Averaging and backreaction in cosmology:
A summary

George Ellis,
University of Cape Town

LAM, Marseilles

Cosmoback Meeting, May 2018

Averaging occurs in all physics:

- Averaging in Fluid mechanics: Batchelor
- Averaging in electrodynamics (E,B) ←→ (D,H)

Averaging in GR: 4 contexts

1. Gravitational radiation: Isaacson
2. Weak Field: Szekeres
3. Observations in Cosmology: Bertotti, Dyer-Roeder
4. Dynamics in Cosmology: Backreaction

    Ellis (GR10, 1984)

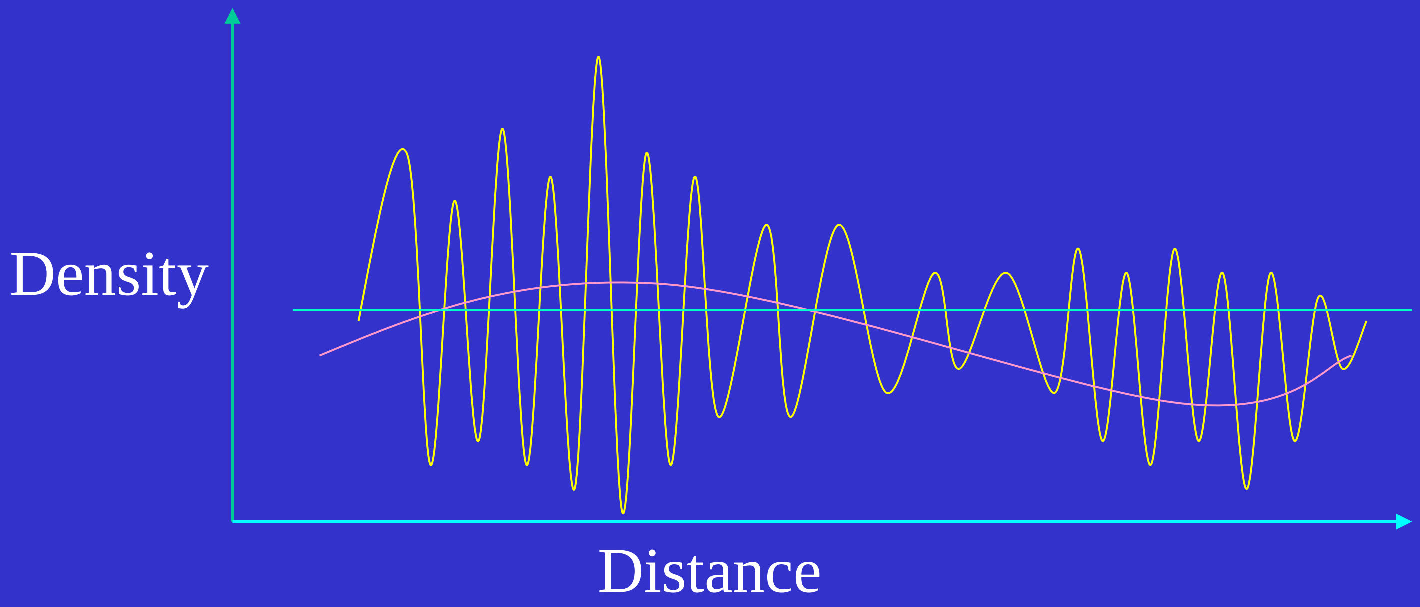
- The people in this room!

1: Local inhomogeneity: description

G.K. Batchelor: An Introduction to Fluid Dynamics, Cambridge University Press (1967).

Multiple scales of representation of ***same*** system

Implicit averaging scale



Molecules in a box of gas

2. Averaging in electrodynamics (E,B) ⇔ (D,H) Polarisation tensor (from Szekeres)

On the other hand, how does refraction make its appearance in electromagnetism? If we wish to treat an electromagnetic field in a material medium the current j^μ undergoes extreme fluctuations as we go from point to point in the medium on account of the intricate molecular structure of the matter. Eqs. (1.4) describe the detailed behaviour of the fields and are consequently called the "microscopic" equations; but it is clearly impossible to obtain any real information from them. To understand the gross behaviour of the field, it is necessary to consider some kind of average current $\langle j^\mu \rangle$. From the conservation identity $\langle j^\mu \rangle_{,\mu} = 0$, this average current may be seen to have the structure

$$\langle j^\mu \rangle = J^\mu - cP^{\mu\nu}{}_{,\nu} \qquad (1.7)$$

where J^μ is the average "free" current residing in the free electrons and molecules and $P^{\mu\nu}$ is a skew tensor, the *polarization tensor*, whose components in a Lorentz frame turn out to be

$$P_{ij} = -\epsilon_{ijk} M_k , \qquad P_{i0} = -P_{0i} = P_i , \qquad (1.8)$$

where **M** is the magnetization and **P** the polarization or average dipole moment of the molecules.

On forming the average of (1.4),

$$\langle F^{\mu\nu} \rangle_{,\nu} = \frac{4\pi}{c} \langle j^\mu \rangle = \frac{4\pi}{c} (J^\mu - cP^{\mu\nu}{}_{,\nu}),$$

Averaging in electrodynamics (E,B) ←→ (D,H)
Polarisation tensor

we obtain the *macroscopic equations*

$$H^{\mu\nu}{}_{,\nu} = \frac{4\pi}{c} J^{\mu}, \tag{1.9}$$

where

$$H^{\mu\nu} = \langle F^{\mu\nu} \rangle + 4\pi P^{\mu\nu}.$$

It is usual to denote the spacelike and timelike components of $H^{\mu\nu}$ by $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$ and $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$ ($\mathbf{E}$ and $\mathbf{B}$ now referring to the average values of the electric and magnetic fields), so that the vector form of the macroscopic equations reads

$$\nabla \cdot \mathbf{D} = 4\pi\rho \qquad \nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}.$$

This underdetermined system must be supplemented with constitutive equations, usually taken to be of the form

$$\mathbf{D} = \varepsilon\mathbf{E}, \qquad \mathbf{B} = \mu\mathbf{H}, \qquad \mathbf{J} = \sigma\mathbf{E}.$$

In the case of a *dielectric* ($\sigma = 0$), one obtains the familiar characteristic velocity $c/\sqrt{\varepsilon\mu}$ for electromagnetic waves.

Gravitational Polarisation Form (flat background)

Peter Szekeres developed a polarization formulation for a gravitational field acting in a medium, in analogy to electromagnetic polarization. He showed that the linearized Bianchi identities for an almost flat spacetime may be expressed in a form that is suggestive of Maxwell's equations with magnetic monopoles.

Assuming the medium to be molecular in structure, it is shown how, on performing an averaging process on the field quantities, the Bianchi identities must be modified by the inclusion of polarization terms resulting from the induction of quadrupole moments on the individual ``molecules". A model of a medium whose molecules are harmonic oscillators is discussed and constitutive equations are derived.

This results in the form:

$$G_{ab} = T_{ab} + P_{ab}\,.\,, \qquad P^{ab} = Q^{abcd}{}_{;cd}$$

that is P_{ab} is expressed as the double divergence of an effective quadrupole gravitational polarization tensor with suitable symmetries:

$$Q^{abcd} = Q^{[ab][cd]} = Q^{cdab}$$

Gravitational waves are demonstrated to slow down in such a medium. Thus the large scale effective equations include polarisation terms, as in the case of electromagnetism

But no backreaction included

P Szekeres: "Linearised gravitational theory in macroscopic media" Ann Phys 64: 599 (1971)

3. Gravitational Radiation (Isaacson)

Gravitational radiation in the limit of high frequency. I. The linear approximation and geometrical optics RA Isaacson - Physical Review, 1968

A formalism is developed for obtaining approximate gravitational wave solutions to the vacuum Einstein equations of general relativity in situations where the gravitational fields of interest are quite strong. To accomplish this we assume the wave to be of high frequency and expand the vacuum field equations in powers of the correspondingly small wavelength, getting an approximation scheme valid for all orders of 1r, for arbitrary velocities up to that of light, and for all intensities of the gravitational field. To lowest order in the wavelength, we obtain a gauge-invariant linearized equation for gravitational waves which is just a covariant generalization of that for massless spin-2 fields in a flat background space. This wave equation is solved in the WKB approximation to show that gravitational waves travel on null geodesics of the curved background geometry with their amplitude, frequency, and polarization modified by the curvature of space-time in exact analogy to light waves.

Gravitational Radiation (Isaacson)

Gravitational radiation in the limit of high frequency. II. Nonlinear terms and the effective stress tensor RA Isaacson - Physical Review, 1968

The high-frequency expansion of a vacuum gravitational field in powers of its small wavelength is continued. We go beyond the previously discussed linearization of the field equations to consider the lowest-order nonlinearities. These are shown to provide a natural, gauge-invariant, averaged stress tensor for the effective energy localized in the high-frequency gravitational waves. Under the assumption of the WKB form for the field, this stress tensor is found to have the same algebraic structure as that for an electromagnetic null field. A Poynting vector is used to investigate the flow of energy and momentum by gravitational waves, and it is seen that high-frequency waves propagate along null hypersurfaces and are not backscattered by the lowest-order nonlinearities. Expressions for the total energy and momentum carried by the field to flat null infinity are given in terms of coordinate-independent hypersurface integrals valid within regions of high field strength. The formalism is applied to the case of spherical gravitational waves where a news function is obtained

Gravitational Radiation (Isaacson)

2. EFFECTIVE STRESS TENSOR FOR GRAVITATIONAL RADIATION

In I, we expanded the vacuum field equations in powers of the wavelength of the gravitational wave. To lowest order, the field equations become $R_{\mu\nu}{}^{(1)}=0$, or, choosing our gauge with the reservations discussed in I, this was shown to reduce to

$$h_{\mu\nu;}{}^{\beta}{}_{;\beta}+2R_{\sigma\nu\mu\beta}{}^{(0)}h^{\beta\sigma}+R_{\sigma\mu}{}^{(0)}h_{\nu}{}^{\sigma}+R_{\sigma\nu}{}^{(0)}h_{\mu}{}^{\sigma}=0\,, \quad (2.1a)$$

$$h^{\mu\nu}{}_{;\nu}=0\,, \quad (2.1b)$$

$$h\equiv\gamma^{\alpha\beta}h_{\alpha\beta}=0\,. \quad (2.1c)$$

These equations determine the gravitational wave $h_{\mu\nu}$ once the background geometry $\gamma_{\mu\nu}$ is given. The second-

Gravitational Radiation (Isaacson)

3. BRILL-HARTLE AVERAGING SCHEME

The high-frequency oscillations of the gravitational waves are seen to produce the background curvature, but we are not really interested in all the fine details of the latter's fluctuations. The situation is somewhat analogous to the problem of finding electric fields in macroscopic dielectrics. While it is in principle possible to take into account all the atomic charge distributions in a dielectric to find the local electric field at any interior point, it is scarcely interesting to arrive at electric fields which fluctuate over a huge range as we move the observation point by 10^{-13} cm, and which require an exact description of the precise location of 10^{23} atoms. This sort of detail is totally irrelevant to the answering of any reasonable question about bulk matter. Rather, we take the field equation $\nabla \cdot \mathbf{E} = 4\pi\rho$ and average it over a region of space which is large compared to the scale of charge fluctuation, but small compared to the dimensions of the material of interest. Then we say that the average field is given as a solution to $\nabla \cdot \mathbf{E}_{av} = 4\pi\langle\rho\rangle$, where $\langle\rho\rangle$ denotes the space-averaged charge distribution.

Gravitational Radiation (Isaacson)

we let the symbol $\langle \cdots \rangle$ denote an average over a region whose characteristic dimension is small compared to the scale over which the background changes, but independent of ϵ [i.e., $O(1)$], and therefore large compared to the wavelength of the radiation in the limit $\epsilon \longrightarrow 0$. Then the averaged approximate field equations can be cast into the final form as given by BH:

$$R_{\mu\nu}{}^{(1)} = 0\,, \tag{3.1a}$$

$$R_{\mu\nu}{}^{(0)} - \tfrac{1}{2}\gamma_{\mu\nu}R^{(0)} = -8\pi T_{\mu\nu}{}^{\mathrm{BH}}\,, \tag{3.1b}$$

where the BH-averaged effective stress tensor is

$$T_{\mu\nu}{}^{\mathrm{BH}} = (\epsilon^2/16\pi)\langle Q_{\mu\nu} - S_{\mu\nu}{}^{\rho}{}_{;\rho}\rangle\,. \tag{3.2}$$

The oscillatory terms neglected by averaging (2.4) serve as a source for *higher-order* corrections to the metric

Gravitational Radiation (Isaacson)

We find that the effective WKB stress tensor is

$$T_{\mu\nu}{}^{\mathrm{WKB}} = (\epsilon^2/32\pi)\mathcal{A}^2\, k_\mu k_\nu \sin^2\phi + O(\epsilon)\,, \quad \text{(WKB)} \qquad (4.5)$$

where $T_{00}{}^{\mathrm{WKB}}$ is positive definite as expected.

Finally, we combine the BH and WKB approximations to obtain the effective averaged high-frequency wave stress tensor in the geometrical-optics form (to lowest order)

$$T_{\mu\nu}{}^{\mathrm{HF}} = q^2 k_\mu k_\nu\,, \quad q^2 = \epsilon^2 \mathcal{A}^2/64\pi\,. \quad \text{(BH-WKB)} \qquad (4.6)$$

Back reaction term! (but not its effects …)

Note: the volume averaging is not explicitly carried out

4. Local inhomogeneity: dynamic effects

Averaging and calculating the field equations do not commute

G. F. R. Ellis: ``Relativistic cosmology: its nature, aims and problems". In *General Relativity and Gravitation*, Ed B Bertotti et al (Reidel, 1984), 215.

Has implications for cosmology (Kolb, Mataresse, Buchert, Wiltshire, Sussman, et al.)

Contribution to dark energy?

Averaging effects

Metric tensor: $g_{ab} \longrightarrow \hat{g}_{ab} = \langle g_{ab} \rangle$

Inverse Metric tensor: $g^{ab} \longrightarrow \hat{g}^{ab} = \langle g^{ab} \rangle$

but not necessarily inverse …

need correction terms to make it the inverse

Connection: $\Gamma^a{}_{bc} \longrightarrow \langle \Gamma^a{}_{bc} \rangle + C^a{}_{bc}$

new is average plus correction terms

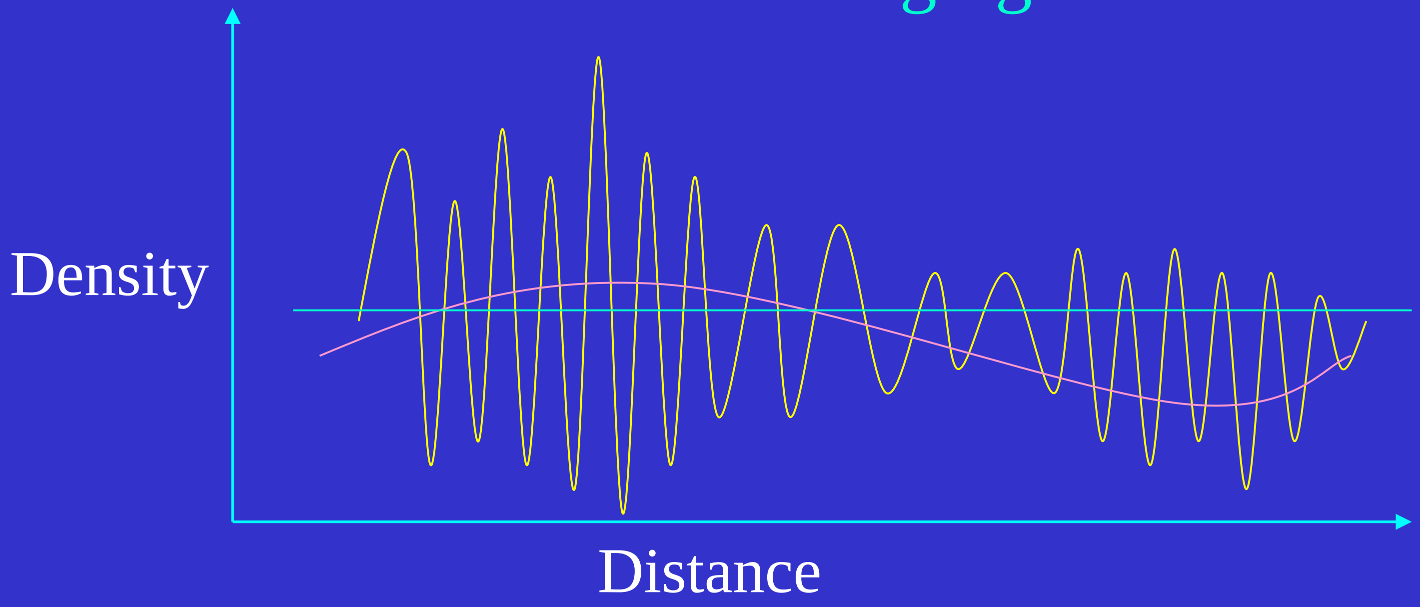
Curvature tensor plus correction terms

Ricci tensor plus correction terms

Field equations $G_{ab} = T_{ab} + P_{ab}$

Cosmology:
Multiple scales of representation of ***same*** system

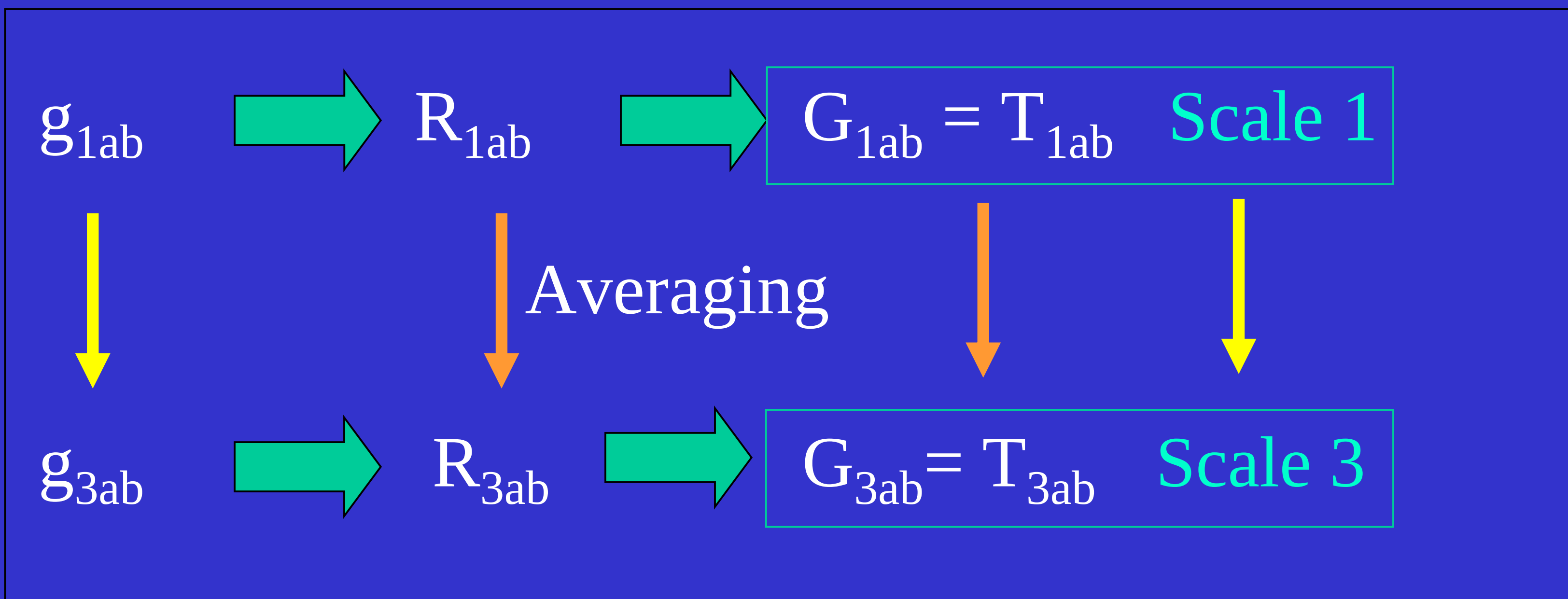
Different averaging scales



Stars, clusters, galaxies, universe

Local inhomogeneity: dynamic effects

Averaging and calculating the field equations do not commute



averaging process

averaging gives different answer

Acceleration due to back reaction from "small scale" inhomogeneities?

Fitting and averaging as related operations

Effect on dynamics?? Occurs – but does it matter?

An ongoing important debate

Modelling and general relativity issues,

Modelling genuinely inhomogeneous models with locally static empty domains in it

- Nature of the Newtonian limit in cosmology
- Domain of validity of quasi-Newtonian coords

Problem of covariant averaging

The problem with such averaging procedures is that they are not covariant. Can’t average tensor fields in covariant way (coordinate dependent results).

They can be defined in terms of the background unperturbed space, usually either flat spacetime or a Robertson--Walker geometry, and so will be adequate for linearized calculations where the perturbed quantities can be averaged in the background spacetime.

But the procedure is inadequate for non--linear cases, where the integral needs to be done over a generic lumpy (non--linearly perturbed) spacetime that are not ``perturbations" of a high--symmetry background. However, it is precisely in these cases that the most interesting effects will occur.

Problem of covariant averaging

* Can use bitensors (Synge) for curvature and matter, but not for metric itself: and leads to complex equations

- R Zalaletdinov "The Averaging Problem in Cosmology and Macroscopic Gravity" *Int. J. Mod. Phys.* **A 23:** 1173 (2008) [arXiv:0801.3256]

* Can average Scalars (Buchert, Coley):

But usually incomplete.

Average basic equations, add Ansatz (Buchert)

so hides some effects

Can we do it for complete set of scalars? (Coley)

The averaging problem in cosmology

Buchert equations for scalars gives modified Friedmann equation

T Buchert "Dark energy from structure: a status report". *GRG Journal* **40:** 467 **(2008) [arXiv:0707.2153].**

Keypoint:

Expansion and averaging do not commute:

in any domain D, for any field Ψ

$$\partial_t\langle\Psi\rangle - \langle\partial_t\Psi\rangle = \langle\theta\Psi\rangle - \langle\theta\rangle\langle\Psi\rangle$$

The averaging problem in cosmology

Buchert equations for scalars gives modified Friedmann and Raychaudhuri equations: e.g.

$$\partial t\langle\Theta\rangle_D = \Lambda - 4\pi G\rho_D + 2\,\langle II\rangle_D - \langle I\rangle_D^2$$

where $II = \Theta^2/3 - \sigma^2$ and $I = \Theta$.

This in principle allows acceleration terms to arise from the averaging process

But relies on an Ansatz for shear evolution:

not the full set of 1+3 equations

Perturbation calculations:

Must be non-linear, need backreaction effects

• What kind of global coordinate system is valid? – doubtful that "quasi-Newtonian" coordinates will do globally (Wald and Ishibara): non-geodesic but conformal

$$ds^2 = -(1+2\Psi)\, dt^2 + a^2(t)\,(1-2\Psi)\, d\sigma^2$$

Truly representing an expanding universe with major voids: locally static on small scale;

50% of global matter density on larger scales

-Take voids seriously! (Wiltshire)

- Note that metric potential may be small but its derivatives not: indeed the latter is essential if it is to represent large density gradients (e.g. as in the Solar System) via EFE

Exact calculations

Krasinski, Ostrowski

Perturbation calculations

Linearised: Many others as discussed here

- Clarkson, Maartens, Umeh
- Clifton, Durrer, Matarrese, Raskanen
- Kaiser, Wald and Green

Systematic schemes

- Roukema
- Buchert
- Wiltshire

Numerical relaticity

- Giblin

N-Body calculations

- Durrer, Adamek

Newtonian calculations

- Fidler, Bertacca

Swiss-cheese (Einstein-Strauss) exact lumpy models

Exact vacuum static domains imbedded in an expanding universe model;

no backreaction! (Birkhoff)

No effect either way

Lattice universes

Lindquist and Wheeler

- No background model: particles and vacuum
- dynamics follows from junction boundary conditions
- ➢ a unique kind of averaging
- ➢ Gives FLRW type expansion

Ferreira and Clifton models

Fleury: there is back reaction effect when we condense fluid to a lattice of point particles in a positively curved universe: formation of structure rescales curvature term

Alternative gravitation

Are we using correct theory of gravity??

- uniqueness of EFE (Lovelock)

[Brax] can get w = -1 [Pace] `designer models'

- Issue of initial value problem

- often equivalent to scalar tensor theories

- Can several scalar fields mimic < -1?

  Or can we get that by adding 2 scalar fields?

- I don't believe so. Example? Proof?

5: Local inhomogeneity: observational effects

Feynmann, Gunn (**Kaiser talk)**

Ricci focusing and Weyl focusing: Ehlers, Sachs, Penrose

B. Bertotti "The Luminosity of Distant Galaxies" ***Proc Royal Soc London.*** **A294, 195 (1966).**

$$d\theta/dv = -R_{ab}K^aK^b - 2\sigma^2 - \theta^2$$

$$d\,\sigma_{mn}/dv = -E_{mn}$$

Θ = expansion

σ = shear

R_{ab} = Ricci tensor, determined pointwise by matter

E_{ab} = Weyl tensor, determined non-locally by matter

Robertson-Walker observations: zero Weyl tensor and non-zero Ricci tensor.

$$d\theta/dv = -R_{ab}K^aK^b - \theta^2$$

$$d\,\sigma_{mn}/dv = 0$$

Actual observations are best described by zero Ricci tensor and non-zero Weyl tensor

$$d\theta/dv = -\,2\sigma^2 - \theta^2$$

$$d\,\sigma_{mn}/dv = -\,E_{mn}$$

This averages out to FRW equations when averaged over whole sky Not obvious! It does not follow from energy conservation (Weinberg) - depends on how area distances average out. But supernova observations are preferentially where there is no matter

Why should it average out?

Weinberg: yes

Ellis Bassett Dunsby: no

Clarkson

Kibble and Lieu

Many others

➔ Kaiser talk and paper with Peacock

Folds and caustics in past light cone

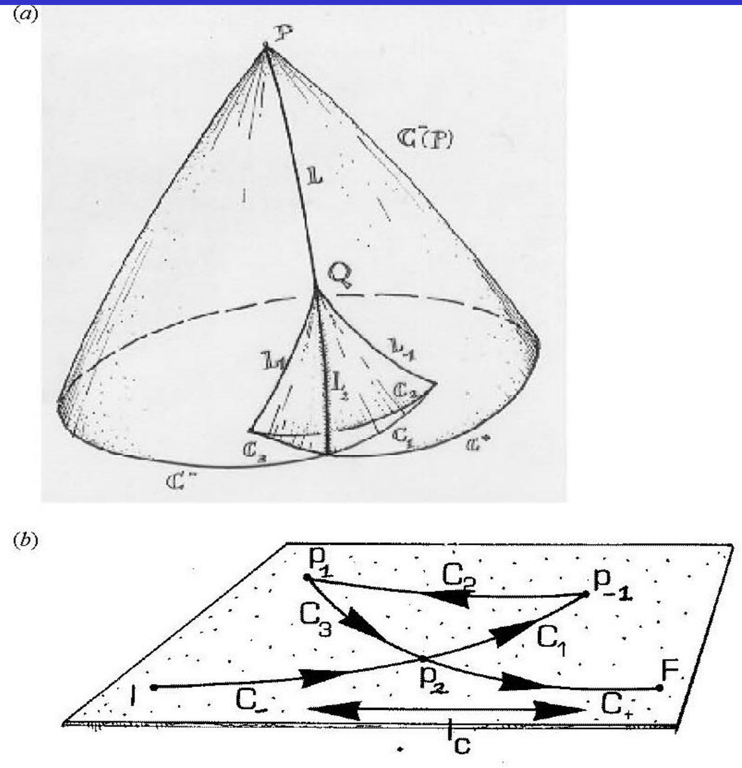


Figure 1. (*a*) A lens L and resulting caustics on the past lightcone $C^-(P)$ (two-dimensional section of the full lightcone), showing in particular the cross-over line L_2 and cusp lines L_{-1}, L_1 meeting at the conjugate point Q. The intersection of the past lightcone with a surface of constant time defines exterior segments C^-, C^+ of the lightcone together with interior segments C_1, C_2, C_3. (*b*) The imaged point moves forward along C_1 from I to the cusp at P_{-1}, backward along C_2 to the cusp at P_1, and then forward along C_3 to F.

CQG 15: 2345 (1998)
Ellis, Bassett, Dunsby

Real past light cone has billions of caustics, hierarchically structured

Observations and averaging

Dyer Roeder equations take matter into account but not shear: allows a fraction of the uniform density

C. C Dyer. & R C Roeder, “Observations in Locally Inhomogeneous Cosmological Models” Astrophysical Journal, Vol. 189: 167 (1974)

NB: must take shear and caustics into account

Note that how this works out depends on how dark matter is clustered. If it is uniform, Dyer-Roeder is good; if dark matter is clustered, it is not so good.

Swiss-cheese (Einstein-Strauss) exact lumpy models can be used to test the observational effects

Exact vacuum static domains imbedded in an expanding universe model; no backreaction! (Birkhoff)

Example: *R. Kantowski "The Effects of Inhomogeneities on Evaluating the mass parameter* Ω_m *and the cosmological constant* Λ*" (1998) [astro-ph/9802208]*

"a determination of Ω_0 made by applying the homogeneous distance--redshift relation to SN 1997ap at z = 0.83 could be as much as 50% lower than its true value."

Lattice universes

Ferreira and Clifton "Archipelagian cosmology: Dynamics and observables in a universe with discretized matter"

Phys. Rev. D **80**, 103503

We consider a model of the Universe in which the matter content is in the form of discrete islands, rather than a continuous fluid. In the appropriate limits the resulting large-scale dynamics approach those of a Friedmann-Robertson-Walker (FRW) universe. The optical properties of such a space-time, however, do not. This incongruity with standard FRW cosmology is not due to the existence of any unexpectedly large structures or voids in the Universe, but only to the fact that the matter content of the Universe is not a continuous fluid.

We see SN in preferred directions

Extremely thin pencil of light rays in vacuum

➔ Else we would not see the SN!

➔ Not a fair sample of the universe

(Mis)interpreting supernovae observations in a lumpy universe

Chris Clarkson, George F. R. Ellis, Andreas Faltenbacher, Roy Maartens, Obinna Umeh and Jean-Philippe Uzan

Mon. Not. R. Astron. Soc. **426,** 1121–1136 (2012)

Outcome depends on clustering of dark matter halos

And so on bias factor (is it constant?)

➔ We do not average over all directions. Preferred directions!!

5. Local inhomogeneity: observational and dynamic effects both occur

Two views: review Clarkson and Maartens: arXiv:1005.2165

Weak field approximation is adequate and shows effect is negligible
- Peebles, Wald, Baumann et al arXiv:1004.2488

Counter claim (Kolb, Wiltshire, Mattarrese):
as there are major voids in the expanding universe a weak-field kind of approximation is not adequate
You have to model (quasi-static) voids and junction to expanding external universe
Maybe: Clarkson, Ananda, Larena: arXiv:0907.3377

Taking both into account: may be enough to bring cosmic concordance into question:
may show universe Is not spatially flat

It exists: BUT is it significant?

Issue: Is the universe well described everywhere by a single linearised coordinate system

- About a FLRW model?

Issue: huge value of $\delta\rho$ can't use linearised theory

➔ Must take non linearities into account

Counter: physics on Earth and Solar system well described by such a system ϕ very small even though second derivative $\delta^2\phi$ is very large

Strong claims

Buchert: influential equations

- - but: needs relation to shear/Weyl tensor
- - needs to relate to N-Body to show acceleration

Wiltshire: timescape

- - very creative
- - but: are effects really that large?

Roukema:

- - issue of virialisation ** key issue **

? Kolb? Matarrese? Ostrowski?

Denials

- Peebles, Rees,
- Kaiser detailed study: but issue of BCs at LSS

- Wald and Green→ Formal theorems Distributional approach, no real averaging

BUT they don't involve averaging

- Ishibashi and Wald

→Assumes one global coordinate system

- **Reply: gr-qc** 1505.07800
- **Is there proof that backreaction of inhomogeneities is irrelevant in cosmology?**
- **Authors:** T. Buchert, M. Carfora, G. F. R. Ellis, E. W. Kolb, M. A. H. MacCallum, J. J. Ostrowski, S. Räsänen, B. F. Roukema, L. Andersson, A. A. Coley, D. L. Wiltshire

Moderate Proposals

Perturbation approaches: yes small

- Clarkson: could work but does not by coincidence
- Clifton: 2-coordinate approach
- Matarrese? Coley? Rasanen? Fleury?

N-body: yes small

- Durrer, Adamek: gives 1% effect on observations

Newtonian+

- Fidler, Bertacca

Issues

1. Differing use of language

   - e.g. what is a velocity?

   - what are the concepts, really?

2. Role of virialization

   - How does it relate to backreaction?

Durrer: 1% effects in (m,z) observations

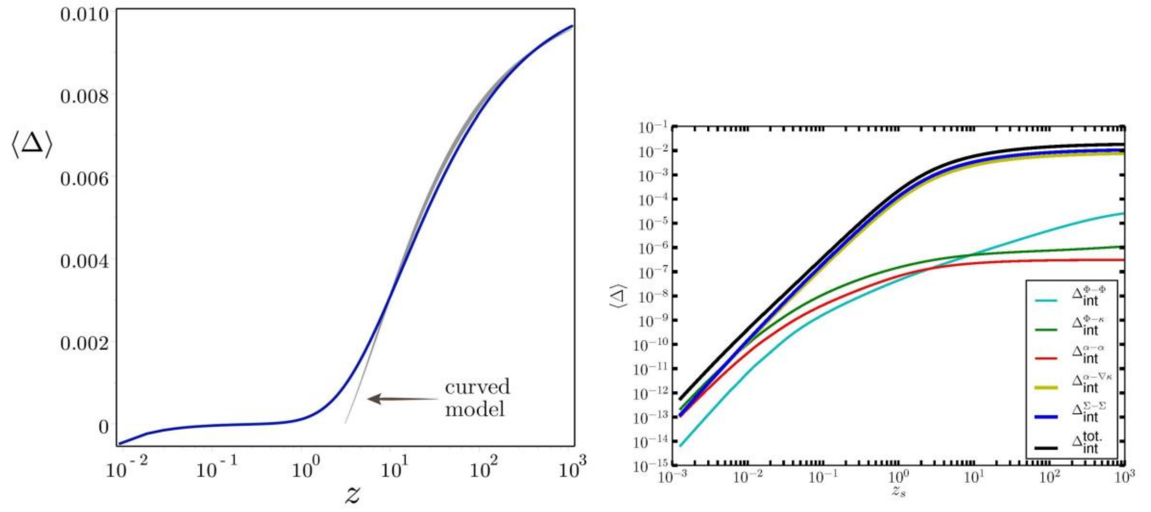


Figure 1: Left: *Fractional correction* $\langle\Delta\rangle(z)$ *to the distance for a fiducial model* $\Omega_m = 0.3, h = 0.68, h^2\Omega_b^2 = 0.0222, w = -1$ *and* $n_s = 0.96$. *The correction is negative for* $z < 0.25$, *purely from the local contribution. At higher redshift the shift arises from the aggregated lensing term. For* $z > 10$ *the corrections grow* $\propto \chi_s^3$, *and are similar to an open* Λ*CDM model with* $\Omega_K^{\rm eff} \approx 0.0066$ *(grey 'curved', shown for high z)* [9]. Right: *The different terms contributing to the distance correction* [8].

Already at first order it was found that the variance of the distance from lensing is of the order of 10^{-3}-10^{-2}, hence much larger than the Bardeen potential $\Psi \approx 10^{-5}$

➔ Cannot be ignored in a precision cosmology era

Real observations

- Key: BOSS, SKA, XMM, Lyα Forest, etc

- Fantastic technology

- Never flux limits:

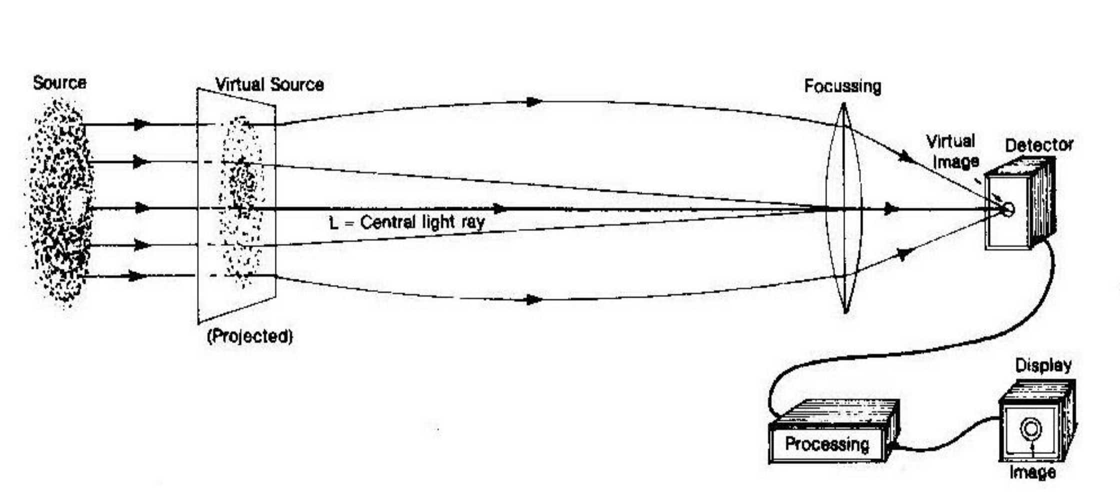
Selection and detection depend on observed (a) surface brightness (b) angular size

⇒ Magnitude + Scale size + redshift/cosmology

Ellis Perry and Sievers *AJ* **89:** 1124 **(**1984)

Never flux limits

•



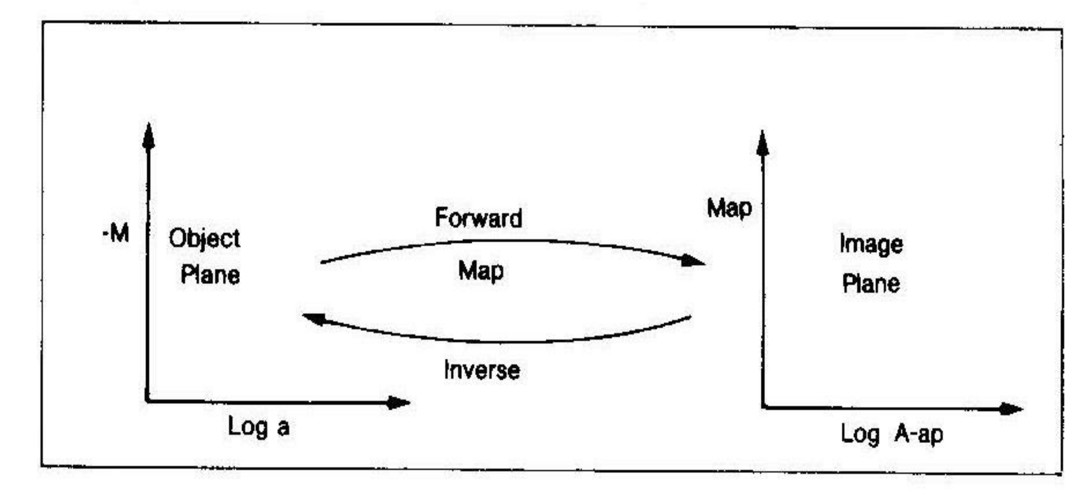
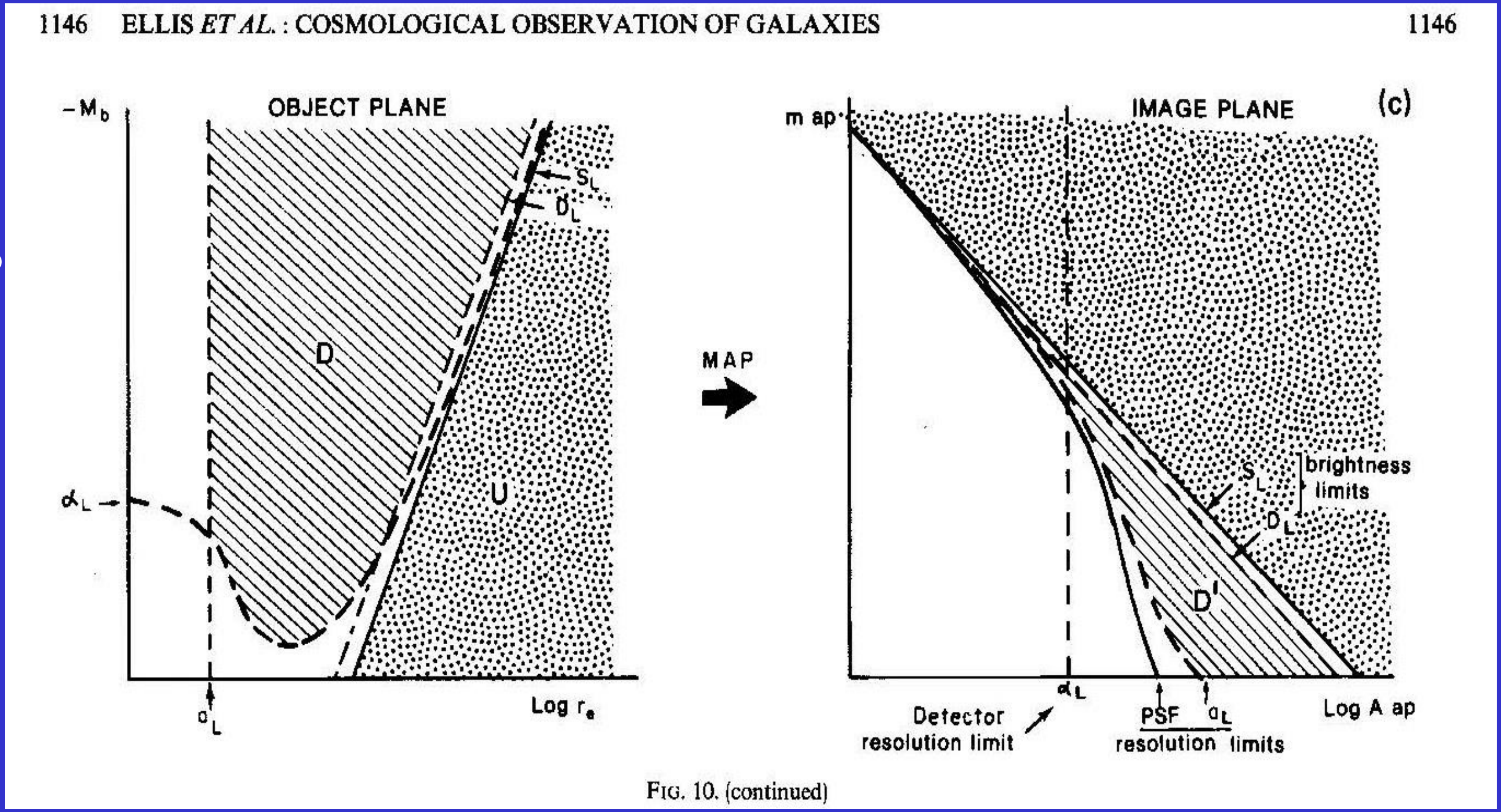


FIG. 2. Observational map from the space of object parameters to the space of image parameters. The source is described by its magnitude M and radius a, while the image is represented by its apparent magnitude M_{ap} and apparent angle A_{ap}. Note that the magnitude scales are inverted in the two planes (the object plane being given with Arp's coordinates).

Never flux limits



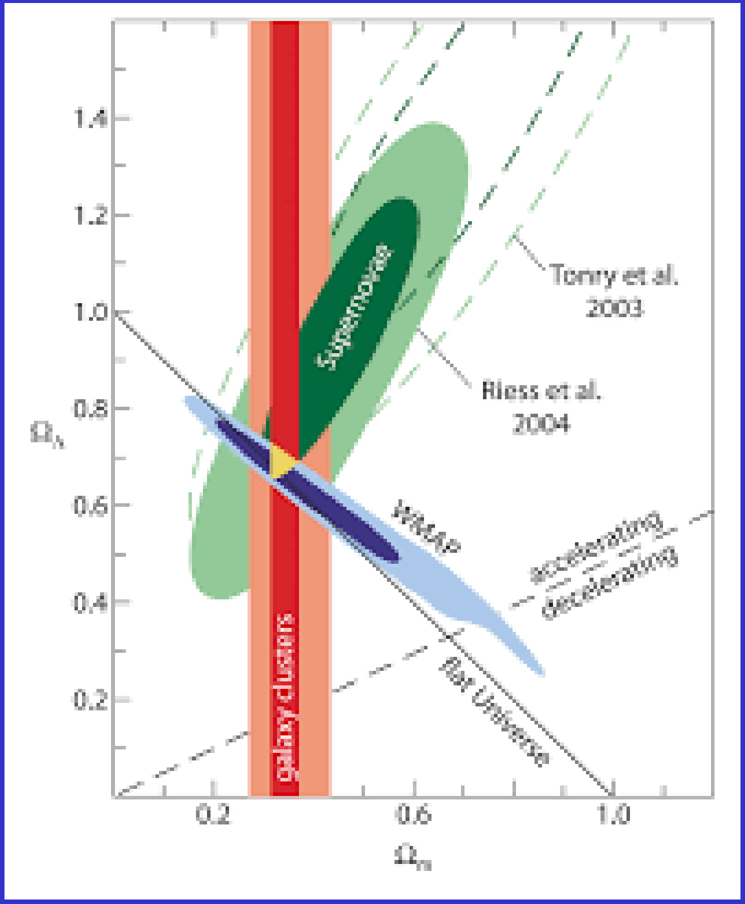
Detection limits in the image plane (right) mapped back into the object plane (left). The area U is unobservable. There are brightness limits, PSF limits, and detector (pixel) limits.

Real observations

- Data compression

- Bayesian Hierarchical Models (Heavens)

Most sensitive tests of cosmology via inhomogeneities ➔ structure formation



Top-down effects of FLRW model on structure formation
➔FLRW parameters can be deduced from the structures that form

Back reaction effects on structure formation??

Most sensitive tests of cosmology

Can back reaction affect structure formation?

➔non-linear feedback: IF it can happen, most powerful effect

➔Yes $\delta\phi$ was 10^{-5}: but these led to structure!

Back-reaction will lead to +ve feedback

Maybe its already there! It IS structure formation!