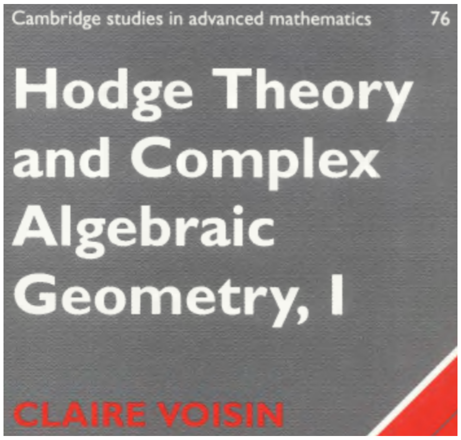

Let $\pi : \mathcal{X} \to B$ be a proper submersive map from one manifold to another. By Ehresmann's theorem 9.3, $\mathcal{X}$ is isomorphic in the neighbourhood of $X_0 = \pi^{-1}(0)$ to $X_0 \times B_0$, where B_0 is a neighbourhood of 0 in B. Consider the sheaves $H_A^k := R^k\pi_* A$, where A is a ring of coefficients (usually $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$), considered as the constant sheaf of stalk A, and $R^k\pi_*$ is the kth derived functor of the functor π_* from the category of sheaves over $\mathcal{X}$ to the category of sheaves over B. In general, it is not difficult to show that $R^k\pi_*\mathcal{F}$ is the sheaf associated to the presheaf $U \mapsto H^k(\pi^{-1}(U), \mathcal{F}_{|\pi^{-1}(U)})$. In our case, as B is locally contractible, we have $H^k(X_0 \times B_0, A) \cong H^k(X_0, A)$ for a fundamental system of neighbourhoods B_0 of 0, and we deduce that $R^k\pi_*A$ is a local system, isomorphic in the neighbourhood of 0 to the constant sheaf of stalk $H^k(X_0, A)$. Note that the stalk of this local system at a point $t \in B$ is canonically isomorphic to $H^k(X_t, A)$ by restriction.

Definition 9.13 *The flat connection*

$$\nabla : \mathcal{H}^k \to \mathcal{H}^k \otimes \Omega_B$$

on the vector bundle associated to the local system H_A^k is called the Gauss–Manin connection.