Black Holes in Supergravity

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Abstract

A review is given on what is known about stationary black hole configurations in $N = 1$ and $N = 2$ supergravity.

Soon after supergravity was discovered, the question arose how the causal structure of spacetime might be influenced by the gravitino (spin-3/2) field. It was Mme. Yvonne Choquet-Bruhat who helped to clarify this question by studying the Cauchy problem in supergravity [1], [2], [3]. It is for me a pleasure to summarize what we have learned since then about the existence of black hole solution in supergravity.

In classical relativity stationary black holes are characterized by a restricted number of "charges" that can be expressed as surface intergrals at spacial infinity (no-hair conjecture). In supergravity the invariance under local supersymmetry transformations give rise to a new conserved spinor charge (supercharge). It is therefore of interest to study whether black hole solution with supercharge exist.

Within the Einstein-Maxwell theory a generic stationary black hole is completely characterized by its mass (M), angular momemtum (a), electric (e) and/or magnetic (q) charge. This theory can be embedded in $N = 2$ supergravity whose field content is: The gravitational vierbein field $e^a = e^a{}_\mu dx^\mu$, the electromagnetic potential one-form $A = A_\mu dx^\mu$ and two Majorana spinor-valued one-forms ψ^j, combined to a complex (Dirac) field $\psi = \psi^1 + i\psi^2 = \psi_\mu dx^\mu$ (Rarita-Schwinger field). All fields are Grassmann-valued, the bosonic fields (e, A) being even elements while ψ is odd.

A possible way to study the existence of black hole solutions in supergravity is to consider pertubation of the spin- 3/2 field on a black hole background.If one retains from the full $N = 2$ field equations term linear in ψ, the system is reduced to the

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Einstein-Maxwell equations plus the (linearized) Rarita-Schwinger equation on a given background:

$$\gamma \wedge \widehat{D}\psi = 0 \tag{1}$$

where

$$\widehat{D} = d + \frac{1}{2}\omega^{ab}\sigma_{ab} - \frac{k}{2}F^{ab}\sigma_{ab}\gamma, \qquad \gamma = \gamma_a e^a. \tag{2}$$

(The explicit form of the field equations , notations and conventions may be found in Ref. 4, especially we have $k^2 = 4\pi G$ and signature $(+---)$.) The local supersymmetry transformation are generated by a complex spinor field $\varepsilon = \varepsilon^1 + i\varepsilon^2$ ($\varepsilon^1, \varepsilon^2$ Majorana). At the linearized level these transformations change the Rarita- Schwinger field by

$$\delta\psi = k^{-1}\widehat{D}\varepsilon \tag{3}$$

while the vierbein and the electromagnetic potential are invariant.

The global conserved quantity associated with this symmetry is the spinorial supercharge

$$\mathcal{S} = -\frac{i}{k}\oint_{S^2_\infty} \gamma_5\gamma \wedge \psi \tag{4}$$

which acts as a generator for the (asymptotic) global supersymmetry transformations. We consider only asymptotically flat configurations and ψ has to fall off like $O(r^{-2})$ in order to render $\mathcal{S}$ finite.

For the background one takes the Kerr-Newman black hole and tries to solve eq.(1) imposing the following conditions:

i) stationarity

ii) fall off at spatial infinity

iii) regularity at and outside the horizon.

Because of the local gauge freedom (3) any field of the form

$$\psi = k^{-1}\widehat{D}\varepsilon \tag{5}$$

is automatically a solution to eq. (1). Moreover, the supercharge (4) is only invariant under (proper) gauge transformations for which the gauge spinor ε tends to zero at spatial infinity. We shall therefore distinguish between gauge-generated and non-gauge configurations (see Fig. 1).

Non-Gauge Fields

Let us first consider non-gauge configurations. A detailed analysis of the spin-3/2 modes on a Kerr-Newman black hole [5] has shown that there are no solutions of Eq. (1) that satisfy the conditions i) – iii) unless the background parameters are such that

$$k^2M^2 = e^2 + q^2 \tag{6}$$

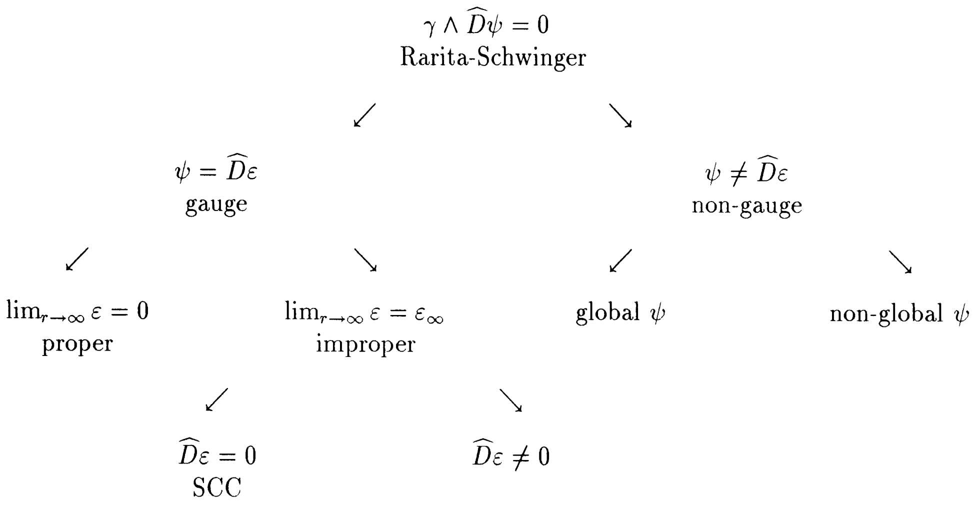

Figure 1: The classification of solutions to the Rarita-Schwinger equation are shown. Of interest are the non-gauge global and non-global (monopole-like) as well as the improper gauge generated (superpartners) configurations.

i.e. the charge of the hole equals its mass (in units where $4\pi G = 1$). This singles out the extreme Reissner-Nordstrøm metric, since a has to vanish in order that a horizon exists. It was shown [6,7] that the extreme Reissner-Nordstrøm field indeed admits spin-3/2 perturbations satisfying the conditions i) – iii). These perturbations are characterized by two complex constants and give rise to a non-zero supercharge. The linear solution was later generalized (by rather tedious calculations) to an exact one, taking into account the full back-reaction of the Rarita-Schwinger field onto the Einstein-Maxwell fields [8]. Embacher [9] has shown that this solution is the most genral one.

The uniqueness of the extreme Reissner-Nordstrøm metric in supporting a spin-3/2 supercharge has a geometrical explanation: Among the Kerr-Newman black holes the extreme Reissner-Nordstrøm metric is singled out by the existence of supercovariantly constant (SCC) spinors χ_{SCC} satisfying

$$\widehat{D}\chi_{SCC} = 0. \tag{7}$$

These spinors give rise to additional gauge invariant Rarita-Schwinger modes. Actually it is possible to construct the linear superhair solutions directly from χ_{SCC} [10].

Further insight into the nature of static configurations with supercharge was gained by considering Rarita-Schwinger fields on multi-black-hole backgrounds. Spacetimes of more than one black hole are expected to be non-static. There exists however a class of spacetimes, the Majumdar-Papapetrou fields, which represent static configurations of several extreme black holes in equilibrium under their mutual gravitational and electrical forces. Moreover, the Majumdar-Papapetrou black holes are the only solutions within the Einstein-Maxwell fields that admit SCC spinors.

Detailled calculations have shown that regular static spin-3/2 perturbations exist also on Majumdar-Papapetrou spacetimes. On a background with n black holes these solutions depend on $2n$ complex parameters which are however, strongly constraint.The constraints reduce the number of free parameter to two, indepent of the underlying spacetime. A direct consequence is that for a two-black hole system the supercharge is proportional to the difference in the mass parameters [11]. Consequently, for two equal black holes the supercharge is zero. One may interpret the constraints as equilibrium conditions for static configurations.This would imply that there is a new kind of interaction between black holes due to the existence of supercharge. A step towards such an interpretation was made in Ref. [12], by relating the supercharge to changes in the mass and charge parameters of the background.

Monopole-Like Solutions

There is another possibility to look at the constraints: Since they follow from global integrability conditions, violating them implies the non-existence of global solutions for ψ on the given background. However, ψ being a gauge field, it is conceivable to consider only its supercovariant field strength $\widehat{D}\psi$ as the fundamental object and to give up global existence of ψ. If one requires regularity for $\widehat{D}\psi$, then ψ may have line-singularities which may be shifted by gauge transformations of the form (3). This is analogous to the Dirac monopole in electrodynamics.

The existence of "strings" is associated with non-zero global quantities:

$$\mathcal{P} = \frac{1}{4\pi i} \oint_{S^2_\infty} \widehat{D}\psi$$

which we call "spinorial magnetic-type charges".

In a paper "Monopole-like solutions on Majumdar Papapetrou backgrounds" [13], we have studied these kind of solutions.At the linearized level it is possible to associate with each black hole a spinorial magnetic charge.Because of the Grassmann character of the fields it is not clear whether these magnetic type charges are of topological origin.

Gauge-Generated Solutions

Consider now ψ-configurations of the type (5) for which the gauge spinor ε tends to a constant ε_∞ at spatial infinity. The gauge-induced supercharge may then be written as:

$$\mathcal{S} = -\frac{i}{k^2} \oint_{S^2_\infty} \gamma_5 \gamma \wedge \widehat{D}\varepsilon = (-i\gamma^a P_a^{ADM} + \frac{i}{k} e + \frac{i}{k} \gamma_5 q)\varepsilon_\infty \qquad (8)$$

whre P_a^{ADM} is the usual ADM four-momentum of the configuration, while e and q is the total electric and magnetic charge, respectively.

We wish to identify all configurations which can be obtained by "proper" gauge transformations ($\varepsilon \rightarrow 0$) from one another thereby dividing the solutions into equivalence classes, which may be parametrized by a constant spinor at infinity. If the background is extreme, satisfying condition (6), the matrix on the r.h.s. of eq.(8) is singular. As a consequence half of the asymptotic spinor space leaves the supercharge $\mathcal{S}$ invariant. This part is associated with the super covariantly constant spinor χ_{SCC}, eq. (7), at spatial infinity. The remaining transformations induce a supercharge via eq. (8).

It was pointed out by Gibbons [14], [15], that in $N = 2$ supergravity the extreme Reissner-Nordstrøm or, more generally, the Majumdar-Papapetrou spacetimes are partially supersymmetric: gauge transformations with χ_{SCC} leave the fields invariant. On the other hand, those transformations that induce a supercharge give rise to "superpartners" to the purely bosonic configurations. Hajicek has emphasized [16] that the extreme Reissner-Nordstrøm spacetimes should be considered also quantum mechanically as stable particle-like (solitons) configurations because they do not show Hawking radiation.

The linear gauge-generated solutions may be itereted to obtain an exact superpartner satisfying the full $N = 2$ supergravity equations [17]. The gauge induced supercharge gives rise to an "intrinsic" angular momentum.Similar to the Kerr-Newmann spacetimes the configurations is stationary with a magnetic moment.

In a series of papers [18], [19], [20], [21], we have formulated a dynamics for these superpartners. Generalizing the work of Hajicek [22] for the Einstein-Maxwell solitons to supergravity one describes the superpartner by collective coordinates associated with its asymptotic degrees of freedom: translational and supertranslational (spinorial) parameters.By freezing all degrees of freedom but these asymptotic ones, one is lead to a particle dynamics in rigid (flat) $N = 2$ superspace. Upon quantizing his theory, one arrives at the basic $N = 2$ hypermultiplet. The next step was to study the dynamics of these particle in an external field. This naturally gave rise to a formulation of the theory in curved superspace. The classical equation of motion is the supersymmetric generalization of the Mathison-Papapetrou equation for a spinning particle in a gravitational field.

The last step in this development was to formulate an effective soliton-soliton interaction in supergravity. This was achieved by choosing a superpartner configuration for the background in which the other particle moves and applying a slow-motion and long-distance approximation. The interaction potential was obtained to lowest order that incorporates the effect of the supercharge. A Dirac-like proceedure was then applied to quantize the supersymmetric two- particle system.

Summary

The study of classical particle-like solutions in supergravity has shown that stationary black holes can carry a new conserved quantity called supercharge. Within $N = 2$ supergravity only the extreme Reissner-Norstrøm can support a supercharge. Exact non-gauge and gauge- induced (superpartner) configurations are known. On the classical level the geometrical interpretation of these black holes are obscured by the fact that all the fields are Grassmann- valued. In a quantum theory especially the superpartners should play an essential role when considering non-perturbative effects.

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