M2-branes, ADE and Lie superalgebras

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http://www.maths.ed.ac.uk/~jmf/CV/Seminars/Hongo.pdf

This talk is based on

- arXiv:0809.1086 [hep-th] (with PdM, EME, PR)
- arXiv:0908.2125 [hep-th] (with PdM, EME)
- arXiv:0909.1063 [hep-th] (with PdM, SG, EME)

where

PdM = Paul de Medeiros

SG = Sunil Gadhia

EME = Elena Méndez-Escobar

PR = Patricia Ritter

Motivation

After more than 15 years we still have not answered this:

Main question

What is M-theory?

- not a theory of strings!
- a theory of membranes?
- maybe, but quantising membranes is difficult!
- AdS/CFT: try to at least understand dual theory

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- In this talk we will learn some of these words:
  1. we will classify $\mathcal{N} \geq 4$ M2-brane geometries in terms of "ADE with a twist"
  2. we will classify $\mathcal{N} \geq 4$ superconformal Chern–Simons+matter theories in terms of metric Lie superalgebras, or if you prefer, metric triple systems
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The M2-brane solution

Definition

The elementary M2-brane:

$$g = H^{-\frac{2}{3}} ds^2(\mathbb{R}^{2,1}) + H^{\frac{1}{3}} ds^2(\mathbb{R}^8)$$
$$F = \mathrm{dvol}(\mathbb{R}^{2,1}) \wedge dH^{-1},$$

where

$$H = \alpha + \frac{\beta}{r^6},$$

for $\alpha, \beta \in \mathbb{R}$ not both equal to zero.

It is half-supersymmetric for .

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It is half-supersymmetric for $\alpha\beta \neq 0$.

Asymptotia

- $\beta \to 0$ (or $r \to \infty$):

$$(g, F) \to (ds^2(\mathbb{R}^{10,1}), 0)$$

  $\therefore$ Minkowski vacuum

- $\alpha \to 0$ (or $r \to 0$):

$$H^{\frac{1}{3}} ds^2(\mathbb{R}^8) = H^{\frac{1}{3}}(dr^2 + r^2 ds^2(S^7)) \to \beta^{\frac{1}{3}} \frac{dr^2}{r^2} + \beta^{\frac{1}{3}} ds^2(S^7)$$

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Killing superalgebra

- Every supersymmetric supergravity background has an associated Lie superalgebra, generated by the Killing spinors: the Killing superalgebra
  FO (1999), FO+Meessen+Philip (2004)
- For $\mathrm{AdS}_4 \times S^7$ it is $\mathfrak{osp}(8|4)$
- The even subalgebra is

$$\mathfrak{so}(8) \oplus \mathfrak{sp}(4, \mathbb{R}) \cong \mathfrak{so}(8) \oplus \mathfrak{so}(3, 2),$$

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- The Killing superalgebra is isomorphic to the conformal superalgebra of the dual theory
- Now $\mathfrak{so}(3,2)$ is the conformal algebra of $\mathbb{R}^{2,1}$ and $\mathfrak{so}(8)$ is the R-symmetry algebra
- In general, three-dimensional conformal field theories admit realisations of the conformal superalgebras $\mathfrak{osp}(\mathcal{N}|4)$, with R-symmetry $\mathfrak{so}(\mathcal{N})$, for $\mathcal{N} \leqslant 8$.
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- field equations $\Longrightarrow$ M is Einstein
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(Notice: here supersymmetry $\Longrightarrow$ field equations)

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Bär's cone construction

Question

Which manifolds admit real Killing spinors?

- The metric cone of a riemannian manifold (M, g_M) is the manifold $C = \mathbb{R}^+ \times M$ with metric $g_C = dr^2 + r^2 g_M$
  e.g., the metric cone of the round sphere S^n is $\mathbb{R}^{n+1} \setminus \{0\}$
- (M, g_M) admits real Killing spinors if and only if (C, g_C) admits parallel spinors — Bär (1993)
- If M is complete, then C is either irreducible or flat — Gallot (1979)

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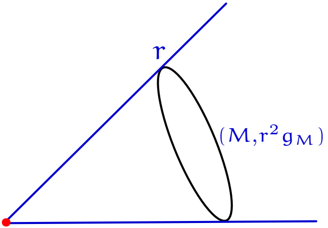

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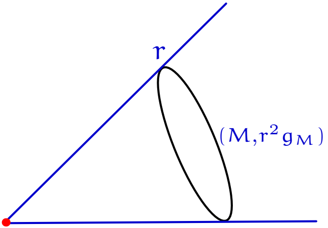

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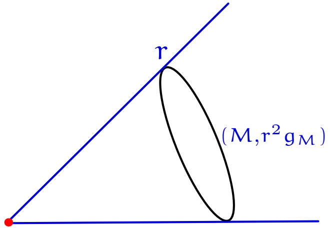

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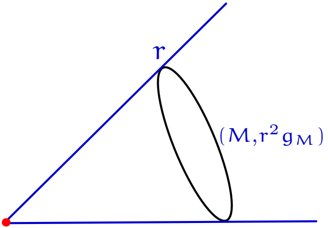

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Irreducible holonomies

Simply-connected 8-manifolds with parallel spinors:

$\mathscr{N}$	Cone holonomy	7-dimensional geometry
8	$\{1\}$	sphere
3	$\mathrm{Sp}(2)$	3-Sasaki
2	$\mathrm{SU}(4)$	Sasaki-Einstein
1	$\mathrm{Spin}(7)$	weak G_2 holonomy

M. Wang (1989)

So generalised supersymmetric M2-brane solutions describe M2 branes at a conical singularity in an 8-manifold with special holonomy.

$\mathscr{N} > 3$ and sphere quotients

- To obtain $8 > \mathscr{N} > 3$ we need to consider quotients S^7/Γ, for $\Gamma \subset \mathrm{SO}(8)$ such that
  - Γ acts freely on S^7 (so that S^7/Γ is smooth)
  - Γ lifts to Spin(8) (for S^7/Γ to be spin)
  - Γ leaves some chiral spinors invariant (for supersymmetry)
- It turns out there is an ADE classification... with a twist!

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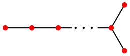
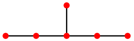
ADE subgroups of $\mathrm{Sp}(1)$

Dynkin diagram	Label	Name	Order
	A_n	cyclic	$n+1$
	$D_{n\geqslant 4}$	binary dihedral	$4(n-2)$
	E_6	binary tetrahedral	24
	E_7	binary octahedral	48
	E_8	binary icosahedral	120

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The $\mathcal{N} > 3$ classification

The backgrounds $\mathrm{AdS}_4 \times M^7$ with $\mathcal{N} > 3$ are those with $M = S^7/\Gamma$ with $\Gamma \subset \mathrm{SO}(8)$ given by pairs (ADE, τ):

$\mathcal{N}$	Groups Γ
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6	$A_{n \geqslant 2}$
5	$D_{n \geqslant 4}, E_6, E_7, E_8$
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If $\tau = 1$ we don't write it and ν is the unique nontrivial outer automorphism of $E_{7,8}$. (The ones in red were not known.)

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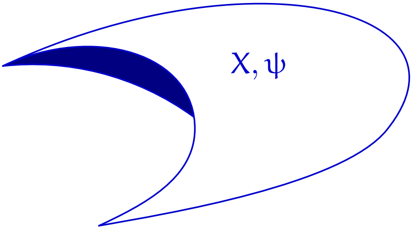
1 M2-branes and AdS/CFT

2 M2-brane geometries and ADE

3 Superconformal Chern–Simons theories

4 Triple systems and Lie superalgebras

(Supersymmetric) M2-brane degrees of freedom



(all fields in $\mathbb{R}^{2,1}$)

- X: real scalars corresponding to transverse excitations
- ψ: real 2-component spinors
- A: $\mathfrak{g}$-valued Chern–Simons gauge fields

- X, ψ are in a unitary representation $\mathfrak{M}$ of the metric Lie algebra $\mathfrak{g}$; that is, $\mathfrak{g}$ has an invariant inner product

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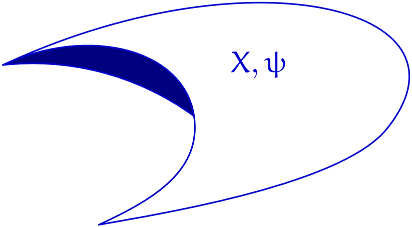

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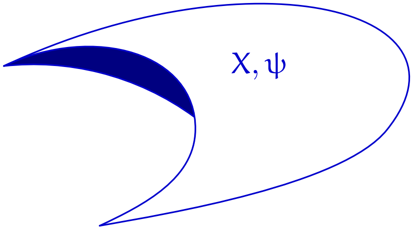

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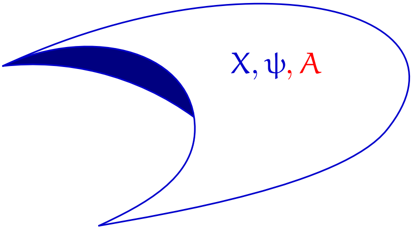

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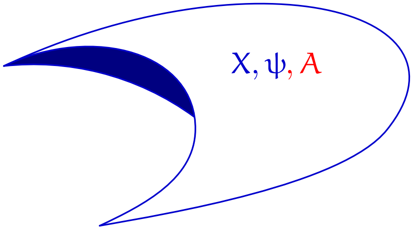

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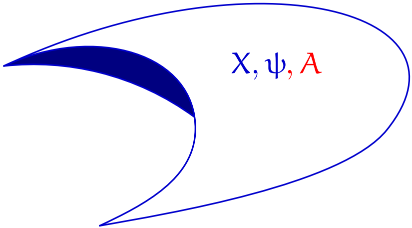

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Superconformal theories in 3 dimensions

- The superconformal algebra of $\mathbb{R}^{2,1}$ is $\mathfrak{s} := \mathfrak{osp}(\mathscr{N}|4)$ with $\mathscr{N} \leqslant 8$,

$$\mathfrak{s}_0 = \mathfrak{so}(\mathscr{N}) \oplus \mathfrak{so}(3,2) \qquad \text{and} \qquad \mathfrak{s}_1 = \mathbb{R}^{\mathscr{N}} \otimes \mathbb{R}^4$$

- The R-symmetry algebra is $\mathfrak{so}(\mathscr{N})$, under which
  - the supercharges transform as vectors [illegible]
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  - so do the fermions [illegible]
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Spinor representations

$\mathscr{N}$	1	2	3	4
$\mathfrak{so}(\mathscr{N})$		$\mathfrak{u}(1)$	$\mathfrak{sp}(1)$	$\mathfrak{sp}(1) \oplus \mathfrak{sp}(1)$
spinors	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H} \oplus \mathbb{H}$

$\mathscr{N}$	5	6	7	8
$\mathfrak{so}(\mathscr{N})$	$\mathfrak{sp}(2)$	$\mathfrak{su}(4)$	$\mathfrak{so}(7)$	$\mathfrak{so}(8)$
spinors	$\mathbb{H}^2$	$\mathbb{C}^4$	$\mathbb{R}^8$	$\mathbb{R}^8 \oplus \mathbb{R}^8$

Matter representations

- The degrees of freedom of any physical theory are fundamentally real
- This determines the type (i.e., $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$) of the matter $\mathfrak{g}$-representation $\mathfrak{M}$ in terms of the type of the R-symmetry representation:
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- If $\mathscr{N} = 2, 6$ we have to take both real and imaginary parts of the fields
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Matter representations

- The degrees of freedom of any physical theory are fundamentally real
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$\mathscr{N} \leqslant 3$ theories

For $\mathscr{N} \leqslant 3$ theories, the matter representation $\mathfrak{M}$ is not constrained beyond its type:

- $\mathscr{N}=1$ theory = $\mathfrak{M} \in \mathrm{Rep}(\mathfrak{g}, \mathbb{R})$ + quartic $\mathfrak{g}$-invariant superpotential
- $\mathscr{N}=2$ theory = $\mathfrak{M} \in \mathrm{Rep}(\mathfrak{g}, \mathbb{C})$ + quartic $\mathfrak{g}$-invariant F-term superpotential
- $\mathscr{N}=3$ theory = $\mathfrak{M} \in \mathrm{Rep}(\mathfrak{g}, \mathbb{H})$!
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$\mathcal{N}$	Matter representation	Remarks
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5	$\Delta^{(5)} \otimes W$	$W \in \mathrm{Irr}(\mathfrak{g}, \mathbb{H})_{\mathrm{aLTS}}$
6	$\Delta_{\pm}^{(6)} \otimes V \oplus \Delta_{\mp}^{(6)} \otimes \overline{V}$	$V \in \mathrm{Irr}(\mathfrak{g}, \mathbb{C})_{\mathrm{aJTS}}$
8	$\Delta_{\pm}^{(8)} \otimes U$	$U \in \mathrm{Irr}(\mathfrak{g}, \mathbb{R})_{\mathrm{3LA}}$

- $\Delta_{(\pm)}^{(\mathcal{N})}$ = ±-chirality (if applicable) spinor of $\mathfrak{so}(\mathcal{N})$
- Irr = irreducible representations
- The subscripts aLTS, aJTS and 3LA stand for certain types of metric triple systems or equivalent for certain types of Lie-embeddable unitary representations (see later)

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Some remarks

- For $\mathcal{N} \geqslant 5$, irreducible representations decouple
- $\mathcal{N} = 7$ theories are automatically $\mathcal{N} = 8$
- Representation theory uniquely determines $\mathcal{N} \geqslant 3$ theories
- $\mathcal{N} \geqslant 4$ theories can be defined in terms of 3-algebras or triple systems, as in the original BLG model
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1 M2-branes and AdS/CFT

2 M2-brane geometries and ADE

3 Superconformal Chern–Simons theories

4 Triple systems and Lie superalgebras

Superalgebras from representations

Slogan

When a Lie algebra admits an invariant inner product, its unitary representations give rise to superalgebras.

- The superalgebra consists of two subspaces
  1. A metric Lie algebra $\mathfrak{g}$ in degree 0
  2. A unitary representation V in degree 1
- and three products
  1. $\mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ is the Lie bracket on $\mathfrak{g}$
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  3. $V \times V \to \mathfrak{g}$ (or $\to \mathfrak{g}_{\mathbb{C}}$) is defined so that the natural inner product on the superalgebra is invariant; equivalently it is the transpose of the $\mathfrak{g}$-action on V
- If the Jacobi identity holds, V is Lie-embeddable
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Triple systems from superalgebras

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The odd subspace of a superalgebra is a triple system.

- $\mathfrak{g} \oplus V$ a superalgebra
- We define a 3-bracket on V by

$$[u, v, w] := [[u, v], w]$$

  where the RHS brackets come from the superalgebra
- For the $\mathscr{N} \geqslant 4$ theories, the relevant triple systems are
  - metric 3-Lie algebras for $\mathscr{N} > 6$
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The odd subspace of a superalgebra is a triple system.

- $\mathfrak{g} \oplus V$ a superalgebra
- We define a 3-bracket on V by

$$[u, v, w] := [[u, v], w]$$

  where the RHS brackets come from the superalgebra
- For the $\mathscr{N} \geqslant 4$ theories, the relevant triple systems are
  - metric 3-Lie algebras for $\mathscr{N} > 6$
  - metric anti-Jordan triple systems for $\mathscr{N} = 6$
  - symplectic anti-Lie triple systems for $\mathscr{N} = 4, 5$

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Metric Lie algebras

- Let $\mathfrak{g}$ be a Lie algebra with an invariant inner product $(-,-)$, not necessarily positive-definite.
- Let (X_i) be a basis for $\mathfrak{g}$:

$$[X_i, X_j] = f_{ij}{}^k X_k \qquad \text{and} \qquad (X_i, X_j) = \kappa_{ij}$$

- Invariance means that $f_{ijk} = f_{ik}{}^\ell \kappa_{\ell k}$ is totally skewsymmetric
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Real unitary representations of a metric Lie algebra

- Let $\mathfrak{U} \in \mathsf{Rep}(\mathfrak{g}, \mathbb{R})$ be a real unitary representation with $\mathfrak{g}$-invariant inner product $\langle -, - \rangle$
- Let (e_a) be a basis for $\mathfrak{U}$ and let $\langle e_a, e_b \rangle = g_{ab}$.
- For every $X_i \in \mathfrak{g}$ we have

$$X_i \cdot e_a = e_b T_i{}^b{}_a$$

- Unitarity means that $T_{iab} = T_i{}^c{}_a g_{bc} = -T_{iba}$

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Classification

Theorem

There is precisely one (up to scale) irreducible positive-definite metric 3-Lie algebra.

(Conjectured in FO+Papadopoulos (2003))
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It follows from the classification of simple 3-Lie algebras
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The metric 3-Lie algebra

- $\mathfrak{g} = \mathfrak{so}(4) \cong \mathfrak{sp}(1)_{-k} \oplus \mathfrak{sp}(1)_k$, with subscripts indicating the multiple of the Killing form
- A priori $k \in \mathbb{R}$ but it is quantised in the quantum theory
- $U = \mathbb{R}^4$ of $\mathfrak{so}(4)$, or $U = \mathbb{H}$ with $\mathfrak{sp}(1) \oplus \mathfrak{sp}(1)$ acting by left- and right-multiplications
- $F_{abcd} = k^{-1} \varepsilon_{abcd}$
- The resulting superconformal Chern–Simons theory is the original BLG model

Bagger+Lambert (2006,2007)
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Complex unitary representations

- Let $V \in \mathrm{Rep}(\mathfrak{g}, \mathbb{C})$ with hermitian inner product $g_{a\bar{b}}$
- On the 3-graded superspace $\overline{V} \oplus \mathfrak{g}_{\mathbb{C}} \oplus V$ define

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$\iff \mathscr{N} = 6$ supersymmetry!
- The sesquibilinear 3-bracket $V \times \overline{V} \times V \to V$

$$[e_a, e_{\bar{b}}, e_c] := [[e_a, e_{\bar{b}}], e_c]$$

turns V into anti-Jordan triple system

Faulkner+Ferrar (1980)

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Classification

- Indecomposable $\mathcal{N} = 6$ theories $\iff$ $V \in \mathrm{Irr}(\mathfrak{g}, \mathbb{C})_{\mathrm{aJTS}}$
- V irreducible $\iff$ embedding Lie superalgebra is simple
  PALMKVIST, FO (2009)
- Two classes of simple 3-graded complex (metric) Lie superalgebras: $A(m,n)$ and $C(n+1)$ KAC (1977)
- $\mathcal{N} = 6$ theories have been classified by other means
  SCHNABL+TACHIKAWA (2008)
- The $A(n,n)$ theory is the ABJM model
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Quaternionic unitary representations

- There are no quaternionic Lie algebras
- so we think of $W \in \mathrm{Rep}(\mathfrak{g}, \mathbb{H})$ as a complex unitary representations with a quaternionic structure:
  - a complex antilinear map $J : W \to W$ obeying $J^2 = -1$
  - J is $\mathfrak{g}$-equivariant
  - J is compatible with the hermitian inner product: $g(Ju, Jv) = g(v, u)$
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$$\omega(u, v) := g(u, Jv)$$

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Symplectic anti-Lie triple systems

- On $\mathfrak{g}_{\mathbb{C}} \oplus W$ we define

$$[e_a, e_b] = \kappa^{ij} T_i{}^c{}_a \omega_{cb} X_j, \quad [X_i, e_a] = T_i{}^b{}_a e_b, \quad [X_i, X_j] = f_{ij}{}^k X_k$$
$$(e_a, e_b) = \omega_{ab} \qquad (X_i, X_j) = \kappa_{ij}$$

- Let $G_{abc}{}^d := \kappa^{ij} T_i{}^e{}_a \omega_{eb} T_j{}^d{}_c$. Then

$$\text{Jacobi} \iff G_{abc}{}^d + G_{bca}{}^d + G_{cab}{}^d = 0 \iff \mathcal{N} = 5!$$

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A comment on the $\mathscr{N}=4$ theories

- Indecomposable $\mathscr{N}=4$ theories of Gaiotto+Witten (2008) — those with matter representations $\Delta_{\pm}^{(4)} \otimes W$ — are also classified by irreducible symplectic anti-Lie triple systems; equivalently by simple metric complex Lie superalgebras with quaternionic odd subspace
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- We classified complete, smooth $\mathcal{N} \geqslant 4$ Freund–Rubin backgrounds $\mathrm{AdS}_4 \times M^7$ by
  1. showing that $M = S^7/\Gamma$ for some freely acting $\Gamma \subset \mathrm{SO}(8)$
  2. classifying such Γ in terms of pairs (ADE, τ) consisting of an ADE subgroup of $\mathrm{Sp}(1)$ together with an automorphism
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Further results

- `arXiv:0908.2125 [hep-th]` contains more results!
- The representation theory also explains naturally the mechanism of supersymmetry enhancements:
  1. $\mathcal{N} = 4$ to $\mathcal{N} = 5$
  2. $\mathcal{N} = 5$ to $\mathcal{N} = 6$
  3. $\mathcal{N} = 6$ to $\mathcal{N} = 8$, in particular why $\mathcal{N} = 7$ implies $\mathcal{N} = 8$
- In the paper we also construct the theories starting from the representation-theoretic data; in particular, we give explicit expressions for the superpotentials
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  2. $\mathcal{N} = 5$ to $\mathcal{N} = 6$
  3. $\mathcal{N} = 6$ to $\mathcal{N} = 8$, in particular why $\mathcal{N} = 7$ implies $\mathcal{N} = 8$
- In the paper we also construct the theories starting from the representation-theoretic data; in particular, we give explicit expressions for the superpotentials
- That in itself is worth another seminar, and luckily PAUL and ELENA will be visiting IPMU from October (^_^)

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