Modal Dependent Type Theory

David Corfield

Modal Dependent Type Theory: New tools for philosophy

At the start of the twentieth century, one of the iconic founding acts of analytic philosophy took place when Bertrand Russell deployed a recently devised logic, extracted "by an analysis of mathematics", to attack the problem of denoting expressions. Here, in 1905, he famously claimed to have resolved a puzzle concerning the use of the definite article, *the*, for terms which have no referent, such as 'the present King of France'. From such a small, yet transformative step, there arose a century of formal work in philosophy. Later, important innovations came via a notable extension of Russell's predicate calculus to modal logic, whose use in philosophy flourished during the second half of the century, especially through the work of Saul Kripke and David Lewis.

Formal treatments of topics in philosophy of language, epistemology and metaphysics had a profound impact on existing and newly forming fields: logic, linguistics, computer science, cognitive science. However, while these latter fields took on a life of their own, radically modifying their formalisms over a period of several decades, philosophers have overwhelmingly chosen to rest content with their own tools. Rather than open themselves up to fruitful exchange with neighbouring fields, they have closed themselves off from an arsenal of power calculi.

The central aim of my research proposal is to demonstrate the enormous potential awaiting the philosopher who avails him or herself of modal dependent type theory, an especially powerful set of calculi, recently developed by mathematicians, logicians and computer scientists. I will achieve this by bringing varieties of *modal dependent type theory* into contact with a diverse range of philosophical ideas and constructions. Once these connections are opened, we may expect an invigoration of philosophy through the importation of streams of ideas from computer science, logic, mathematics, mathematical physics, cognitive science and linguistics.

Background

Categorical logic, the development of logic in terms of category theory, dates back over fifty years now to the seminal work of Lawvere in the 1960s. Over the intervening decades, category theorists, computer scientists and logicians have developed a wide range of novel systems along Lawvere's lines, or at least compatible with them. In particular, a strong connection has been forged between categorical logic and *dependent type theory*. These disciplines now act as mutually influencing fields, with ideas being passed in each direction. This practice is now so widespread that a new, somewhat fanciful, term has been coined to describe it: *computational trinitarianism.* This doctrine asserts that for a construction in either programming, logic or mathematics to be deemed important, it must make good sense in each of the three disciplines. Under this association, programs meeting a specification, elements of a type (including proofs of propositions), and mappings to an object in a category are identified.

A good case in point of this convergence is the concept of a *monad.* It first appeared in mathematics in the late 1950s and was subsequently rediscovered under different names. Category theorists soon realised that all monads factorise via *adjunctions*, the workhorses of their discipline, and so capture the process of mediating between two domains of entities or two inference regimes. From the 1970s, categorical logicians observed that these monads, and the dual *comonads*, behave like modal operators, such as those for possibility and necessity.

Computer scientists working in functional programming needed a way to represent side effects, that is, actions carried out by programs which are not merely the passing on of a calculation, such as printing or writing to memory. Monads were found by Moggi (1991) to be ideal for this job, and are thus widely used in functional programming, especially by the Haskell community. Dually, in the category-theoretic sense, there are comonads. These have been taken up in computer science to represent resource management.

Since these monads and comonads behave like the modalities of modal logic, those working on the functional programming of effects and coeffects were led to develop kinds of modal type theories. Monads have also been taken up in linguistics. Very much like the side effects considered in computer science, linguists can capture the pragmatic effects of speech (Asudeh and Giorgolo, 2016) which go beyond declarations of states of affairs.

Dependent type theory with the addition of modalities makes for a very powerful formalism. In February 2020 I brought out a book, *Modal homotopy type theory: The prospect of a new logic for philosophy,* with Oxford University Press, which offers a range of ways that philosophers might think about the component parts: types, dependency, homotopy, modality. Dependent type theory provides a logic which goes well beyond multi-sorted first-order logic. Its type discipline is excellent for warding off category mistakes and for capturing the subtleties of context-dependence. Furthermore, with this type theory emerging from the proof-theoretic tradition of Gentzen's Natural Deduction, this new logic lends itself to clarify and extend ways of thinking in the *inferentialist* tradition of Sellars, Brandom and others.

Two chapters of my book are devoted to the modal variety of homotopy type theory, where I make sense of modalities as capturing variation over domains, situations, worlds, times, and so on, and modalities as capturing geometric cohesion in mathematics. However, this work marks a solitary and very early philosophical foray into this field; the time is right for a much more profound study of its possibilities.

Aims and objectives

The aim of this research proposal is to establish modal dependent type theory as an essential tool for philosophy. The objectives of this work correspond to the achievement of this aim for each of six systems:(1) Dependent type theory; (2) General modal type theory; (3) Graded modal type theory; (4) Probabilistic type theory; (5) Temporal type theory; (6) Equivariant type theory.

Methodology

Each of the six studies involves a low-risk portion, constituted by exposition of existing theories in the area. Aside from the first study, this will be completely new material to almost all philosophers, since these systems have been developed to their current state by mathematicians, logicians and computer scientists. Next, for each case study, increasingly bold connections will be made with topics in philosophy, some developing from applications in linguistics and computer science. I shall take a 'bilingual' approach, using the twin languages of type theory and category theory.

1. Dependent type theory

(a) I will provide an account of the *intensional* form of this calculus, developed by the logician-philosopher, Per Martin-Löf, and discuss its use in natural language (cf. Ranta 1994, and succeeding work) and in informal mathematical language, '"laboratory conditions" for the study of semantics and pragmatics' (Ranta 2020, p. 122).

(b) Presuppositions and context-dependence: R.G. Collingwood and Sir Peter Strawson, in their different ways, formulated theories concerning the presuppositions of a proposition. Martin-Löf formulated his type theory in terms of 'contexts' of assumptions. I will develop the claim briefly made in my book that this device makes sense of presuppositions. This idea of the dependence of a proposition's meaning on its context is made literal in the type theory, through dependent types depending on types presupposed within the context. David Lewis's 'scorekeeping' will be examined through the lens of contexts to understand his claim that 'presupposition evolves in a more or less rule-governed way during a conversation' (1979, p. 340). Claims of those philosophers, such as Jason Stanley in *Language in Context* (2007), who argue that the truth-conditional effects of extra-linguistic context can be traced to logical form, receive great support from the type-theoretic outlook.

(c) Category mistakes: Ofra Magidor (2013) has given a thorough development of the concept of a category mistake, initially considered by Gilbert Ryle. I claim that dependent type theory is perfectly suited to provide the necessary discipline to avoid category mistakes, while remaining sufficiently expressive. For instance, we need to allow for predicates such as *large* which is only appropriate for certain types of entity, and which is defined relative to each of these types.

2. General modal type theory

(a) I start with Licata, Shulman and Riley's treatment of modalities in a series of increasingly powerful type theories (2017). The semantics here is provided by systems of categories linked by adjunctions. We may think of these as interlinked arenas of reasoning where results of inference may be passed between arenas. The collection of arenas corresponds to a category of modes, attached to each of which is a type system.

(b) The idea of mediating between different inferential frameworks has already appeared in philosophical logic. The logician Haskell Curry (1957) understood modalities as ways of relating inference taking place in weaker or stronger reasoning environments. Further back, we find Charles Peirce's gamma system of his Existential Graphs inscribed on a plurality of coloured sheets. Where both Peirce's Alpha and Beta systems of graph have been given a category-theoretic reading, and are beginning to appear in computer science (Bonchi et al. 2018), his Gamma system of existential graphs written on differently tinted sheets remains largely untouched, and yet Peirce was explicit that these served modal purposes.

(c) Robert Brandom claims that "I can be said to understand your remark insofar as I can compute its inferential significance both for you and for me, and navigate successfully back and forth across the two perspectives on its content constituted by the background of auxiliary hypotheses drawn from your collateral commitments and the ones drawn from mine." (2007, p. 667). I shall bring this point of view into accord with Curry's picture above and adapt David Lewis's ideas on scorekeeping (1b) to allow different scores for participants.

(d) In their *Behavioral Mereology*, Fong, Myers and Spivak (2018) employ what they call intermodalities to examine systems and their parts through the types of behavior they can exhibit. For instance, one may speak of the necessity of one subsystem to be in a set of states to allow the possibility of another subsystem to be in a desired state. These intermodalities arise from the present perspective when there are systems of interlocking modalities. I shall explore the extension of their ideas to dependent types.

3. Graded modal types

(a) Computer scientists have augmented modal type theory by the addition of a 'grading' on the modalities. Now we have an indexed family of modal operators, where the indices interact according to the structure placed on the grading. This is done to provide a more subtle formalism to treat side effects and resource usage. They are looking to extend this formalism to graded *dependent* type theory.

(b) In the early 1970s, two philosophers, Lou Goble and Kit Fine, were looking to provide a more expressive modal language, one which could, for instance, say of a proposition that it holds not only of some possible world, but of a number of possible worlds. Their different formalisms have been shown by my Kent colleague, Dominic Orchard (private communication), to be instances of his graded modal type theory. I shall investigate this connection.

(c) The linguist Daniel Lassiter (2017) has described the use of phrases such as: *more likely than*, *quite possible*, and *very good* as cases of graded modality. I propose an examination of his work in terms of graded modal type theory.

4. Probabilistic type theory

(a) Lassiter's work (3c) points to a connection between modality and probability and expected value. Recently a range of probabilistic type theories have appear, adopting a wide array of approaches, although most relying on the monad defined by Giry (1980) that assigns the collection of probability distributions on a space to that space, see, e.g. Jacobs and Zanesi (forthcoming).

(b) Various attempts have been made by philosophers to blend logic and probability theory, see e.g. Jon Williamson's Inductive logic (2017). I will scrutinise such accounts through the lens of probabilistic type theory.

(c) I will examine aspects of inductive reasoning in the light of probabilistic type theory. The Ravens' paradox suggests that we must take care with contrasting classes, for example, what to count as non-black and as a non-raven. In a typed setting, opposite properties must be taken relative to the type on which the property is defined.

5. Temporal type theory

(a) The version of temporal type theory constructed by Schulz and Spivak (2017) is expressive enough to allow the embedding of existing forms of temporal logic, such as Linear and Metric Temporal Logic. A sketch of a version of *dependent* temporal type theory is given in my (2020). More detail can now be provided of the structure of time to be used as a temporal index, including, branching futures, dense orderings, unboundedness, and so on.

(b) Temporal and tense logics have been used widely in philosophy, developed in particular by Arthur Prior (1967). In my (2020), I show how we might approach aspects of event structure using temporal type theory. I propose to bring temporal type theory systematically into contact with related work in philosophy of language and metaphysics, and with the use of temporal logic in linguistics and computer science.

6. Equivariant type theory

(a) In homotopy type theory, to work in the *context* of the type corresponding to a group of symmetries is to operate in a reasoning space where everything is acted upon by that group. As outlined in my book, this is the right environment in which to understand variation and invariance under symmetries.

(b) Philosophers and psychologists have been intrigued by our ability to detect an object as it moves or in changing lighting conditions, or a tune across transcription of key. Some, such as Cassirer (1944), Sheperd (1984), and Pizlo (2008), have looked to solve this problem in terms of invariance under a group of transformations.

(c) Noether's theorem establishes the correspondence between symmetries in the equations of motion of a system and conservation laws operating there. I will explore how the treatment by Urs Scheiber (forthcoming) of how equivariant type theory can make sense of this result.

Outputs

Each of my case studies will take 6 months to research and write up. At the end of the three years of the Fellowship, I will have ready for publication a monograph composed of six chapters, one for each case study. Oxford University Press has first option to publish this work, since it is a follow-up to my 2020 book with them. I will also make available extensive notes at the nLab, a wiki that I helped found in 2008. This considerable resource already contains 14000 pages and is visited over 2 million times a year. The Fellowship will also allow me the opportunity to prepare myself for future work which looks to understand the place of modal homotopy type theory in theoretical physics, right up to string theory (Schreiber, forthcoming).

Bibliography

Asudeh, A. and Giorgolo, G. 2016. Perspectives. Semantics and Pragmatics 9(21), pp. 1-57.

Bonchi, F., Seeber, J., Sobocinski, P. 2018. Graphical Conjunctive Queries, ()

Brandom, R. 2007. Inferentialism and Some of Its Challenges, Philosophy and Phenomenological Research Vol. LXXIV No. 3, pp. 651-676.

Cassirer, E. 1944. The concept of group and the theory of perception. Philosophy Phenomenological Research, 5, pp. 1–35.

Curry, H. 1957. A theory of formal deducibility. Vol 6 of Notre Dame Mathematical Lectures, Notre Dame, 2nd edition.

Fine, K., 1972. In so many possible worlds. Notre Dame Journal of Formal Logics 13, pp. 516–520.

Fong, B., Myers, D., Spivak, D. Behavioral Mereology. ()

Giry, M. 1982, A categorical approach to probability theory, Categorical aspects of topology and analysis, Lecture Notes in Math. 915 Springer, pp. 68–85.

Goble, L. 1970. Grades of Modality. Logique Et Analyse 13, pp. 323-334.

Jacobs, B. and Zanasi, F. Forthcoming. The Logical Essentials of Bayesian Reasoning. To appear in: G. Barthe and J.-P. Katoen and A. Silva (eds), Probabilistic Programming, Cambridge Univ. Press.

Lassiter, D. 2017, Graded Modality: Qualitative and Quantitative Perspectives, OUP.

Lewis, D. 1979. Scorekeeping in a language game, Journal of Philosophical Logic 8(1), pp. 339-359.

Licata, D., Shulman, M., and Riley, M. 2017. A Fibrational Framework for Substructural and Modal Logics. FSCD, 25, Leibniz-Zentrum fuer Informatik, pp. 1-22.

Magidor, O. 2013. Category Mistakes. Oxford University Press.

Moggi, 1991. Notions of Computation and Monads. Information and Computation, 93(1), pp. 55–92.

Pizlo, Z. 2008. 3D Shape: Its Unique Place in Visual Perception. MIT Press.

Prior, A. 1967. Past, Present and Future, Oxford University Press.

Ranta, A. 1994. Type-theoretical grammar, Oxford University Press.

Ranta, A. 2020. Some remarks on pragmatics in the language of mathematics, Journal of Pragmatics 160, pp. 120-122.

Schreiber, U. (forthcoming). Higher Pre-quantized Geometry. In G. Catren and M. Anel (eds.) New

Spaces for Mathematics and Physics, Vol.2, Cambridge University Press.

Schultz, P., Spivak, D. 2017. Temporal Type Theory: A topos-theoretic approach to systems and behavior,

Shepard, R. 1984 'Ecological constraints on internal representation: resonant kinematics of perceiving, imagining, thinking, and dreaming', Psychological Review 91:417-47

Stanley, J. 2007. Language in Context: Selected essays. Oxford University Press.

Williamson, J. 2017. Lectures on Inductive Logic. Oxford University Press.