Instanton wave and M-wave in multiple M5-brane system

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In collaboration with Chong-Sun Chu (NTHU)

This talk is based on

- Chu-Isono arXiv:1305.6808

references :

- Chu-Ko arXiv:1203.4224
- Chu-Ko-Vanichchapongjaroen arXiv:1207.1095
- Chu-Vanichchapongjaroen arXiv:1304.4322

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- For single M5, there have been various proposals for M5-brane action.
- For multiple M5, WE DON'T KNOW !

Why M5-brane is mysterious

- M5-brane : (2,0) supersymmetry (#=16) with tensor multiplet

  scalars : for transverse directions in 11D : 5

  2-form tensor field B_2 with self-duality $H_3 = *_6 H_3$: 3

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- I am not sure how much the action will be of practical use.

equation of motion rather than action

- But, equations of motion may still be useful since classical solutions will describe non-perturbative excitations on M5-branes.
- Such solutions would correspond to known configurations of BPS objects in M-theory. This would be a nontrivial check of M5-brane EoM.

What we have done

- We consider low energy equations of motion of non-abelian self-dual gauge field on multiple M5-branes proposed by Chu-Ko [1203.4224]

- We found a new exact solution, which is a wave with instanton configuration as the amplitude.

- We argue that this solution corresponds to M-wave/M5 system.

self-dual non-abelian tensor gauge fields and EoM

- problem : interactions needed $H_3 = dB_2 + ??$ cf. $F_2 = dA + A^2$

- proposal [Chu-Ko 1203.4224] :
  —— generalisation of Perry-Schwarz

  2-form gauge field $B_{\mu\nu}$ + 1-form Yang-Mills field A_μ

- coordinate dependence :

$$B_{\mu\nu}(x^\mu, x^5), \;\; A_\mu(x^\mu) \quad (\mu, \nu = 0, 1, \cdots, 4)$$

- 1-form Yang-Mills field A_μ has to be auxiliary

self-dual non-abelian tensor gauge fields and EoM

- covariant derivative $D(A)_\mu B_{\nu\lambda} := \partial_\mu B_{\nu\lambda} + [A_\mu, B_{\nu\lambda}]$
- field strength $H_{\lambda\mu\nu} := D_\lambda B_{\mu\nu} + D_\mu B_{\nu\lambda} + D_\nu B_{\lambda\mu}$
- proposal for equations of motion [Chu-Ko 1203.4224]

  self-duality + F as an auxiliary field

$$\widetilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu} \qquad F_{\mu\nu} = \int dx^5 \, \widetilde{H}_{\mu\nu}$$

$$(\widetilde{H}_{\mu\nu} = \frac{1}{6}\epsilon_{\mu\nu\rho\sigma\tau} H^{\rho\sigma\tau}, \; F = dA + A^2)$$

self-dual non-abelian tensor gauge fields and EoM

- All fields are in the adjoint rep. of YM gauge group G
- The group G turns out to be the gauge group of YM theory on D4-branes, which is the dim. reduction of the M5-branes.
- The system has tensor gauge symmetry too.
- When $G = \mathrm{U}(1)$, the equation of motion is $\widetilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}$, which is Perry-Schwarz's EoM for single M5.

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  M2-M5 [Chu-Ko-Vanichchapongjaroen, Chu-Vanichchapongjaroen]

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- In this talk, I would like to present solutions for MW-M5

Classical Solutions and F

- M2-M5 solutions : $F =$ Wu-Yang monopole
- Let's try to find a solution based on $F =$ 4D YM instanton solution

Instanton wave solution : [Chu-HI 1305.6808]

$$\widetilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu} \qquad F_{\mu\nu} = \int dx^5 \, \widetilde{H}_{\mu\nu}$$

- ansatz : $A_0 = B_{0a} = 0, \quad \partial_0 A_a = 0 \quad (a, b, \cdots = 1, 2, 3, 4)$

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- **Instanton wave solutions**

$$B_{ab} = F_{ab}(x^c) f(x^0 \pm x^5)$$

4D self-dual field strength $F_{ab} \longleftrightarrow f(x^0 + x^5)$

4D anti-self-dual field strength $F_{ab} \longleftrightarrow f(x^0 - x^5)$

and $f(\infty) - f(-\infty) = 1$.

MW-M5 ? : dimensional reduction to IIA string

- dim. reduced EoM $\longrightarrow$ identity $\boldsymbol{D^{\mu} F_{\mu\nu} = -\pi R \epsilon_{\nu\alpha\beta\gamma\delta}[F^{\alpha\beta}, B^{\gamma\delta}]}$

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- According to string theory, the instanton is realized by **D0 on D4**
- The D0-D4 is exactly the **dimensional reduction of MW-M5**

MW-M5 ?: D0-branes and KK momentum

- instanton number of F $=$ the number of $D0$

  due to WZ term of D4 action : $\int C_1 \wedge \mathrm{tr}(F \wedge F)$

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  Translating the WZ term in our M5-brane language, we get

$$\int \boldsymbol{C_1} \wedge \mathbf{tr}(\boldsymbol{F} \wedge \boldsymbol{F}) \propto \int \boldsymbol{g_{\mu 5}}\, \mathbf{tr}(\boldsymbol{H}^{\boldsymbol{\mu\alpha\beta}} \boldsymbol{H}^{\mathbf{5}}{}_{\boldsymbol{\alpha\beta}}) \equiv \int \boldsymbol{g_{\mu 5}} \boldsymbol{T}^{\boldsymbol{\mu 5}} \in \boldsymbol{S}_{\mathbf{M5}}$$

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- We can show $T_{M5} \equiv \mathrm{tr}(H_{M\mu\nu} H_5{}^{\mu\nu})$ is conserved : $\partial_M T^{M5} = 0$

  conjecture : this is (lowest order of) the energy-momentum tensor.

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- **the number of D0** $\sim \int T_{05} \sim$ **KK momentum of MW on M5**

  This is the relation which is one of the motivations of M-theory

discussions and open problems

- inclusion of other matter fields & supersymmetrisation
- coupling to background C-field $\longrightarrow$ noncommutative geometry ??
  Fact : If all [,] in 6D EoMs are replaced by [,] with the Moyal product with $\theta_{\mu\nu} = C_{\mu\nu 5}$, dim. reduction yields 5D YM eqn with the Moyal product.
- other solutions for BPS M-theory objects such as MW-M2-M5
- relation to 4D SYM theories, e.g. relation to AGT
- 4D YM eqn has integrability : ADHM, Nahm
  We have shown our SD relation also has solutions based on ADHM, Nahm $\longrightarrow$ our SD relation also has integrability ?

evidence of the EoMs 4 : degrees of freedom

$$\widetilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu} \qquad F_{\mu\nu} = \int dx^5 \, \widetilde{H}_{\mu\nu}$$

- d.o.f counting in the "free level"

  $\Longrightarrow$ the EoMs become a collection of abelian SD : $*_5 dB = \partial_5 B$

- d.o.f is $3 \times \mathbf{dim}(G)$ ["3" from abelian SD B (Perry-Schwarz)]

- Problem : separation of free part from interacting parts would be subtle.

properties indicating MW-M5 : 2. supersymmetry

1. If we assume covariant SUSY transformation

$$\delta\Psi = H^{LMN}\Gamma_{LMN}\epsilon + (\text{ other matters })$$

then the solution yields $1/2$-BPS condition $(1 - \Gamma^0\Gamma^5)\epsilon = 0$.