Universidade Estadual de Campinas

IMECC

Explicit Constructions over the Exotic 8-sphere

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Outline

$\star$-Bundles

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Bredon-Durán-Gromoll-Meyer-Rigas

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A Pull-back diagram for the Gromoll-Meyer construction

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A Pull-back diagram for the Gromoll-Meyer construction

Results on the 8-sphere

- Construction:
- Isotopy
- Linear S^7-bundles

Definition and Main Theorem

Definition ($\star$-bundle):

Let M be a G-manifold and $M = \cup U_i$, U_i equivariant. Let $\phi_{ij} : U_i \cap U_j \to G$ be conjugation equivariant, i.e.:

$$\phi_{ij}(g \cdot x) = g\phi_{ij}(x)g^{-1},$$

and $P = \cup_{f_{\phi_{ij}}} U_i \times G$ be a bundle. Then P is called a $\star$-bundle with transition maps ϕ_{ij}.

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Theorem $\star$:

If $P \overset{\pi}{\to} M$ is a $\star$-bundle then the action $g \star (x, q) = (g \cdot x, qg^{-1})$ is well-defined on P, free, and has quotient $M' = \cup_{\widehat{\phi_{ij}}} U_i$.

Here $f_{\phi_{ij}}(x, q) = (x, q\phi_{ij}(x))$ and $\widehat{\phi_{ij}}(x) = \phi_{ij}(x) \cdot x$. The proof is based on the involution $(x, q) \mapsto (q \cdot x, q^{-1})$.

Equivariant maps

Proposition 1 (Equivariant maps):

The following maps are smooth and conjugation equivariant:

$$\theta : S^3 \times S^3 \to S^3 \qquad (x, y) \mapsto xy^{-1}$$

$$b : S^6 \to S^3 \qquad (\xi, w) \mapsto \frac{w}{|w|} e^{\pi \xi} \frac{\bar{w}}{|w|}$$

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I.e.:

$$\theta(qx\bar{q}, qy\bar{q}) = q\theta(x, y)\bar{q}$$

$$b(q\xi\bar{q}, qw\bar{q}) = qb(\xi, w)\bar{q}.$$

Durán's Theorem

Theorem 1 ([Durán 01]):

Furthermore, $D^4 \times S^3 \times S^3 \cup_{f_\theta} S^3 \times D^4 \times S^3$, $D^7 \times S^3 \cup_{f_b} D^7 \times S^3$ and $Sp(2)$ are equivariantly diffeomorphic

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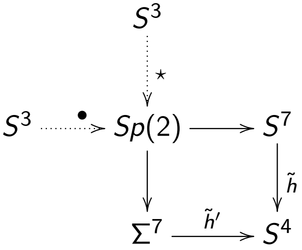
$$(q_1, q_2) \cdot (q_2 \cdot \vec{x}, q_1 q \bar{q}_2).$$

and

$$(q_1, q_2) \cdot \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} q_2 a \bar{q}_2 & q_2 c \bar{q}_1 \\ q_2 b \bar{q}_2 & q_2 d \bar{q}_1 \end{pmatrix}$$

on $Sp(2)$.

Picture



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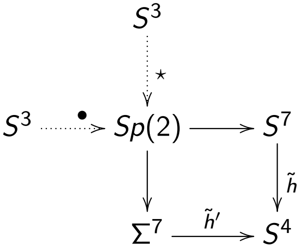

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Corollary:

The spaces $D^4 \times S^3 \cup_{\hat{\theta}} S^3 \times D^4$ and $D^7 \cup_{\hat{b}} D^7$ are diffeomorphic to the Gromoll-Meyer sphere Σ^7 (a generator of $\theta^7 \approx \mathbb{Z}_{28}$).

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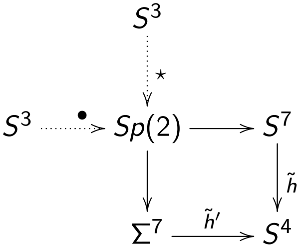

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Remark: $b \in \mathcal{C}^\omega$, generates $\pi_6 S^3 \approx \mathbb{Z}_{12}$ and $D^7 \cup_{\hat{b}^k} D^7 = \#_k \Sigma^7$.

$Sp(2)$ via pull-back

$$h : S^7 \to S^4$$
$$(x, y) \mapsto (|x|^2 - |y|^2, 2x\bar{y})$$

$$S^7 = D^4 \times S^3 \cup_{(x,q)\mapsto(x,qx)} D^4 \times S^3$$

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Restatement of Theorem 1:

$Sp(2)$ is a $\star$-bundle over S^7 with the action $q \cdot (x, y) = (qx\bar{q}, qy\bar{q})$ with trivialization maps θ or b and quotient Σ^7.

Pulling-back $\star$-bundles

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Theorem (Pull-back):

For $V_i = f^{-1}(U_i)$ we have that $N' = \cup_{\widehat{\phi_{ij} f}} V_i$ and for each i a commutative diagram:

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$\Rightarrow \Sigma^7 \overset{-h'}{\to} S^4$ is a linear S^3-bundle.

8-sphere:

Construction

$$\begin{matrix} \mathbb{R} \\ \mathbb{H} \\ \mathbb{H} \end{matrix} \ni \begin{pmatrix} \lambda \\ qx\bar{q} \\ y\bar{q} \end{pmatrix} \overset{f=E^5\eta}{\mapsto} \begin{pmatrix} q(\lambda + y^{-1}iy)\bar{q} \\ qx\bar{q} \end{pmatrix}$$

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Theorem (exotic 8-sphere):

$\Sigma^8 = \sigma_{3,4}(1,1) \neq 0 \in \theta^8$, where $\sigma_{3,4} : \pi_3 SO(4) \otimes \pi_4 SO(3) \to \theta^8$ is the Milnor's pairing.

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8-sphere: an order 2 generator

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Theorem (isotopy):

Let $t \in [0, \pi]$ and $\chi_t : S^7 \to S^3$ be given as

$$\chi_t \begin{pmatrix} x \\ y \end{pmatrix} = b \begin{pmatrix} y^{-1}(\cos ti + \sin tj)y \\ x \end{pmatrix} = \frac{y}{|y|} e^{\pi y^{-1}(\cos ti + \sin tj)y} \frac{\bar{y}}{|y|}.$$

Then $\hat{\chi}_t : S^7 \to S^7$ is an isotopy from a generator of θ^8 to its inverse. Furthermore, it indeuces an explicit diffeomorphism $\Sigma^8 \# \Sigma^8 \to S^8$.

8-sphere: linear S^7-bundles

Remark: $q \cdot (\lambda, x, y) = (\lambda, qx\bar{q}, y\bar{q})$ is in G_2 so $H : S^{15} \to S^8$ preserves it.

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Theorem (Exotic Hopf map):

Σ^{15} is diffeomorphic to S^{15}. Furthermore $H' : \Sigma^{15} \to \Sigma^8$ defines a linear S^7-bundle over Σ^8 with total space diffeomorphic to S^{15}.

Proof.

Use the framming of $H^{-1}(1,0) \subset S^{15}$ induced by H and note that $\Sigma^{15} = D^8 \times S^7 \cup S^7 \times D^8$ glued by this framming composed with $(X, Y) \mapsto (X, b(f(X)) \cdot Y)$. But $X \mapsto (Y \mapsto b(f(X)) \cdot Y) \in SO(7)$ has order 2 in $\pi_7 SO(7) \approx \mathbb{Z}$ so it does not affect any diffeomorphism class. The same can be proved for the element correspondent to the framming. □

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Remark: One can replace $H: S^{15} \to S^8$ by any linear S^7-bundle over S^8 with total space homeomorphic to the 15-sphere. A classification result gives:

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Theorem (linear bundles over Σ^8):

A homotopy 15-sphere fibers over the exotic 8-sphere with linear S^7 as fibers if and only if it fibers in the same way over S^8.

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