Narratives category theorists tell themselves

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Imre Lakatos

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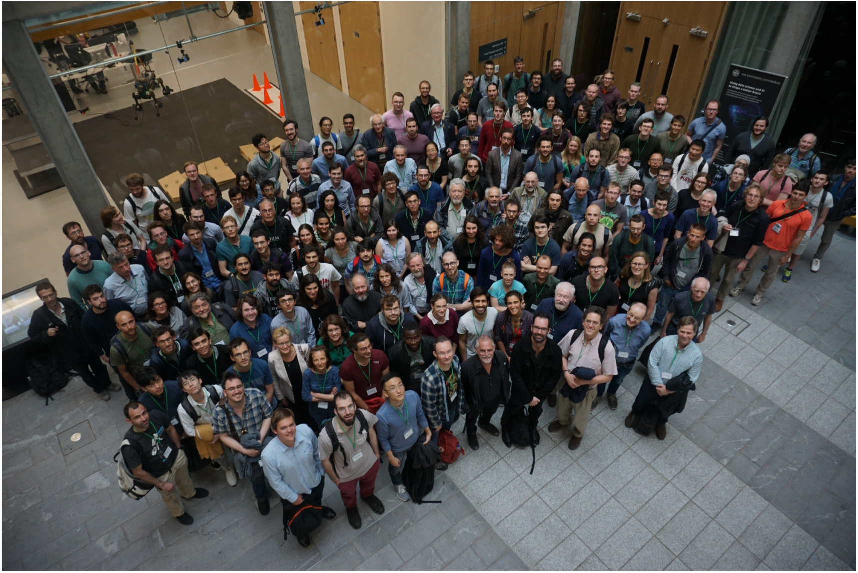
Other than by means of narrative accounts, how could we describe the development of such programmes? How could participants of such programmes form an understanding of what has been done, and what is still to do?

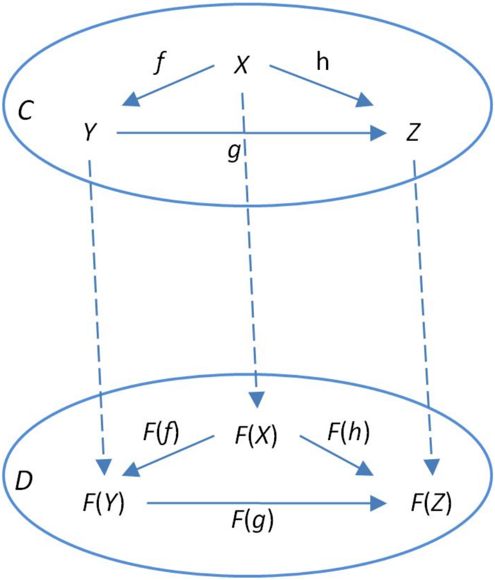
Category theorists at CT2019



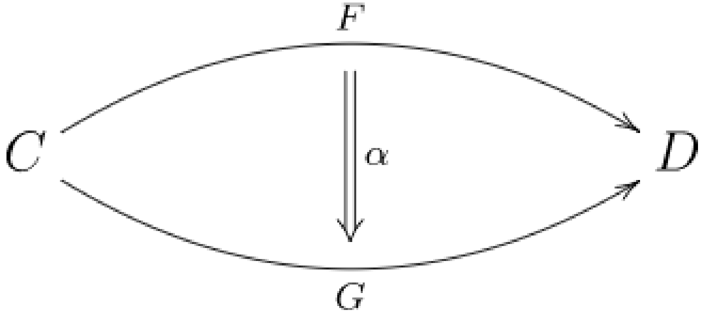
Some early milestones

- 1945 Publication by Eilenberg and Mac Lane, *General theory of natural equivalences*.
- 1952 Put to use in Eilenberg and Steenrod, *Foundations of algebraic topology*.
- 1958 Kan on adjoint functors
- 1960s Grothendieck revises algebraic geometry.
- 1960s Lawvere proposes Category theory as a foundational language.
- 1971 Mac Lane publishes *Categories for the working mathematician*.

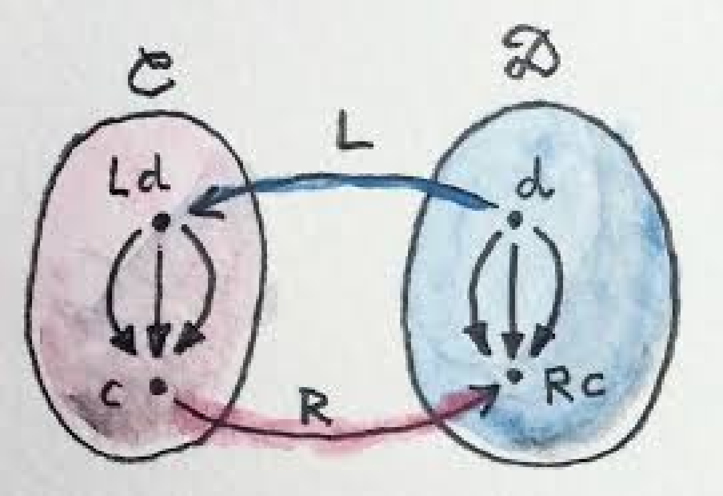
Categories and functors



Natural transformation



Adjunction - key to duality



(Bartosz Milewski)

Grothendieck and the reconstruction of algebraic geometry

- A polynomial becomes a functor from all (commutative) rings to sets.

Grothendieck and the reconstruction of algebraic geometry

- A polynomial becomes a functor from all (commutative) rings to sets.
- Slogan: Better a category extended with 'bizarre' objects so as to have good properties, than a category of standard objects which has few good properties.

Grothendieck and the reconstruction of algebraic geometry

Contrast cracking a nut with a hammer and chisel with...

> *...immersing the nut in some softening liquid, and why not simply water? From time to time you rub so the liquid penetrates better, and otherwise you let time pass. The shell becomes more flexible through weeks and months — when the time is ripe, hand pressure is enough, the shell opens like a perfectly ripened avocado!*

Lawvere

In the mathematical development of recent decades one sees clearly the rise of the conviction that the relevant properties of mathematical objects are those which can be stated in terms of their abstract structure rather than in terms of the elements which the objects were thought to be made of. The question thus naturally arises whether one can give a foundation for mathematics which expresses wholeheartedly this conviction concerning what mathematics is about, and in particular in which classes and membership in classes do not play any role...A foundation of the sort we have in mind would seemingly be much more natural and readily-useable than the classical one when developing such subjects as algebraic topology, functional analysis, model theory of general algebraic systems, etc. (The Category of Categories as a Foundation for Mathematics 1966)

Concerns about its value

A number of sophisticated people tend to disparage category theory as consistently as others disparage certain kinds of classical music. When obliged to speak of a category they do so in an apologetic tone, similar to the way some say, "It was a gift – I've never even played it" when a record of Chopin Nocturnes is discovered in their possession. For this reason I add to the usual prerequisite that the reader have a fair amount of mathematical sophistication, the further prerequisite that he have no other kind. (Barry Mitchell, Theory of Categories, 1965)

Usage normalized in many fields today

Category theory takes a bird's eye view of mathematics. From high in the sky, details become invisible, but we can spot patterns that were impossible to detect from ground level. How is the lowest common multiple of two numbers like the direct sum of two vector spaces? What do discrete topological spaces, free groups, and fields of fractions have in common? We will discover answers to these and many similar questions, seeing patterns in mathematics that you may never have seen before. (Tom Leinster, Basic Category Theory, 2014)

Usage normalized in many fields today

Atiyah described mathematics as the “science of analogy.” In this vein, the purview of category theory is mathematical analogy. Category theory provides a cross-disciplinary language for mathematics designed to delineate general phenomena, which enables the transfer of ideas from one area of study to another. The category-theoretic perspective can function as a simplifying abstraction, isolating propositions that hold for formal reasons from those whose proofs require techniques particular to a given mathematical discipline. (Emily Riehl, Category theory in context, 2016)

It's still growing

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- Computer science: computational trinitarianism
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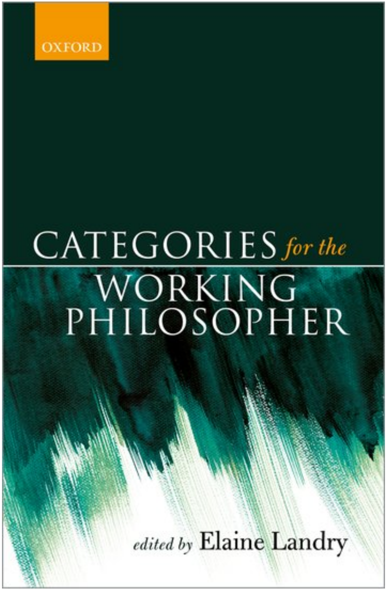
- Philosophy?


OXFORD
CATEGORIES for the
WORKING
PHILOSOPHER
edited by Elaine Landry

Why 'working'?

Saunders Mac Lane, Categories for the **working** mathematician, 1971

General abstract nonsense

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Grothendieck himself can't necessarily be blamed for this since his own use of categories was very successful in solving problems. (Miles Reid, Undergraduate Algebraic Geometry, p.116).

MathOverflow question

In which active research areas of (pure) mathematics is no (or only minimal) knowledge in category theory required ?

Tentative user

As a (slowly) recovering category-phobe, allow me to suggest that you change the way you think of category theory. Specifically, don't think of category theory as a "theory"... To learn a theory is to understand the proofs of the main theorems and how to apply them to the examples.

Category theory is different: there is an incredibly rich supply of definitions and examples, but very few theorems compared to other established "theories" like group theory or algebraic topology. Moreover, the proofs of the theorems are almost trivial (the Yoneda lemma is one of the most important theorems in category theory and it is not even called a "theorem"). A consequence of this is that you don't have to sit around reading a category theory book before you make contact with the language of categories: the very act of understanding how people express results from "ordinary" mathematics in the language of categories and functors **is learning category theory**. *(Paul Siegel)*

Lawvere - visionary

It is my belief that in the next decade and in the next century the technical advances forged by category theorists will be of value to dialectical philosophy, lending precise form with disputable mathematical models to ancient philosophical distinctions such as general vs. particular, objective vs. subjective, being vs. becoming, space vs. quantity, equality vs. difference, quantitative vs. qualitative etc. (1992)