CHAIRMAN: S. J. BRODSKY

Scientific Secretaries: O. Hrycyna, S. Slizovskiy

DISCUSSION I

- Buividovich:

I would like to ask about the manifestations of conformal symmetry in lattice gauge theories. Usually conformal invariance in lattice theories corresponds to phase transitions, so is that true in our case? Can it also be present in QCD? Can it be, for example, a crossover or a phase transition or something like this?

- Brodsky:

In my lecture, I showed recent lattice gauge theory results from Furui and Nakajima. They find that the QCD coupling becomes strong and flat in the low momentum region. There is no sign of a non-analytic change of behavior when you go from the asymptotic high momentum regime to the infrared regime. There is also no evident discontinuity between physics in the two domains in the Dyson-Schwinger approach. In fact, since we are looking at zero temperature, one does not expect any phase transition. It is also important to mention that there is no signal for what is often called ``IR slavery" -- there is no sign that that the QCD coupling blows up at low momentum. I believe that Gribov was the first to emphasize this point. I also presented an argument due to Cornwall that the vacuum polarization contribution to the beta-function has to vanish in the IR because of an effective nonzero gluon mass. There is thus no evidence that color confinement is associated with the infrared behavior of the QCD running coupling; in fact phenomenologically, it is more reasonable that confinement resides in the scalar potential rather than in the vector channel.

- Buividovich:

Actually, I asked because this plateau of the running coupling constant in the limit of small momenta points to something like a crossover transition in lattice gauge theories.

- Brodsky:

If there is a maximum wavelength for confined gluons, then the gluonic loops associated with the gluon self–energy decouple at virtuality below the effective two-gluon cut. I don't believe there is an actual phase transition; however, if the gluon does acquire mass, then there could be some related phenomena associated with its acquiring its longitudinal degree of freedom.

- Kenway:

The conformal window that you have described in QCD is similar to walking technicolor. Are the AdS techniques used in that case too?

- Brodsky:

Yes, there is a connection to the Banks,- Zaks approach for determining zeroes of the beta function in general gauge theories. For example, in walking technicolor, representations of different particles balance which causes the beta function to vanish at a given scale. One can also identify the conformal contribution to perturbative QCD formulae by expanding the series about the Banks-Zaks point. The terms which remain – the "conformal series" -- are independent of the terms which are associated with the running of the QCD coupling. The conformal series is well-behaved: it has no factorial growth. The separation of conformal and nonconformal terms provides an elegant and useful representation of the perturbation series. In fact, one can systematically modify the conformal series at each order by incorporating all of the contributions of the nonzero beta-function into the scale of the running coupling. You then retain the elegant properties of the conformal series, such as the Crewther relation, but now the scale of the QCD coupling is properly set at each order of perturbation theory. In effect this is how one sets the renormalization scale in QED; in QCD, the same method determines the renormalization scale, as in the BLM procedure.

- Benincasa:

Maldacena's conjecture, as originally stated, defines a correspondence between a gravity theory on AdS_5 and a gauge theory in the large N_c limit. Why do you say that it does not matter to you to be in the large N_c limit?

- Brodsky:

AdS_5 is a mathematical tool; we simply use the fact that SO(4,2) has a mathematical representation in a space with a fifth compact dimension.

- Benincasa:

So why does the large N_c limit not matter for you?

- Brodsky:

In the work I presented with Guy de Teramond, we only need the observation that a conformal theory in 3+1 dimensions has a mathematical mapping to a theory defined in AdS_5 space. It is thus not essential to have a large number of colors. As I have discussed, the running coupling, as seen by lattice simulations, shows that QCD has an IR regime where it is large and constant. This "conformal window" is all we really need to apply the AdS_5 procedure.

- Benincasa:

Well, but the fact that SO(4,2) is an isometry group of AdS_5 is not enough to state the correspondence between the N=4 Super-Yang-Mills and the type IIB superstring theory; I mean, you should also specify the gravity theory you are working on.

- Brodsky:

Yes, that's right. Maldacena's approach provides a duality between two different specific theories, N=4 Super-Yang-Mills and supergravity on the $AdS_5 x S^5$ – an elegant result. But for our applications to QCD, we do not need to actually find the dual theory in the higher space.

- Benincasa:

Then how do you define the correspondence?

- *Brodsky:*

We should distinguish what is called the "bottom-up" approach from the "top-down" approach, as in the Maldacena duality. In the top-down approach one attempts to find the actual dual of QCD to a string theory, for example by utilizing various types of branes. In contrast, de Teramond and I use the less ambitious bottom-up approach -- we start with the AdS_5 as a tool based on the conformal window of QCD and then we modify the metric in the fifth dimension to simulate confinement. This is clearly just a model, not a derivation! This approach seems to simulate hadron physics in a very elegant way -- it predicts hadron wave-functions and spectra which appear realistic. Our ultimate goal is to use the complete and orthonormal AdS eigensolutions as basis functions for diagonalizing the true Hamiltonian of QCD. This is in contrast to the discretized light-cone quantization (DLCQ) approach where one uses plane-waves rather than bound-state solutions as the initial basis. The hope is that if one starts with a basis which is already close to reality, then the convergence to the true theory will be rapid and efficient. AdS/QCD thus provides an initial approximation to QCD which in principle is improvable.

- *Maiani:*

Can you put heavy quarks in your conformal window, and if so, can you say a few words about that?

- *Brodsky:*

Yes, I shall explain this in some detail in the next lecture. In brief, we will show a close correspondence between AdS and the light-front Hamiltonian formulation of quantum field theories. The zero-mass LF kinetic energy operator for the constituents can then be improved by adding appropriate mass terms. The resulting LF equation has the possibility of describing hadrons with nonzero quark mass.

- *Reiter:*

Referring to an earlier remark about renormalization scale fixing: if one applies BLM-scale fixing – there is no ambiguity and therefore no scale uncertainty. What is your recommendation to a phenomenologist on how to put error bars to a prediction?

- *Brodsky:*

That is one of my favorite subjects! The typical procedure in QCD phenomenology is to choose an arbitrary renormalization scale and then vary it by factor of 2 to characterize the error in the perturbation theory. I believe that such a procedure is untenable. First of all, the initial guess may be very far from the true renormalization scale. Secondly, the error estimate does not represent anything more than the variation generated by the non-conformal terms in the series. In fact, there is an unambiguous way to determine the renormalization scale at any given order of perturbation theory. For example, in the case of electron-electron scattering which I discussed in my lecture, the t and u channel amplitudes are characterized by alpha(t) and alpha(u), respectively. One can start from any initial scale, but the final result, after summing an infinite number of vacuum polarization diagrams, always returns the physical scales t and u. This is clearly the correct scale assignment, since the t and u channel amplitudes must have the correct cut structure at the lepton pair mass thresholds. Although the initial guess for the renormalization scale is arbitrary, the final answer has no uncertainty. Thus, as in QED, the renormalization scale in QCD is always properly set so that beta-dependent terms are incorporated into the running coupling. The resulting prediction is scheme independent and satisfies the transitivity property of the renormalization group. Convergence can be estimated by comparing results at different orders. If I could change the world, no one would ever show a curve where they vary the renormalization scale to characterize the uncertainty in a QCD prediction. However, it is correct to vary the factorization scale in order to optimize the matching of soft and hard contributions associated with the parton distributions and hard subprocesses.

- *'t Hooft:*

Returning to your transparency #40 where you argue that $\alpha_s \rightarrow$ constant in the infrared limit, so that QCD becomes conformally invariant. How can that be if all physical states, the mesons and the baryons, are far from scale invariance there. In the world of mesons and baryons there is nothing even remotely like a scale transformation that transforms the world of states into itself.

- *Brodsky:*

Absolutely true! Let me review our AdS/QCD approach. As a first step one identifies the conformal window where the running coupling is flat and assumes zero-mass quarks. At this point one has a conformally invariant theory; however as you note, this cannot represent the real world. What is missing, of course, is confinement which we put in by hand -- either by the hard-wall boundary condition, which resembles the bag model, or by explicitly putting in a quadratic $\kappa^4 z^2$ potential. Recall that in the non-covariant bag model, structure functions did not vanish at x=0 or x=1. Now we have a frame-independent AdS/QCD LFmodel which solves such problems. Note that the Regge-trajectories are not linear in the hard-wall model; they have a slight curvature. In fact, if one extrapolates Regge trajectories to large negative t, they must curve over in order to be consistent with the dimensional counting rules for fixed-CM angle scaling. This poses a possible problem for the linear Regge trajectories of soft-wall model.

- *Garbrecht:*

How about the η' ? Does the AdS-CFT correspondence in principle also accounts for mass generation by instantons? And how does it look in AdS?

- *Brodsky:*

This is a difficult question; to my knowledge, instantons have not been identified in AdS/QCD; in fact it is not clear in the AdS approach how to distinguish between perturbative and non-perturbative effects. Some argue that the η' is a glueball mixed with a strange-antistrange Fock state. The glueball spectrum is reproduced well by AdS/QCD in that it fits well to the lattice gauge theory results. The equations for bound states of gluons are similar to those for quarks. It will be interesting to see results from AdS/QCD when one includes massive strange quarks and allows mixing with the glueball states.

CHAIRMAN: S.J. BRODSKY

Scientific Secretaries: A. M. Teixeira, S. Pacetti

DISCUSSION II

- Wu:

Do you have anything to say concerning the "unparticle physics" recently proposed by Georgi?

- Brodsky:

I understand that Howard Georgi assumes the existence of a conformal invariant theory with a nontrivial IR fixed point, as in AdS/CFT. However, he also requires nonzero anomalous dimensions in order to create "unparticle" phenomena. This is in contrast to the semi-classical application of AdS/QCD which I have discussed, where one assumes canonical integer-valued dimensions of the operators which create hadrons at short distance. I might also mention a related point that one can see how the algebra of supersymmetry emerges from the integrability structure of AdS/QCD, as can be seen when one compares the forms of the spin ½ and spin 0 ladder operators.

- Buividovich:

You have mentioned that the AdS/CFT approach to QCD reproduces well the glueball spectrum. However, since glueballs are basically just infinitely small Wilson loops, does such modification of the AdS space also give the correct predictions for Wilson loops in real world QCD? And what about the macroscopic Wilson loops?

- Brodsky:

Colin Morningstar and others have computed the glueball spectrum using lattice gauge theory. The AdS/QCD approach give similar results, as shown by Colangelo et al., and others. The structure of the glueball and mesonic quark-antiquark wavefunctions are quite similar in AdS/QCD, so I don't see a connection with infinitely small Wilson loops. Wilson loops were investigated in AdS by Maldacena and by Rey and Yee to determine the potential between heavy quarks in QCD. I agree it would be interesting to study the general structure of Wilson loops in AdS/QCD.

- Zhou:

By using the AdS/CFT approach to QCD you get results larger than those obtained in pQCD (by a factor of 16/9), when compared to asymptotic form. Has this result been confirmed by NLO or NNLO QCD? Since it is hard to calculate NNLO corrections in QCD, how can we confirm these results? Concerning the difficulty in computing the NNLO, is it possible to use your method to do this computation?

- Brodsky:

(Focusing on page 99 of the slide presentation)

The analysis of Choi and Ji uses the LO pQCD formula for the pion form factor where the renormalization scale of the running coupling in the hard scattering amplitude is set at NLO using the BLM method. The broader distribution amplitude from AdS/QCD leads to an increase of 16/9 relative to the asymptotic solution to the ERBL evolution equation. Nobody, to my knowledge, has ever computed the NNLO correction to an exclusive hard process. One also should consistently apply QCD evolution, assuming the AdS/QCD form at some Q_0. Another important observable is the photon-to-pion form factor. It would be interesting to see, if one can fit simultaneously the elastic and transition form factors as well as the pion decay constant. These are the simplest exclusive processes in QCD, so it is definitely worth the effort to be as precise as possible. It is hard to compute the analog of NNLO corrections in AdS/CFT since the basic formalism is semi-classical i.e., it ignores quantum loops.

- *Slizovskiy:*

How is the chiral symmetry breaking realized in your model?

- *Brodsky:*

The implementation of chiral symmetry breaking with vector and axial currents in AdS in the bottom-up approach was introduced by Erlich, Katz, Son and Stephanov (EKSS) and by Da Rold and Pomarol. Lebed and Radyushkin and collaborators have used the model of EKSS to compute the pion FF in this framework. In this approach, the axial and vector currents become the primary entities, as in effective chiral theory. In the AdS/QCD approach that de Teramond and I have discussed, all mesons are quark-antiquark composites. In the soft-wall model of color confinement, the theory admits a zero-mass pion, which is a crucial feature of chiral symmetry. In contrast, in the hard-wall model, the masses of the π and the ρ are degenerate. I also talked in my lecture about implementing heavy quark masses in AdS/QCD; one simply modifies the light-front Hamiltonian using the expression for the LF kinetic energy for massive quarks, together with the standard interaction kernel. In the top-down, string-theory approach, heavy quarks are localized on a new brane, the so-called D7-brane which leads to the mass generation. Both approaches are worth investigating.

- *Rogan:*

When you add mass in "by hand", by adding to the kinetic energy operator in the light front equations, what does this correspond to in AdS space? Intuitively what, if anything, does it correspond to?

- *Brodsky:*

Once one identifies the light-front equations which are equivalent to AdS/CFT, you can analyze the theory in ordinary 3+1 space; in effect you can discard the string aspects of AdS. This procedure allows one us to include quark masses into the kinetic energy operator, as in atomic physics. As you suggest, one can make an ansatz for introducing nonzero quark masses in the higher-dimension space. There are many possible schemes for this, such as placing the heavy quarks on separate -branes as in Lisa Randall's lectures. It would be interesting to study the relation between these top-down and bottom-up approaches.

- *Garbrecht:*

When introducing complex quark masses, is it possible to observe the neutron electric dipole moment?

- *Brodsky:*

In my first lecture I described the approach by Dae Sung Hwang and S. Gardner and myself. which leads to an exact expression for the CP-violating form factors as overlaps of light-front wavefunctions. One then finds a direct relation between the F_2 and F_3 form factors, independent of the origin of the CP-violating phase φ (see slide presentation, page 22). As you note, this phase could arise from the assumption of a complex quark mass which could generate an anomalous relative phase between the initial and final-state wavefunctions. I emphasize that the light-front formula gives a universal relation between the Pauli form factor and F_3, independently of the mechanism for CP violation. An example can be found by Feng, Matchev and Shadmi, where they calculate the muon electric dipole moment from a CP-violating broken supersymmetric theory.

- *Ferrara:*

I would like to make an historical remark on the application of conformal symmetry to hadron processes such like Bjorken scaling, the operator product expansion technique, and form factors. This approach goes back to the early 70's, to works of Migdal, Polyakov, Mecu, and Symanzik, and the Frascati group, including Gatto, Grillo, Parisi, and myself. The application of AdS and the realization that AdS space had the unique property of having boundary excitations (which at that time were called singletons) goes back to a French Physicist - Moshe Flato - and to a Norwegian Physicist - Christian Fronsdal. The Maldacena conjecture went along the prospective of integrating this idea into string theory, but the mathematical fact that in AdS space there are these exceptional excitations, which are confined to the boundary, does go back to the latter persons. This kind of early development, which goes back 20 or 30 years ago, should be brought to the attention of the young community.

- *Brodsky:*

I appreciate your valuable historical remarks. There is a related early literature such as the publications by Farrar and myself and Matveev, Muradyan, and Tavkhelidze which used conformal symmetry to prove dimensional counting rules. There was also early work by Frishman, Lepage, Sachrajda, and myself, and later Vlodya Braun, on the direct application of conformal symmetry to the structure of hadronic amplitudes in QCD. For instance, conformal symmetry immediately yields the eigensolutions of evolution equations for the distribution amplitudes of hadrons given by Lepage and myself and Efremov and Radyushkin.

- *Ferrara:*

Let me remark that the basic fact that the three-point function for conformal operators is fixed by conformal invariance is actually due to Polyakov's and our groups, in 1972-73.

- *Brodsky:*

Thank you for reminding us of these pioneering papers.