Baryons at finite temperature

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Introduction

from hadronic to quark-gluon plasma

- thermodynamics: pressure, entropy, fluctuations
- symmetries: confinement, chiral symmetry

spectroscopy

- quarkonia
- light mesons
- baryons

real time

- transport
- far from equilibrium
Introduction

from hadronic to quark-gluon plasma

- thermodynamics: pressure, entropy, fluctuations
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- quarkonia
- light mesons
- BARYONS

real time

- transport
- far from equilibrium
Mesons in a medium

mesons in a medium very well studied

- hadronic phase: thermal broadening, mass shift
- QGP: deconfinement/dissolution/melting
- quarkonia survival as thermometer
- transport: conductivity/dileptons from vector current
- chiral symmetry restoration

relatively easy on the lattice

- high-precision correlators

what about baryons?
Baryons in a medium

lattice studies of baryons at finite temperature very limited

- screening masses  De Tar and Kogut 1987
- ... with a small chemical potential  QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators  Datta, Gupta, Mathur et al 2013

not much more ...

holographic studies of baryons at finite temperature?
Outline

baryons across the deconfinement transition:

- some basic thermal field theory
- lattice QCD – FASTSUM collaboration
- baryon correlators
- in-medium effects below $T_c$
- parity doubling above $T_c$
- spectral functions
Baryons

correlators

\[ G^{\alpha\alpha'}(x) = \langle O^{\alpha}(x) \overline{O}^{\alpha'}(0) \rangle \]

in this work – \( N, \Delta, \Omega \) baryons

\[ O_{\alpha}^{N}(x) = \epsilon_{abc} u_{a}^{\alpha}(x) \left( d_{b}^{T}(x) C \gamma_{5} u_{c}(x) \right) \]

\[ O_{\Delta,i}^{\alpha}(x) = \epsilon_{abc} \left[ 2 u_{a}^{\alpha}(x) \left( d_{b}^{T}(x) C \gamma_{i} u_{c}(x) \right) + d_{a}^{\alpha}(x) \left( u_{b}^{T}(x) C \gamma_{i} u_{c}(x) \right) \right] \]

\[ O_{\Omega,i}^{\alpha}(x) = \epsilon_{abc} s_{a}^{\alpha}(x) \left( s_{b}^{T}(x) C \gamma_{i} s_{c}(x) \right) \]
Baryons

- essential difference with mesons: role of parity

\[ \mathcal{P}O(\tau, x)\mathcal{P}^{-1} = \gamma_4 O(\tau, -x) \]

- positive/negative parity operators

\[ O_{\pm}(x) = P_{\pm}O(x) \quad P_{\pm} = \frac{1}{2}(1 \pm \gamma_4) \]

- no parity doubling in Nature: nucleon ground state

positive parity: \( m_+ = m_N = 0.939 \text{ GeV} \)
negative parity: \( m_- = m_{N^*} = 1.535 \text{ GeV} \)

- thread: what happens as temperature increases?
Spectral properties in a medium

- euclidean correlators $G(x) = \langle O(x)O^\dagger(0) \rangle$

- dispersion relation

$$G(i\omega_n, p) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega, p)}{\omega - i\omega_n}$$

- imaginary part of retarded correlator

$$\rho(\omega, p) = 2\text{Im} G(i\omega_n \to \omega + i\epsilon, p)$$

- back to euclidean time

$$G(\tau, p) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, p)$$
Spectral properties: mesons/bosons

\[ G(\tau, p) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, p) \]

- **Bosonic operators**  
  \( (\tilde{\tau} = \tau - 1/2T) \)

\[ K_{\text{boson}}(\tau, \omega) = T \sum_n \frac{e^{-i\omega_n \tau}}{\omega - i\omega_n} = \frac{\cosh(\omega \tilde{\tau})}{\sinh(\omega / 2T)} \]

- Kernel symmetric around \( \tau = 1/2T \), odd in \( \omega \)

- Spectral decomposition

\[ \rho_B(p) = \frac{1}{Z} \sum_{n,m} \left( e^{-k_n^0 / T} - e^{-k_m^0 / T} \right) |\langle n | O(0) | m \rangle|^2 \frac{\delta(4)}{(2\pi)^4} \delta(p + k_n - k_m) \]

- If \( O^\dagger = \pm O \) \( \Rightarrow \) \( \omega \rho(\omega, p) \geq 0 \) positivity
Spectral properties: baryons/fermions

\[ G^{\alpha\alpha'}(\tau, p) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho^{\alpha\alpha'}(\omega, p) \]

with

\[ G^{\alpha\alpha'}(x - x') = \langle O^\alpha(x) \overline{O}^{\alpha'}(x') \rangle \]
\[ \rho^{\alpha\alpha'}(x - x') = \langle \{ O^\alpha(x), \overline{O}^{\alpha'}(x') \} \rangle \]

fermionic Matsubara frequencies

\[ K(\tau, \omega) = T \sum_n \frac{e^{-i\omega_n \tau}}{\omega - i\omega_n} = \frac{e^{-\omega \tau}}{1 + e^{-\omega/T}} = e^{-\omega \tau} [1 - n_F(\omega)] \]

kernel not symmetric, instead

\[ K(1/T - \tau, \omega) = K(\tau, -\omega) \]
Kernels

- bosons

\[ K_{\text{boson}}(\tau, \omega) = \frac{\cosh(\omega \tilde{\tau})}{\sinh(\omega/2T)} = [1 + n_B(\omega)] e^{-\omega \tau} + n_B(\omega) e^{\omega \tau} \]

- fermions: even and odd terms

\[ K(\tau, \omega) = \frac{1}{2} [K_e(\tau, \omega) + K_o(\tau, \omega)] , \]

\[ K_e(\tau, \omega) = \frac{\cosh(\omega \tilde{\tau})}{\cosh(\omega/2T)} = [1 - n_F(\omega)] e^{-\omega \tau} + n_F(\omega) e^{\omega \tau} \]

\[ K_o(\tau, \omega) = -\frac{\sinh(\omega \tilde{\tau})}{\cosh(\omega/2T)} = [1 - n_F(\omega)] e^{-\omega \tau} - n_F(\omega) e^{\omega \tau} \]

- no singular behaviour \( 2T/\omega \) for fermions, no transport subtlety
Spectral decomposition: Positivity

\[ \rho(x) = \sum \gamma_\mu \rho_\mu(x) + \mathbb{1} \rho_m(x) \]

- take trace with \( \gamma_4 \), \( P_\pm = (\mathbb{1} \pm \gamma_4)/2 \):

\[
\rho_4(p) = \frac{1}{Z} \sum_{n,m,\alpha} \left( e^{-k_n^0/T} + e^{-k_m^0/T} \right) \frac{1}{4} |\langle n | O^\alpha(0) | m \rangle|^2 (2\pi)^4 \delta^{(4)}(p + k_n - k_m)
\]

\[
\rho_\pm(p) = \frac{\pm 1}{Z} \sum_{n,m,\alpha} \left( e^{-k_n^0/T} + e^{-k_m^0/T} \right) \frac{1}{4} |\langle n | O^\alpha_\pm(0) | m \rangle|^2 (2\pi)^4 \delta^{(4)}(p + k_n - k_m)
\]

- \( \rho_4(p), \pm \rho_\pm(p) \geq 0 \) for all \( \omega \)

- take trace with \( \mathbb{1} \)

\[
\rho_m(p) = [\rho_+(p) + \rho_-(p)]/4
\]

not sign definite
Charge conjugation

charge conjugation symmetry (at vanishing density):

\[ G_\pm(\tau, p) = -G_{\mp}(1/T - \tau, p) \quad \rho_\pm(-\omega, p) = -\rho_{\mp}(\omega, p) \]

- relates pos/neg parity channels

using \( G_+ (\tau, p) \) and \( \rho_+ (\omega, p) \)

- positive- (negative-) parity states propagate forward (backward) in euclidean time

- negative part of spectrum of \( \rho_+ \leftrightarrow \) positive part of \( \rho_- \)

example: single state

\[ G_+ (\tau) = A_+ e^{-m_+ \tau} + A_- e^{-m_- (1/T - \tau)} \]

\[ \rho_+ (\omega)/(2\pi) = A_+ \delta(\omega - m_+) + A_- \delta(\omega + m_-) \]
Chiral symmetry

**propagator**

\[ G(x) = \sum_{\mu} \gamma_\mu G_\mu(x) + \mathbb{1} G_m(x) \]

**chiral symmetry** \( \{ \gamma_5, G \} = 0 \Rightarrow G_m = 0 \)

**hence**

\[ G_+(\tau, p) = -G_-(\tau, p) = G_+(1/T - \tau, p) = 2G_4(\tau, p) \]

degeneracy of ± parity channels

\[ \rho_+(p) = -\rho_-(p) = \rho_+(-p) = 2\rho_4(p) \]

**parity doubling**

in Nature at \( T = 0 \): no chiral symmetry/parity doubling
Baryons in a medium

questions:

• in-medium effects below $T_c$?

• relevant for heavy-ion phenomenology?

• emergence of parity doubling?

• connection to deconfinement transition, chiral symmetry?

• chiral symmetry $\Leftrightarrow$ parity doubling
anisotropic $N_f = 2 + 1$ Wilson-clover ensembles

FASTSUM collaboration

GA (Swansea)
Chris Allton (Swansea)
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Maria-Paola Lombardo (Frascati)
Sinead Ryan (Trinity College Dublin)
Don Sinclair (Argonne)
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Tim Harris (TCD->Mainz->Milan)
Benjamin Jaeger (Swansea->ETH)
Aoife Kelly (Maynooth)
Bugra Oktay (Utah->)
Kristi Praki (Swansea)
Davide de Boni (Swansea)
This work

GA, Chris Allton, Simon Hands, Jonivar Skullerud

Davide de Boni, Benjamin Jäger, Kristi Praki


in preparation
FASTSUM ensembles

- $N_f = 2 + 1$ dynamical quark flavours, Wilson-clover
- many temperatures, below and above $T_c$
- anisotropic lattice, $a_s/a_\tau = 3.5$, many time slices
- strange quark: physical value
- two light flavours: somewhat heavy $m_\pi = 384(4)$ MeV

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>24</th>
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<td>$N_\tau$</td>
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<td>32</td>
<td>28</td>
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<td>16</td>
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<td>$T/T_c$</td>
<td>0.24</td>
<td>0.76</td>
<td>0.84</td>
<td>0.95</td>
<td>1.09</td>
<td>1.27</td>
<td>1.52</td>
<td>1.90</td>
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<tr>
<td>$N_{cfg}$</td>
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<td>500</td>
<td>500</td>
<td>1000</td>
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<tr>
<td>$N_{src}$</td>
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<td>4</td>
<td>4</td>
<td>2</td>
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- tuning and $N_\tau = 128$ data from HadSpec collaboration
Baryons in a medium

technical remarks

- studied various interpolation operators
- Gaussian smearing for multiple sources and sinks
- same smearing parameters at all temperatures
Lattice correlators

- nucleon

\[ G(\tau) \]

- positive parity
- negative parity

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- pos/neg parity channels nondegenerate
- more $T$ dependence in negative-parity channel
Lattice correlators

\[ \Delta \]

positive parity

negative parity

\[ \frac{G(\tau)}{T/T_c} \]

\[ \Delta \]

pos/neg parity channels nondegenerate

more \( T \) dependence in negative-parity channel
Lattice correlators

\[ \Omega \]

\[ G(\tau) \]

- **positive parity**
- **negative parity**

\[ \frac{T}{T_c} \]
- 1.90
- 1.52
- 1.27
- 1.09
- 0.95
- 0.84
- 0.76
- 0.24

- **pos/neg parity channels nondegenerate**
- **more** \( T \) dependence in negative-parity channel
Baryons in the hadronic phase
determine masses of pos/neg-parity groundstates

\begin{table}[h]
\centering
\begin{tabular}{lcccc|c}
\hline
$T/T_c$ & 0.24 & 0.76 & 0.84 & 0.95 & PDG ($T = 0$) \\
\hline
$m^+_N$ & 1158(13) & 1192(39) & 1169(53) & 1104(40) & 939 \\
m$^+_N$ & 1779(52) & 1628(104) & 1425(94) & 1348(83) & 1535(10) \\
\hline
$m^+_\Delta$ & 1456(53) & 1521(43) & 1449(42) & 1377(37) & 1232(2) \\
m$^+_\Delta$ & 2138(114) & 1898(106) & 1734(97) & 1526(74) & 1710(40) \\
\hline
$m^+_\Omega$ & 1661(21) & 1723(32) & 1685(37) & 1606(43) & 1672.4(0.3) \\
m$^+_\Omega$ & 2193(30) & 2092(91) & 1863(76) & 1576(66) & 2250–2380–2470 \\
\hline
$\delta_N$ & 0.212(15) & 0.155(35) & 0.099(40) & 0.100(35) & 0.241(1) \\
$\delta_\Delta$ & 0.190(31) & 0.110(31) & 0.089(31) & 0.051(28) & 0.162(14) \\
$\delta_\Omega$ & 0.138(9) & 0.097(23) & 0.050(23) & -0.009(25) & 0.147–0.175–0.192 \\
\hline
\end{tabular}
\caption{Masses of Baryons in the Hadronic Phase}
\end{table}

masses in MeV

$\delta = \frac{m_- - m_+}{m_- + m_+}$
Baryons in the hadronic phase

masses, normalised with $m_+$ at lowest temperature

- emerging degeneracy around $T_c$
- negative-parity masses reduced as $T$ increases
- positive-parity masses nearly $T$ independent
Baryons and parity partners

- distinct temperature dependence in hadronic phase
- relevant for heavy-ion phenomenology?

model studies of the role of chiral symmetry

eexample: parity doublet model

deTar & Kunihiro 89

- chiral invariant contribution $m_0$ equal for $N$ and $N^*$
- mass splitting due to chiral symmetry breaking
- degeneracy emerges as chiral symmetry is restored
  $m_0 \sim 500 - 800$ MeV

holographic predictions?
Baryon channels in QGP

- no clearly identifiable groundstates: baryons dissolved
- instead: parity doubling
- study correlator ratio

\[ R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)} \]

if

- no parity doubling and \( m_- \gg m_+ \): \( R(\tau) = 1 \)
- parity doubling: \( R(\tau) = 0 \)

note

- \( R(1/T - \tau) = -R(\tau) \) and \( R(1/2T) = 0 \)
Nucleon channel

- ratio close to 1 below $T_c$, decreasing uniformly
- ratio close to 0 above $T_c$, parity doubling
Quasi-order parameter

\[ R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)} \]

- integrated ratio

- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: \( m_q \neq 0 \)

- effect of heavier \( s \) quark visible
Parity doubling

- clear signal for parity doubling even with finite quark masses
- crossover behaviour, coinciding with transition to QGP
- visible effect of heavier $s$ quark
  what about other strange baryons?

lattice technical remark:

- Wilson fermions break chiral symmetry at short distances
  what about chiral lattice fermions?
Spectral functions

extract same information from spectral functions

\[ G_{\pm}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\pm}(\omega) \]

\[ K(\tau, \omega) = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}} \]

- *ill-posed* inversion problem
- use Maximum Entropy Method (MEM)
- featureless default model
- construct \( \rho_{+}(\omega) \geq 0 \) for all \( \omega \)
- \( \rho_{-}(\omega) = -\rho_{+}(-\omega) \)
Baryon spectral functions

- nucleon

groundstates below $T_c$

degeneracy emerging above $T_c$
Baryon spectral functions

\[ \Delta \]

\[ \tilde{\rho}(\omega) \]

\[ \omega [\text{GeV}] \]

- groundstates below \( T_c \)
- degeneracy emerging above \( T_c \)
Baryon spectral functions

- groundstates below $T_c$
- degeneracy emerging above $T_c$, finite $m_s$
Baryon spectral functions

all channels: low and high temperature

- groundstates below $T_c$
- degeneracy emerging above $T_c$
Baryon spectral functions

- results consistent with correlator analysis
- latter is on firmer ground, due to inversion uncertainties
- effect of heavier $s$ quark visible

expectation at very high temperature

- compute baryon spectral functions at $g^2 \to 0$
- similar to computation of meson spectral functions

Karsch et al 03, GA & Martínez Resco 05
Free spectral functions

lowest order in perturbation theory

\[ G(x) = \langle O(x) \overline{O}(0) \rangle \quad O(x) \sim uu^T C \gamma_5 d(x) \]

two-loop diagram \((c = 4, i, m)\)

\[ \rho_c(\omega) = 3 \int_{k_{1,2,3}} d\Phi_{123} \sum_{s_j=\pm} 2\pi \delta \left( \omega + \sum_j s_j \omega_{k_j} \right) \left[ \text{stat.} \right] f_c(\omega, s_i, k_i) \]

with

\[ d\Phi_{123} = \prod_{j=1}^{3} \frac{d^3 k_j}{(2\pi)^3 2\omega_{k_j}} (2\pi)^3 \delta(k_1 + k_2 + k_3) \]

\[ \left[ \text{stat.} \right] = n_F(s_1 \omega_{k_1}) n_F(s_2 \omega_{k_3}) n_F(s_3 \omega_{k_3}) \]

\[ + n_F(-s_1 \omega_{k_1}) n_F(-s_2 \omega_{k_3}) n_F(-s_3 \omega_{k_3}) \]
Free spectral functions

\[ \rho_4(\omega) \]

\[ \rho_m(\omega) \]

- decay: \( \omega > 3m \) with \( m \) quark mass
- at \( T > 0 \) scattering contributions for all \( \omega \)
- large \( \omega \): thermal contributions suppressed
- \( \rho_m(\omega) \) not positive definite
Free spectral functions

\[ \rho_4(\omega) = \frac{5\omega^5}{2048\pi^3} \left( 1 + \frac{112\pi^4}{3} \frac{T^4}{\omega^4} + \ldots \right) \]

\[ \rho_m(\omega) = \frac{7m\omega^4}{512\pi^3} \left( 1 - 4\pi^2 \frac{T^2}{\omega^2} + \ldots \right) \]

\[ \omega \gg T \gg m \]
Free spectral functions

\[ \rho_{\pm}(\omega) = \frac{1}{2} \left[ \rho_m(\omega) \pm \rho_4(\omega) \right] \]

- thermal enhancement at \( \omega \sim T \sim m \)
- apparent peak depends on presentation/normalisation
- exponentially suppressed as \( \omega \to 0 \)
- \( \pm \rho_{\pm}(\omega) \geq 0 \)
- \( \rho_-(\omega) = -\rho_+(-\omega) \)
Lattice free spectral functions

- lattice dispersion relation, sum over Brillouin zones
- maximal energy $\omega = 3\omega_{k,\text{max}}$
- similar to mesons (Karsch et al 03, GA & Martínez Resco 05)
- no cusps due to two-loop Brillouin sum
Summary: baryons in medium

in hadronic phase

- pos-parity groundstates mostly $T$ independent
- stronger $T$ dependence in neg-parity groundstates
  - reduction in mass, near degeneracy close to $T_c$
- relevant for heavy-ion phenomenology?

in quark-gluon plasma

- pos/neg parity channels degenerate: parity doubling
- linked to deconfinement transition and chiral symmetry restoration
- correlator and spectral function analysis consistent
- effect of heavier $s$ quark noticeable
Outlook: baryons in medium

lattice

- Wilson fermions: no chiral symmetry at short distances
- manifestly chiral fermions?

physics

- strangeness dependence
- physical light quarks
- phenomenology

understanding

- models?
- holography?