Manifestations of the D1-D5 system

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Abstract

These lectures provide an overview of the correspondence between gravity and field theory in the D1-D5 system. The notes are based on lectures given at the International Workshop on "Recent Advances in String Theory", Graduate School of Mathematical Science, The University of Tokyo, Japan, December 1999.

Contents

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1 Introduction

The D1-D5 system has catalyzed a lot of recent progress in string theory, beginning with Strominger and Vafa's calculation of extremal black hole entropy using the elliptic genus of a certain two dimensional conformal field theory [1]. Their result was extended to near extremal black holes [2, 3], and to other near extremal brane systems, such as D3-branes [4]. On the other hand, Banks, Fischler, Shenker, and Susskind proposed [5] that the quantum mechanics of D0-branes contains a description of the gravitational dynamics of M-theory. Maldacena [6] gave a vast generalization of this idea, conjecturing that many brane systems can be decoupled from gravity; and the dynamics of string theory in certain situations can be described by a gauge theory (or some appropriate generalization) whose dynamics does not contain intrinsically gravity within it. This circle of ideas has come to be known as the AdS/CFT correspondence.

In this lecture, we revisit the D1-D5 system to give a brief overview of the AdS/CFT correspondence. In section 2, we describe this brane system and the dynamics on it, and then present the basic dual description as a solution to general relativity in the low energy limit [7, 8]. According to the Maldacena conjecture [6], these two different descriptions are contained in the moduli space of a single theory, and we check in section 3 that quantities calculated on both sides agree. In particular, we illustrate the correspondence of (super) conformal symmetry [9] and the BPS states [10, 11, 12]. Then we consider some more intricate dynamics, that of wrapped branes and the moduli space of vacua of the theory [13]; these provide some rather sophisticated information about the correspondence. Finally in section 4, we discuss some uses of the correspondence [14]. The field theory is a non-linear sigma model on the moduli space of instantons in the gauge theory, *i.e.* the target space of the sigma model is the moduli space of solutions to the instanton equations in the gauge theory. There are a lot of singularities coming from the instantons shrinking to zero size. We will be able to use the correspondence to obtain some information about the singularities using the dual language.

A note to students: the references below are intended as a representative guide to further reading, rather than an exhaustive compendium of the extant literature, or an authoritative genealogy of ideas. For that, the reader is referred to [7].

2 Outline of the correspondence

2.1 Field theory on the branes

We consider the system consisting of Q_1 D1-branes and Q_5 D5-branes. The directions transverse to both such branes are $x_\perp = x_6, \ldots, x_9$, the common direction is x_5 and the directions parallel to the D5-branes and transverse to the D1-branes are $x_{\|} = x_1, \ldots, x_4$. We compactify the system on T^4 in the $x_{\|}$ direction with the coordinate size Σ_i, $i = 1,2,3,4$ and on a circle (radius R) in the common direction x_5. In this situation, there are a number of different types of strings we can consider, the open strings which stretch from the D1-branes back to themselves and the open strings which purely attach to the D5-branes, *etc.*.

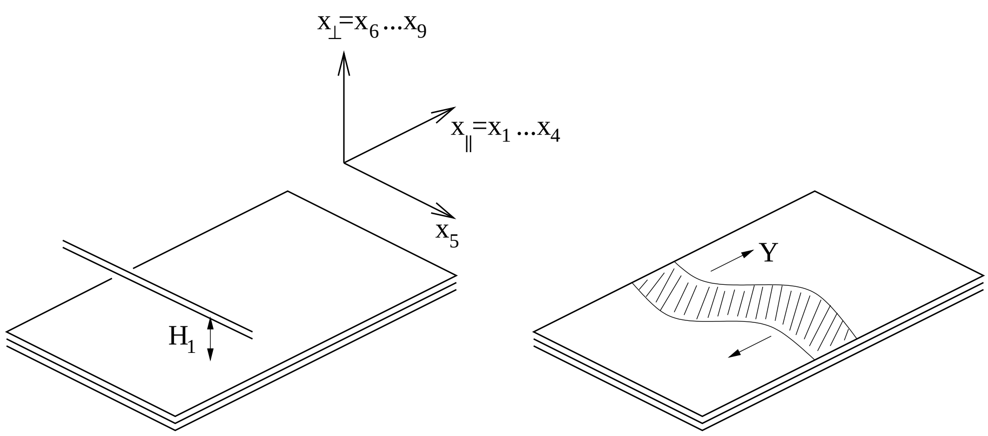
The lowest mass states of the open string sector consist of the following field content. The strings that stretch between D1-branes and D5-branes are the quanta of 1-5 hypermultiplet fields Y which belong to the $(Q_1, \bar{Q}_5)$ representation of the gauge group (and their conjugates). Also there are 1-1 strings that describe the motion of the D1-branes along $x_\perp$, described by a hypermultiplet H_1 in the adjoint representation of $U(Q_1)$. There is a similar field of 5-5 strings, a $U(Q_5)$ adjoint hypermultiplet H_2 describing the motion of the D5-branes along $x_\perp$.

There are also vector multiplets. The gauge dynamics on the D1-branes involves the $U(Q_1)$ vector multiplet V_1; the scalars in the vector multiplet describe the motions of the D1-branes along $x_{\|}$. And among the 5-5 strings, there is the $U(Q_5)$ vector multiplet V_5 giving the gauge dynamics of the D5-branes.

In the end, we are going to simplify the system dramatically. In general, the gauge dynamics is some 5+1 dimensional gauge theory which couples to some kind of 1+1 dimensional defects. We want to take a limit where the directions parallel to the D5-branes, transverse to the D1-branes (that is Σ_i) are to be the size of string scale l_s and this scale is going to zero, while the other scale R is to be fixed with respect to the energy scale of interest. For instance, $E \cdot R$ is held fixed, whereas $E \cdot \Sigma_i$ is taken to zero. In this limit, the dynamics is effectively reduced to 1+1 dimensions.

Now in the 1+1 dimensional dynamics, scalar fields do not have expectation values; the scalar fields fluctuate and in contrast to the higher dimensional gauge theory, what we talk about are the regions of the configuration space where the wave function is concentrated on large H_1 or large Y. These are called branches. There is one branch, where the separations between D1-branes and D5-branes have non zero values, conventionally called the "Coulomb branch"; $H_1 \neq 0$ parametrizes the separation of D1-branes from D5-branes. The other branch is obtained by bringing the D1-branes on top of the D5-branes. As the energy cost of string stretching between them goes to zero, fields Y fluctuate dramatically and tend to smear the wave function of the D1-brane. This is conventionally called the "Higgs branch" and $Y \neq 0$.

According to Douglas [15], if we look at the equations for zeros of the scalar potential, those are the same as the "ADHM equations" for Q_1 instantons in $U(Q_5)$ gauge theory on the T^4. In addition to the zero momentum sector, we can of course allow the solution to these equations to slowly oscillate. For instance, the location of the instanton can move a



(a) Coulomb branch

(b) Higgs branch

Figure 1: The Coulomb branch and Higgs branch configurations.

little bit in the $x_{||}$ directions as we run along x_5. Thus we get 1+1 dimensional fields and the low energy dynamics of the Higgs branch is a non-linear sigma model whose target space is the instanton moduli space. To motivate this a little bit, we should note that the Yang-Mills coupling $g_{\rm YM}$ of the D5-brane gauge theory is related to the parameters of the string theory (string coupling g_s and string length l_s) by $g_{\rm YM}^2 \sim g_s l_s^2$, and this is the same as the inverse tension of a D1-brane. Moreover, the instanton is a codimension 4 object in the gauge theory, thus it is point-like in four directions; therefore the instanton in 5+1 dimensional gauge theory is a solitonic string whose tension is determined by the Yang-Mills coupling.

The dimension of the instanton moduli space is $4Q_1Q_5$ up to order one corrections (which will be discussed later). The instanton moduli space is a hyperKähler space and the brane dynamics is supersymmetric. Thus the sigma model on the hyperKähler space has four left moving and four right moving supersymmetries and such a sigma model has vanishing beta-function. The resulting infrared conformal field theory has central charge $c_{IR} = 6Q_1Q_5$.

There are of course many mixed branches. For instance, one can make some bound states (Q_1', Q_5') separated in the transverse directions from another stack of the branes with (Q_1'', Q_5'') so long as the charges are conserved. This is called a "mixed Coulomb-Higgs branch", where some of the components of Y get nontrivial values and some of the components of H_1 get nontrivial values compatible with the moduli space of the low energy theory.

2.2 Gravity solution

On the closed string side, D-branes serve as sources for gravity. The supergravity

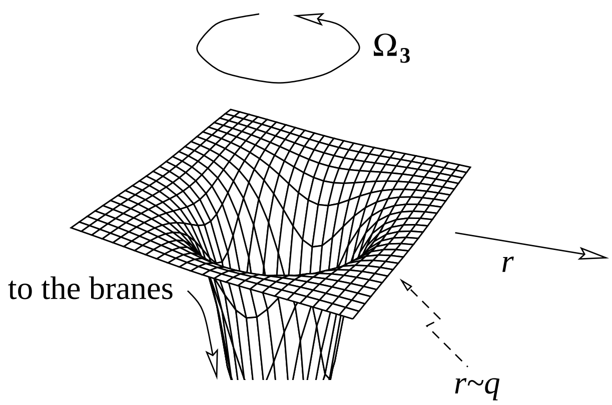


Figure 2: A rough picture of the geometry in the directions transverse to the branes. The D-branes serve as sources for gravity.

solution corresponding to this source (see Figure 2) is given by

$$ds^2 = (H_1H_5)^{-\frac{1}{2}}(-hdt^2 + dx_5^2) + \left(\frac{H_1}{H_5}\right)^{\frac{1}{2}} d\vec{x}_{||}^{\,2} + (H_1H_5)^{-\frac{1}{2}}(h^{-1}dr^2 + d\Omega_3^2), \qquad (1)$$

where the harmonic functions are given by

$$H_i = 1 + \left(\frac{q_i}{r}\right)^2 \quad i = 1,5; \qquad h = 1 - \left(\frac{r_0}{r}\right)^2 . \qquad (2)$$

In the near-extremal limit, the charge radii of the branes q_i, $i = 1,5$ are given approximately by

$$q_1^2 \simeq g_s l_s^6 Q_1/V_4, \quad q_5^2 \simeq g_s l_s^2 Q_5, \qquad (3)$$

where $V_4 = \Sigma_1\Sigma_2\Sigma_3\Sigma_4$ is the coordinate volume of T^4 and r_0 is the horizon radius. Allowing the branes to be slightly non-extremal, one obtains a non-trivial h and the geometry is the corresponding black D-brane solution.

Now there are a lot of parameters, the two charge radii q_i, $i = 1,5$, the horizon radius r_0, various coordinate lengths Σ_i of parallel directions and the radius R of x_5 direction. Thus we have many ways of taking scaling limits, namely what parameters to hold fixed as we send l_s to zero. Here we consider the scaling limit described by Maldacena [6]:

$$l_s \to 0, \quad \text{with } g_6 = \frac{g_s l_s^2}{V_4^{1/2}}, \ \frac{\Sigma_i}{l_s}, \ ER, \ E\frac{l_s^2}{r} \ \text{ fixed}, \qquad (4)$$

where we denote by E the energy scale.[2] In this limit, energy scale is fixed relative to R, while the coordinate size Σ_i is going to zero. Thus the momenta along the other four directions possess more and more energy and decouple. Therefore the effective dynamics

[2]Another consistent limit is $l_s \to 0$, with g_6, Σ_i/R, and ER held fixed; here the dynamics is 5+1d – we keep all the box sizes fixed relative to the instanton string tension. This limit is relevant to the DLCQ limit of fivebranes [16, 17].

becomes 1+1 dimensional. Moreover, going down the throat, due to the redshift one does not have enough energy to explore the things that are far away and the dynamics essentially decouples from the asymptotically flat region of the spacetime.

In this limit, we can neglect 1's in H_i, $i = 1, 5$ ($r \ll q_i$ and so we are far down the throat in figure 2) and the metric (1) becomes

$$\frac{ds^2}{l_s^2} = (g_s^2 Q_1 Q_5)^{-\frac{1}{2}} \left(\frac{r}{l_s^2}\right)^2 (-hdt^2 + dx_5^2) + \left(\frac{Q_1}{Q_5}\right)^{\frac{1}{2}} \left[\frac{d\vec{x}_{\|}^2}{V_4^{1/2}}\right] + (g_6^2 Q_1 Q_5)^{\frac{1}{2}} \left[h^{-1}\left(\frac{dr}{r}\right)^2 + d\Omega_3^2\right]. \tag{5}$$

The geometry is locally $AdS_3 \times S^3 \times T^4$ and the radius of S^3 is $R_{AdS} = l_s(g_6^2 Q_1 Q_5)^{1/4}$ and the characteristic proper size of T^4 is $l_s(Q_1/Q_5)^{1/4}$.

The analogue on the gravity side of the mixed Coulomb-Higgs branch is the multi-centered solution, where we write the harmonic function as

$$H = \sum_\alpha \frac{q_\alpha^2}{|\vec{x} - \vec{x}_\alpha|^2}, \quad h = 0, \tag{6}$$

where $\vec{x} = \vec{x}_\perp$. Then to keep all of these multi-center collections talking to each other, we need to make sure that the energy cost associated to communicating a piece of information from one to the other does not go to infinity in the scaling limit. Therefore for all α and β we also need to take

$$|\vec{x}_\alpha - \vec{x}_\beta| \to 0, \quad \text{like} \quad l_s^2 E \tag{7}$$

in the limit. We can also expect approximate non-extremal solutions for $r_0^{(\alpha)} \ll |\vec{x}_\alpha - \vec{x}_\beta|$ for all β.

The next thing we should ask is where this solution becomes a valid approximation to string theory. One thing we should worry about is whether the curvature of the geometry ever becomes of order the string scale. At this scale, we will expect α' corrections to the gravity equations. This scale (curvature $\sim l_s^{-2}$) is called the correspondence point. Then from the radius of curvature (the radius of S^3), we can see $g_6^2 Q_1 Q_5 \sim 1$ at the correspondence point and we can trust the supergravity when $g_6^2 Q_1 Q_5 \gg 1$. On the other hand, the perturbative sigma model field theory description should be a good description when $g_6^2 Q_1 Q_5 \ll 1$.

We should also worry about whether the string coupling is sufficiently small and thus the effective description needs $g_s \leq 1$. The volume of T^4 is Q_1/Q_5 in the string units and the coupling g_6 is proportional to the string coupling divided by the square root of the volume of T^4 and therefore the condition $g_s \leq 1$ becomes

$$g_6 \leq \sqrt{\frac{Q_5}{Q_1}}\,. \tag{8}$$

If this condition is not satisfied, we should perform an S-duality transformation and a D-string becomes a fundamental string and a D5-brane becomes a NS5-brane.

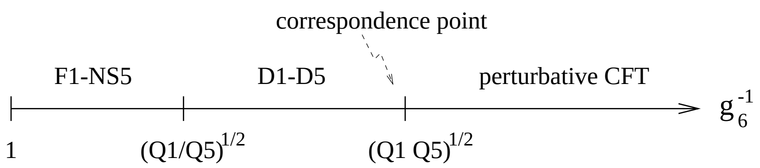


Figure 3: The valid description for each region of g_6^{-1}.

Finally, there is an U-duality symmetry that takes this solution into itself. This is the automorphism $g_6 \to 1/g_6$; the specific transformation is $ST_{1234}S$, which preserves Q_1 and Q_5. The S-duality takes D1-D5 to F1-NS5, the T-duality does not affect these charges, and S-duality just takes us back.

Thus we can draw a picture of what the theory looks like as a function of the parameter g_6^{-1} (see Figure 3). $g_6^{-1} = 1$ is the point at which the theory is taken back to itself under the above U-duality, and in the region $1 < g_6^{-1} < \sqrt{\frac{Q_1}{Q_5}}$ there is a low energy description in terms of F-strings and NS5-branes as the background charges. In the region $\sqrt{\frac{Q_1}{Q_5}} < g_6^{-1} < \sqrt{Q_1 Q_5}$, a D1-D5 supergravity solution is valid and beyond that there is a regime where the perturbative sigma model should be a valid description, and thus $g_6^{-1} \sim \sqrt{Q_1 Q_5}$ is the correspondence point.

Indeed, g_6^{-1} is the characteristic scale, namely (the square root of) the volume of the sigma model on the instanton moduli space (see section 3.3). The sigma model perturbation theory is the expansion in the volume of the target space in the units of the string scale. Thus the perturbative sigma model is valid at large volume (large g_6^{-1}) precisely as seen in Figure 3.

Unfortunately this parameter is only a small part of the full parameter space of the sigma model. We know that the moduli space of type II string theory compactified on the T^4 is

$$\Gamma \setminus SO(5,5;\mathbf{R})/SO(5) \times SO(5), \tag{9}$$

where $\Gamma = SO(5,5;\mathbf{Z})$ is the duality group. Thus one of the things we will try to do is to extend the descriptions of the sigma model to this full moduli space. As claimed by Maldacena, what we will see is that the gravitational description in terms of supergravity and the field theoretical description are different dual descriptions of the same physics.

3 Checks of the correspondence

3.1 Conformal symmetry

When we take the limit (4), the near horizon geometry is $AdS_3 \times S^3 \times T^4$ on the gravitational side and we have a (4,4) superconformal sigma model on the other side of the correspondence. Thus the question is whether we can find the infinite dimensional conformal algebra of this two dimensional field theory on the gravitational side.

First of all, $AdS_3 \times S^3$ is isomorphic to the group manifold $SL(2,\mathbf{R}) \times SU(2)$ and that has left and right actions of the group. Thus the global symmetry is

$$[SL(2,\mathbf{R}) \times SU(2)]_L \times [SL(2,\mathbf{R}) \times SU(2)]_R. \tag{10}$$

This is the global bosonic part of infinite dimensional superconformal algebra and the supersymmetric completion is the supergroup $SU(1,1|2)$ (just the AdS_3 version of the $SU(2,2|4)$ for D3-branes).

We can also investigate the local part [9, 18, 19, 20]. We deform the asymptotic metric in the AdS_3 part as

$$ds^2_{AdS_3} \sim \left(\frac{r}{R_{AdS}}\right)^2 (-dudv) + R^2_{AdS}\left(\frac{dr}{r}\right)^2 + \frac{6}{c}T(u)du^2 + \frac{6}{c}T(v)dv^2, \tag{11}$$

where we use the natural light cone coordinates on t and x_5

$$u = t + x_5, \quad v = t - x_5. \tag{12}$$

First two terms are the AdS_3 parts of the metric (5) and the last two terms are the deformations which are sub-leading in the expansion in r/R_{AdS}. $T(u)$ is the analytic function of u and $c = 6Q_1Q_5$. Similarly for $T(v)$ in the obvious fashion.

This metric is invariant under the following transformations [20]

$$\begin{aligned} u &= f(u'), && (13)\\ r &= r'(\partial' f)^{-1/2}, && (14)\\ v &= v' - \tfrac{1}{2}(r')^{-2}\frac{\partial'^2 f}{\partial' f}, && (15) \end{aligned}$$

and most importantly

$$T'(u') = T(u)(\partial' f)^2 - \frac{c}{12}\{f, u'\}. \tag{16}$$

This is exactly the anomalous Virasoro transformation law. After a little work, one can indeed show that $T(u)$ satisfies Poisson bracket algebra which is the infinite dimensional conformal algebra with the central charge $c = 6Q_1Q_5$.

From this correspondence of the sub-leading term in the metric with the modes of Virasoro algebra (after Fourier transformation in u), let us identify

$$L_0 = \tfrac{1}{2}(E + P_5), \quad \bar{L}_0 = \tfrac{1}{2}(E - P_5), \tag{17}$$

where L_0 and $\bar{L}_0$ are zero modes of T and $\bar{T}$; E is the energy and P_5 is the momentum along the x_5 direction. There is a nice matching, once we allow the solution to be non-extremal [18]. The non-extremality in the metric appears as the sub-leading term in powers of r/R_{AdS} in the coefficient of $(dt)^2$ and $(dx_5)^2$. One can check that if we convert the quantity r_0 in equation 5 into the expression of energy E, the following is true. The entropy of the black hole solution given by the proper area of the horizon in the metric given as

$$S = \frac{\text{Area}}{4\pi G_N} = 2\pi\left(\sqrt{\frac{c}{6}L_0} + \sqrt{\frac{c}{6}\bar{L}_0}\right). \tag{18}$$

This expression can be compared with the conformal field theory for the asymptotic density of the states $\rho(E, P_5)$ [1, 2, 3]. Therefore the entropy is

$$S \sim \log[\rho(E, P_5)]. \tag{19}$$

Due to the Cardy's formula [21], one finds that this is exactly the same expression for the asymptotic density of the states as in the conformal field theory.

Note that everything in this subsection is at lowest order in the semi-classical expression of supergravity. The quantum corrections of supergravity are in powers of $1/Q_1Q_5 \sim (l_p/R_{AdS})^4$ in terms of the six dimensional Planck scale, and so the entropy of the supergravity solution should be regarded as an approximate expression in the expansion in terms of $1/Q_1Q_5$.

3.2 BPS states

The next thing to check is that the perturbations of the geometry from any of the 256 modes of supergravity match the corresponding deformations of the conformal field theory [10, 11, 12].

On the geometrical side, AdS_3 is the $SL(2, \mathbf{R})$ group manifold. Therefore the wave operators (Laplacian) involve the quadratic Casimir of $SL(2, \mathbf{R})$

$$L^2 = \frac{1}{2}(L_1L_{-1} + L_{-1}L_1) - L_0^2, \tag{20}$$

and similar for $\bar{L}^2$. The lowest energy state for a given mass is the primary state in the language of the two dimensional conformal field theory. That is the state which satisfies

$$L_1\psi = \bar{L}_1\psi = 0, \quad L_0\psi = h\psi, \quad \bar{L}_0\psi = \bar{h}\psi. \tag{21}$$

In terms of that data, $h + \bar{h}$ is related to the mass of supergravity field via

$$h + \bar{h} = 1 + \sqrt{m^2 R_{AdS}^2 + 1} \tag{22}$$

with $h - \bar{h} = s = AdS_3$ spin. Of course we can then act with $L_{-1}^n \bar{L}_{-1}^m$, which generate the Fourier modes of AdS_3 waves for the given mass and the spin.

The perturbative field theory and the geometry are appropriate descriptions in different parts of the moduli space, and therefore we have to compare the quantities that are invariant across the moduli space. Thus we should look at the states that are protected from getting corrections as a function of the moduli and these are the BPS states. The BPS condition for the superconformal symmetry is simply that the left moving $SU(2)$ spin is the left moving dimension and the right moving $SU(2)$ spin is the right moving dimension:

$$j_{SU(2)} = h, \qquad \bar{j}_{SU(2)} = \bar{h} \ . \tag{23}$$

What we would like to do is to compare this with the field theory side.

Now we need to know something about the moduli space of instantons. On the quantum field theory side, the moduli space of Q_1 instantons in $U(Q_5)$ gauge theory on T^4 has the topology

$$(T^4)^{Q_1 Q_5}/S_{Q_1 Q_5} \times T^4, \tag{24}$$

which is the analogue of the moduli space of instantons used in [1]. The extra T^4 is just coming from the Wilson lines of the overall $U(1)$ in $U(Q_5)$ on the T^4. In general, the non-linear sigma model is not metrically that orbifold space but that is good enough because in the sigma model, BPS states come from the cohomology of the target space. Therefore the BPS states are in one-to-one correspondence with the orbifold cohomology of the target space. One can show that the quantum numbers of the Kaluza-Klein reduction of supergravity on $AdS_3 \times S^3 \times T^4$ are exactly matched by this cohomology.

We roughly prove this statement as follows. First of all, the cohomology of the T^4 in the tensor product has the following facts:

- i) T^4 has 16 cohomology elements.
- ii) BPS states which are known as ultrashort multiplets of $SU(1,1|2)$ have 16 elements = 8 bosons + 8 fermions;

and then the other piece of the cohomology is that of the symmetric product:

- iii) Generators of $S_{Q_1 Q_5}$ are cycles of length n, $2 \leq n \leq Q_1 Q_5$.

Therefore we can take any of the cohomology states of i), and each is a short-multiplet ii). Thus we have $16 \times 16 = 256$ states. These are identified with the 256 fields of supergravity. The generator of a cycle of length n in the orbifold carries $SU(2)_R$ R-charge $j = n/2$. But R-charge of $SU(2)_R$ is just the angular momentum on the S^3 in the target space geometry and fills out the Kaluza-Klein modes on S^3 (recall that $L_{-1}^n \bar{L}_{-1}^m$ fills out the modes on AdS_3).

Upshot: All supergravity states with any $SL(2,\mathbf{R}) \times SU(2) \simeq AdS_3 \times S^3$ quantum numbers have the quantum numbers found in the sigma-model with the target space $(T)^{Q_1 Q_5}/S_{Q_1 Q_5}$.

Now you might be wondering that in our list of states we found only a match between the BPS states of the field theory and the states of supergravity for Fourier modes which have no excitation on the T^4. That is, we got the states with arbitrary quantum numbers on $AdS_3 \times S^3$ but not states with arbitrary quantum numbers on the T^4. The reason for that is the translations on the T^4 are not global parts of a current algebra extension of the superconformal group.

Therefore the next task is to resolve that discrepancy. In fact, there are a lot of BPS states which have the quantum numbers of not only Kaluza-Klein modes on the T^4 but of the branes that we can wrap on the T^4.

3.3 Wrapped branes

There is more to string theory on $AdS_3 \times S_3 \times T^4$ than we have reproduced so far. What about supergravity modes with momentum on the T^4, and various branes wrapping on the T^4? Thus we would like to discuss the structure of BPS states of K.K. momentum and wrapped branes on the T^4 [13].

The point is that such states are BPS before taking the scaling limit. But we are interested in what happens after taking the limit :

$$\Sigma_i \sim l_s \to 0 \quad , \quad R \gg \Sigma_i. \tag{25}$$

Let us write down the list of objects which carry nontrivial quantum numbers on the internal space:

$$\begin{aligned} M_{K.K} &= \frac{1}{\Sigma_i},\ \frac{1}{R} \qquad M_{F1} = \frac{\Sigma_i}{l_s^2},\ \frac{R}{l_s^2} \\ M_{D1} &= \frac{\Sigma_i}{g_s l_s^2},\ \frac{R}{g_s l_s^2} \qquad M_{D3} = \frac{\Sigma_i\Sigma_j\Sigma_k}{g_s l_s^4}\left(= \frac{v_4 \epsilon^{ijkl}}{g_s \Sigma_l}\right),\ \frac{\Sigma_i\Sigma_j R}{g_s l_s^4} \\ M_{D5} &= \frac{R v_4}{g_s l_s^2} \qquad M_{N5} = \frac{R v_4}{g_s^2 l_s^2}. \end{aligned} \tag{26}$$

where $v_4 \equiv \Sigma_1\Sigma_2\Sigma_3\Sigma_4/l_s^4$. Let us see how things scale in the limit. We can classify their behavior into 3 types : $M \sim l_s^0,\ l_s^{-1},\ l_s^{-2}$.

- There are 10 objects (wrapped branes) which are heaviest (scale as l_s^{-2}). There are 10 tensor fields in 6D supergravity (IIB/T^4) and they should couple to strings. Thus we can identify these 10 objects as such strings before we further compactify on S^1, which is obvious from the fact that $M \propto R$.
- The number of objects (wrapped branes and K.K. momentum) which scale as l_s^{-1} is 16. They are point like in 6D. They couple to 16 vector fields in 6D maximal supergravity.

- The one object which scales as l_s^0 is the K.K. momentum on x^5. This is the momentum in the sigma model, therefore its energy is finite. It is also momentum in $AdS_3 \times S_3$ and it couples to the graviton.

Now we consider the D1-D5 system as a background. If we try to add other charges of the heaviest type, we would change the vacuum in the low energy theory. Thus we would like to consider only perturbations of D1-D5 system and their energy scales as $\sim l_s^{-1}$.

Then we have the following puzzle. These excitations in isolation have naively infinite energy in the scaling limit and are invisible in the Hilbert space of the low energy theory. But that neglects the fact that they form a bound state with the background branes and the binding energy can cancel the large bare energy. To see that this is the case, let us consider first of all the momentum and winding. The total energy of the D1-D5 system carrying K.K. momentum and winding is

$$l_s M = \left[\left(Q_1 \frac{R}{l_s g_s} + P_5 \frac{l_s}{R}\right)^2 + \sum_{i=1}^{4}\left(w^i \frac{\Sigma_i}{g_s l_s} + P_i \frac{l_s}{\Sigma_i}\right)^2\right]^{\frac{1}{2}} + Q_5 \frac{R v_4}{g_s l_s}. \tag{27}$$

Further if we subtract the background energy, we find

$$\begin{aligned} l_s E &= l_s M - Q_5 \frac{R v_4}{g_s l_s} - Q_1 \frac{R}{l_s g_s} \\ &\simeq P_5 \frac{l_s}{R} + \frac{l_s}{2Q_1 R} \sum_{i=1}^{4}\left(w^i \frac{\Sigma_i}{\sqrt{g_s} l_s} + P_i \frac{\sqrt{g_s} l_s}{\Sigma_i}\right)^2. \end{aligned} \tag{28}$$

Note that E is finite in the scaling limit. If we multiply by $\frac{R}{l_s}$, it becomes [3]

$$RE = P_5 + \frac{1}{2Q_1}\left(\sqrt{g_s}\vec{P} + \frac{1}{\sqrt{g_s}}\vec{w}_{D1}\right)^2, \tag{29}$$

and it is obvious that l_s disappears in this equation.

Further we can add other excitations because if we perform a T-duality on the T^4, it exchanges Q_1 for Q_5, $\vec{w}_{D1}$ for $\vec{w}_{D3}$ and $\vec{P}$ for $\vec{w}_{F1}$. Thus we obtain[4]

$$RE = P_5 + \frac{1}{2Q_1}\left(\sqrt{g_s}\vec{P} + \frac{1}{\sqrt{g_s}}\vec{w}_{D1}\right)^2 + \frac{1}{2Q_5}\left(\sqrt{\frac{g_s}{v_4}}\vec{w}_{F1} + \sqrt{\frac{v_4}{g_s}}\vec{w}_{D3}\right)^2. \tag{30}$$

We should note the following two points.

- This is a BPS formula and is exact in the scaling limit.
- We can measure the radius of the sigma model target from the above energy formula by comparing it to the standard form of energy in string theory; we find $\frac{\Sigma_i}{\sqrt{g_s} l_s} \sim g_6^{-\frac{1}{2}}$. This verifies the previous claim from section 2.2 regarding the volume of the sigma model target.

[3] Here we defined $\vec{P} = \frac{l_s}{\Sigma_i} P_i$, $\vec{w}_{D1} = \frac{\Sigma_i}{l_s} w_i$.

[4] Note that $g_s \to \frac{g_s}{v_4}$ under the T-duality.

In this way, we have shown that all of the previous 16 charged objects have finite energy and therefore we should find them in the dual sigma model.

As discussed in [22], one can also show in supergravity that if we look at the gauge field that couples to each charge and do the dimensional reduction down to AdS_3, we have the following Chern-Simons terms in the Lagrangian for the 16 U(1) vectors

$$\pm Q_{1,5} \int_{AdS_3} A \wedge dA. \tag{31}$$

These generate level Q_1, Q_5 current algebras on the boundary of AdS_3 as

$$U(1)^{16} \quad \rightarrow \quad U(1)^8_L \times U(1)^8_R. \tag{32}$$

The wrapped branes are part of the BPS structure. But once we take the scaling limit, not all of these excitations couple to the superconformal algebra. These states carry $U(1)^{16}$ current algebra energy, but they carry no angular momentum on the S^3 and have energy $h \neq j_{SU(2)}$ (not BPS !).

BPS states do not depend on the moduli of the CFT. But we here have much more detailed information about the moduli space, because the energetics of $U(1)$ current algebra depends on the moduli.

We can also look at non-extremal black holes. The energy cost of U(1) current algebra and angular momentum excitations is subtracted from the total energy, and the entropy of the remaining excitations is reduced. Thus the entropy formula is

$$S \sim 2\pi \sqrt{\frac{c}{6}\left(h - h_{U(1)} - \frac{j^2}{4Q_1Q_5}\right)} + 2\pi \sqrt{\frac{c}{6}\left(\bar{h} - \bar{h}_{U(1)} - \frac{\bar{j}^2}{4Q_1Q_5}\right)}, \tag{33}$$

where $h_{U(1)}, \bar{h}_{U(1)}$ is the $U(1)^{16}$ current algebra part of the energy and $j, \bar{j}$ is the angular momentum on S^3. This entropy is related to the density of states as

$$S \;=\; log\ \rho[E, P_5, \vec{q}_{U(1)}, j, \bar{j}]. \tag{34}$$

To construct corresponding solutions in supergravity would be a useful exercise as a further check of the correspondence.

If we look at the extremal case and set the right moving energy to zero, then it is easy to check that the result agrees with the $E_{6(6)}$ invariant extremal black hole entropy formula [23].

Let us now return to the sigma model description. As we have seen, there should be a $U(1)^8_L \times U(1)^8_R$ current algebra in that CFT. If we were to assume that the target space of the sigma model is indeed metrically $(T^4)^N/S_N \times T^4$ ($N = Q_1Q_5$), we can get 8 U(1) level Q_1Q_5 current algebras from the diagonal T^4 in the symmetric product and 8 U(1) level 1 current algebras from the extra T^4. Then we have a puzzle because the previous mass formula (30) and Chern-Simons terms in supergravity (31) predict that the level of each U(1) current algebra is Q_1 or Q_5 and the spectrum is naively only compatible with the sigma model for $Q_5 = 1$.

But all of the different choices (Q'_1, Q'_5) $(Q'_1 Q'_5 = N)$ are related to $Q_5 = 1$ by duality transformations, and therefore the above statement is too naive. Later we will return to this point.

3.4 The moduli space of vacua

The next task is to investigate the structure of the moduli space for this CFT. The moduli space of type IIB string on the T^4 is given in (9). The generic charge vector $\vec{q}$ is given in terms of the 10 heavy background charges as

$$\vec{q} = (f_1, n_5, Q_1, Q_5, d_3^{ij}) \quad . \tag{35}$$

Further we can define

$$\vec{q}^{\,2} = f_1 n_5 - Q_1 Q_5 - d_3^{ij} d_3^{kl} \epsilon^{ijkl}, \tag{36}$$

which is invariant under the U-duality group $SO(5,5;\mathbf{Z})$.

It is sufficient for our purposes to concentrate on the four dimensional subspace $d_3^{ij} = 0$. Then we have a nice representation of the remaining charges as follows

$$\mathbf{Q} = \begin{pmatrix} f_1 & Q_1 \\ Q_5 & n_5 \end{pmatrix} \qquad \vec{q}^{\,2} = \det\ \mathbf{Q} \quad . \tag{37}$$

A subgroup $SO(2,2;\mathbf{Z}) = SL(2,\mathbf{Z})_L \times SL(2,\mathbf{Z})_R$ of the whole U-duality group acts on $\mathbf{Q}$ as

$$\mathbf{Q} \ \rightarrow \ g_L \ \mathbf{Q} \ g_R^{-1} \quad . \tag{38}$$

Using this action, we can transform as follows [5]

$$\mathbf{Q} = \begin{pmatrix} 0 & Q_1 \\ Q_5 & 0 \end{pmatrix} \ \rightarrow \ \hat{\mathbf{Q}} = \begin{pmatrix} 0 & Q_1 Q_5 \\ 1 & 0 \end{pmatrix} \quad . \tag{39}$$

Therefore all possible choices which have the same central charges of CFT are related by the U-duality group. But we should be careful because not only the charges but also the moduli transform under the U-duality. Let us fix the convention of charges that we always map to the canonical frame specified by $\hat{\mathbf{Q}}$, that is

$$\vec{q} = (0, 0, N, 1, 0) \quad . \tag{40}$$

[5]Here we assume that Q_1 and Q_5 are relatively prime.

What are the moduli after the scaling limit? First we must discuss an important phenomenon called the attractor mechanism [24, 25], where some of the scalar expectation values are fixed in the near-horizon limit. The moduli in supergravity on T^4 are

$$\{\ g_{ij}\ ,\ B_{ij}\ ,\ g_s = e^{\phi}\ ,\ \chi\ ,\ C_{ij}\ ,\ A^{+}_{ijkl}\ \}. \tag{41}$$

Let us look at the mass of the background which has Q_1 D1-brane and Q_5 D5-brane charges [13], as a function of these moduli.

$$\begin{aligned} \frac{G_N^{(6)} M^2}{R^2} &= \frac{1}{v_4}\left[Q_1 + (v_4 - B\wedge B)\, Q_5\right]^2 \\ &\quad + \frac{g_s^2}{v_4}\left[\chi(Q_1 - B\wedge B Q_5) - \left(A_4 - \frac{1}{2}B\wedge C\right) Q_5\right]^2 \\ &\qquad + \left[v_4 \frac{1}{8} B^2 + 2B\wedge B\right] Q_5^2 \end{aligned} \tag{42}$$

The branes themselves exert tension and such attractive force can be seen as negative pressure. But the branes couple to antisymmetric tensor fields and their flux lines repel one another and therefore exert (positive) pressure. The supergravity is trying to decide which one to favor and minimize the energy. As we go from infinity toward the source, the scalar fields attract to the values which minimize the mass formula. One finds the attractor is at

$$v_4 + B\wedge B = \frac{Q_1}{Q_5}\quad ,\quad B = B^*\quad ,\quad v_4\chi = A_4 - \frac{1}{2}B\wedge C \tag{43}$$

These five conditions reduce the moduli space in the near horizon low energy limit to

$$\mathcal{H}_{\vec{q}}\backslash SO(5,4)/SO(5)\times SO(4)\quad . \tag{44}$$

Here $\mathcal{H}_{\vec{q}}$ is some subset of the "little group" of a charge vector, $SO(5,4;\mathbf{Z})$. $\mathcal{H}_{\vec{q}}$ is generated (for canonical $\hat{Q}$) by

1) $ST_\alpha S$ where $T_\alpha \in SO(4,4;\mathbf{Z})$ is T-duality of T^4.

2) $\Gamma_0(N = Q_1 Q_5) \subset SL(2,\mathbf{Z})_L \times SL(2,\mathbf{Z})_R$,

where $g \in \Gamma_0(N)$ acts on the usual type IIB coupling constant τ as

$$\tau = \chi + \frac{i}{g_s}\ \rightarrow\ \frac{a\tau + b}{c\tau + d}\quad (c \equiv 0 \mod N,\quad ad - bc = 1)\ . \tag{45}$$

Let us take as an example the $N = 6$ case (the canonical charge is $\vec{q} = (0,0,6,1,0)$), and investigate the moduli space. Here we project the whole moduli space into the particular two (τ) of the 20 moduli for simplicity and in that case we can regard $\mathcal{H}_{\vec{q}}$ as $\Gamma_0(N)$. A picture of the fundamental domain is shown in Figure 4. Note that this includes the familiar $SL(2,\mathbf{Z})$ fundamental domain which would constitute the moduli space in the absence of the brane background.

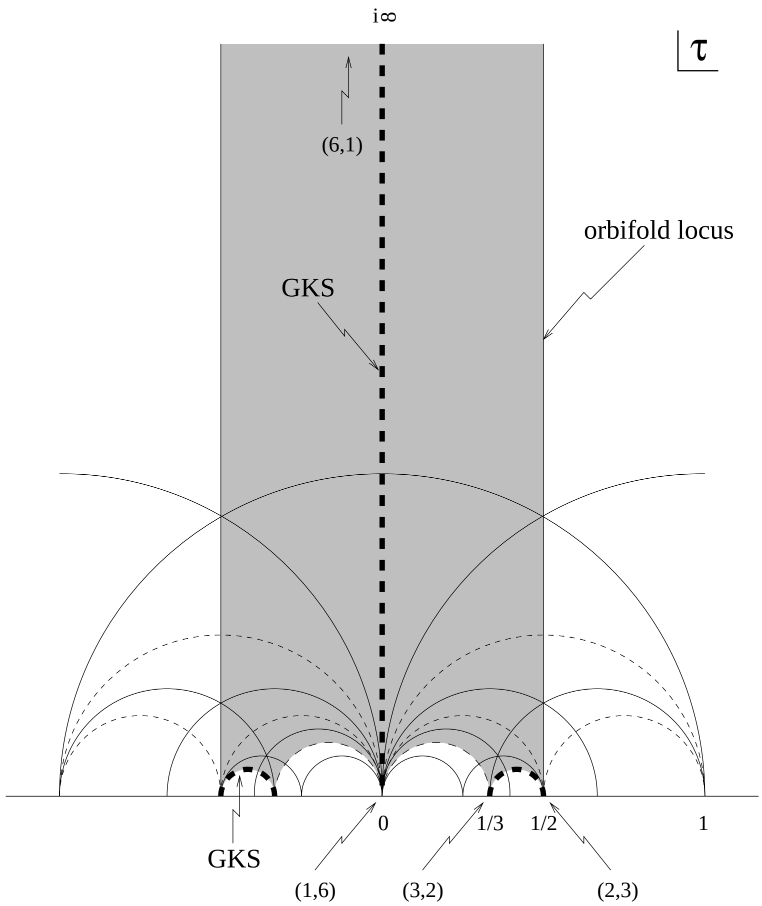


Figure 4: Fundamental domain of $\Gamma_0(6)$. Shaded region is the fundamental domain. Thick dotted lines denote the subspace where the spacetime CFT becomes singular.

The weak coupling limit of the sigma model for $(Q_1, Q_5) = (6,1)$ or $(1,6)$ is the upper end ($\text{Im}\ \tau \to \infty$) or the lower end ($\text{Im}\ \tau \to 0$)[6]. The other rational cusps (aligned along $\text{Re}\ \tau = 0$) correspond to the weak coupling limits for $(Q_1, Q_5) = (2,3), (3,2)$ as can be seen from $SO(2,2;\mathbf{Z})$ transformation. The lesson is that all these different (Q_1, Q_5) charges are continuously connected in the moduli space. Note that we have in this setting a cartoon of the usual picture of the moduli space of M-theory, in which different domains described by weak-coupling perturbation expansions (in this case sigma models on Q_1 instanton moduli space in $U(Q_5)$ gauge theory) are continuously connected in the moduli space through regions of strong coupling, where supergravity is a valid low-energy description.

Now the puzzle about the level of U(1) current algebras is resolved. The point is that if we move in the moduli space, identification of the level will change and thus "The diagonal U(1) of $(T^4)^{Q_1 Q_5}/S_{Q_1 Q_5}$ is level $Q_1 Q_5$" is a misleading statement.

Let us mention some theories which we can associate to different regions of the moduli space. When $g_6 > \sqrt{\frac{Q_5}{Q_1}}$ (strong coupling region of the sigma model), we should perform an S-duality to the F1-NS5 system. In such case there is a nice description of string theory on $AdS^3 \times S^3 \times T^4$, which is due to Giveon, Kutasov, and Seiberg (so-called GKS formalism) [26]. Thus we would like to know where that is in this moduli space. We must turn off RR fields to use the perturbative string description. That means $\chi = 0$ or $\Re\ \tau = 0$ in Figure 4, or its images under $SL(2,\mathbf{Z})$ that change the background charges; these are the thick dotted lines on the figure.

Another theory is the orbifold CFT $(T^4)^{Q_1 Q_5}/S_{Q_1 Q_5} \times T^4$, which describes $(Q_1, Q_5) = (N, 1)$. A particular element of the symmetric group is transposition: $T^4_{(i)} \leftrightarrow T^4_{(j)}$. This action has a fixed point along the diagonal, which is roughly speaking an $R^4/\mathbf{Z}_2$ singularity. It is well-known from the study of orbifolds that its metric is singular, but the CFT makes sense on such a space because of a discrete flux of B-field through the singularity ($b = \frac{1}{2}$). Thus this orbifold theory corresponds to the region $\chi = \frac{1}{2}$ in the moduli space.

We have been able to check symmetry algebras, BPS states, wrapped branes, and moduli spaces. There is a nice analysis in a paper [27] of Dijkgraaf, where the explicit map between the moduli space of supergravity and moduli of the hyperKähler sigma model is given. So perhaps is is time to stop questioning the correspondence, and ask what we can do with it.

[6]Note that $\frac{g_s^2}{v_4} = g_6^2$ by definition and $v_4 = \frac{Q_1}{Q_5}$ by the fixed scalar equation.

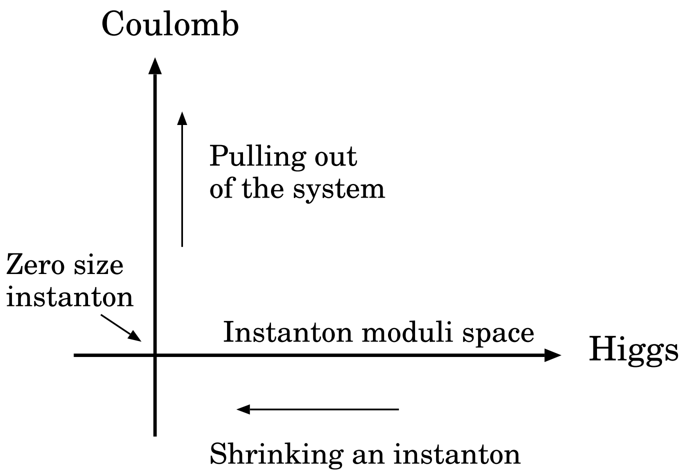


Figure 5: The process of shrinking an instanton.

4 Application of the correspondence

One application of the correspondence [14][7] is to investigate the Coulomb branch of the D1-D5 system. Most of this lecture has been spent talking about the Higgs branch. Therefore the question is what happens if we try to separate the branes into the clusters (the mixed Coulomb-Higgs branches) in the following way

$$Q_1 = Q_1' + Q_1'', \qquad Q_5 = Q_5' + Q_5''. \tag{46}$$

The fixed scalar conditions that can be obtained by minimizing the mass formula in this (Q_1, Q_5) background are given in (43). If we start pulling branes apart to two separate subsystems, we have to satisfy the fixed scalar conditions for each subsystem. Because we assume that Q_1, Q_5 are relatively prime, it is generically impossible to satisfy the fixed scalar conditions for each system. Thus if we try to pull the system apart into two pieces, it costs us some energy in the generic background. But there are some places in the moduli space in which the degeneracy conditions are satisfied. Namely, in the codimension four subspace [8] :

$$B = \chi = A_4 = 0, \tag{47}$$

the conditions are satisfied. Roughly speaking, this situation is represented as Figure 5. The transition from the Higgs branch to the Coulomb branch corresponds to the shrinking instanton (zero size instanton) singularity.

Note that the orbifold CFT is not singular because $\chi = \frac{1}{2}$ and it is away from such a singularity. However it is a problem for the GKS string description [26] because $\chi = 0$ and

[7]The present discussion represents an interpretation of [14] developed in discussions with A. Strominger.

[8]Note that we have already had the constraint (43).

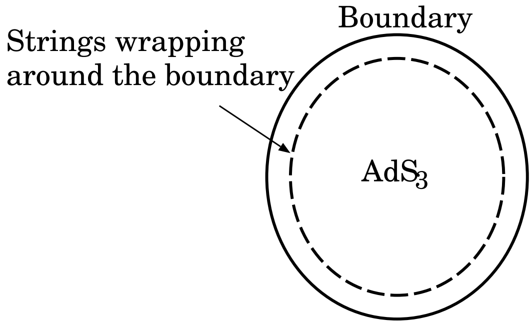


Figure 6: Strings wrapping around the boundary.

therefore it is sitting on the singular region. Surprisingly, their original paper described precisely such strings which wrap around the entire AdS_3 and are living near the boundary (see Figure 6). Now we can interpret such strings as fundamental strings pulled far away from the source.

In the gauge theory which describes the D1-D5 system, the problem appears as follows. The potential is essentially $U \sim \int \; |H_1 Y|^2$, and there is a Coulomb branch ($Y \neq 0$) and Higgs branch ($H_1 \neq 0$). If we turn off the fields Y, the instanton shrinks to zero size and the degeneracy occurs at that point. A piece of the configuration space a where small instanton develops is a region of high curvature. The metric of the moduli space becomes singular at such a point; thus there is a singular CFT in codimension 4 subspace of the moduli.

Now let us consider pulling out a single D1-brane and investigate the previous singularity dynamically by using a probe D1-brane. There are a number of ways of describing such a brane as follows:

a) Start with the gauge theory and try to integrate out the fields Y assuming that H_1 is large and the brane is far away. (*c.f.* Douglas, Polchinski, and Strominger [28])

b) Do a supergravity analysis. (*c.f.* Seiberg and Witten [14])

c) Do S-duality and look at the fundamental string. (*c.f.* GKS [26] or Callan, Harvey, and Strominger [29])

Any of these is sufficient and the effective action of the probe D1-brane is given by (see also [30])

$$S_{probe} = \frac{Q_5}{2} \int d^2\sigma \; \frac{(\partial H_1)^2}{H_1^2} + [SU(2)\mathrm{WZW}] + [T^4 \text{ part}] + [\text{fermion terms}]. \tag{48}$$

The angular mode is SU(2) WZW with the level $Q_5 - 2$ and $\log(H_1^2)$ is an effective Liouville mode which corresponds to the radial coordinate of AdS_3. The energy scale of mass for Y is $\sim H_1$ and therefore this effective action is only good at large H_1 (in scaled units, equation 7). There is an R-symmetry of this effective theory inherited from the original Higgs branch CFT, which rotated the angular S^3 transverse to the branes.

Thus it includes the bosonic SU(2) currents and rotation of fermions.[9] The total level is $(Q_5-2)+1=Q_5-1$ including the fermion contribution. There is also a T^4 part because the D1-brane can move in that direction.

What is the central charge after we pull the brane out? Before we pull it out, we find[10]

$$c_{Higgs}=6(Q_1Q_5+1)\ . \tag{49}$$

After we pull out the brane, we find[11]

$$\begin{aligned} c_{Coulomb} &= 6[(Q_1-1)Q_5+1]+6(Q_5-1)+6 \\ &= 6(Q_1Q_5+1)=c_{Higgs}\ . \end{aligned} \tag{50}$$

Thus the central charge does not change in such a process.

It is a simple exercise to generalize to all of the other mixed branches. If we consider the case

$$Q_1=Q_1'+Q_1'' \qquad Q_5=Q_5'+Q_5''\ , \tag{51}$$

and integrate out the heavy modes from the string between two clusters, we obtain

$$S_{throat}=\frac{Q_1'Q_5''+Q_5'Q_1''}{2}\int\frac{(\partial H)^2}{H^2}+\cdots\ . \tag{52}$$

It is again easy to check that

$$c_{Higgs'}+c_{Higgs''}+c_{throat}=c_{Higgs}\ . \tag{53}$$

Seiberg and Witten [14] argued (following Maldacena, Michelson, and Strominger [31]) that this effective theory is in some sense a dual description of the singularity of the original Higgs branch sigma model when the instanton shrinks to zero size. A rough analogy is $N=2$ 4D Yang-Mills theory, where the Coulomb branch meets with the Higgs branch in a nonperturbative regime.

As we have seen, the hyperKähler sigma model becomes singular at small instantons because it has large curvature. On the other hand the above effective theory describes a slow separation of two systems and has small corrections when H_1 is large. The problem here is that we do not have any expectation values in the 1+1 dimensional field theory. Therefore if one effective description becomes singular, we do not have the option of

[9] A second possibility, that the SU(2) R-symmetry only rotates the fermions, arises in a different scaling limit; the sigma model has $c=6$ instead of $c=6(Q_5-1)$. Unfortunately, this sigma model is also called the 'Coulomb branch' of the effective dynamics; but in the scaling limit, the region of the geometry being described decouples from the Higgs branch, unlike the above model that arises in the scaling limit 7.

[10] $6Q_1Q_5$ is from $(T^4)^{Q_1Q_5}/S_{Q_1Q_5}$ and 6 is from the extra T^4.

[11] $6(Q_5-1)$ is from the "throat" part of the CFT and 6 is from the T^4 part.

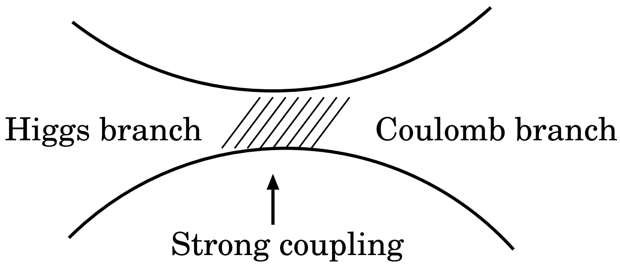


Figure 7: A rough picture of the configuration space of the 2D effective theory.

replacing the theory with a new theory. All we can do is to say that when the wave function is in the region of singularity, this is an approximate effective description of the dynamics. It is a little bit different from the usual field theory duality where one theory in weak coupling is dual to another theory in strong coupling. A rough picture is given in Figure 7.

The singular behavior only occurs in codimension 4 in the moduli space. Thus we should be able to identify in this probe theory the perturbations which we can turn on to lift the degeneracy. Such a perturbation of $N = 4$ super Liouville model is the Liouville area term. There is a quartet of these because any marginal field in $N = 4$ theory has highest field $(\frac{1}{2}, \frac{1}{2})$ under $SU(2)_R \times SU(2)_L$. The area term $(\sim \int\ H^{\alpha})$ gives an energy cost and therefore if we turn on this term, the degeneracy lifts. These Liouville area terms correspond to 4 scalar fields χ , B. Thus there is a rather detailed correspondence again between various pictures.

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