text{cost}\, \eta_{bm} \eta_{cn} \epsilon^{amnpq} R^{\otimes}_{\cdot pq} \\
R^{a}_{\cdot \otimes m} &= \text{cost}\, \eta^{ab} R^{\otimes}_{bm} \\
R^{\otimes}_{\cdot \otimes m} &= 0 \\
R^{rs}_{\cdot \otimes k} &= \text{cost}\, \epsilon^{rsabc} R^{it}_{\cdot bc} \eta_{ik} \eta_{at}
\end{aligned} \tag{36}$$

These equations tell us that the independent curvature components are $R^{\otimes}{}_{pq}$ and $R^{ab}{}_{\cdot pq}$. All the other components can be expressed in terms of these. Such an occurrence is indeed what we named rheonomic symmetry and it guarantees that the original theory (35) restricted to the inner subspace II (spanned by the $5 V^a$) admits an extra U(1) symmetry whose infinitesimal form is:

$$
\begin{aligned}
\delta V^{a} &= \text{cost} \times \epsilon^{\otimes} V^{c} R^{\otimes}{}_{\cdot bc}\, \eta^{ab} \\
\delta B &= d\epsilon^{\otimes} \\
\delta \omega^{ab} &= \text{cost} \times \epsilon^{\otimes} V^{k} \epsilon^{abrst} R^{ij}{}_{st}\, \eta_{ik} \eta_{rj}
\end{aligned}
\tag{37}
$$

The meaning of this symmetry transformation becomes apparent when one turns from the first to the second order formalism for the space-time restriction of the theory (35). The transition to the second order description is obtained by the feed-back into (35) of Eqs (36) which can be solved for the spinor connection ω_{μ}^{ab} in terms of V_{μ}^{a}, B_{μ} and their derivatives. Once this is done the resulting second order action is

$$
\begin{aligned}
\mathcal{A}^{(2nd\ order)} = \int \Big\{ &(\det V)\, R^{\mu\nu}_{\mu\nu} + \text{cost}\, F_{\mu\nu} F^{\mu\nu} + \\
&+ \text{cost}'\, \epsilon^{\lambda\mu\nu\rho\sigma} F_{\lambda\mu} F_{\nu\rho} B_{\sigma} \Big\}\, d^5x
\end{aligned}
\tag{38}
$$

where $R^{\mu\nu}_{\mu\nu}$ is the usual curvature scalar and $F_{\mu\nu} = 1/2(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu})$. This theory is obviously invariant under the transformation

$$
\delta V^{a}_{\mu} = \text{cost}\ \epsilon^{\otimes}\, V^{a|\nu} F_{\nu\mu}\,; \quad \delta B_{\mu} = \partial_{\mu} \epsilon^{\otimes}
$$

which is the component transcription of (37) and quite remarkably exhibits the trilinear coupling of the spin 1 field which is a well-established feature of D = 5 supergravity[10].

The conclusion is that the cohomology argument has reproduced the correct bosonic sector of D = 5 supergravity in the same way as it has reproduced D = 4 supergravity. It is therefore very surprising that the complete theory based on the most general cohomology class (34) admits only the vacuum solution (all components of the curvature equal to zero) for all values of the parameters.

This result, which will be fully discussed in Ref. 4), seems to suggest that some of the existing second order supersymmetric theories have no first order parents on the group manifold and this might be the explanation why no action in superspace has been found for them.

REFERENCES

1. Y. Ne'eman and T. Regge, Rivista del Nuovo Cimento 1, number 5, p.1 (1978).
2. A. D'Adda, R. D' Auria, P. Frè and T. Regge, "Geometrical Formulation of Supergravity Theories on Orthosymplectic Supergroup Manifolds", to be published in Rivista del Nuovo Cimento (1980).
3. A. D' Adda, R. D'Auria, P. Frè and T. Regge, "Gauge Theories of Supergravity on Orthosymplectic Supergroup Manifolds", Proceedings Firenze Workshop (June 1979), to be published by J. Hopkins University;
   A. D'Adda, R. D'Auria, P. Frè and T. Regge, "Geometrical Formulation of Supergravity as a Theory on a Supergroup Manifold", in "Supergravity", Proc. of the Stony Brook Workshop, eds P. van Nieuwenhuizen and D.Z. Freedman, North-Holland Publ. Co., Amsterdam,(1979), p. 85
4. R. D'Auria, P. Frè and T. Regge,"Graded Lie Algebra Cohomology and Supergravity", forthcoming paper.
5. C. Chevalley and S. Eilenberg, Trans. Amer. Math. Soc. 63 (1948) 85.
6. E. Cremmer, B. Julia and J. Scherk, Phys. Letters 76B (1978) 409.
7. P. Freund and I. Kaplanski, J. Math. Phys. 17 (1976) 228.
8. J. Lukierski, Wroclaw preprint 436 (1978)
9. R. D'Auria and P. Frè, Torino preprint IFTT 323, Nucl. Phys. B, in press (1980).
10. E. Cremmer, J. Scherk and J.H. Schwarz, Phys. Letters 84B (1979) 83.