Universal accelerating cosmologies from 10d supergravity

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Based on:

* *Paul Marconnet & DT*, JHEP 01 (2023) 033

Introduction

- There has been a lot of recent effort in obtaining realistic 4d cosmologies from the 10d/11d supergravities that capture the low-energy limit of string/M-theory.
- In the early 21st century accelerating 4d cosmologies from compactification were thought to be as difficult as 4d Sitter
- The famous no-go excludes acceleration, provided:
  - absence of sources, no (or mild) singularities
  - compactness
  - two-derivative actions
  - the Strong Energy Condition is obeyed by the 10d/11d theory

* *Gibbons*, 1984
* *Maldacena & Nuñez*, 2000

Introduction

- Consider a compactification of the form

$$\hat{g}_{MN}\mathrm{d}X^M\mathrm{d}X^N = \Omega^2(y)\Big(g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}(y)\mathrm{d}y^m\mathrm{d}y^n\Big)$$

  where the 4d factor is of FLRW form,

$$g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu = -\mathrm{d}T^2 + S^2(T)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \;;\quad R(\gamma)_{ij} = 2k\gamma_{ij}$$

- In particular we have

$$\hat{R}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}\Big(\nabla^2\ln\Omega + 8(\partial\ln\Omega)^2\Big)$$

- The SEC

$$\hat{R}_{00} = \kappa^2\left(T_{00} - \tfrac{1}{8}\,\hat{g}_{00}\,T_L{}^L\right) \geq 0$$

  then implies

$$\ddot{S}(T) \leq 0$$

Introduction

- Time-dependent compactifications, however, can evade the no-go!

$$\Omega = \Omega(y;T) \ ; \quad g_{mn} = g_{mn}(y;T)$$

* *Townsend & Wohlfarth*, 2003

- Transient acceleration is in fact generic in flux compactifications!
- de Sitter space is still ruled out by the SEC (if the 4d Newton's constant is time-independent in the conventional sense)
- Late-time acceleration is not ruled out by the SEC (although no known examples from 10d/11d compactifications, if we require non-vanishing acceleration asymptotically)

* *Russo & Townsend*, 2018; 2019

Introduction

- Reexamine these statements within the framework of *universal* 10d/11d compactifications
  - Type II supergravity 10d solutions with a 4d FLRW factor
  - Compactification on 6d Einstein, Einstein-Kähler, or CY
  - Solutions independent of the compactification details
  - All solutions are obtainable from a 1d action (consistent truncation) of 3 time-dependent scalars (the dilaton and 2 warp factors). All fluxes appear as constant coefficients in the potential.
  - In certain cases there is a 4d consistent truncation to 2 scalars

Introduction

- Many analytic solutions
  - Always possible if a single excited species of flux.
  - Examples with up to four excited species of flux.
- Autonomous dynamical system if 2 excited species of flux
  - Intuitive description of the cosmological features of the cosmologies (trajectories), in particular the condition of accelerated expansion
  - Fixed points and trajectories on phase-space boundary correspond to analytic cosmological solutions
  - Several novel (top down) examples of (semi-)eternal inflation; cosmologies with parametric control of e-foldings; rollercoaster cosmologies

Type IIA supergravity

- Action

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big(- R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2 + \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2 + \frac{1}{2}m^2 e^{5\phi/2}\Big) + S_{\mathrm{CS}}$$

- Bianchi identities

$$\mathrm{d}F = mH \ ; \quad \mathrm{d}H = 0 \ ; \quad \mathrm{d}G = H\wedge F$$

Metrics & times

- The 10d Einstein-frame metric

$$\mathrm{d}s_{10}^2 = e^{2A(t)}\left[e^{2B(t)}(-\mathrm{d}t^2 + \mathrm{d}\Omega_k^2) + g_{mn}(y)\mathrm{d}y^m\mathrm{d}y^n\right]$$

where

$$\mathrm{d}\Omega_k^2 = \gamma_{ij}(x)\mathrm{d}x^i\mathrm{d}x^j \quad ; \quad R_{ij}^{(3)} = 2k\gamma_{ij}$$

- The 4d Einstein-frame metric

$$\mathrm{d}s_{4E}^2 = -S^6\mathrm{d}\tau^2 + S^2\mathrm{d}\Omega_k^2$$

where

$$S = e^{4A+B} \quad ; \quad \frac{\mathrm{d}t}{\mathrm{d}\tau} = S^2$$

- The cosmological time

$$\frac{\mathrm{d}T}{\mathrm{d}\tau} = S^3 \quad ; \quad \mathrm{d}s_{4E}^2 = -\mathrm{d}T^2 + S^2\mathrm{d}\Omega_k^2$$

Flux Ansätze examples

- Calabi-Yau

$$m = 0 \ ; \quad F = 0 \ ; \quad H = h + \mathrm{d}\chi \wedge J + \tfrac{1}{2} b_0 \mathrm{Re}\Omega \ ;$$

$$G = \varphi \mathrm{vol}_4 + \tfrac{1}{2} c_0 J \wedge J - \tfrac{1}{2} \mathrm{d}\xi \wedge \mathrm{Im}\Omega - \tfrac{1}{2} \mathrm{d}\xi' \wedge \mathrm{Re}\Omega$$

solution of form equations and Bianchi identities

$$\varphi = e^{-\phi/2-2A+4B} c_\varphi \ ; \quad h = c_h \mathrm{vol}_3 \ ;$$

$$\mathrm{d}_t \chi = c_\chi e^{\phi-4A-2B} \ ; \quad (\mathrm{d}_t \xi)^2 + (\mathrm{d}_t \xi')^2 = 2 {c_{\xi\xi'}}^2 e^{-\phi-4A-4B}$$

- Einstein-Kähler with internal 2-form

$$m = 0 \ ; \quad H = 0 \ ; \quad G = \varphi \mathrm{vol}_4 \ ;$$

$$F = c_f J \ ; \quad R_{mn} = \lambda \, g_{mn}$$

solution of form equations and Bianchi identities

$$\varphi = e^{-\phi/2-2A+4B} c_\varphi$$

The 1d consistent truncation

- The remaining equations of motion (Einstein & dilaton)

$$\mathrm{d}_\tau^2 A = -\tfrac{1}{48}\left(\partial_A U - 4\partial_B U\right)$$
$$\mathrm{d}_\tau^2 B = \tfrac{1}{12}\left(\partial_A U - 3\partial_B U\right)$$
$$\mathrm{d}_\tau^2 \phi = -\partial_\phi U$$

- Constraint

$$72(\mathrm{d}_\tau A)^2 + 6(\mathrm{d}_\tau B)^2 + 48\mathrm{d}_\tau A\mathrm{d}_\tau B - \tfrac{1}{2}(\mathrm{d}_\tau \phi)^2 = U$$

The 1d consistent truncation

- They are derivable from

$$S_{1\mathrm{d}} = \int \mathrm{d}\tau \Big\{ \frac{1}{\mathcal{N}} \Big(-72(\mathrm{d}_\tau A)^2 - 6(\mathrm{d}_\tau B)^2 - 48\mathrm{d}_\tau A \mathrm{d}_\tau B + \tfrac{1}{2}(\mathrm{d}_\tau \phi)^2 \Big) - \mathcal{N} U(A, B, \phi) \Big\}$$

where

$$U = \begin{cases} \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}c_h^2 e^{-\phi+12A} + \frac{3}{2}c_\chi^2 e^{\phi+4A} + c_\xi^2 e^{-\phi/2+6A} - 6k e^{16A+4B} & \text{CY} \\ 72 b_0^2 e^{-\phi+12A+6B} + \frac{3}{2}c_0^2 e^{\phi/2+10A+6B} & \text{CY} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}m^2 e^{5\phi/2+18A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \text{E} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{1}{2}c_h^2 e^{-\phi+12A} + \frac{3}{2}c_\chi^2 e^{\phi+4A} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \\ \frac{3}{2}c_0^2 e^{\phi/2+10A+6B} + \frac{1}{2}m^2 e^{5\phi/2+18A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \\ \frac{1}{2}c_\varphi^2 e^{-\phi/2+6A+6B} + \frac{3}{2}c_f^2 e^{3\phi/2+14A+6B} - 6k e^{16A+4B} - 6\lambda e^{16A+6B} & \text{EK} \end{cases}$$

The 1d consistent truncation

- The 10d origin of the constants

m	zero-form (Romans mass)
c_f	internal two-form
c_h	external three-form
b_0	internal three-form
c_χ	mixed three-form
c_φ	external four-form
c_0	internal four-form
$c_{\xi\xi'}$	mixed four-form
k	external curvature
λ	internal curvature

The 1d consistent truncation

- The terms in the potential are of the form

$$\text{const} \times e^{\alpha A + \beta B + \gamma \phi}$$

where (for RR forms)

$$\alpha = 18(1 - n_t) - 2(-1)^{n_t}(n_s + n_i) \ ;$$

$$\beta = 6(1 - n_t) - 2(-1)^{n_t} n_s \ ;$$

$$\gamma = (-1)^{n_t} \frac{5 - (n_t + n_s + n_i)}{2}$$

with n_t, n_s, n_i the number of legs along the time, 3d space, internal directions

Minimal solution (zero flux)

- Warp factors and dilaton

$$A = c_A \tau + d_A \ ; \quad B = c_B \tau + d_B \ ; \quad \phi = c_\phi \tau + d_\phi$$

- Constraint

$$\frac{c_A}{c_B} \leq -\tfrac{1}{2} \text{ or } \frac{c_A}{c_B} \geq -\tfrac{1}{6}$$

with constant dilaton if either inequality is saturated

- 4d Einstein metric

$$\mathrm{d}s^2_{4E} = -\mathrm{d}T^2 + T^{\frac{2}{3}} \mathrm{d}\vec{x}^2$$

- e^A may collapse, decompactify or stay constant as $T \to 0, \infty$

The 4d consistent (cosmological) truncation

- The equations of motion are derivable from

$$S_{4\mathrm{d}} = \int \mathrm{d}^4x\sqrt{g}\Big(R - 24g^{\mu\nu}\partial_\mu A\partial_\nu A - \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(A,\phi)\Big)$$

where

$$V = \begin{cases} 72b_0^2e^{-\phi-12A} + \frac{3}{2}c_0^2e^{\phi/2-14A} & \text{CY with internal 3- and 4-form fluxes} \\ \frac{1}{2}c_\varphi^2e^{-\phi/2-18A} + \frac{1}{2}m^2e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{E with external 4-form flux} \\ \frac{3}{2}c_0^2e^{\phi/2-14A} + \frac{1}{2}m^2e^{5\phi/2-6A} - 6\lambda e^{-8A} & \text{EK with internal 4-form flux} \\ \frac{1}{2}c_\varphi^2e^{-\phi/2-18A} + \frac{3}{2}c_f^2e^{3\phi/2-10A} - 6\lambda e^{-8A} & \text{EK with internal 2-form, external 4-form} \end{cases}$$

- In the CY case: a sub-truncation, to the metric and two scalars, of the consistent truncation to the universal sector

* *Robin Terrisse & DT*, 2019 ; *DT*, 2020

Dynamical system analysis

- Consider the case of a two-term potential

$$U=\sum_{i=1}^{2} c_i e^{E_i} \; ; \quad E_i := \alpha_i A + \beta_i B + \gamma_i \phi$$

- The eom's become a dynamical system, with $\mathrm{d}\sigma := e^{E_1/2}\mathrm{d}\tau$

$$\begin{aligned}
\mathrm{d}_\sigma v =& -\left(\tfrac{1}{8}\alpha_2-\tfrac{1}{2}\beta_2\right)u^2-\left(\alpha_2+\tfrac{1}{2}\beta_1-4\beta_2\right)uv-\left(\tfrac{1}{2}\alpha_1+\tfrac{3}{2}\alpha_2-6\beta_2\right)v^2-\tfrac{1}{2}\gamma_1 vw \\
&-\left(-\tfrac{1}{96}\alpha_2+\tfrac{1}{24}\beta_2\right)w^2+\tfrac{1}{48}c_1\left[\alpha_2-\alpha_1+4\left(\beta_1-\beta_2\right)\right] \\
\mathrm{d}_\sigma u =& -\tfrac{1}{2}\left(-\alpha_2+b_1+3\beta_2\right)u^2-\left(\tfrac{1}{2}\alpha_1-4\alpha_2-12\beta_2\right)uv-\left(-6\alpha_2+18\beta_2\right)v^2-\tfrac{1}{2}\gamma_1 uw \\
&-\left(\tfrac{1}{24}\alpha_2+\tfrac{1}{8}\beta_2\right)w^2+\tfrac{1}{12}c_1\left[\alpha_1-\alpha_2+3\left(\beta_2-\beta_1\right)\right] \\
\mathrm{d}_\sigma w =& -6\gamma_2 u^2-48\gamma_2 uv-72\gamma_2 v^2-\tfrac{1}{2}\beta_1 uw-\tfrac{1}{2}\alpha_1 vw-\tfrac{1}{2}\left(\gamma_1-\gamma_2\right)w^2+c_1\left(\gamma_2-\gamma_1\right)
\end{aligned}$$

where $v=e^{-E_1/2}\mathrm{d}_\tau A \; ; \quad u=e^{-E_1/2}\mathrm{d}_\tau B \; ; \quad w=e^{-E_1/2}\mathrm{d}_\tau \phi$

- The constraint takes the form

$$72v^2+6u^2+48vu-\tfrac{1}{2}w^2=c_1+c_2 e^{E_2-E_1}$$

Dynamical system analysis

- The phase space can be compactified using

$$x = \frac{2v}{4v+u} \; ; \quad y = \frac{w}{2\sqrt{3}(4v+u)} \; ; \quad z = \frac{\sqrt{c_1}}{\sqrt{6}(4v+u)}$$

- The eom's become an autonomous dynamical system

$$x' = \frac{1}{4}\Big([\alpha_2 + 2\beta_2(-2+x)](-1+x^2+y^2+z^2) + [-\alpha_1 - 2\beta_1(-2+x)]z^2\Big)$$

$$y' = \frac{1}{2}\Big((2\sqrt{3}\gamma_2 + \beta_2 y)(-1+x^2+y^2+z^2) - (2\sqrt{3}\gamma_1 + \beta_1 y)z^2\Big)$$

$$z' = \frac{1}{4}z\Big(\alpha_1 x + 4\sqrt{3}\gamma_1 y - 2\beta_1(-1+2x+z^2) + 2\beta_2(-1+x^2+y^2+z^2)\Big)$$

where $f' = \mathrm{d}_\omega f$ and $\mathrm{d}\omega := \frac{\sqrt{c_1}}{\sqrt{6}z}\mathrm{d}\sigma$

- Relation to the other time parameters

$$\mathrm{d}\omega = \frac{\sqrt{c_1}}{\sqrt{6}z} e^{E_1/2}\mathrm{d}\tau \; ; \quad \mathrm{d}T = \frac{\sqrt{6}z}{\sqrt{c_1}} e^{12A+3B-E_1/2}\mathrm{d}\omega$$

Dynamical system analysis

- The constraint takes the form
  $c_1(1 - x^2 - y^2 - z^2) = c_2\, z^2 e^{E_2 - E_1}$
  restricting to either the interior or the exterior of the unit sphere
- The unit sphere is an invariant surface

$$\frac{1}{2}\left(x^2 + y^2 + z^2\right)' = \frac{1}{4}\left(-1 + x^2 + y^2 + z^2\right) \times \left(\alpha_2 x + 4\sqrt{3}\gamma_2 y - 2\beta_1 z^2 + 2\beta_2\left[(-2 + x)x + y^2 + z^2\right]\right)$$

- The equatorial disc (at $z = 0$) is an invariant surface
- The equator (at $x^2 + y^2 = 1$ and $z = 0$) is a circle of fixed points
- The plane $ax + by + c = 0$ is an invariant surface, where

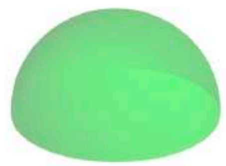
$$(\alpha_2 - 4\beta_2)a + 4\sqrt{3}\gamma_2 b - 2\beta_2 c = 0$$

$$\left[\alpha_2 - \alpha_1 - 4(\beta_2 - \beta_1)\right] a + 4\sqrt{3}(\gamma_2 - \gamma_1)b - 2(\beta_2 - \beta_1)c = 0$$

Dynamical system analysis

- The condition for expansion, $\dot{S}(T) > 0$, is equivalent to $z > 0$
- The flow is invariant under $(z, \omega) \to -(z, \omega)$
  so trajectories in the northern and southern hemispheres are paired
- The condition for acceleration, $\ddot{S}(T) > 0$, is equivalent to

$$(\beta_1 - \beta_2) z^2 - \beta_2 \left(x^2 + y^2\right) + \beta_2 - 4 > 0$$

β_1 \ β_2	0	4	6
0	$\varnothing$	$\varnothing$	
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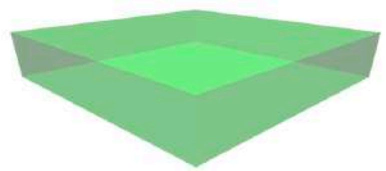
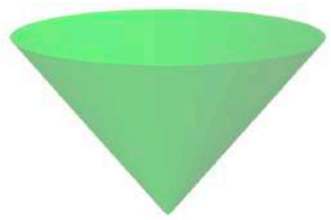
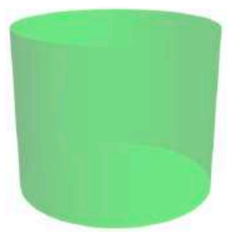
6			

Dynamical system analysis

- The flow parameter is related to the scale factor via

$$\omega = \ln \frac{S}{S_0}$$

so the number of e-foldings is given by

$$N = \int \mathrm{d}\omega$$

- The cosmological time reads

$$T(\omega) = \sqrt{\frac{6}{c_1}} \int^{\omega} \mathrm{d}\omega' z(\omega') \exp\left[\left(12 - \frac{\alpha_1}{2}\right) A(\omega') + \left(3 - \frac{\beta_1}{2}\right) - \frac{\gamma_1}{2}\phi(\omega')\right]$$

- This can be inverted to obtain the scale factor $S(T)$ via

$$\omega(T) = \ln S$$

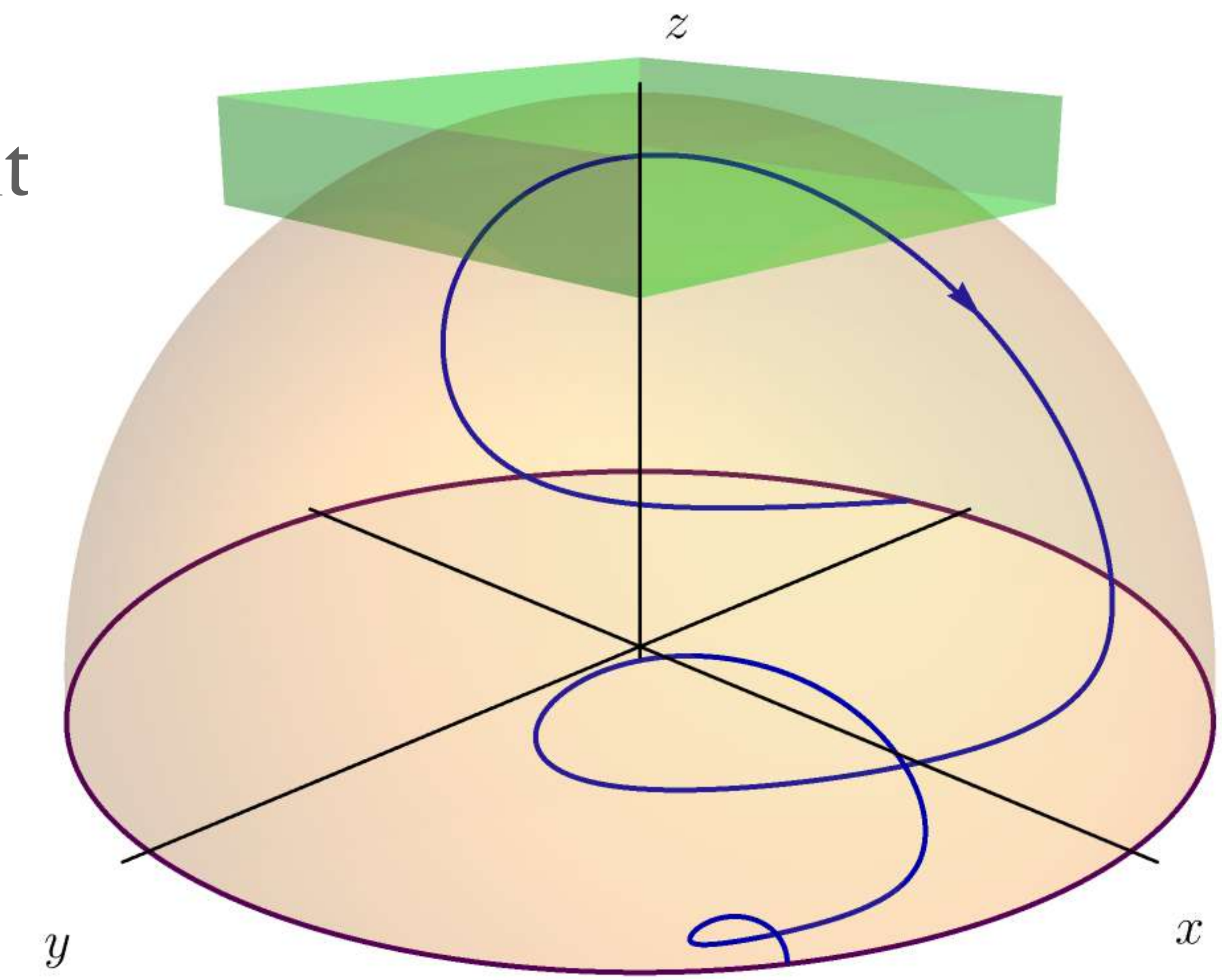
and similarly for all other cosmological parameters

Recap

- Many analytic solutions
  - Always possible if a single excited species of flux.
- Autonomous dynamical system if 2 excited species of flux
  - 3 first-order equations and a constraint
  - Solutions correspond to trajectories in phase-space
  - Compactification of phase-space to (the interior of) a 3d ball
- *Sonner & Townsend*, hep-th/0608068
  - The equatorial disc and the 2d sphere boundary are invariant surfaces of the dynamical flow
  - There is always an additional invariant plane
  - Fixed points and trajectories on the sphere boundary or on the disc correspond to analytic solutions

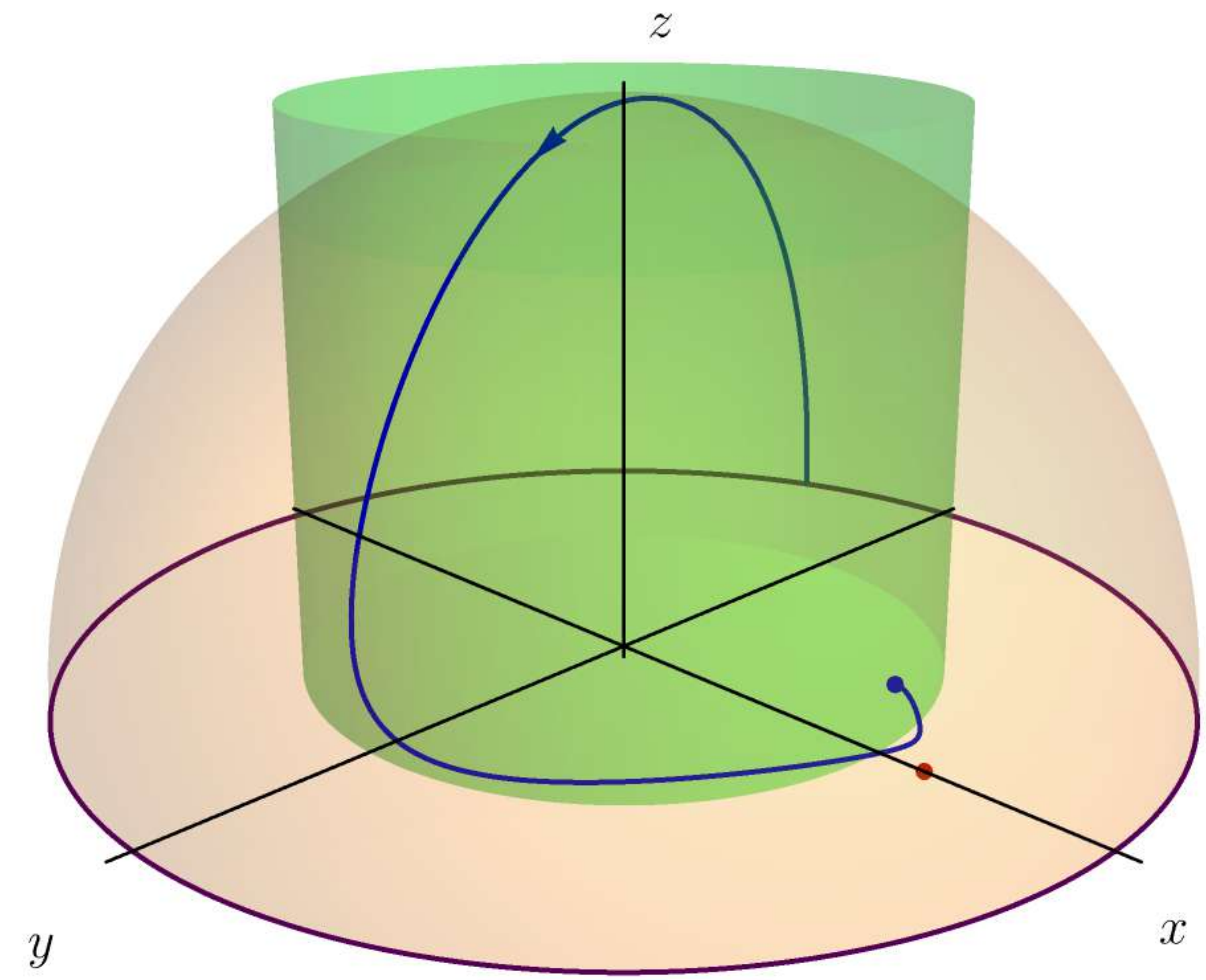
Recap

- Rephrasing the question of accelerated expansion
  - Expanding cosmologies correspond to trajectories in the northern hemisphere (interpolating between two fixed points)
  - Acceleration is possible whenever there is a non-empty *acceleration region* (determined by the type of excited fluxes)
  - This explains why transient accelerated expansion is generic: it corresponds to trajectories in the northern hemisphere, passing through the accelerated region.



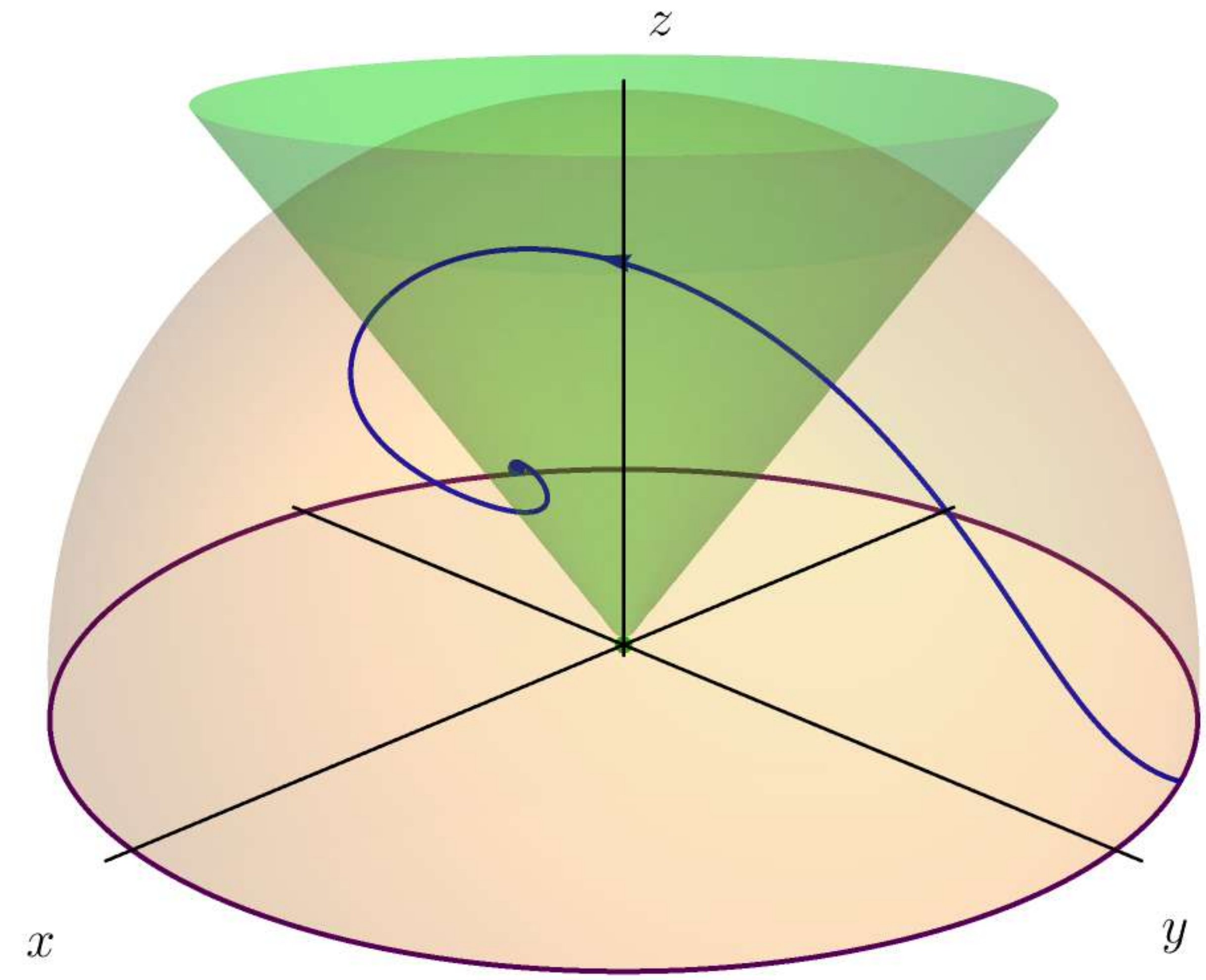
Results

- Fixed points correspond to scaling cosmologies: $S(T) \sim T^a$
  - The equator is a circle of fixed points with $a = \frac{1}{3}$
  - Fixed points on the boundary of the acceleration region have $a = 1$
    They correspond to a regular (singular) Milne universe if the fixed point is (not) the origin of the sphere.
  - There are no eternally accelerating scaling cosmologies, *i.e.* $a \leq 1$
  - There are fixed points with $a = \frac{3}{4}, \ \frac{19}{25}, \ \frac{9}{11}$



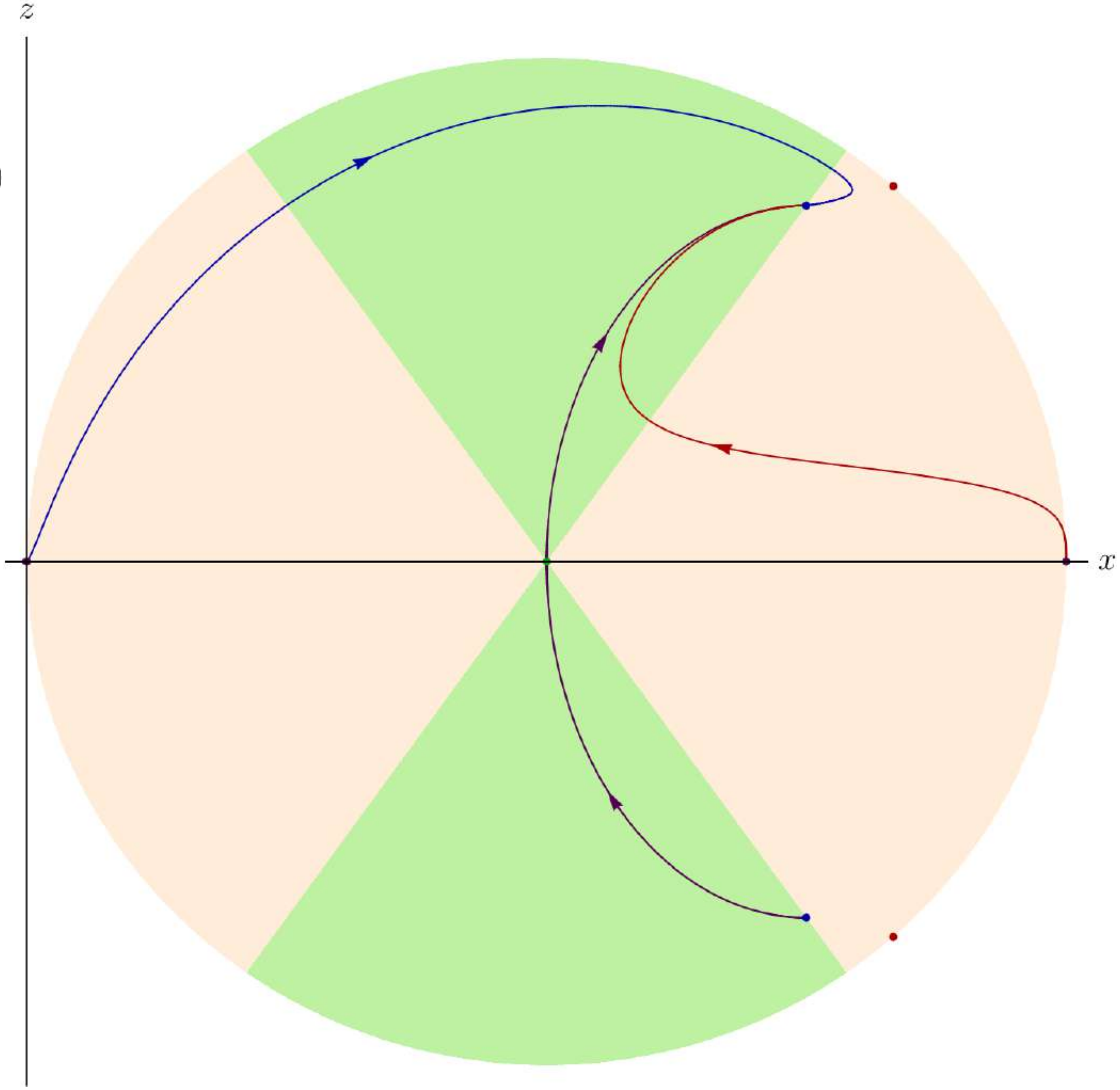
Results

- Many examples of (semi-)eternal inflation, and cosmologies with a parametric control of the number of e-foldings
  - They have $k = -1$ and non-vanishing $\lambda, \ m, \ c_f, \ c_0$ or c_φ
  - They have a fixed point on the boundary of the accelerated region



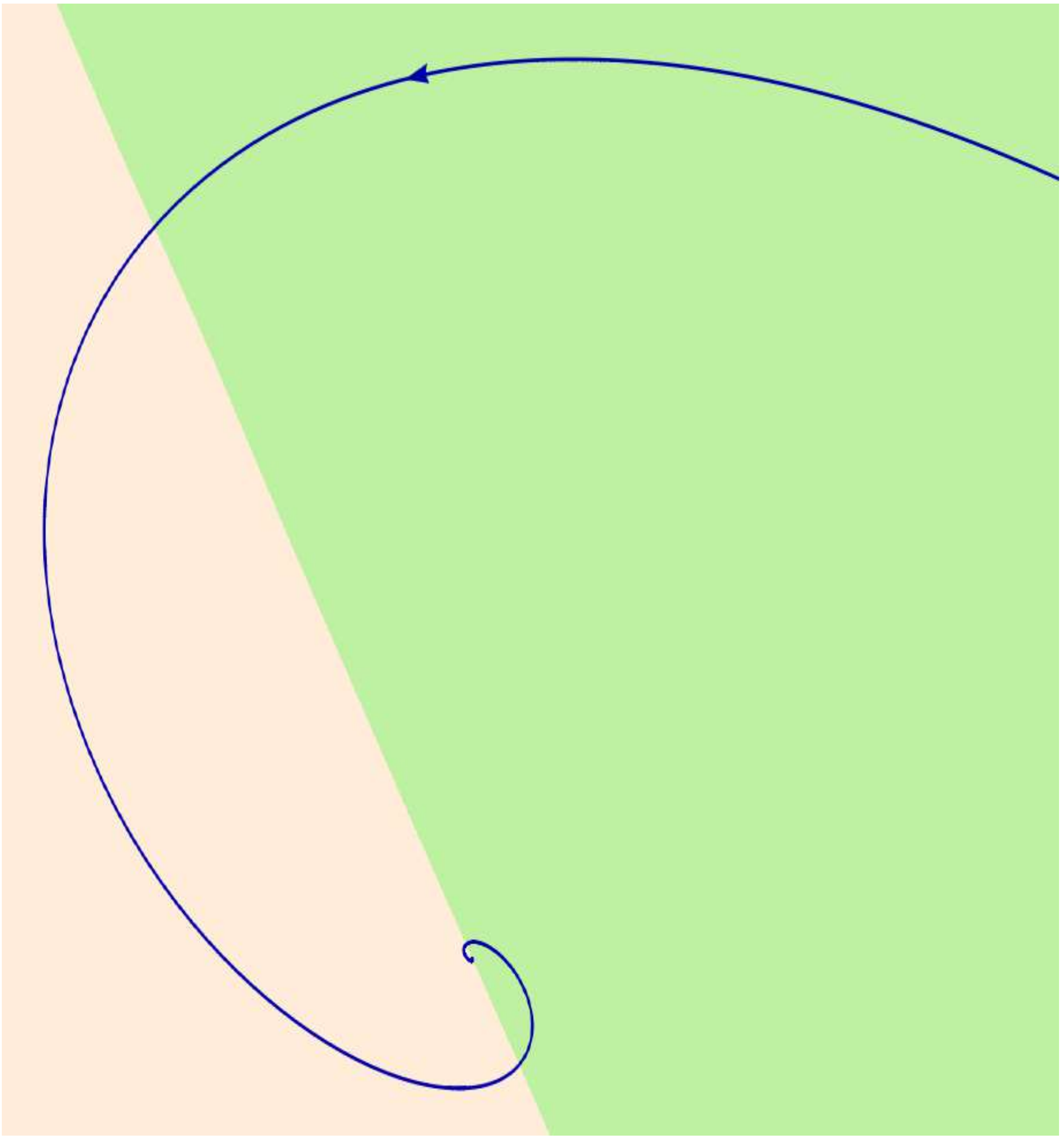
Results

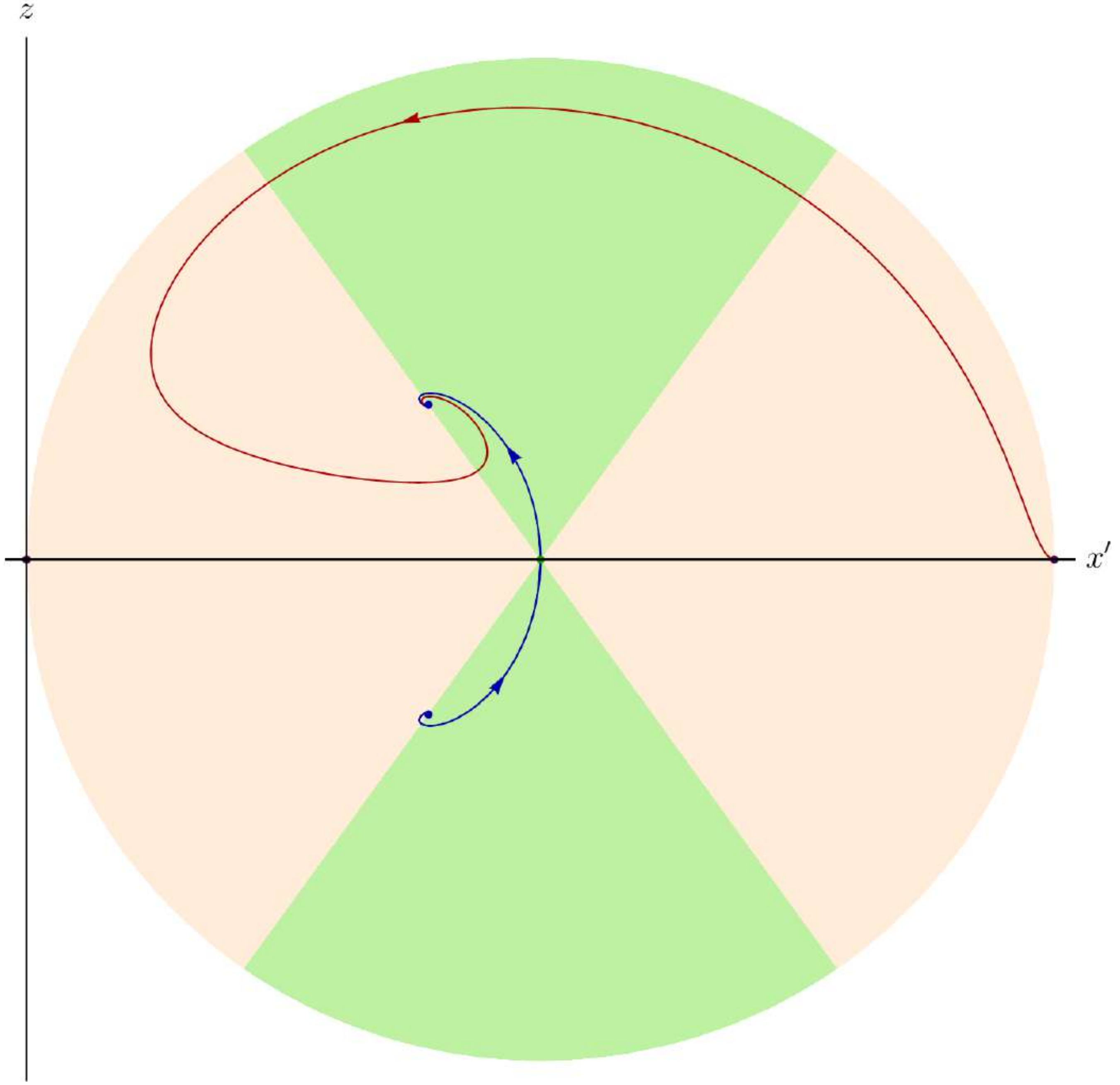
- Examples of semi-eternal inflation, and cosmologies with a parametric control of the number of e-foldings
- An example of eternal inflation without Big-Bang singularity
  - Accelerated contraction (expansion) for $T < 0$ ($T > 0$)
  - de Sitter in the neighborhood of $T = 0$



Results

- Several examples of solutions with infinite cycles of accelerated and decelerated expansion (rollercoaster cosmology)
- Example without Big-Bang singularity





Conclusions

- We confirm that transient acceleration is generic in flux compactifications (universal, top-down models)
- Cosmologies featuring (semi-)eternal acceleration, or a parametric control on the number of e-foldings also seem generic !
  - They have $k = -1$ and asymptotically vanishing acceleration
- Examples of spiraling cosmologies with an infinite number of cycles alternating between accelerated and decelerated expansion (rollercoaster cosmology)
- Comparison with the effective 4d approach, swampland
- Extend the dynamical system analysis to more than 2 fluxes
- Inclusion of sources (orientifolds), higher derivatives
- Realistic cosmologies ?