The Numerical Challenge

Backreaction in Relativistic N-body Simulations of Cosmic Structure Formation

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CosmoBack

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The Quest for a Complete Answer

Heuristic arguments can motivate studies of backreaction, but are only a first step towards a complete understanding

Exactly solvable systems give interesting insights, but are always toy models that miss some aspects of reality

Perturbative methods allow quantitative statements but have limited domain of validity

Numerical simulations are a unique tool to study scenarios that resemble reality more closely

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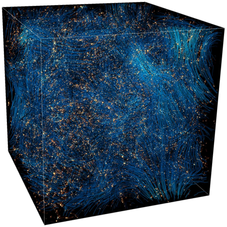
Numerical Challenge: include all relevant aspects of reality

- Allow for fully nonlinear evolution of matter
- Consistently solve the evolution of perturbed geometry
- Compute *actual* observables

A Brief Overview of *gevolution*

gevolution, a general relativistic N-body code
Adamek, Daverio, Durrer & Kunz, Nature Phys. **12** (2016) 346–349



spin-1 metric perturbation with *gevolution*

- based on weak-field expansion (in Poisson gauge)
- for any given T^{μ}_{ν} computes the six metric d.o.f. (Φ, Ψ, B_i, h_{ij})
- N-body particle ensemble evolved using relativistic geodesic equation

https://github.com/gevolution-code/gevolution-1.1.git

Strategy

- choose ansatz for the metric (perturbed FLRW)

$$ds^2 = a^2(\tau)\left[-e^{2\Psi}d\tau^2 + e^{-2\Phi}\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j - 2B_i dx^i d\tau\right]$$

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- stress-energy tensor is determined by solving the EOM's of all sources of stress-energy

$$T^{\mu\nu}_{\mathrm{m}} = \sum_n m_{(n)} \frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_{(n)})}{\sqrt{-g}} \left(-g_{\alpha\beta}\frac{dx^\alpha_{(n)}}{d\tau}\frac{dx^\beta_{(n)}}{d\tau}\right)^{-\frac{1}{2}} \frac{dx^\mu_{(n)}}{d\tau}\frac{dx^\nu_{(n)}}{d\tau}$$

Canonical Momentum

One-particle action ⇒ canonical momentum

$$\mathcal{S} = -m \int \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau \qquad \Rightarrow \qquad \mathbf{q} = \frac{\partial \mathcal{L}}{\partial \mathbf{v}}$$

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Geodesic equation

$$\frac{dq_i}{d\tau} = -\frac{\partial}{\partial x^i}\left(e^\Psi \sqrt{\mathbf{q}^2 e^{2\Phi} - q^j q^k h_{jk} + m^2 a^2} + q^j B_j\right)$$

$$\frac{dx^i}{d\tau} = \frac{\partial}{\partial q_i}\left(e^\Psi \sqrt{\mathbf{q}^2 e^{2\Phi} - q^j q^k h_{jk} + m^2 a^2} + q^j B_j\right)$$

Stress-energy tensor

$$T^0_0 = -\delta^{(3)}(\mathbf{x} - \mathbf{x}_{(n)}) \frac{e^{3\Phi}}{a^4}\left(\sqrt{\mathbf{q}^2 e^{2\Phi} - q^i q^j h_{ij} + m^2 a^2} + q^i B_i\right)$$

Einstein's Equations

$$-\frac{a^2}{2}G^0_0 = \frac{3}{2}e^{-2\Psi}\left(\mathcal{H}-\Phi'\right)^2 + e^{2\Phi}\left[\Delta\Phi - \frac{1}{2}\left(\nabla\Phi\right)^2\right]$$

$$\frac{a^2}{2}G^0_i = e^{-\Psi}\nabla_i\left[e^{-\Psi}\left(\mathcal{H}-\Phi'\right)\right] - \frac{1}{4}\Delta B_i$$

$$a^2\left(G^i_j - \frac{1}{3}\delta^i_j G^k_k\right) = \left(\delta^{ik}\delta^l_j - \frac{1}{3}\delta^i_j\delta^{kl}\right)\left[e^{\Phi+\Psi}\nabla_k\nabla_l e^{\Phi-\Psi} - 2e^{2\Phi}\left(\nabla_k\Psi\right)\left(\nabla_l\Psi\right) + B'_{(k,l)} + 2\mathcal{H}B_{(k,l)} + \frac{1}{2}h''_{kl} + \mathcal{H}h'_{kl} - \frac{1}{2}\Delta h_{kl}\right]$$

Here I dropped quadratic and higher-order terms only with B_i or h_{ij}.

For computational efficiency the exponentials can be expanded (weak-field expansion).

Features

Version 1.1 (public)

- Multiple particle species (CDM, baryons, neutrinos)
- Initial condition generation “on the fly”
- Auto- and cross-power spectra
- Linear perturbations in the radiation field
- Newtonian mode compatible with radiation perturbations (using N-body gauge)
- Massive neutrinos can be treated as linear perturbations and/or as particles

Version 1.2 (upcoming)

- Particle & metric light cones for ray tracing and post-processing
- Linear dark energy fluids (w-c_s-parametrization)

Ray Tracing

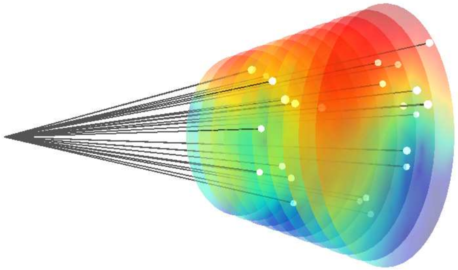
Instead of keeping *snapshots* $= \{\mathsf{data} \mid \tau = \tau_{\mathsf{snap}}\}$, we store a *thick light cone* $= \{\mathsf{data} \mid \tau - \tau_o + r \in [-\Delta\tau, \Delta\tau]\}$, where $\Delta\tau$ is chosen such that the *perturbed light cone* $\subset$ *thick light cone*.

In a post-processing step, we integrate backwards in time (without approximation):

null geodesic equation

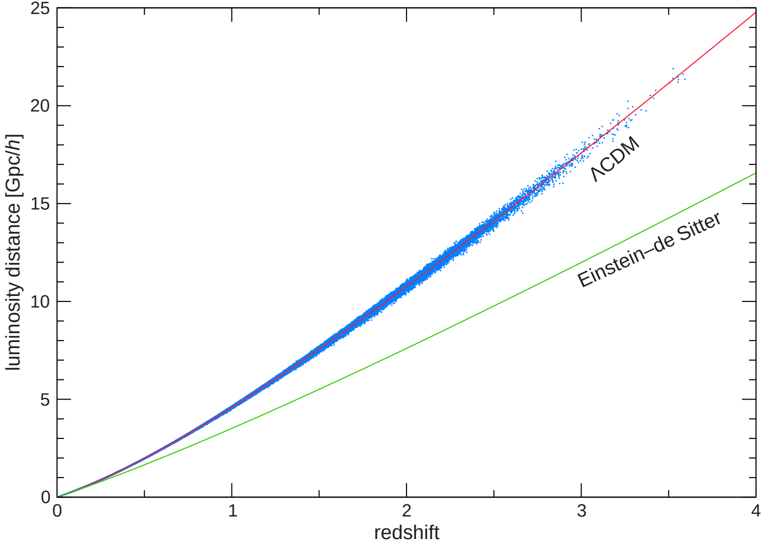
$\Downarrow$

observed angles & redshifts



This allows us to construct the statistics of observed sources.

Preliminary Results



work in progress with: Clarkson, Coates, Durrer & Kunz

Concluding Remarks

Backreaction is a real phenomenon that…

- can be quantified accurately with numerical experiments
- quantitatively cannot explain observed data without dark energy
- may nevertheless be relevant for precision cosmology with future surveys

Our insights from this debate will help us to test gravity on cosmological scales

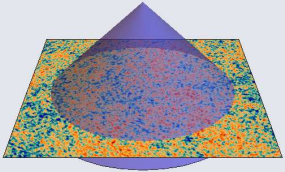
Discussion

- What about boundary conditions?
- What about black holes?
- Is this “full GR” yet?
- How does this compare to Numerical Relativity fluid simulations?
- What is wrong with arguments that give large effects on H_0?
- Do we need a mathematical proof about backreaction?
- *Add your own question here*

Discussion

- What about boundary conditions?

For sufficiently large box, the Cauchy data for a light cone fits into the simulation volume. Data outside the light cone is irrelevant and can be chosen to fulfill periodic boundary conditions.



Discussion

- What about black holes?

Black holes fall the same way as stars or other matter (**equivalence principle**)
$\Rightarrow$ gravitational dynamics of large-scale structure is unaffected.

Discussion

- Is this "full GR" yet?

Yes it is.

. . . to the extent that for a particular class of solutions (i.e. cosmological ones) we obtain an accurate rendition of spacetime and the matter configuration in it. We do not miss any relativistic aspects.

Discussion

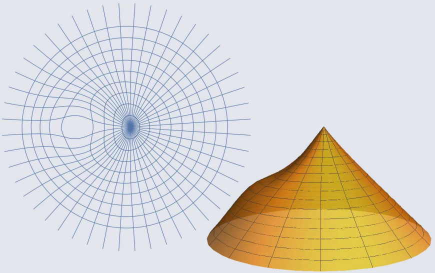
- How does this compare to Numerical Relativity fluid simulations?

Good agreement expected (but further studies warranted)
East, Wojtak & Abel, Phys. Rev. **D97** (2018) 043509

Comparison needs to be done based on observables
Adamek, Gosenca & Hotchkiss, Phys. Rev. **D93** (2016) 023526

NB: Fluid simulations often use coordinates in which the light cone is heavily distorted!

Naive interpretation easily fails.



Fluid simulations have *no access* to the clustering / multistream regime!

- What is wrong with arguments that give large effects on H_0?

In an inhomogeneous universe the notion of H_0 becomes ambiguous: one can define multiple quantities that coincide with H_0 in the FLRW limit, some of which can be very sensitive to the presence of inhomogeneities.

Such arguments can be **misleading**.

The correct, unambiguous procedure is to predict the statistics of observed sources.

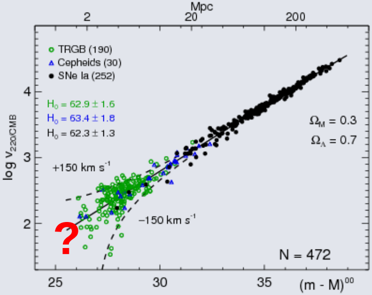


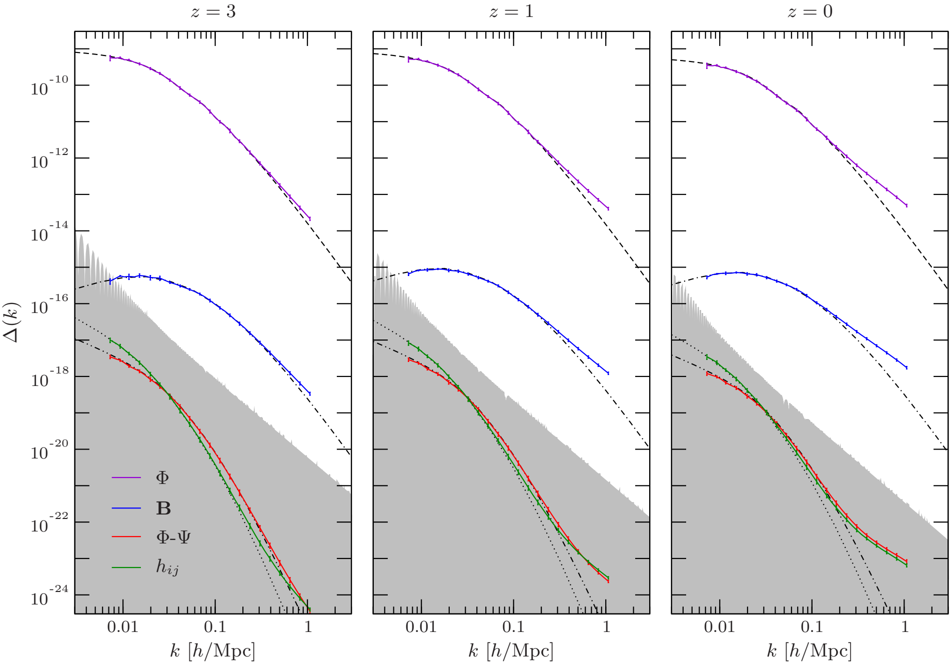
figure: Tammann & Reindl, IAU Symp. **289** (2013) 13

Discussion

- Do we need a mathematical proof about backreaction?

No (even though I appreciate the effort).

We need a satisfactory understanding of how observed phenomena arise from laws of nature.



$z = 3$
$z = 1$
$z = 0$
$\Delta(k)$
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
Φ
$\mathbf{B}$
Φ-Ψ
h_{ij}
0.01
0.1
1
k [h/Mpc]