High-energy collective modes in fractional quantum Hall liquids: Rise of the parton

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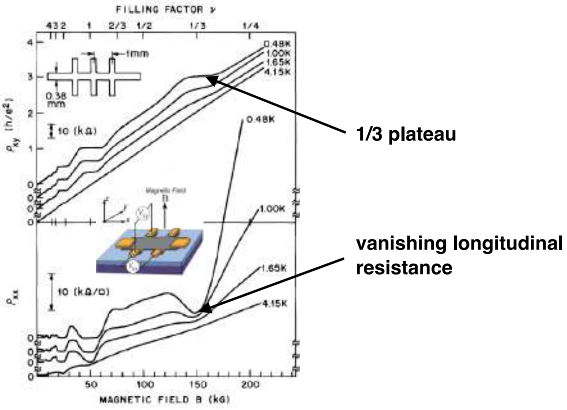
Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. X **12**, 021008 (2022)

THE ROYAL SOCIETY

Plan of the talk

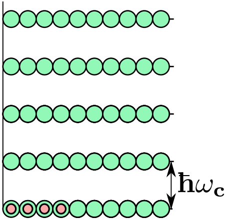
- Fractional quantum Hall effect (FQHE) in the Lowest Landau level (LLL): composite fermion (CF) theory
- New collective mode in certain FQHE states: parton mode
- Emergent SUSY at 5/2
- Conclusion and outlook

Advent of FQHE: plateau at $h/(\frac{1}{3}e^2)$



D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982)

FQHE arises from electron-electron interactions



fractional filling ($\nu = 2/5$)

- Electrons interacting via Coulomb forces:

$$\mathcal{H} = \frac{e^2}{\epsilon} \sum_{i<j} \frac{1}{|r_i - r_j|}$$

- Quantum mechanics $\rightarrow$ *lowest Landau level* constraint for $B \rightarrow \infty$
- Interactions $\rightarrow$ a unique state from the degenerate manifold

Laughlin's ansatz for $\nu = 1/m$

- assumed a Jastrow (pairwise) correlated state.

$$\Psi_{1/m}^{\text{Laughlin}} = \prod_{i<j}(z_i - z_j)^m$$

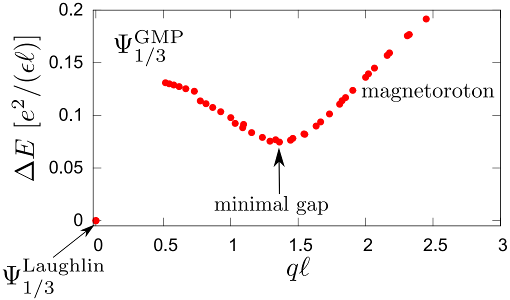
- fermionic wave functions must be antisymmetric, hence m is odd integer
- fluid with fractionally charged particles obeying fractional braid statistics

R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983)

Density-mode ansatz for the neutral excitation

$$\Psi_\nu^{\rm GMP} = \bar{\rho}_{\vec{q}}\Psi_\nu \text{ [single-mode approximation (SMA)]}$$

$\bar{\rho}_{\vec{q}}$ is the LLL projected density operator

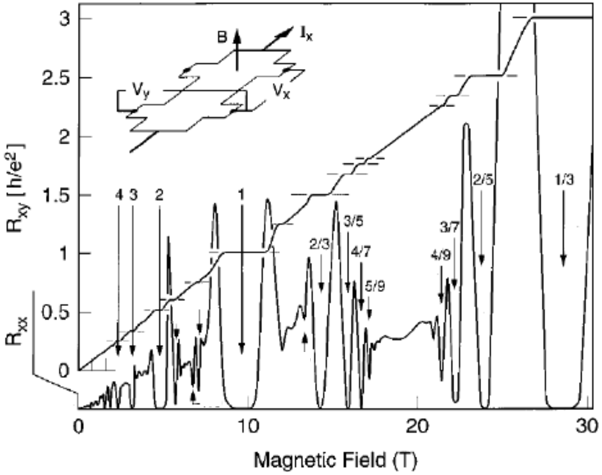


Girvin, MacDonald, and Platzman (GMP), Phys. Rev. Lett. **54**, 581 (1985), Phys. Rev. B **33**, 2481 (1986)

Experimentally observed using light-scattering

A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West, Phys. Rev. Lett. **70**, 3983 (1993)

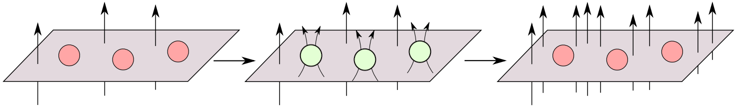
Zoo of fractions in the $\nu = n/(2pn \pm 1)$ sequence



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

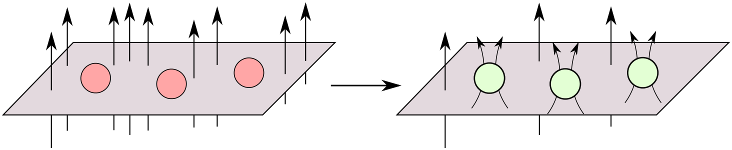
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE as IQHE of composite fermions

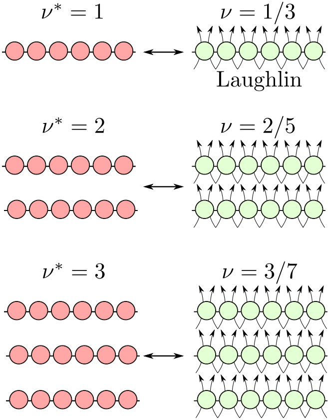


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = hc/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

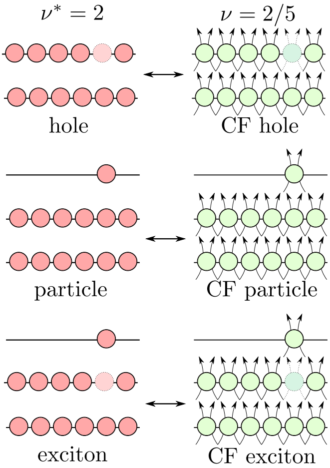
J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE excited states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions can be built from IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\mathrm{CF}}_{\nu=\frac{n}{2pn\pm 1}} = \mathcal{P}_{\rm LLL}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

Φ_n wave function of n filled LLs.

$\mathcal{P}_{\rm LLL}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Overlaps of CF states with LLL Coulomb ground states

overlaps obtained from direct projected states

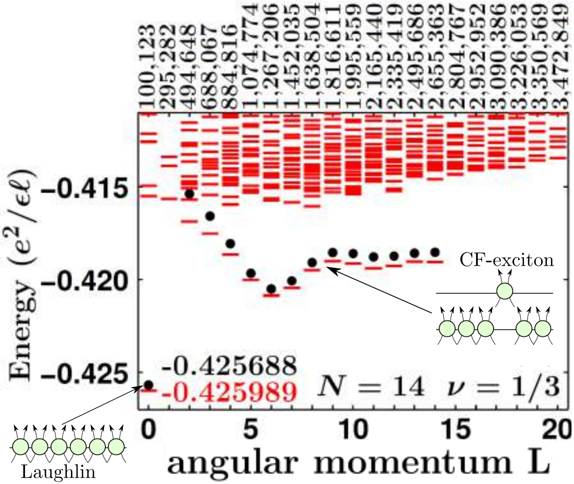
ν	N	Hilbert space dimension	$\lvert\langle\Psi^{0\mathrm{LL}}\vert\Psi^{\mathrm{CF}}\rangle\rvert$
1/3	15	2×10^9	0.9876 (Laughlin)
1/5	11	4×10^8	0.9413 (Laughlin)
2/5	12	3×10^5	0.9971
3/7	12	6×10^4	0.9988
2/9	10	1×10^7	0.9744

$\lvert\Psi^{0\mathrm{LL}}\rangle$ is obtained by brute-force exact diagonalization

Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)
B. Yang and Ajit C. Balram, New J. Phys. **23**, 013001 (2021)

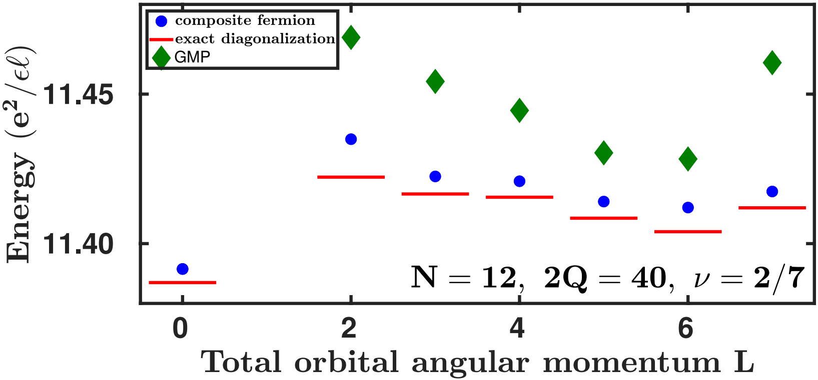
Ajit C. Balram, SciPost Phys. **10**, 083 (2021)

Magnetoroton merges with continuum for $q \to 0$ at $\nu = 1/3$



Ajit C. Balram, Arkadiusz Wójs, and Jainendra K. Jain, Phys. Rev. B **88**, (2013)

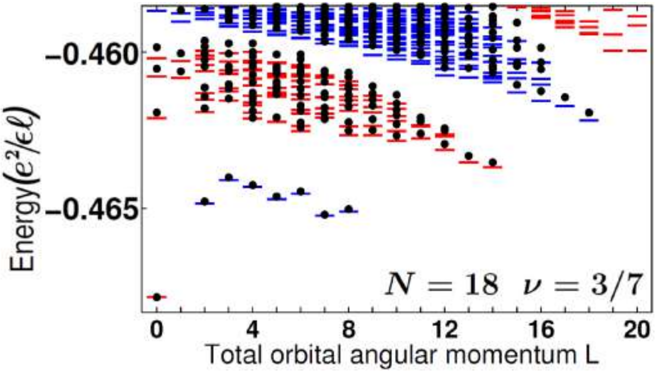
CF-exciton (CFE) is more accurate than the GMP



At $\nu = 1/(2p+1)$, in the $q \to 0$ limit, CFE and GMP are identical

R. K. Kamilla, X. G. Wu, and J. K. Jain, Phys. Rev. B **54**, 4873 (1996)

CF theory is extremely accurate in the lowest Landau level



dashes are obtained by brute-force exact diagonalization

$\sim 10^6$ states at each total orbital angular momentum L

Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

A puzzle in the $n/(4n \pm 1)$ secondary Jain states

- Present composite fermion field theories for the secondary Jain state states violate the so-called "Haldane bound," which places a lower bound on the coefficient of q^4 in the static structure factor ($\langle\bar{\rho}\bar{\rho}\rangle$) set by the shift.

  requires lowest Landau level (LLL) projection

  H. Goldman and E. Fradkin, Phys. Rev. B **98**, 165137 (2018).

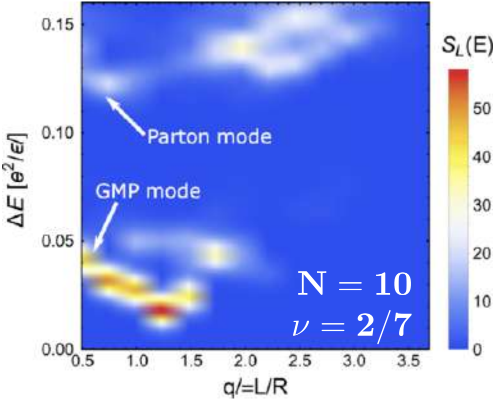
- Nguyen and Son proposed that, in addition to the GMP mode, there exist additional, higher energy collective modes, which resolves this contradiction.

  Dung Xuan Nguyen and Dam Thanh Son, Phys. Rev. Research **3**, 033217 (2021)

- We give a microscopic justification and construct a trial wave function to capture this additional "parton" mode.

Parton mode is seen in the dynamical structure factor

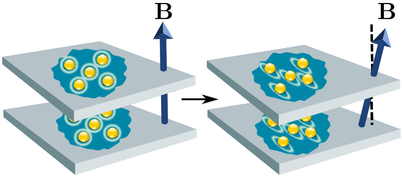
$$S_L(E) = \sum_n |\langle E_n|\bar{\rho}_L|0\rangle|^2 \delta(E_n - E_0 - E)$$



Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

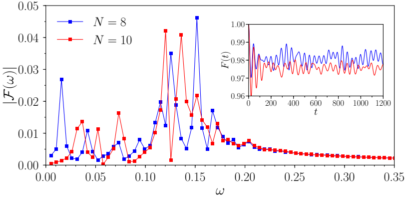
see also Nguyen *et al.*, Phys. Rev. Lett. **128** 246402 (2022)

Parton mode is observed in quench dynamics



Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. B **98**, 155140 (2018)

Zhao Liu, Ajit C. Balram, Zlatko Papić, and Andrey Gromov, Phys. Rev. Lett. **126**, 076604 (2021)



Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

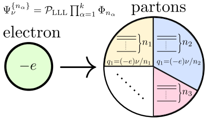
Parton states: product of fermionic states

- break each electron into fictitious partons, place partons into IQH (or any) fermionic states, fuse the partons back to recover the electron

$$\Psi_\nu^{\{n_\alpha\}} = \mathcal{P}_{\rm LLL} \prod_{\alpha=1}^{k} \Phi_{n_\alpha}(\{z_i\})$$

- k is odd for fermions

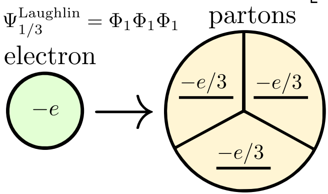
$$q_\alpha = (-e)\frac{\nu}{n_\alpha}, \quad \nu^{-1} = \sum_{\alpha=1}^{k} n_\alpha^{-1}$$



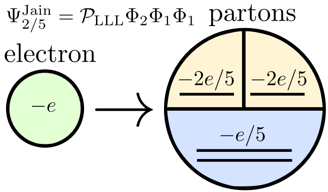
J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

Laughlin and Jain states are parton states

- Laughlin state is a "$111\cdots$" parton state $\left[\Phi_1 \equiv \prod_{i<j}(z_i - z_j)\right]$



- Jain/CF states are "$n11\cdots$" parton states



J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

secondary Jain $\sim$ primary Jain $\times$ bosonic Laughlin

- for $n, p \geq 2$

$$\Psi^{\rm Jain}_{n/(2pn\pm 1)} = \mathcal{P}_{\rm LLL}\Phi_1^{2p}\Phi_{\pm n} \sim \mathcal{P}_{\rm LLL}\Phi_1^{2}\Phi_{\pm n} \times \Phi_1^{2(p-1)} \sim \Psi^{\rm Jain}_{n/(2n\pm 1)} \times \Psi^{\rm Laughlin}_{1/[2(p-1)]}$$

The two collective modes are

- CF-exciton/GMP mode of the primary Jain state $\Psi^{\rm parton}_{n/(2pn\pm 1)} = \Psi^{\rm CFE}_{n/(2n\pm 1)} \times \Psi^{\rm Laughlin}_{1/[2(p-1)]}$
- CF-exciton/GMP mode of the bosonic Laughlin state $\Psi^{\rm parton}_{n/(2pn\pm 1)} = \Psi^{\rm Jain}_{n/(2n\pm 1)} \times \Psi^{\rm CFE}_{1/[2(p-1)]}$

Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Factorization hardly changes the microscopic wave function

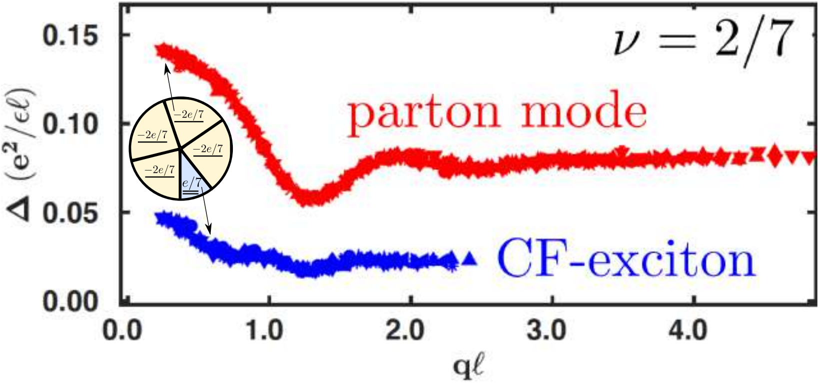
overlaps between different versions of the projected Jain states

ν	N	dimension	$\lvert\langle\Psi^{Jain}_{n/(4n\pm1)}\lvert\Psi^{Jain}_{n/(2n\pm1)}\times\Psi^{Laughlin}_{1/2}\rangle\rvert$
2/7	8	5×10^4	0.9925
2/9	8	1×10^5	0.9999

Different versions of the Laughlin wave function have unit overlap with each other since there is no projection involved in the Laughlin states.

Ajit C. Balram and J. K. Jain, Phys. Rev. B **93**, 235152 (2016)

Trial wave function for the parton mode is accurate

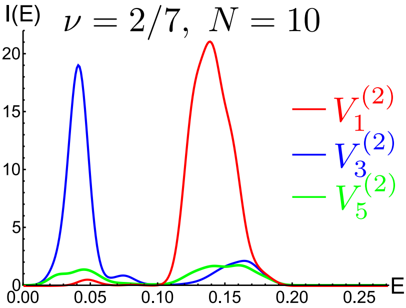


Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

Clustering properties of the two modes are different

$\Psi_{2/7}^{\text{parton-mode}} = \Psi_{2/3}^{\text{Jain}} \times \Psi_{1/2}^{\text{CFE}} \propto r$ high-energy

$\Psi_{2/7}^{\text{CFE}} = \Psi_{2/3}^{\text{CFE}} \times \Psi_{1/2}^{\text{Laughlin}} \propto r^3$ low-energy $\because \langle V_1 \rangle_{\Psi_{2/7}^{\text{CFE}}} = 0$



States also have different chiralities

Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

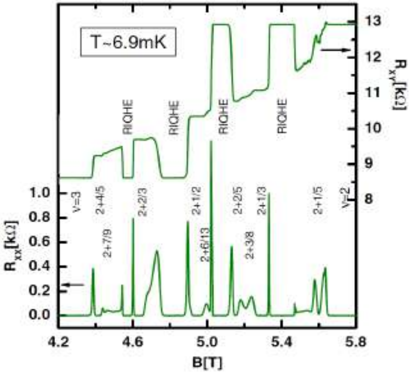
Parton mode is absent in primary Jain states

- Numerics show only one mode in the primary Jain states.
- All Laughlin fractions, in particular $\nu = 1/3$, can support only one mode since the state is made up of only one kind of parton, as each parton fills one LL.
- By particle-hole conjugation in the LLL, $\nu = 2/3$ also hosts only one mode.
- The Jain wave function for $2/5$ is analogous to that at $2/3$ and thus since $2/3$ has only one mode $2/5$ is expected to have only one mode.
- Projection removes the parton-like mode $\mathcal{P}_{\rm LLL}\Phi_{\pm n}\Phi_1\Phi_1^{exciton}$.
- A different argument based on geometric fluctuation of conformal Hilbert spaces.

Yuzhu Wang and Bo Yang, Nat. Comm. **14**, 2317 (2023)

Ajit C. Balram *et al.*, Phys. Rev. X **12**, 021008 (2022)

FQH states in the second Landau level



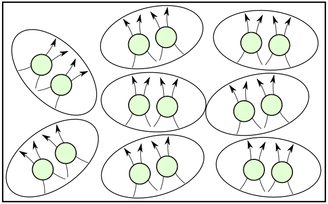
- appearance of *even* denominator fractions, in particular, a robust plateau at $\nu = 5/2$

A. Kumar *et al.* Phys. Rev. Lett. **105**, 246808 (2010)

Moore-Read state is a good candidate for $\nu = 5/2$

$$\Psi^{\rm MR}_{\nu=1/2} = {\rm Pf}\left[\frac{1}{z_i - z_j}\right] \prod_{i<j}(z_i - z_j)^2$$

G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991)



non-Abelian p-wave paired state of composite fermions

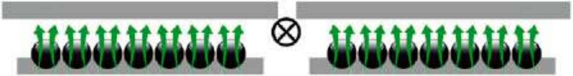
N. Read and Dmitry Green, Phys. Rev. B **61**, 10267 (2000)

N	Hilbert space dimension	$\|\langle \Psi^{\rm 1LL} \| \Psi^{\rm MR}_{\nu=1/2}\rangle\|$
20	4×10^8	0.6736

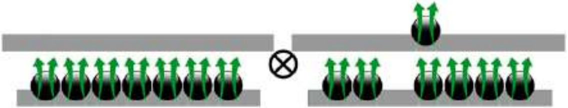
Ajit C. Balram and A. Wójs, Phys. Rev. Research **2**, 032035(R) (2020)

Moore-Read state supports two collective modes

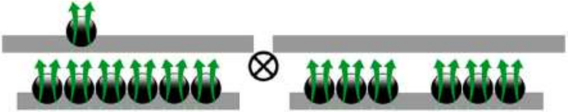
Moore-Read state



magnetoroton: starts from L=2 on the sphere "graviton"



neutral fermion: starts from L=3/2 on the sphere "gravitno"



Sreejith *et al.* Phys. Rev. Lett. **107**, 136802 (2011) Möller *et al.* Phys. Rev. Lett. **107**, 036803 (2011)

Hints of emergent SUSY from numerics on small systems

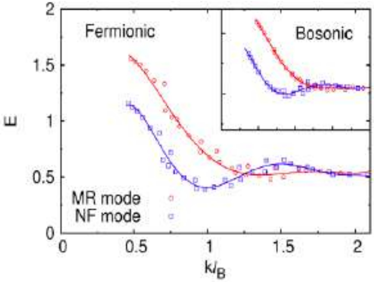


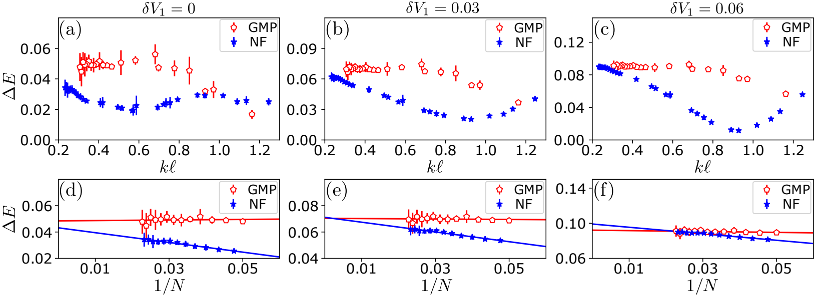
FIG. 2 (color online). The variational energy of the model wave functions for the magnetoroton (MR) mode and the neutral fermion (NF) mode, evaluated against the three-body Hamiltonian. The data are generated from system sizes ranging from 5 to 17 electrons, where the odd number of electrons contribute to the NF mode, and the even number of electrons contribute to the magnetoroton mode. (The inset shows the same plot for the bosonic Moore-Read state.)

Yang *et al.* Phys. Rev. Lett. **108**, 256807 (2012), see also Möller *et al.* Phys. Rev. Lett. **107**, 036803 (2011)

Stronger signatures of emergent SUSY at $\nu=5/2$

Wave functions that can be evaluated for large systems ($N \lesssim 50$)

A. Gromov *et al.*, Phys. Rev. Lett. **125**, 077601 (2020)



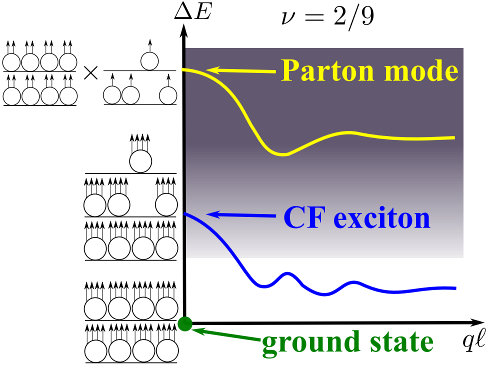
Songyang Pu, Ajit C. Balram, Mikael Fremling, Andrey Gromov, and Zlatko Papić, PRL **130**, 176501 (2023)

Outlook

- Very high-energy excitations in the Jain states that are not described by composite fermions but lend themselves to a description in terms of partons.
- How do we experimentally probe these excitations that lie deep in the continuum?
- Can these excitations help identify the underlying topological order in exotic, in particular non-Abelian, states?

Thanks

Partons are "real" quasiparticles of quantum Hall fluids and like quarks in QCD, show their true "color" only at high energies.



Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, Phys. Rev. X **12**, 021008 (2022)