Philosophical Applied Category Theory

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Topics in the study of categorical systems and processes

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- Directed/undirected wiring
- Internal representation within systems
- Emergence
- Fundamental languages for ACT

1. Directed vs undirected paradigms

Composition paradigms for wiring interfaces of systems.

- How significant is this difference?
- What does it say about time?
- What does it say about causality?
- Do we live with both? How do they relate?

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> *[T]he reason why physics has ceased to look for causes is that in fact there are no such things. The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm. (Bertrand Russell, 1913)*

> *In the last century, causality survived not just as a convenient fiction, suitable for throw-away explanations given to undergraduates: it is deeply embedded in conventional thinking about interacting systems across many fields. Our work shows that, for linear dynamical systems, discarding it is beneficial in several ways (The Calculus of Signal Flow Diagrams I, Filippo, Pawel, Fabio)*

The majority position in philosophy retains causality. Ordinary, legal, medical, biological discourse is saturated in causal language.

But physics?

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Note the 'Time-neutral process theories' of Generalised Process Theories, and the operadic approach to AQFT of factorization algebras and time-orderable prefactorization algebras.

Cf. attempts to merge wiring paradigms (e.g., An Algebra of Resource Sharing Machines, Sophie) and to translate between them (e.g., Bayesian Networks, Markov Networks, Moralisation, Triangulation: a Categorical Perspective, Antonio, Fabio).

In general, symmetric monoidal loose right modules work well when you have a global notion of time (aka synchronous); algebras for double operads work well when you have a distributed (aka concurrent or asynchronous) notion of time. This is to get representablility for algebras for double operads, you don't only have to give some clocks; you have to give ways those clocks may combine to form new clocks. (David Jaz)

2. Internal representations of other systems

Idea: for a system to function well in the world, it needs to be able to represent (have a model of) features of its environment. (Mental representations)

Presumably, only features of the world that are relevant to the system.

- The world statically responding to your outputs
- The world autonomously changing in the way it responds to your outputs
- The world also dynamically changing according to the history of your outputs.

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Exploit the idea of a dual interface in the ACT sense:

- (A, B)-Moore machine has the dual interface to a (B, A)-Mealy machine
- (B, A)-Mealy machine as a (B, A^B)-Moore machine.

3. Emergence

Emergent properties

> *The characteristic properties of the whole $R(A, B, C)$ [where R marks their structural arrangement] cannot, even in theory, be deduced from the most complete knowledge of the properties of A, B, and C in isolation or in other wholes which are not of the form $R(A, B, C)$. (C. D. Broad 1925)*

Focus has shifted to the question of whether there are emergent phenomena that are both dependent on and yet autonomous with respect to underlying physical structures.

A double category theoretic outlook:

- Loose: Composition of subsystems. Laxness of functors.
- Tight: Coarse-graining

Both directions: How does the coarse-graining of the composite compare with the composite of the coarse-grainings.

(Compare with Leo and Fabio on Layered Monoidal Theories.)

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We may also consider coarse-graining to optimize for control (Erik Hoel). E.g., in a stochastic double category, Markov category in loose direction:

- Split epi = Each effect has a sufficient cause.
- Split mono = Each cause is necessary for its effects.
- Together, these conditions entail an isomorphism.

Fundamental languages

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Logical pluralism opposed by, e.g., One True Logic: A Monist Manifesto (Griffiths and Paseau).

- Dependent type theory
- HoTT
- Modal HoTT
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Perhaps we need to inhabit a polyglottal world.