Blending Philosophy of Mathematics and Cognitive Science

David Corfield

Max Planck Institute for Biological Cybernetics, Tübingen, Germany

Cognitive Blending - Psychology

Psychoanalysis

- Dreamwork (condensation and displacement, metaphor and metonymy)
- Loss/lack (dents are only possible for blenders)
- Phantasy (a child is being beaten, someone is being sent away to improve)
- Attunement theory (as triggering some forms of interpersonal blending)
- Identification (emotions are not just predictable responses to stimuli)

Cognitive Blending - Psychology

Discourse Analysis

- Battle for control of the blend. (He's a jealous type. That behavior would make anyone jealous.)
- Positioning theory (who's allowed to dictate the 'right' frames and blends)

Philosophy of Mathematics

Cognitive Science

Image schemas, blending

?

Traditions of enquiry

Philosophy of Mathematics

Automated theorem proving I

Generation of new syntactical strings assessed for novelty and proximity to desired result.

- (1) x + y = y + x (commutativity)
- (2) (x + y) + z = x + (y + z) (associativity)
- (3) n(n(x) + y) + n(n(x) + n(y)) = x (Huntington equation)
- (4) n(n(x + y) + n(x + n(y)) = x (Robbins equation)

Need to derive an equation of the form $\mathbf{C} + \mathbf{D} = \mathbf{C}$ from (1), (2) and (4).

Automated theorem proving I

5 (wt=5) [] -(x+y = x).

7 (wt=13) [] n(n(n(x)+y)+n(x+y)) = y.

10 (wt=18) [para(7,7)] n(n(n(x+y)+n(x)+y)+y) = n(x+y).

11 (wt=19) [para(7,7)] n(n(n(n(x)+y)+x+y)+y) = n(n(x)+y).

29 (wt=21) [para(11,7)] n(n(n(n(x)+y)+x+y+y)+n(n(x)+y)) = y.

54 (wt=29) [para(29,7)] n(n(n(n(n(x)+y)+x+y+y)+n(n(x)+y)+z)+n(y+z)) = z.

217 (wt=34) [para(54,7)] n(n(n(n(n(x)+y)+x+y+y)+n(n(x)+y)+n(y+z)+z)+z) = n(y+z).

674 (wt=42) [para(217,7)]

n(n(n(n(n(n(x)+y)+x+y+y)+n(n(x)+y)+n(y+z)+z)+z+u)+n(n(y+z)+u)) = u.

6736 (wt=49) [para(10,674)] n(n(n(n(x+x+x)+x)+n(x+x+x))+n(n(n(x+x+x)+x)+x+x+x+x+x))

= n(n(x+x+x)+x).

8855 (wt=27) [para(6736,7),demod([54]),flip(1)] n(n(n(x+x+x)+x)+x+x+x+x+x) = n(x+x+x).

8865 (wt=43) [para(8855,7)] n(n(n(n(x+x+x)+x)+n(x+x+x)+x+x)+n(x+x+x)) =

n(n(x+x+x)+x)+x+x.

8866 (wt=19) [para(8855,7),demod([11])] n(n(n(x+x+x)+x)+n(x+x+x)) = x.

8870 (wt=27) [para(8866,7)] n(n(n(n(x+x+x)+x)+n(x+x+x)+y)+n(x+y)) = y.

8871 (wt=17) [back demod(8865),demod(8870),flip(1)] **n(n(x+x+x)+x)+x+x = x+x.**

The Eliza Effect?

EQP's proof is much more than a calculation. The proof depends upon a successful search among a realm of possibilities and the skillful application of pattern recognition and the application of axioms. This is very close to the work of a human mathematician. EQP, being able to handle many possibilities and great depths of parentheses has an advantage over her human colleagues. I understood EQP's proof with an enjoyment that was very much the same as the enjoyment that I get from a proof produced by a human being. (Kauffman 2000: 4)

Automated theorem proving II

- Attempt to create analogical proof relying on syntactical transformation of variables in proof plan.
- "Analogy is the heart and soul of all intelligent behavior, especially mathematical behavior. Why have we made so little use of analogy in ATP [automated theorem proving]? We predict no substantial advance until our provers begin to effectively use Analogy with the help of an adequate MKB [mathematical knowledge base]." (Woodie Bledsoe)

Automated theorem proving II

- Challenge: automatically analogize the Heine-Borel theorem from 1 dimension to 2 dimensions.
- **Heine-Borel theorem in one dimension:**
- For any closed interval of the reals, [a, b], any covering by open sets has a finite subcover.

Automated theorem proving II

Proof plan:

- For a reductio, assume that there is an interval and a cover which has no finite subcover.
- Divide this interval into two equal parts and select the leftmost one which cannot be finitely covered.
- Iterate this process to achieve an infinite sequence of nested intervals, each of which is half the length of its predecessor and cannot be finitely covered.
- Intersection of such a sequence must contain a point.
- This point is contained in the initial interval, so must belong to some member of the open cover.

Automated theorem proving II

Proof plan:

- This open set must contain a closed interval centred on the point.
- Far enough along the sequence the nested intervals are narrow enough to fit inside this closed interval.
- They have a one element cover.
- Contradiction of the fact that they could not be finitely covered.

Automated theorem proving II

- Caution: When showing that eventually members of the sequence of nested rectangles are contained in a rectangle centered on the point contained in the intersection of the sequence, we must ensure that we are far enough along the sequence that both length and breadth are small enough to fit inside that rectangle.
- **Moral: Don't expect double blending to be captured by simplex blending.**

Image Schemas - OVER

- Schema 1: ‘The bird flew over the hill’. ‘She lives over the hill.’
- Schema 2: ‘The painting is over the fireplace.’
- Schema 3: ‘A veil covered her face,’ ‘The sky clouded over,’ ‘Daisies grew all over the lawn.’
- Schema 4: ‘The log rolled over’ or ‘The lamp fell over.’
- Schema 5: ‘Socrates has overdone it’
- Schema 6: ‘He did it over and over again’

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Over-paid, over-sexed, and over here!

Image schemas - OVER

Mathematics:

- Variables vary over a domain (Schema 3 with the paths taken as a multiplex entity).
- Sheaf (agricultural): Vertical stalks rooted in the base space. ‘Stalks of wheat are growing all over the field’ (Schema 3)
- Sheaf (paper) (Possibly) many sheeted surface sitting over the base space. ‘Paper over a wall with many layers.’ (Schema 3)

Image schemas - OVER

Mathematics (cont.):

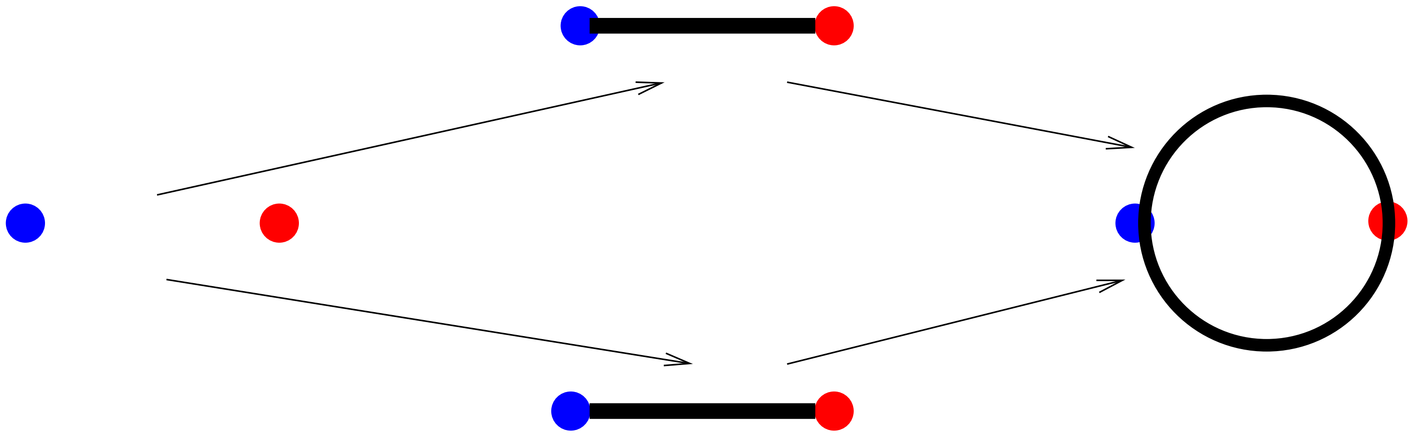
- We adjoin an algebraic number to the rationals to obtain a finite field over Q, the rationals. Q is called the ground field of the extension.
- Then we can build a tower of extensions where each field is of finite degree over the one below it. 'The picture hangs over the fireplace.' Schema 2.

Loose analogies

"There are exactly four normed division algebras: the real numbers, complex numbers, quaternions, and octonions. The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The quaternions, being noncommutative, are the eccentric cousin who is shunned at important family gatherings. But the octonions are the crazy old uncle nobody lets out of the attic: they are nonassociative." (John Baez)

Tight Blending - Pushouts I

How to reinvent the wheel:



Mirror blend

Tight Blending - Pushouts II

Equivalence relation

?

Group

Tight Blending - Pushouts II

Equivalence relation

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Group

- Collection of objects divided into mutually exclusive, exhaustive classes, there being one way an object can be related to itself.

Tight Blending - Pushouts II

Equivalence relation

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- One object, there being possibly many ways it can be transformed to itself.

Tight Blending - Pushouts II

Equivalence relation

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- Collection of objects divided into mutually exclusive, exhaustive classes, there being one way an object can be related to itself.
- One object, there being possibly many ways it can be transformed to itself.
- Collection of objects divided into mutually exclusive, exhaustive classes, there being possibly many ways one thing can be related to itself.

Tight Blending - Pushouts II

Equivalence relation

Groupoid

Group

- Collection of objects divided into mutually exclusive, exhaustive classes, there being one way an object can be related to itself.
- One object, there being possibly many ways it can be transformed to itself.
- Collection of objects divided into mutually exclusive, exhaustive classes, there being possibly many ways one thing can be related to itself.

Tight Blending - Pushouts II

Alternative blends:

- Cosets,
- Conjugacy classes of elements,
- Disjoint unions of groups,
- Equivalence relation on groups, e.g those of same order,
- Symmetries of an equivalence relation.

(Latter 3 are reasonable but not exciting.)

Tight Blending - Pushouts II

Bad blends:

- Many arrows on C(A,A) but single arrows on C(A,B).
- ...

Looser Mathematical Blending

Surveying surfaces embedded in 3D

Intrinsic geometry of surfaces

Treating 3D geometry intrinsically,
survey triangles between mountains,
and between Earth and distant star.

Our immersion in 3D

Compression

Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through some process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compressions one of the real joys of mathematics.

After mastering mathematical concepts, even after great effort, it becomes very hard to put oneself back in the frame of mind of someone to whom they are mysterious. (William Thurston 1990)

Good blends

Teachers need to unpack good blends, while stopping students blending badly.
How to characterize good/bad blends?

- Blends satisfying the governing principles are more likely to be good cognitive blends.
- But this 'good' must include those that advertising companies are keen to develop.
- Mathematical entities have been defined as 'Mental objects with reproducible properties'.
- Can you have misleading good mathematical blends?

Good blends in art and mathematics

The creation of hitherto inconceivable conceptions by the combination of hitherto incompatible features is not, of course, restricted to art, poetry, and religious myth. It is a commonplace in mathematics and modern physics as well. But the imaginary entities created by means of the integration of incompatibles in art and myth go beyond those imaginary entities created from incompatibles in mathematics and physics. The latter are acceptable as natural integrations; the former, by contrast, must be called transnatural. Integrations of the sort one finds in mathematics and physics, though the incompatibles involved actually remain incompatible - in the sense that we can never bring their elements into the perfect logical identity in which we can see that A is B because B is logically identical to A - come nevertheless to seem to be naturally...

Good blends in art and mathematics

...compatible. At the time of their discovery, these integrations required considerable and obvious imaginative power to bring their parts together; but once we have become accustomed to working with them in the ordinary day-to-day world of our practical concerns, they seem quite "natural" to us. They "work" in our mundane world. The integrations of art, poetry, and myth, however, do not enter into a practical way into our ordinary lives. They do not "work" in such a sphere. They are, as we have said, detached from our daily lives. And their incompatibles remain incompatible. They must be joined by a new act of imagination every time we contemplate them. They thus appear to us to be meaningful and coherent but nevertheless to have meanings that go quite beyond the "natural". (M. Polanyi)

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Traditions of enquiry

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The way we think

The way we think
well mathematically
(ought implies can)

Justification in mathematics

Philosophy of Mathematics

Cognitive Science

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Justification in mathematics

Philosophy of Mathematics

- Applicability? Blending points to a restriction on what kinds of mathematical concept are thinkable.
- Why has mathematics been so powerful in physics, but so weak in biology?