Lamb Shift

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Introduction

the **Lamb shift**, named after Willis Lamb (1913–2008), is a small difference in energy between two energy levels $^2S_{1/2}$ and $^2P_{1/2}$ of the hydrogen atom in quantum electrodynamics (QED). According to the Dirac equation, the $^2S_{1/2}$ and $^2P_{1/2}$ orbitals should have the same energy. However, the interaction between the electron and the vacuum (which is not accounted for by the Dirac equation) causes a tiny energy shift which is different for states $^2S_{1/2}$ and $^2P_{1/2}$.

this measurement provided the stimulus for renormalization theory to handle the divergences.

Lamb won the Nobel Prize in Physics in 1955 for his discoveries related to the Lamb shift.

Wikipedia free encyclopedia

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Fine Structure of the Hydrogen Atom by a Microwave Method* **

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THE spectrum of the simplest atom, hydrogen, has a fine structure[1] which according to the Dirac wave equation for an electron moving in a Coulomb field is due to the combined effects of relativistic variation of mass with velocity and spin-orbit coupling. It has been considered one of the great triumphs of Dirac's theory that it gave the "right" fine structure of the energy levels. However, the experimental attempts to obtain a really detailed confirmation through a study of the Balmer lines have been frustrated by the large Doppler effect of the lines in comparison to the small splitting of the lower or $n=2$ states. The various spectroscopic workers have alternated between finding confirmation[2] of

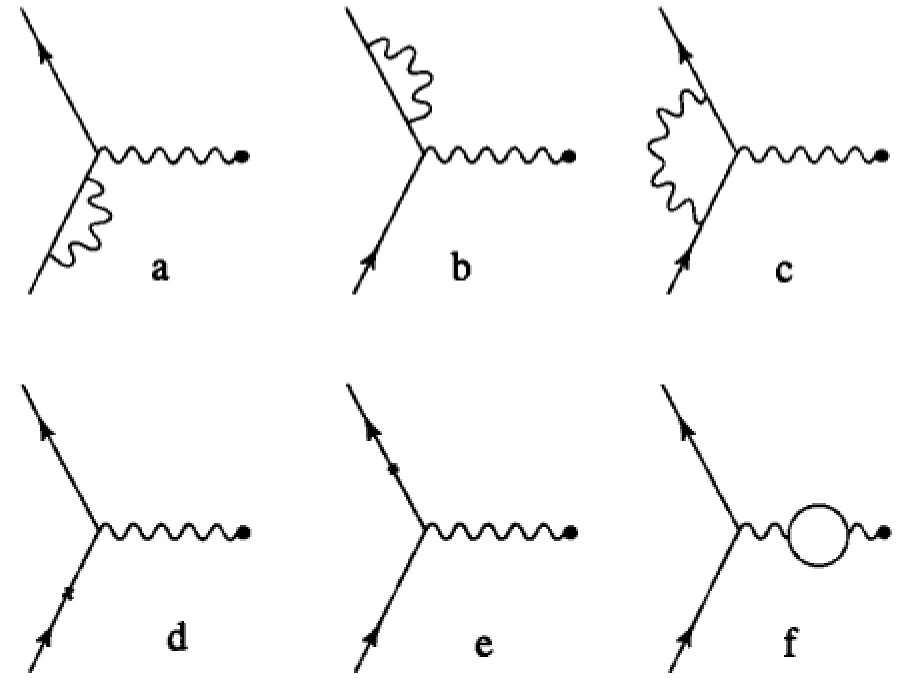
population and the high background absorption due to electrons. Instead, we have found a method depending on a novel property of the $2^2S_{1/2}$ level. According to the Dirac theory, this state exactly coincides in energy with the $2^2P_{1/2}$ state which is the lower of the two P states. The S state in the absence of external electric fields is metastable. The radiative transition to the ground state $1^2S_{1/2}$ is forbidden by the selection rule $\Delta L=\pm 1$. Calculations of Breit and Teller[4] have shown that the most probable decay mechanism is double quantum emission with a lifetime of 1/7 second. This is to be contrasted with a lifetime of only 1.6×10^{-9} second for the non-metastable 2^2P states. The metastability is very

The calculation of the lamb shift is rather intricate, because we are dealing with the hydrogen atom as a bound-state problem .

We must sum over <u>all radiative corrections</u> to the electron interacting with coulomb potential that modify the naïve $\bar{u}\gamma_0 u A^0$ vertex.

These corrections :

1) Vertex correction
2) The anomalous magnetic moment
3) The self energy of the electron
4) The vacuum polarization graph
5) Even infrared divergence



The various higher-order graphs that contribute to the Lamb shift: (a) and (b) the electron self-energy graphs; (c) the vertex correction; (d) and (e) the electron mass counterterm; (f) the photon self-energy correction.

Because of the intricate nature of the calculation , we will only sketch the highlights of the calculation.

We first see that the vacuum polarization graph can be attached to the photon line , changing the photon propagator to :

$$D_{\mu\nu} = -\frac{g_{\mu\nu}}{k^2}\left(1 - \frac{e^2}{60\pi^2}\frac{k^2}{m^2} + \mathcal{O}(k^4)\right)$$

Analyzing the zeroth component of this propagator , we see that the coupling of the electron to the coulomb potential changes as follows :

$$ie_0^2\frac{\bar{u}\gamma_0 u}{q^2} \rightarrow ie^2\frac{\bar{u}\gamma_0 u}{q^2}\left(1-\frac{\alpha q^2}{15\pi m^2}+\mathcal{O}(\alpha^2)\right)$$

$$momentum\ space \xrightarrow{fourier\ transform} x - space$$

$$\frac{1}{q^2}\sim\frac{1}{r}$$

$$\left(1-\frac{\alpha}{15\pi m^2}\nabla^2\right)\frac{e^2}{4\pi r}=\frac{e^2}{4\pi r}+\frac{e^2}{60\pi^2 m^2}\delta^3(x)$$

There is a correction to Coulomb's law given by QED.

Let us generalize this discussion to include the other correction to the calculation of the lamb shift .

Our method :

1) Calculation the corrections to the vertex function $\bar{u}\gamma_\mu u$

2) Take the zeroth component

3) Take the low-energy limit

From Anomalous magnetic moment correction :

$$\Lambda_\mu \sim \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^2}{2m} F_2(q^2)$$

& Infrared divergence correction :

$$\gamma_\mu + \Lambda_\mu^c(p',p) \sim \gamma_\mu \left[1 + \frac{\alpha}{3\pi}\frac{q^2}{m^2}(\log\frac{m}{\mu} - \frac{3}{8}\right] + \frac{\alpha}{8\pi m}[\gamma^\nu q_\nu, \gamma_\mu]$$

$$|q^2| \gg m^2 \Rightarrow \gamma_\mu + \Lambda_\mu^c(p',p) \sim \gamma_\mu \left\{1 - \frac{\alpha}{\pi}\log\frac{m}{\mu}\left[\log\left(-\frac{q^2}{m^2}\right) - 1 + \mathcal{O}\left(\frac{m^2}{q^2}\right)\right]\right\}$$

If we add the various contributions to the vertex correction , we find :

$$\bar{u}\gamma_\mu u \;\longrightarrow\; \bar{u}\left\{\gamma_\mu\left[1-\frac{\alpha q^2}{3\pi m^2}\left(\log\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}\right)\right]+\frac{i\alpha}{4\pi m}\sigma_{\mu\nu}q^\nu\right\}u$$

Now take the low-energy limit of this expression .

The $\sigma_{\mu\nu}q^\nu$ term reduces down to a spin-orbit correction , and we find the effective potential given by :

$$\Delta V_{eff} \sim \frac{4\alpha^2}{3m^2}\left(\log\frac{m}{\mu}-\frac{3}{8}-\frac{1}{5}+\frac{3}{8}\right)\delta^3(x)+\frac{\alpha^2}{4\pi m^2 r^3}\vec{\sigma}.\vec{L}$$

- The vertex correction → a correction of 1010 MHz
- The anomalous magnetic moment of the electron contributes 68 MHz
- The vacuum polarization graph → contributes -27.1 MHz

- We find Lowest loop level → arrive at Lamb shift to within 6MHz, accuracy.

- The higher –order correction have been calculated , so the difference between experiment and theory has been reduced to 0.01 MHz .
- Theoretically the $^2S_{1/2}$ level is above the $^2P_{1/2}$ energy level by 1057.864 ± 0.014 MHz
- The experiment result is 1057.862 ± 0.020 MHz .

This is an excellent indicator of the basic correctness of QED.

References :

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Thanks for your attention