When formulated in the language of *sieves*, descent for an n-category valued presheaf G over a cover $Y \to X$ is controlled by the n-category of n-functors from Y to G, after Y is conceived as a presheaf itself.

As n grows beyond very small values, realizing this idea requires a choice of formalization of ∞-category in order to make sense of n-functors and their higher homotopies. Ross Street has given a definition of the descent ∞-category in the context of presheaves with values in *strict* ∞-categories, but without explicitly relating that definition to the notion of ∞-functors from the cover regarded as a sieve to the ω-presheaf in question.

The following is a remark on how Street's defintion of descent can be regarded as being a formalization of ∞-functors from sieves into ω-presheaves.

Let C be some site and assume that all covers $\pi : Y \to X$ are regular epimorphisms, so that the corresponding simplicial C-objects $Y^\bullet := (\cdots Y \times_X Y \times_X Y \underset{\pi_2}{\overset{\pi_1}{\rightrightarrows}} Y \times_X Y \xrightarrow{\pi} Y)$ exist.

Let $\mathsf{Spaces} := \mathsf{Sets}^{C^{\mathrm{op}}}$ be the category of presheaves on C and notice that ω-categories internal to Spaces are the same as ω-category-valued presheaves on C

$$\omega\mathsf{Categories}(\mathsf{Spaces}) \simeq \omega\mathsf{Categories}^{C^{\mathrm{op}}}$$

Fix some cosimplicial ω-category

$$O : \Delta \to \omega\mathsf{Categories}(\mathsf{Spaces})$$

and consider the induced ω-nerve $N : \omega\mathsf{Categories}(\mathsf{Spaces}) \to \mathsf{SimplicialSpaces}$ and its left adjoint $F : \mathsf{SimplicialSpaces} \to \omega\mathsf{Categories}(\mathsf{Spaces})$, the free ω-category with respect to O of a simplicial space S

$$F(S) := \int^{[n]\in\Delta} O([n]) \cdot S^n \,.$$

Street chooses the orientals for O, though I think one should keep in mind that these give the right answer for descent only in the case that the ω-category valued presheaves for which one considers descent happen to take values in *ω-groupoids*. More generally I think one should take $O([n])$ to be for instance the free ω-groupoid on the n-simplex, which is denoted $\Pi(\Delta^n)$ by Ronnie Brown (the fundamental ω-groupoid of the standard n-simplex regarded as a filtered space with the canonical filtering).

For my main point below this issue is secondary, it becomes relevant when we want to form $F(N(A))$ for an ω-groupoid A and regard that as a cofibrant replacement (wrt the folk model structure) of A, which is related to the notion of descent but shall not further concern me here, except for the observation that for A an ω-category, strict ω-functors out of cofibrant replacements of A are the same as *weak* (pseudo) ∞-functors out of A. For $(n = 2)$-categories it is a theorem by Lack that this notion of pseudo functor reproduces the known one.

With that in mind, the ω-category valued presheaf (the sieve) which corresponds to (a suitable replacement of) the cover $Y \to X$ should be

$$F(Y^\bullet) = \int^{[n]\in\Delta} O(\Delta^n) \cdot Y^{[n+1]}$$

and for $G : C^{\mathrm{op}} \to \omega\mathsf{Catgegories}$ an ω-category valued presheaf the corresponding descent ω-category should be

$$\mathrm{Hom}_{\omega\mathsf{Cat}(\mathsf{Spaces})}(F(Y^\bullet), G)\,.$$

Using the fact that the contravariant Hom takes colimits to limits this is

$$\cdots \simeq \int_{[n]\in\Delta} \mathrm{Hom}(O([n]) \cdot Y^{[n+1]}, G)\,.$$

Then using the Hom-adjunction (essentially the definition of the tensor $\cdot$ appearing here) this is

$$\cdots \simeq \int_{[n]\in\Delta} \mathrm{Hom}(O([n]), \mathrm{Hom}(Y^{[n+1]}, G))\,.$$

Finally with Yoneda this becomes

$$\cdots \simeq \int_{[n]\in\Delta} \mathrm{Hom}(O([n]), G(Y^{[n+1]}))\,.$$

But this last expression (my thanks to Dominic Verity for discussion of this point) is indeed equivalent to Street's definition of the descent ω-category

$$\cdots \simeq \mathrm{Desc}(Y, G)\,.$$