

Strongly Interacting Topological Phases

Jason Alicea (Caltech),
Matthew P. A. Fisher (University of California, Santa Barbara),
Marcel Franz (University of British Columbia),
Yong-Baek Kim (University of Toronto)

September 20–25, 2015

1 Overview of the Field

Topology and physics comprise two ostensibly very different fields. The former represents a branch of mathematics associated with invariance of form, while the latter quantifies the behavior of nature that, fundamentally, arises from geometric laws. These disciplines have nevertheless become increasingly intertwined with the advent of ‘topological phases’—wherein topological properties *emerge* from purely geometric rules in many-particle systems. Their behavior often defies intuition and conventional theories of matter in stunning ways, necessitating the development of new physics paradigms.

The order present in a topological phase, for instance, is characterized not by a local observable (*e.g.*, magnetization) but rather a topological invariant that signifies global ‘knotting’ in a system’s wavefunction. One measurable implication is that topological phases often exhibit metallic states at their boundary that are intrinsically resilient to imperfections such as impurities—in sharp contrast to standard metallic systems. Moreover, many topological phases support exotic ‘fractionalized’ excitations whose properties bear little resemblance to those of the constituent electrons and ions in a material. These characteristics, fascinating in their own right, endow topological phases with great technological promise in areas ranging from low-dissipation electronics to topological quantum computation.

Over the last several years the field has matured at an astonishing pace—particularly in the case of *weakly correlated* topological phases, whose universal properties do not rely on inter-particle interactions in an essential way. The latter are indeed now fairly well-understood due to the wealth of analytical and numerical tools available for treating weakly interacting electrons. A prominent case in point is provided by ‘topological insulators’, which were theoretically discovered in 2005 to arise in effectively non-interacting crystalline solids featuring strong coupling between the electrons’ spin and orbital motion. Such systems possess an electrically inert interior yet necessarily conduct at their edges provided certain symmetries such as time-reversal are present. Soon after, a vast number of candidate topological insulator materials were identified via standard band structure methods, whose predictive power is outstanding in weakly correlated settings. Indeed, following such predictions many 2D and 3D topological insulators have now been observed experimentally using electron spectroscopy and transport techniques. The detailed phenomenology of topological insulators—including their unusual electromagnetic response properties, stability to symmetry-breaking perturbations, entanglement structure, *etc.*—can also be efficiently addressed theoretically owing to the weakly interacting nature of the problem.

Other weakly correlated topological phases enjoy a similar level of tractability. In fact an exhaustive classification of topological phases for free-electron systems in *all* spatial dimensions is now in place, which

captures topological insulators, integer quantum Hall systems, and topological superconductors in a unified framework. (For free-boson systems the classification is trivial since without interactions Bose-Einstein condensation necessarily results.) All such weakly correlated topological phases either find intrinsic realization in specific compounds, or can be ‘engineered’ by judiciously combining conventional materials to essentially force electrons into nontrivial topological states. The complete experimental realization of phases in this classification remains an outstanding problem, but a concrete roadmap nonetheless exists and efforts in this direction are well underway.

2 Recent Developments and Open Problems

Strongly correlated topological phases—which require inter-particle interactions to exist—pose much greater challenges to both theory and experiment. Methods that have proven spectacularly successful for dealing with non-interacting systems (*e.g.*, band structure and K -theory among others) indeed are wholly inadequate in this context. Many profound questions therefore remain only partially answered here: What is the full classification of topological phases in the interacting case, both for fermionic and bosonic systems? (Interacting bosons can exhibit topological phases.) What mathematical and numerical tools allow one to efficiently extract their essential universal properties? Which materials are likely ‘intrinsic’ candidates for strongly correlated topological phases? Can they be ‘engineered’ in a reliable way? How can their topological properties be unambiguously observed in an experiment? And how does the physics survive when realistic ingredients such as disorder and thermal fluctuations are incorporated?

Exploring such questions promises to push the boundary of our understanding of quantum mechanics and the organizing principles of nature, yet is by no means purely academic. In particular, certain strongly interacting topological phases can provide a platform for a universal quantum computer that exhibits intrinsic immunity against decoherence—the primary bottleneck that for decades has stymied efforts at fabricating a scalable quantum computer. Thus the study of highly correlated topological phases may ultimately help to solve one of the great outstanding technological problems in physics, providing immense complementary motivation for the problem.

The topic of strongly interacting topological phases has, consequently, captivated diverse segments of the physics community. Theorists are attempting a complete classification of topological phases for interacting bosons and fermions. Complementing this challenging endeavor, solvable toy models for correlated topological phases with increasing levels of realism are being constructed, and new schemes for engineering their physics in the laboratory are being developed. Another active area of theoretical research concerns the interplay between interactions and strong randomness, which may, counterintuitively, enable topological properties to survive in regimes where they are absent in an otherwise clean system. Quantum information scientists grapple with the question of how correlated topological phases can be used for applications ranging from robust quantum memory to universal quantum computation. On the experimental side, measurements suggest that solid-state systems including GaAs quantum wells, graphene, heavy fermion compounds, and magnetic insulators host correlated topological phases whose precise nature remains to be understood. Concomitantly, cold atoms researchers have devised ingenious schemes for realizing models known to support interacting topological phases in optical lattice setups.

The workshop was particularly inspired by five areas:

1) Classification of topological phases. Strong correlation is known to produce topological phases, such as fractional quantum Hall states and spin liquids, that are absent in the free-particle classification. Bosonic topological phases provide an extreme example where in fact *all* such states originate from interactions. Conversely, phases that are topologically distinct in the free-particle limit can, in select cases, actually lose their distinction when strong interactions are present. Recent studies have even shown that interactions can generate novel topological phases that can *only* exist at the boundaries of a higher dimensional system. These examples strikingly underscore the need for a more general classification of topological phases that extends into the strong correlation regime. Specific aims of the workshop for this area included (*i*) exploring mathematical methods necessary for this classification; (*ii*) applying them to characterize possible topological phases with and without symmetry requirements, as well as to boundary topological phenomena; (*iii*) constructing general topological invariants for these phases; and (*iv*) enumerating their universal physical properties.

2) Strongly correlated topological materials. Recently there has been great progress in connecting theoretical lattice models for interacting topological phases with real materials. The underlying idea is to use both strong spin-orbit coupling and Coulomb interactions to generate highly entangled many-body quantum states. Particularly fruitful are systems defined on geometrically frustrated lattices, wherein all pairwise couplings between particles cannot be satisfied simultaneously. On these lattices—which often consist of corner-sharing simplexes such as triangles and tetrahedra—the interplay between spin-orbit coupling, Coulomb repulsion, and frustration can stabilize a plethora of new interacting topological phases. Examples include correlated analogues of topological insulators that feature gapless fractionalized excitations at their boundary, ‘axion insulators’ exhibiting topological electromagnetic response, and Weyl semi-metals in which gapless excitations are described by two-component Dirac fermions. Several iridium-based compounds provide possible sources for such phenomena, and are therefore presently the subject of intense experimental investigation. The goal of the workshop in this area of research was to (i) understand the universal structure of lattice models that may possess topological phases; (ii) make precise connections between topological properties of the many-body wavefunction and the geometry of the underlying lattices; and (iii) obtain more precise theoretical or mathematical criteria that can be used to identify such phases in both models and real materials.

3) ‘Engineered’ platforms for interacting topological phases. While nature provides numerous promising host materials, many (and perhaps most) possible strongly correlated topological phases may be difficult to realize by relying solely on a system’s internal dynamics. A powerful alternative method is to seek ‘designer’ interacting topological phases by combining well-understood ingredients that are already widely available and well-understood. Such an approach is routinely used in the context of cold atoms, where proposals exist for generating nontrivial correlated phases via artificial gauge fields, flat bands with non-trivial Chern numbers, spin models with enlarged symmetries, *etc.* In the solid state realm, heterostructures formed out of systems such as (Abelian) fractional quantum Hall phases and superconductors have been predicted to host exotic non-Abelian defects that can be used for universal topological quantum computation. These examples illustrate the potential power behind engineered topological phases but are by no means exhaustive. The workshop sought to address possible ways of enhancing the feasibility of existing proposals, identify means of detecting the predicted correlated topological phases and harnessing their properties for applications, and work towards more general schemes for designing arbitrary interacting topological phases.

4) Many-body localization. Most topological phases are characterized by an energy gap to bulk excitations. This energy gap is critical in protecting topological degeneracies that can serve as decoherence-free quantum memory or as the basis for fault-tolerant quantum computation. When temperatures are well below this energy gap, unwanted ‘stray’ excitations can in principle be exponentially suppressed leaving the protected manifold essentially untouched. But at higher temperatures thermally induced excitations can be disastrous, destroying quantum memory or dephasing a quantum computation. In many potential topological quantum systems, avoiding such thermal dephasing effects will, in practice, be exceedingly challenging. Will such ‘thermalizing’ effects always be a showstopper? Perhaps not. Remarkably, when an isolated quantum system is subject to a quenched random potential (disorder), a Many-Body-Localized (MBL) phase can exist within which thermalization is simply not operative. In an MBL phase energy can become localized by the disorder and the system cannot self-thermalize, even when the system possesses a finite energy density. The implications of many-body localization for topological phases is in its infancy, and many questions need addressing. Is it possible to stabilize quantum memory in an MBL phase? Can one characterize topological order in an MBL phase which has no energy gap? What are the implications for braiding and quantum computation in the MBL setting?

5) Quantum information applications. One of the holy grails of the field is to utilize correlated topological phases to construct a universal, decoherence-free quantum computer. (The key idea is that certain correlated topological phases allow storage and processing of quantum information in a non-local manner that is intrinsically immune to local environmental perturbations that ordinarily cause decoherence.) The workshop will provide a venue for discussing many fundamental questions related to this ultimate goal. For instance, what implications does many-body localization have for the stability of qubits? What is the ‘optimal’ correlated topological phase for this application? More precisely, what phase of matter allows one to run a given quantum algorithm with the fewest number of qubits, and using the smallest number of elementary operations? On a more practical level, how can such an ‘ideal’ correlated topological phase be constructed experimentally and manipulated for quantum computation? As the technology develops, what initial applications can be realized when the number of qubits is relatively low? Addressing such questions can provide

valuable long-term direction for the field.

3 Presentation Highlights

Our workshop gathered a broad spectrum of researchers studying many facets of the problem. The following broad themes were explored: Classification of topological phases; frontiers in Majorana-zero-mode platforms, with an emphasis on interaction-driven physics; non-Abelian defects in topological phases; the interplay between symmetry and topology; new developments in the half-filled Landau level and connections to topological insulators; and topological materials including Weyl semimetals, magnetic insulators, and a new type of ‘cohomological insulator’. Five extended overview talks highlighted progress and open challenges in these areas:

1. Lukasz Fidkowski (Stony Brook), ‘Symmetry and Topological Phases: an overview’

In this talk I will review the recent progress in topological phases with symmetries. A central focus will be Symmetry Protected Phases (SPTs), and I will discuss various approaches to studying these, including microscopic models, group cohomology, effective Chern-Simons field theories, and non-linear sigma models. I will emphasize open issues, including SPTs in higher dimensions and those beyond the cohomology classification, and SPTs built out of fundamental fermion degrees of freedom.

2. Alexey Gorshkov (JQI) ‘Topological Phases in Atomic, Molecular, and Optical Systems’

We will first review schemes for taking advantage of the tremendous degree of control recently achieved in AMO (atomic, molecular, and optical) systems to realize topological phenomena. In particular, we will emphasize unique features of AMO systems such the abundance of bosonic platforms, accessibility of far-out-of-equilibrium dynamics, and natural occurrence of interactions decaying as tunable power laws. We will then focus on a few examples such as SPT phases with ion crystals, various fractional quantum Hall states with dipoles, and parafermionic zero modes with ultracold neutral bosons.

3. Charlie Marcus (University of Copenhagen), ‘Majorana update’

This talk surveyed experimental progress in synthesizing topological superconductors fashioned from semiconductor wire/superconductor hybrid architectures. Breakthroughs on both the fabrication and measurement end were discussed, including new signatures of Majorana zero modes in ultra-clean, hard-gap devices.

4. Todadri Senthil (MIT), ‘Half-filled Landau level, topological insulator surfaces, and three dimensional quantum spin liquids’

We synthesize and partly review recent developments relating the physics of the half-filled Landau level in two dimensions to correlated surface states of topological insulators in three dimensions. The latter are in turn related to the physics of certain three dimensional quantum spin liquid states. The resulting insights provide an interesting answer to the old question of how particle-hole symmetry is realized in composite fermion liquids. Specifically the metallic state at filling $\nu = \frac{1}{2}$ - described originally in pioneering work by Halperin, Lee, and Read as a liquid of composite fermions - was proposed recently by Son to be described by a particle-hole symmetric effective field theory distinct from that in the prior literature. We show how the relation to topological insulator surface states leads to a physical understanding of the correctness of this proposal. We develop a simple picture of the particle-hole symmetric composite fermion through a modification of older pictures as electrically neutral “dipolar” particles. We revisit the phenomenology of composite fermi liquids (with or without particle-hole symmetry), and show that their heat/electrical transport dramatically violates the conventional Wiedemann-Franz law but satisfies a modified one. We also discuss the implications of these insights for finding physical realizations of correlated topological insulator surfaces.

5. Xiao-Gang Wen (MIT) ‘2+1D Bosonic/Fermionic topological orders with/without symmetry’

2+1D bosonic/fermionic topological orders with/without symmetry can be described in a unified way by the so called “non-degenerate braided fusion categories (UBFC) over a symmetric category”; where the symmetric category describes a fermionic product state without symmetry or a fermionic/bosonic product state with symmetry G . I will describe those mathematical notion in simple terms and discuss the table of bosonic/fermionic topological orders. For example, we find that, up to invertible $p + ip$ fermionic topological orders, there are only four fermionic topological orders with one nontrivial topological excitation: (1) the $K = [-10; 02]$ fractional quantum Hall state, (2) a Fibonacci bosonic topological order $2_{14/5}^B$ stacking with a fermionic product state, (3) the time-reversal conjugate of the previous one, (4) a primitive fermionic topological order that has a chiral central charge $c = \frac{1}{4}$, whose only topological excitation has a non-abelian

statistics with a spin $s = \frac{1}{4}$ and a quantum dimension $d = 1 + \sqrt{2}$.

Numerous shorter, more specialized talks were also presented by experts in the field:

1. Maissam Barkeshli (Station Q), ‘Superconductivity Induced Topological Phase Transition at the Gapless Edge of Even Denominator Fractional Quantum Hall States’

We show that every even-denominator fractional quantum Hall (FQH) state possesses at least two robust, topologically distinct gapless edge phases if charge conservation is broken at the boundary by coupling to a superconductor. The new edge phase allows for the possibility of a direct coupling between electrons and emergent neutral fermions of the FQH state. This can potentially be experimentally probed through geometric resonances in the tunneling density of states at the edge, providing a probe of fractionalized, yet electrically neutral, bulk quasiparticles. Other measurable consequences include a charge e fractional Josephson effect, a charge $e/4q$ quasiparticle blocking effect in filling fraction $p/2q$ FQH states, and modified edge electron tunneling exponents. We also discuss similar phenomena in quantum spin liquids, which suggests new probes of fractionalization in such states.

2. Erez Berg (Weizmann), ‘Topological phenomena in periodically driven systems: the role of disorder and interactions’

Periodically driven quantum systems, such as semiconductors subject to light and cold atoms in optical lattices, provide a novel and versatile platform for realizing topological phenomena. Some of these are analogs of topological insulators and superconductors, attainable also in static systems; others are unique to the periodically driven case. I will describe how periodic driving, disorder, and interactions can conspire to give rise to new robust steady states, with no analogues in static systems. In disordered two-dimensional driven systems, a phase with chiral edge states and fully localized bulk states is possible; this phase can realize a non-adiabatic quantized charge pump. In interacting one dimensional driven systems, current carrying states with excessively long life times can arise.

3. Andrei Bernevig (Princeton), ‘Cohomological insulators’

This talk presented theory and experiment for a new type of topological insulator exhibiting novel topological features in the band structure over *extended* regions of momentum space.

4. Parsa Bonderson (Station Q), ‘Symmetry, Defects, and Gauging of Topological Phases’

We examine the interplay of symmetry and topological order in 2+1 dimensional topological phases of matter. We describe how the global symmetries of the microscopic system act on the emergent topological degrees of freedom. A general framework is provided to classify symmetry fractionalization in topological phases, including phases that are nonAbelian and symmetries that permute the quasiparticle types and/or are anti-unitary. We develop a theory of extrinsic defects (fluxes) associated with elements of the symmetry group. This provides a general classification of symmetry-enriched topological phases derived from a topological phase of matter with unitary on-site symmetry. The algebraic theory of the defects, known as a G-crossed braided tensor category, allows one to compute many properties, such as the number of topologically distinct types of defects associated with each group element, their fusion rules, quantum dimensions, zero modes, braiding exchange transformations, a generalized Verlinde formula for the defects, and modular transformations of the G-crossed extensions of topological phases. We also examine the promotion of the global symmetry to a local gauge invariance, wherein the extrinsic G-defects are turned into deconfined quasiparticle excitations, which results in a different topological phase.

5. Fiona Burnell (University of Minnesota), ‘Correlated topological and symmetry-breaking order: geometrical frustration and anyon condensation on the lattice’

A number of solvable topological orders admit phase transitions in which quasiparticles condense to bring the system to a new phase with reduced, but non-trivial, topological order. I will describe a transition of this type carried out on a lattice, in which the condensed phase is geometrically frustrated and spontaneously breaks translation symmetry. This symmetry breaking coexists with a residual Z_2 (abelian) topological order, leading to an interesting interplay between quasiparticles and fluctuations in the local order parameter.

6. Jennifer Cano (Princeton), ‘Chirality-Protected Majorana Zero Modes at the Gapless Edge of Abelian Quantum Hall States’

We show that the $\nu = 8$ integer quantum Hall state can support Majorana zero modes at domain walls between its two different stable chiral edge phases without superconductivity. This is due to the existence of an edge phase that does not support gapless fermionic excitations; all gapless excitations are bosonic in this edge phase. Majorana fermion zero modes occur at a domain wall between this edge phase and the more conventional one that does support gapless fermions. Remarkably, due to the chirality of the system,

the topological degeneracy of these zero modes has exponential protection, as a function of the relevant length scales, in spite of the presence of gapless excitations, including gapless fermions. These results are compatible with charge conservation, but do not require it. We discuss generalizations to other integer and fractional quantum Hall states, and classify possible mechanisms for appearance of Majorana zero modes at domain walls.

7. Yong Chen (Purdue), ‘Transport experiments in topological insulators’

I will discuss our recent transport experiments on high-quality topological insulators thin films and nanowires with insulating bulk and surface-dominated conduction. We reveal a number of unique transport properties of spin-helical Dirac fermion topological surface states, such as the half-integer quantum Hall effect, helical spin-polarized current, and half-integer Aharonov-Bohm effect. If time allows, I may also discuss measurements probing proximity induced superconductivity and interaction effects.

8. Paul Fendley (Oxford), ‘Strong Zero Modes and Eigenstate Phase Transitions’

Gapless edge or zero modes surviving the presence of disorder are common in a topological phase of matter. “Weak” zero modes, guaranteeing ground-state degeneracy, necessarily survive throughout a topological phase. A more dramatic effect occurs in the Ising chain/Majorana wire: “strong” edge zero modes result in identical spectra in even and odd fermion-number sectors, up to exponentially small finite-size corrections. There is a presumption that disorder is necessary to stabilize strong zero modes in the presence of interactions, but I show that their presence in a clean system is not a free-fermionic fluke. In this talk I construct an explicit strong zero mode in the XYZ chain/coupled Majorana wires; this operator possesses some remarkable structure apparently unknown in the integrability literature. I also present evidence for strong zero modes in the parafermionic case, implying the existence of an unconventional eigenstate phase transition where the strong zero mode disappears, leaving only the weak one.

9. Josh Folk (University of British Columbia), ‘Trivial edges in a topological material: investigations of InAs/GaSb quantum wells’

Double quantum wells in InAs/GaSb are believed to host a highly tunable quantum spin Hall state when front and backgate voltages align the bottom of the conduction band (InAs) below the top of the valence band (GaSb). To date, the most direct evidence of the quantum spin Hall state in this material has been the observation of conducting edges when bulk is insulating. The conductance properties of those edges are close to those expected for topologically-protected edges, but no direct measurement of edge helicity has been reported and the weak magnetic field dependence of the edge conductance remains a puzzle. Here, we report measurements of edge conductance in InAs/GaSb quantum wells that are clearly tuned to a trivial (non-topological) regime with gate voltages. This observation is robust, in the sense that it is seen in every sample, independent of processing conditions or wafer details, though at a quantitative level edge conductance varies by almost an order of magnitude when considering many samples over a wide range of temperatures. In some samples, these trivial edges are found to become highly resistive below 100mK with a temperature dependence suggestive of variable range hopping, raising hopes that new processing schemes may be developed to eliminate unwanted non-topological edge conduction from this otherwise-promising material.

10. Marcel Franz (University of British Columbia), ‘New phases from interacting Majorana fermions’

Vortices in the Fu-Kane model (describing a superconducting surface of a 3D topological insulator) are known to host Majorana zero modes. By adjusting a single system parameter – the global chemical potential – the zero modes can be tuned to the regime of strong interactions. In this talk I will describe the simplest interacting system that can be built from these ingredients: a 1D Majorana chain with nearest neighbor hopping and the most local 4-fermion interaction. The system exhibits a complex phase diagram with interesting phases and phase transitions between them. These include a gapless Ising phase for attractive interactions separated from a doubly degenerate gapped phase at strong coupling by a quantum critical point in the tricritical Ising universality class. For weak repulsive interactions we find an interesting gapless phase with coexisting Luttinger liquid and Ising degrees of freedom. The latter is separated from a 4-fold degenerate gapped phase at strong coupling by a novel generalization of the commensurate-incommensurate transition.

11. Liang Fu (MIT), ‘Bit from it: building a robust quantum computer from Majorana fermions’

This talk described a new way of implementing surface codes for quantum computation based on Majorana networks.

12. Roman Lutchyn (Station Q), ‘Interplay between Kondo and Majorana Interactions in Quantum Dots’

We study the properties of a quantum dot coupled to a topological superconductor and normal leads and discuss the interplay between Kondo- and Majorana-induced couplings in quantum dots. The latter appears

due to the presence of Majorana zero-energy modes localized, for example, at the ends of the one-dimensional topological superconductor. We investigate the phase diagram of the system as a function of Kondo and Majorana interactions using a renormalization-group analysis, a slave-boson mean-field theory, and numerical simulations using the density-matrix renormalization-group method. We show that, in addition to the well-known Kondo fixed point, the system may flow to a new fixed point controlled by the Majorana-induced coupling, which is characterized by nontrivial correlations between a localized spin on the dot and the fermion parity of the topological superconductor and the normal lead. We compute full counting statistics of charge fluctuations, which highlights some peculiar features characteristic to this Majorana fixed point.

13. Mike Hermele (CU Boulder), ‘The flux-fusion anomaly test and bosonic topological crystalline insulators’

I will describe a method, the flux-fusion anomaly test, to test for anomalous symmetry fractionalization in some two-dimensional ($d=2$) symmetry-enriched topological (SET) phases, where the symmetry may include spatial symmetries and/or time reversal. The anomalous fractionalization patterns thus identified cannot occur in strictly $d=2$ systems, but are realized as surface theories of $d=3$ symmetry-protected topological phases. This leads to several new examples of $d=3$ bosonic topological crystalline insulators, and some understanding of physical properties at their surfaces. Time permitting, I will briefly mention applications of the anomaly test to $d=3$ SET phases, and to Z_2 spin liquids in $d=2$ Heisenberg antiferromagnets.

14. Taylor Hughes (UIUC), ‘Abelian Topological Phases: Symmetries, Defects, and Entanglement’
In this talk we will review the quasi-particle lattice of 2+1-d Abelian topological phases described by a K-matrix. From the lattice structure we will discuss the classification of global anyonic symmetries and their related twist defects, and briefly mention the consequences of gauging the anyonic symmetries of a parent topological phase. Finally, we will discuss a new development for the entanglement of Abelian phases which allows for modifications to the sub-leading correction to the area law. These modifications stem from a non-uniqueness in the generation of the topological phase, which we explicitly show using a coupled-wire construction.

15. Michael Levin (University of Chicago), ‘Bulk-boundary correspondence for 3D symmetry-protected topological phases’

Symmetry-protected topological (SPT) phases can be thought of as generalizations of topological insulators. Just as topological insulators have robust boundary modes protected by time reversal and charge conservation symmetry, SPT phases have boundary modes protected by more general symmetries. In this talk, I will discuss the relationship between bulk and boundary properties of 3D SPT phases with unitary symmetries.

16. Nate Lindner (Technion), ‘The Ising bagel: Non-Abelian statistics enriched by defects and their zero modes’

Non-Abelian topological phases of matter can be utilized to encode and manipulate quantum information in a non-local manner, such that it is protected from imperfections in the implemented protocols and from interactions with the environment. The condition that the non-Abelian statistics of the anyons supports a computationally universal set of gates sets a very stringent requirement which is not met by many topological phases. We consider the possibility to enrich the possible topological operations supported by a non-Abelian topological phase by introducing defects into the system. We show that such defects bind zero modes which form a unique algebra that goes beyond the parafermionic algebra describing defects in Abelian phases. Furthermore, we show that by coupling zero modes, one can obtain a set of topological operations that implements a universal set of gates. We also discuss lattice models of interacting defects and their implications to edge phases of non-Abelian topological phases.

17. Yuan-Ming Lu (Ohio State University), ‘Measuring symmetry fractionalization in quantum spin liquids’

Motivated by mounting numerical evidence for spin liquid ground states in the many two-dimensional frustrated spin models, here we develop systematic methods to measure the global and crystal symmetry quantum numbers of fractional excitations. We show that the symmetry fractionalization patterns in a quantum spin liquid can be measured by a dimensional reduction regime, which relates the two-dimensional symmetric topological orders to one-dimensional symmetry protected topological phases. This general framework is directly applicable to numeric results obtained in 2d DMRG studies, and can be generalized to other gapped topological orders in two dimensions.

18. Max Metlitski (KITP), ‘Particle-vortex duality of 2D Dirac fermion from electric-magnetic duality of 3D topological insulators’

Particle-vortex duality is a powerful theoretical tool that has been used to study bosonic systems. Here we propose an analogous duality for Dirac fermions in 2+1 dimensions. The physics of a single Dirac cone is proposed to be described by a dual theory, QED3 with a dual Dirac fermion coupled to a gauge field. This duality is established by considering two alternate descriptions of the 3d topological insulator (TI) surface. The first description is the usual Dirac cone surface state. The second description is accessed via an electric-magnetic duality of the bulk TI coupled to a gauge field, which maps it to a gauged topological superconductor. This alternate description ultimately leads to a new surface theory - dual QED3. The dual theory provides an explicit derivation of the T-Pfaffian state, a proposed surface topological order of the TI, which is simply the paired superfluid state of the dual fermions. The roles of time reversal and particle-hole symmetry are exchanged by the duality, which connects some of our results to a recent conjecture by Son on particle-hole symmetric quantum Hall states.

19. Julia Meyer (Grenoble), ‘Topological Josephson φ_0 -junctions’

We study the effect of a Zeeman field on the current-phase relation of a topological Josephson junction, based on the edge states of a two-dimensional quantum spin Hall insulator or a nanowire with strong spin-orbit coupling. We show that, in the helical regime, the Zeeman field along the spin quantization axis of the states at the Fermi level results in a large anomalous Josephson effect that allows for a supercurrent to flow in the absence of superconducting phase bias. We relate the associated field-tunable phase shift in the Josephson relation of such a φ_0 -junction to the existence of a so-called helical superconductivity, which may result from the interplay of the Zeeman field and spin-orbit coupling.

20. Roger Mong (Pittsburgh), ‘Dirac composite fermions in the half-filled Landau level’

One of the most spectacular experimental findings in the fractional quantum Hall effect is evidence for an emergent Fermi surface when the electron density is nearly half the density of magnetic flux quanta ($\nu = 1/2$). The seminal work of Halperin, Lee, and Read (HLR) attributed this to the formation of composite fermions, bound states of an electron and a pair of vortices. We use infinite cylinder DMRG to provide compelling numerical evidence for the existence of a Fermi sea of composite fermions for realistic interactions between electrons at $\nu = 1/2$. Moreover, we show that the phase is particle-hole symmetric, in contrast to the theory of HLR. Instead, our findings are consistent if the composite fermions are massless Dirac particles, at finite density, similar to the surface state of a 3D topological insulator.

21. Stevan Nadj-Perge (Delft), ‘Majorana fermions in atomic chains on a superconductor’

Majorana fermions are zero-energy excitations predicted to localize at the edge of a topological superconductor, a state of matter that can form when a ferromagnetic system is placed in proximity to a conventional superconductor with strong spin-orbit interaction. With the goal of realizing a one-dimensional topological superconductor, we have fabricated ferromagnetic iron atomic chains on the surface of superconducting lead [1]. Using high-resolution spectroscopic imaging techniques, we show that the onset of superconductivity, which gaps the electronic density of states in the bulk of the chains, is accompanied by the appearance of zero-energy end-states. This spatially resolved signature, corroborated by other observations and theoretical modeling [2], provides evidence for the formation of a topological phase and edge-bound Majorana states in this system. Our results demonstrate that atomic chains are viable platform for future experiments to manipulate Majorana bound states [3] and to realize other 1D and 2D topological superconducting phases. [1] S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, *Science* 346, 602 (2014). [2] Jian Li, Hua Chen, Ilya K. Drozdov, A. Yazdani, B. Andrei Bernevig, A.H. MacDonald, *Phys. Rev. B* 90, 235433 (2014). [3] Jian Li, Titus Neupert, B. Andrei Bernevig, Ali Yazdani, *ArXiv:1404.4058* (2014).

22. Masaki Oshikawa (University of Tokyo), ‘Symmetry protection of critical phases and global anomaly in 1+1 dimensions’

Classification of gapless quantum phases remains very much open. Symmetries are naturally expected to play an important role here, as in the case of gapped quantum phases. In this talk, we argue that there is a protection of bulk gapless critical phases by discrete symmetry. We demonstrate this for the SU(2)-symmetric quantum antiferromagnetic chains and their effective field theory, SU(2) Wess-Zumino-Witten (WZW) theory as an example. The SU(2) WZW theory is characterized by a natural number k , which is called level. In the presence of the SU(2) and a certain discrete Z_2 symmetry, they are classified into the two ‘‘symmetry-protected’’ categories: one corresponds to even levels and the other to odd levels.

23. Dima Pikulin (University of British Columbia), ‘Strongly Interacting Majorana Fermions on the Topological Insulator Surface’

We discuss how to engineer strongly interacting phases of Majorana fermions on the surface of a topological insulator in contact with an s-wave superconductor. We suggest that tuning single parameter, surface chemical potential, we can go between weakly- and strongly-interacting regimes. For a special value of the chemical potential the symmetry class BDI is realised. This allows us to suggest an experimental test for the Fidkowski-Kitaev Z_8 periodicity, a realisation of the interacting-enabled topological crystalline phase first suggested by Lapa, Teo, and Hughes, as well as its generalisations to higher dimensions.

24. Shinsei Ryu (UIUC), ‘Bulk/boundary correspondence in SPT phases’

Many of (but not all of) interesting physical (in particular topological) properties of topological phases and symmetry protected topological phases can be “inferred” from their boundary (end, edge, surface, ..) field theories. In particular, the presence of quantum anomalies in boundary field theories (or lack thereof) gives a way to diagnose bulk topological properties. I will discuss such bulk/boundary correspondence in various examples in 2d and 3d.

25. Kirill Shtengel (UC Riverside), ‘Quantum infidelity’

This talk issues related to quantum information in Majorana systems.

26. Dam Son (University of Chicago), ‘Particle-hole symmetry and the nature of the composite fermion’

I will describe the new picture of the half-filled Landau level, according to which the composite fermion is a massless Dirac fermion in the limit of vanishing Landau-level mixing. Such a fermion is characterized by a Berry phase of π around the Fermi disk. Physical consequences of the new picture are outlined.

27. Ady Stern (Weizmann), ‘Current at a distance and resonant transparency in Weyl semi-metals’

This talk discussed a new way of detecting Weyl semi-metals using remarkably accessible transport experiments.

28. Hidenori Takagi (Max Planck), ‘Exotic electronic states produced by strong spin-orbit coupling in complex Ir oxides’

In 5d Iridium oxides, a large spin-orbit coupling of ~ 0.5 eV, inherent to heavy 5d elements, is not small as compared with other relevant electronic parameters, including Coulomb U , transfer t and crystal field splitting D , which gives rise to a variety of exotic magnetic ground states. In the layered perovskite Sr_2IrO_4 , spin-orbital Mott state with $J_{\text{eff}}=1/2$ is realized due to the novel interplay of those energy scales [1-3]. Despite the strong entanglement of spin and orbital degrees of freedom, $J_{\text{eff}}=1/2$ iso-spins in Sr_2IrO_4 was found to be surprisingly isotropic, very likely due to a super-exchange coupling through almost 180 degree Ir-O-Ir bonds [4]. The temperature dependence of in-plane magnetic correlation length of $J_{\text{eff}}=1/2$ iso-spins, obtained from inelastic x-ray resonant magnetic scattering, was indeed well described by that expected for two-dimensional $S=1/2$ Heisenberg antiferromagnet [5]. The three-dimensional analog of Sr_2IrO_4 , SrIrO_3 perovskite is very close a band insulator due to lattice distortion but a Dirac semimetal protected by crystalline symmetry [6]. Upon increasing effective Coulomb U , magnetism emerges and creates a gap at Dirac nodes, giving rise to a semimetal to magnetic insulator transition. This can be realized by controlling the dimensionality and hence the effective U in $(\text{SrIrO}_3)_m/\text{SrTiO}_3$ (m : number of SrIrO_3 layer) super-lattice structure [7]. With reducing m , a transition to an insulator, accompanied with magnetism was clearly observed. At $m=1$, single layer, the transport remains insulating even above the magnetic ordering temperature, indicative of the increased Mott character. When $J_{\text{eff}}=1/2$ iso-spins interact with each other through 90 degree Ir-O-Ir bonds, very anisotropic bond dependent ferromagnetic coupling is expected, unique to strong SOC system. Complex Ir oxides with honeycomb and more recently identified hyper-honeycomb lattices [8], where x-, y- and z- 90 degree Ir-O-Ir bonds are realized, may be candidates for quantum spin liquid expected for the Kitaev model. Very likely due to the superposition of additional magnetic couplings not included in the Kitaev model [9], in reality, a long range magnetic ordering emerges at low temperatures in those compounds. Hyper-honeycomb $\text{b-Li}_2\text{IrO}_3$, though eventually show a marginal ordering, appears to be located at the critical vicinity to the Kitaev spin liquid. With application of pressure of > 2 GPa, in fact, the long range ordering fades out and replaced by an inhomogeneous spin liquid-like state. 1) B. J. Kim et al., Phys. Rev. Lett. 101, 076402 (2008). 2) B. J. Kim et al., Science 323, 1329 (2009). 3) S. Fujiyama et al., Phys. Rev. Lett. 112, 016405 (2014). 4) G. Jackeli and G. Khaliullin, Phys. Rev. Lett. 102, 017205 (2009). 5) S. Fujiyama et al., Phys. Rev. Lett. 108, 247212 (2012). 6) Y. Chen et al., Nat. Commun. 6:6593 doi: 10.1038/ncomms7593 (2015). 7) J. Matsuno et al., Phys. Rev. Lett. 114 247209(2015). 8) T.Takayama, et al., s Phys. Rev. Lett.114, 077202 (2015) 9) A.Kitaev, Annals of Physics 312 2 (2006).

29. Ashvin Vishwanath (Berkeley), ‘Particle-vortex duality of Dirac fermions: Linking topological insulators and superconductors to the half filled Landau level’

We derive a dual description of a 2+1 dimensional Dirac fermion, analogous to the duality of bosons to vortices. The dual theory is found to be QED_3 , which is also composed of a single Dirac fermion, coupled to a gauge field. We connect the 3+1d topological insulator, which displays a Dirac surface state, and connect it to a topological superconductor, whose surface states supplies the dual description. All known surface phases of topological insulators are shown to emerge from the dual description. Remarkably, this has consequences for particle hole symmetric states in the half-filled Landau level. In particular, the composite Fermi liquid is shown to be closely related to the surface state of a topological insulator. An alternate to the Moore-Read Pfaffian, which respects particle hole symmetry, is also identified.

30. Amir Yacoby (Harvard), ‘Controlled Finite Momentum Pairing and Spatially Varying Order Parameter in Proximitized HgCdTe Quantum Wells’

This talk reported on measurements in long HgTe Josephson junctions subjected to an in-plane magnetic field.

31. Norman Yao (Berkeley), ‘Quantum control in the many-body localized phase’

In thermal phases, the quantum coherence of individual degrees of freedom is rapidly lost to the environment. A number of recent works have postulated that the manybody localize phase may be promising for quantum information applications owing to the slow decay of local coherences. I will try to sharpen this intuition and describe what it means to locally “manipulate” a many-body system. Working with a specific one dimensional model of interacting spins at infinite temperature, I will describe a protocol that allows one to encode quantum information, perform a universal set of gates, and readout the resulting state. Our protocol utilizes protected qubits that emerge at the boundary between a symmetry-protected topological and trivial phase.

4 Outcome of the Meeting

On the broadest level, one of our chief objectives was to provide a venue whereby diverse topics in strongly interacting topological phases could be seamlessly unified. For instance, we hoped to relate questions on the formal mathematical physics end to experiment and even applications. A related goal we hoped to achieve was to foster new interdisciplinary collaborations, which would undoubtedly produce exciting breakthroughs in the field.

We believe that our workshop succeeded in fulfilling these objectives. Participants included both experimentalists and theorists with expertise spanning a wide range of complementary areas. Ample informal discussion time allowed participants to engage in extensive dialogue concerning material presented during the program. Particularly spirited interactions revolved around issues concerning the half-filled Landau level, dualities for the topological insulator surface, Majorana platforms including semiconducting wires and quantum spin Hall materials, Weyl semimetals, classification of phases and defects, and spin liquids. We expect that the conference will both influence ongoing work (many talks at the conference presented unpublished results) and produce new collaborations.