What Long-Range Forces Are Allowed?

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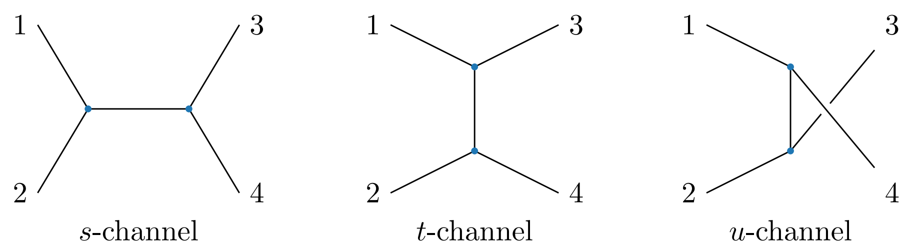
Long-range forces determine the structure of the physical world. In quantum field theory, these forces arise from the exchange of massless particles of integer spin. What are the allowed forms of these forces?

The Four-Particle Test

The simplest interactions involve three particles. For massless particles with spin, the form of these interactions is uniquely fixed by *Lorentz symmetry* and *little group scaling*. The latter means that three-point amplitude has to transform in the expected way when we rotate the polarizations of the three particles.

The challenge then is to construct consistent four-particle interactions out of these three-particle interactions. There is no guarantee that the resulting particle exchange will be *local*.

Consider two incoming particles 1 and 2 scattering into two outgoing particles 3 and 4. There are three ways in which this can happen:



In the *s-channel*, the particles 1 and 2 combine into an intermediate particle I with four-momentum $s \equiv p_I^2 = (p_1 + p_2)^2$, which then decays into the particles 3 and 4. If the intermediate particle was a real particle it would have to have $s = 0$ (because $p_I^2 = m^2 = 0$). By the uncertainty principle, a virtual particle is allowed to have $s \neq 0$, but only for a short amount of time. The particle will live longer if it is close to being real, and the amplitude will be largest. In fact, the amplitude diverges as $1/s$ in the limit $s \to 0$. This singularity corresponds to the intermediate particle propagating a large distance in spacetime. In this limit, the four-particle amplitude must become the product of two three-particle amplitudes connected by the $1/s$ propagator

$$\lim_{s\to 0} A_4 = \frac{A_3 \times A_3}{s} \,.$$

We see that this limit is determined by the unique three-particle amplitudes that we started with. Using those amplitudes, one finds that $A_3 \times A_3 = 1/t^S$, where S is the spin of the particle and $t \equiv p_J^2 \equiv (p_1 - p_3)^2$ is the momentum of the exchange particle in the *t-channel*. We see that the s-channel amplitude has a secret singularity for $t \to 0$. This is only allowed if this singularity can be interpreted as the propagation of the intermediate particle in the t-channel.

Performing a similar analysis in the t-channel, one finds

$$\lim_{t \to 0} A_4 = \frac{A_3 \times A_3}{t} ,$$

where the product of the three-particle amplitudes now leads to $A_3 \times A_3 = 1/u^S$, where $u \equiv p_K^2 = (p_1 - p_4)^2$ is the exchange momentum in the u-channel. We see that the t-channel amplitude has a secret singularity for $u \to 0$. This must be interpreted as the propagation of the intermediate particle in the u-channel. Finally, the same analysis in the u-channel has a secret singularity for $s \to 0$.

It is really hard to make all these singularities consistent, i.e. any singularity must be interpreted as the propagation of a physical particle. No scattering amplitude should have poles in s, t, u that are higher than first order. Such higher-order poles correspond to an unphysical acausal propagation of the intermediate particle in spacetime. Especially for higher-spin particles these higher-order poles seem to be unavoidable.

Discovering YM and GR

The four-particle test is very restrictive and highly constrains the allowed interactions of massless particles with spin:

- $\boldsymbol{S = 1}$: Consider a single massless spin-1 particle. Call it the photon. In that case, the sum of the s, t, u-channels *cannot* be made consistent. We have discovered that photons are not self-interacting. Maxwell's equation are linear. This is why light waves don't scatter off each other and we can see over macroscopic distances.

  Now, consider multiple spin-1 particles. Call them gluons. This passes the four-particle test, but only if the self-couplings of the gluons satisfy the so-called *Jacobi identity*. Consider further the Coulomb scattering of two gluons and two scalars (or fermions). This scattering is only consistent if the couplings of the gluons and the matter particles satisfy a *Lie algebra*. We have discovered the structure of *Yang-Mills theory*.

- $\boldsymbol{S = 2}$: Consider a single massless spin-2 particle. Call it the graviton. Unlike the photon, the four-particle test now can be satisfied (barely). The answer is unique and corresponds to *general relativity*.

  Consider further the Coulomb scattering of two gravitons and two scalars. The four-particle test is only successful if the coupling to the scalars is universal and equal to the graviton self-couplings. We have discovered the *equivalence principle* (without ever thinking about falling elevators).

  At this point, we have recovered the structure of all known long-range interactions. What about new interactions? What is possible?

  For example, consider multiple spin-2 particles. The four-particle test now fails. There is no Yang-Mills analog of gravity. There is only one graviton and it is that of GR.

- $\boldsymbol{S \geq 3}$: You are doomed. All massless particles with spin greater than 2 fail the four-particle test. Such theories are not consistent with both Lorentz symmetry and locality.

What else is out there?

We have found that higher-spin particles can't be self-interacting (like photons), but maybe they still exist and can interact with other particles:

- Consider the scattering of a photon and a spin-S particle. The four-particle test fails if $S \geq 3/2$. There are no massless charge particles with spin greater than $3/2$.

- Similarly, consider the scattering of a graviton and a spin-S particle. Now, the four-particle test fails if $S \geq 5/2$. There are no massless particles with spin greater than $5/2$ that interact consistently with the graviton. That is essentially equivalent to saying they don't exist.

- Consider the scattering of two scalars (spin 0) and two gravitinos (spin $3/2$). The scattering in the s-channel is mediated by the exchange of a graviton (spin 2). This has a singularity for $t \to 0$. However, this singularity can only be interpreted as propagation in t-channel if we introduce a new fermion (spin $1/2$) as the exchange particle. Moreover, the coupling of this fermion must be the same as the gravitational coupling. We have a theory of massless spin $0, 1/2, 3/2, 2$ particles with a universal coupling. This is *supergravity.*

References

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