Quantum Hall effect in Bose-Einstein condensates

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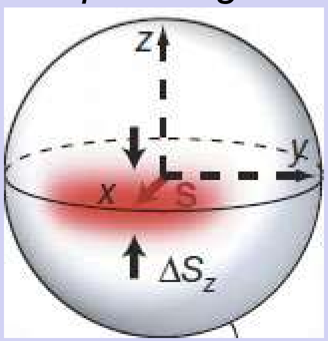
TB and Dowling Phys. Rev. A **92**, 023629 (2015)
Chen and TB Phys. Rev. B 99, 184427 (2019)

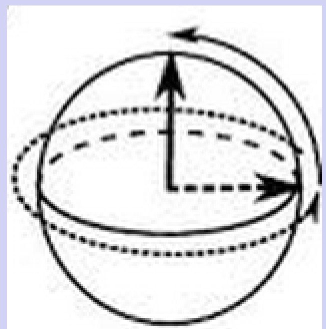
Quantum information using ensembles/BECs

Quantum metrology

Squeezing



Interferometry

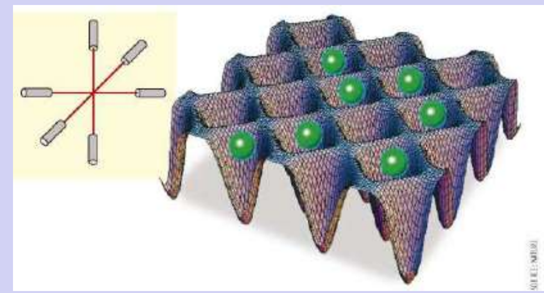


Heisenberg $\phi \propto 1/N$

Statistics $\phi \propto 1/\sqrt{N}$

Gross J. Phys. B AMO Phys. 45, 103001 (2012)

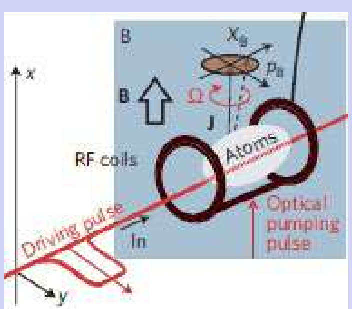
Quantum simulation



Understanding quantum many body problems

M. Greiner et al. Nature **415**, 39 (2002)

Quantum information



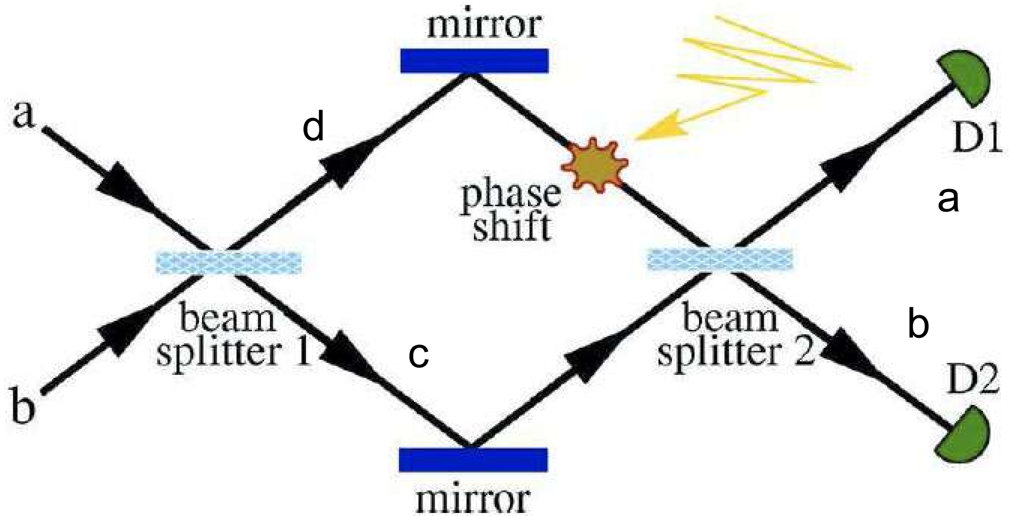
Entanglement and teleportation ensembles

Julsgaard Nature 413, 400 (2001); Krauter Nature Phys. 9, 400 (2012)

Quantum computing

Lukin Phys. Rev. Lett. 87, 037901 (2001); Brion Phys. Rev. Lett. 99, 260501 (2007);

Quantum assisted interferometry



Standard trick to obtain quantum enhancement

Standard quantum limit:

$$|a\rangle \to |c\rangle + |d\rangle \to |c\rangle + e^{i2\theta}|d\rangle \to \cos\theta|a\rangle + \sin\theta|b\rangle \qquad \theta \propto 1/\sqrt{N}$$

Heisenberg limit:

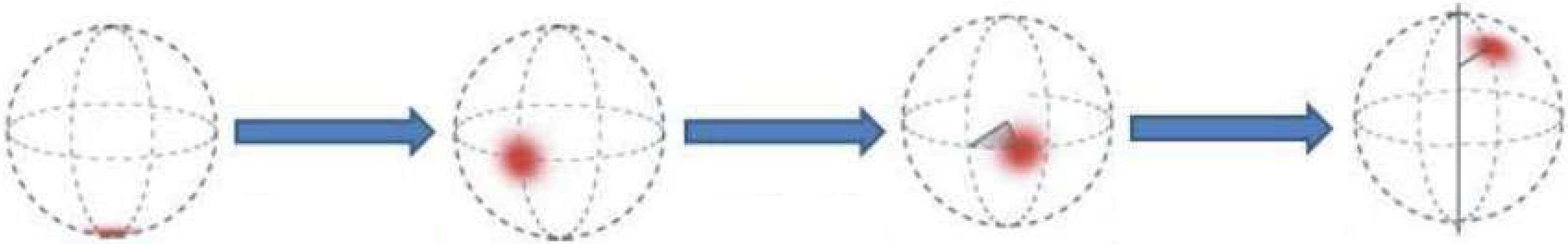
$$|Na\rangle \overset{NL}{\to} |Nc\rangle + |Nd\rangle \to |Nc\rangle + e^{i2N\theta}|Nd\rangle \overset{NL}{\to} \cos N\theta|Na\rangle + \sin N\theta|Nb\rangle$$

$$\theta \propto 1/N$$

Enhancement by squeezing

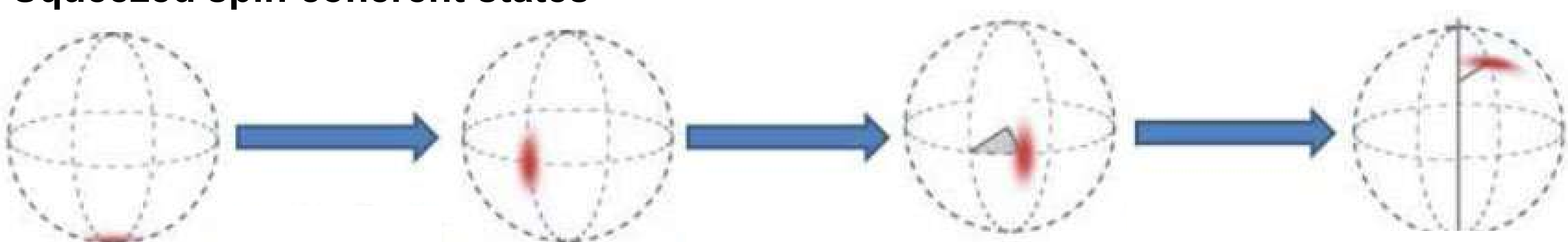
Spin coherent states

"Standard quantum limit" $\sim 1/\sqrt{N}$



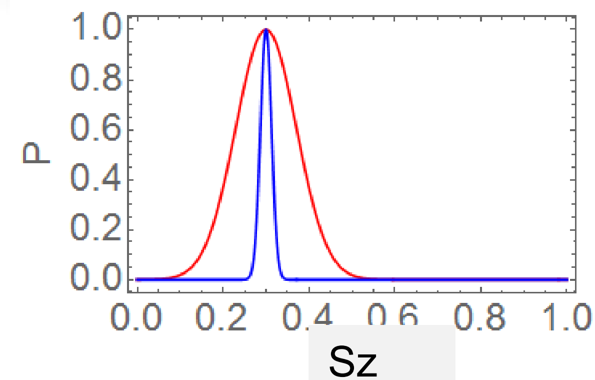
$\left|0,1\right\rangle\!\rangle$ $\left|\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\rangle\!\rangle$ $\left|\frac{1}{\sqrt{2}},\frac{e^{i\phi}}{\sqrt{2}}\right\rangle\!\rangle$ $\left|\cos\phi, i\sin\phi\right\rangle\!\rangle$

Squeezed spin coherent states



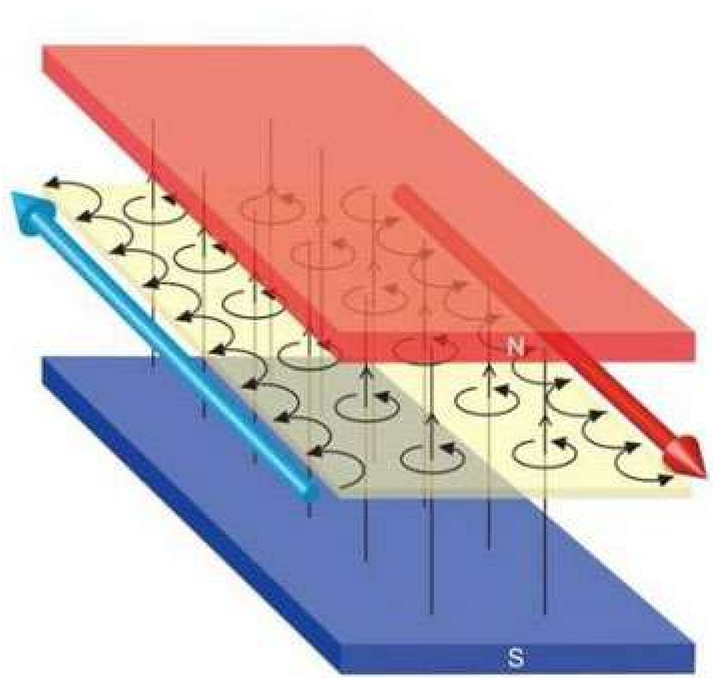
"Heisenberg limit" $\sim 1/N$

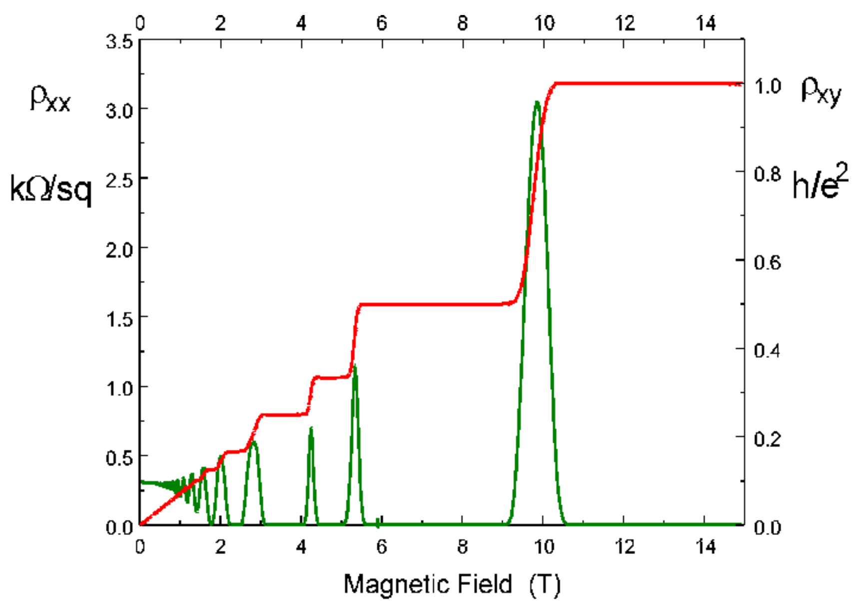
$$\left|\alpha,\beta\right\rangle\!\rangle \equiv \frac{1}{\sqrt{N!}}\left(\alpha a^{\dagger}+\beta b^{\dagger}\right)^{N}\left|0\right\rangle$$



Topological quantum states for metrology

The quantum Hall effect (QHE) forms the standard for electrical resistance, due to its extremely precise quantization of resistance





Extremely precise quantization to ~10^-10

Conductance: $\sigma = \frac{I_{\text{channel}}}{V_{\text{Hall}}} = \nu \frac{e^2}{h}$ $\nu =$ Integer

Observing the (bosonic) quantum Hall effect in ultracold atoms

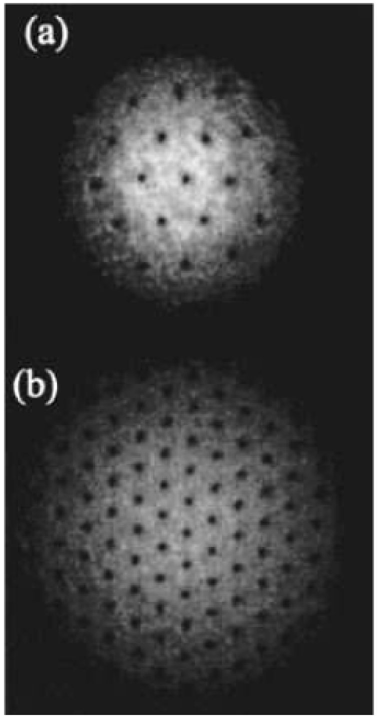
It has been the aim for some time to observe quantum Hall type experiments in ultracold atoms:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 - \boldsymbol{\Omega}\cdot\boldsymbol{L}$$

Can be rewritten as

$$H = \frac{(\boldsymbol{p} - m\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2m} + \frac{1}{2}m(\omega^2 - \Omega^2)r^2$$

$$\boldsymbol{A} = \boldsymbol{\Omega}\times\boldsymbol{r} \qquad \boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A} = 2\boldsymbol{\Omega}$$



Coddington PRA 70, 063607 (2004)

i.e. rotation = effective magnetic field + anti-trapping

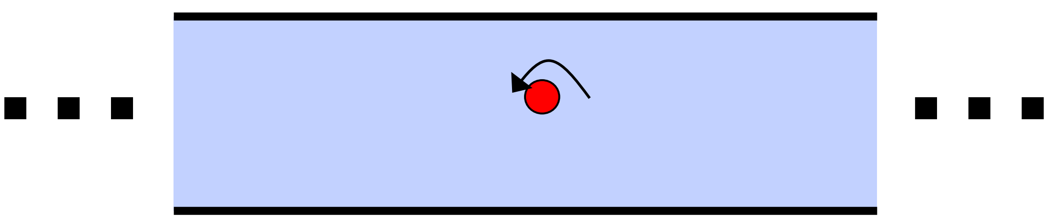
Mass plays the role of charge!

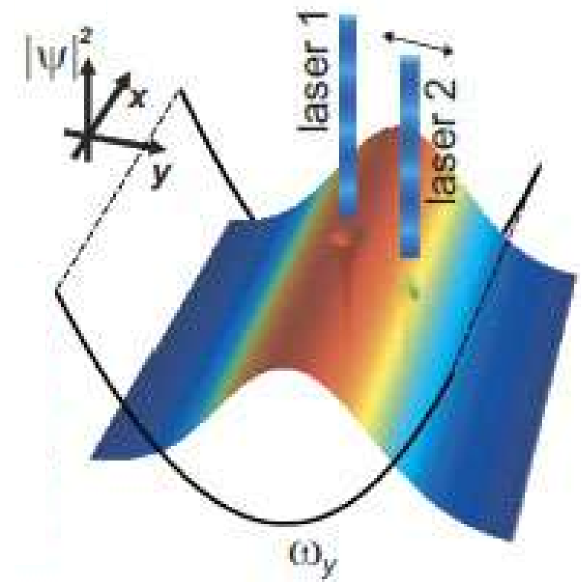
Fetter Rev. Mod. Phys. **81**, 647 (2009)
Aidelsburger Nature Phys. **11**, 162 (2014).

Full realization of QHE remains elusive

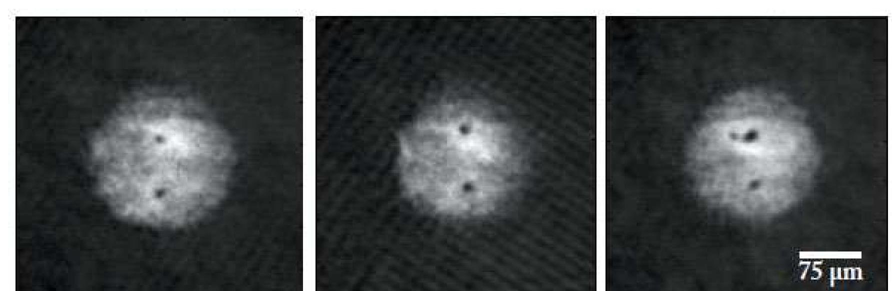
Another approach

Consider a BEC in a long channel, with the presence of vortices



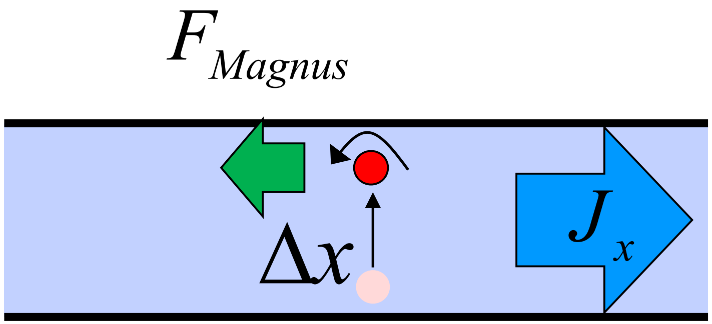


Assume that the vortices are pinned to a particular location, and they can be controlled with blue-detuned lasers



E. C. C. Samson, PRA 93, 023603 (2016).

Magnus force



$$\vec{F}_{magnus} = \rho_s \vec{K} \times (\vec{v}_{vortex} - \vec{v}_s)$$

Thouless Phys. Rev. Lett. 76, 3758 (1996)

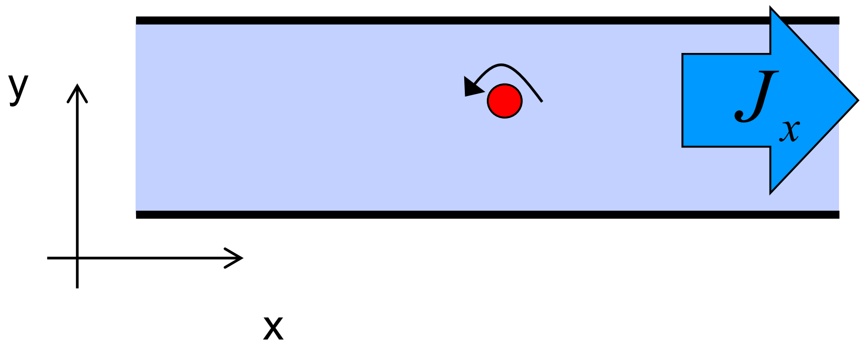
If the vortex is moved, it will experience a Magnus force. For a pinned vortex, it will push the BEC to the right.

The amount of current that is created is a topological invariant, as we show.

Connections between the Berry phase and the magnus force has been known for some time

Ao Phys. Rev. Lett.70, 2158 (1993)

Single vortex



The total current in the x-direction is $$J_x = \frac{\langle p_x \rangle}{m} = \int d\boldsymbol{r}\, j_x(\boldsymbol{r})$$

Local current: $$\boldsymbol{j}(\boldsymbol{r}) = -\frac{i\hbar}{2m}\left[\psi(\boldsymbol{r})^* \boldsymbol{\nabla}\psi(\boldsymbol{r}) - \psi(\boldsymbol{r})\boldsymbol{\nabla}\psi(\boldsymbol{r})^*\right]$$

Now make assumption: $$\psi(x, y) = \sqrt{\rho(y)}e^{iS(x,y)}$$

i.e. the density dependence is uniform along the channel

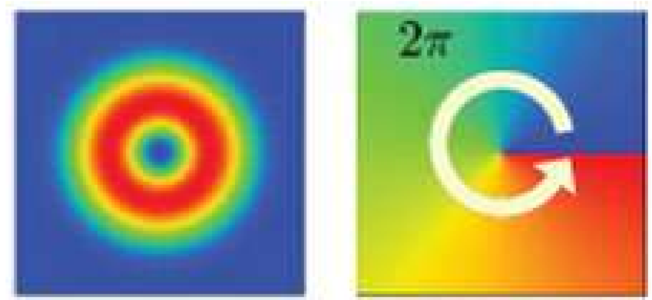
Current for one vortex

Under the assumption, since $\boldsymbol{j} = \frac{\hbar}{m}\rho\boldsymbol{\nabla}S$

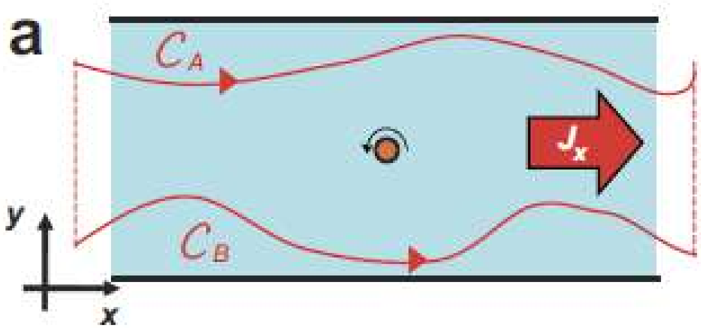
$$J_x = \frac{\hbar}{m}\int_{-\infty}^{\infty} dy\rho(y)\int_{-\infty}^{\infty} dx\frac{\partial S(x,y)}{\partial x}$$

The x-integral can be evaluated exactly!

$$I(C) = \oint \boldsymbol{\nabla}S \cdot d\boldsymbol{l} = 2\pi$$



The phase around a vortex is 2π independent of the path

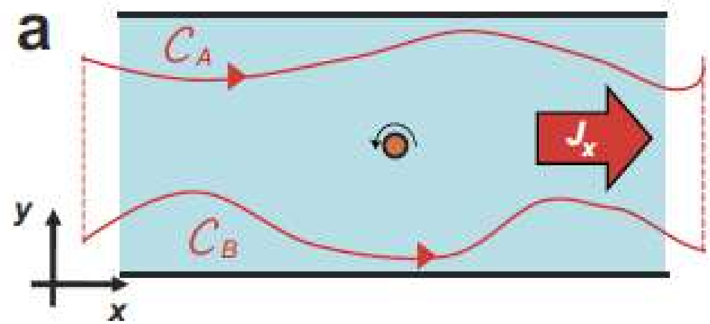


$$I(C_A) - I(C_B) = 2\pi$$

At $x = \pm\infty$ the phase change is 0

$$I(C_A) = k_0 - \pi \qquad \text{(above vortex)}$$

$$I(C_B) = k_0 + \pi \qquad \text{(below vortex)}$$



This gives

$$J_x = \frac{\hbar k_0 \mathcal{N}}{m} + \frac{h}{2m} A_y$$

Current-asymmetry proportionality

Asymmetry parameter:

$$A_y = \int_{-\infty}^{y_1} dy \rho(y) - \int_{y_1}^{\infty} dy \rho(y)$$

(Number of particles above the vortex – Number of particles below the vortex)

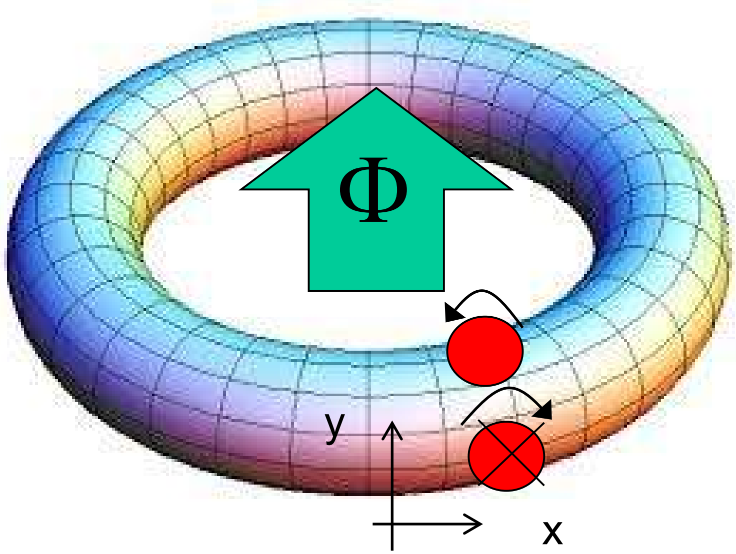
$\mathcal{N}$ = Total number of particles in BEC

Current-vortex position relation

For an infinitesimal vortex displacement

$$\delta J_x = \frac{h}{2m}\left[A_y(y_1+\delta y) - A_y(y_1)\right] = \frac{h}{m}\int_{y_1}^{y_1+\delta y} dy\rho(y) \approx \frac{h}{m}\delta y\rho(y_1).$$

$\rho(y_1) =$ density at vortex position



Agrees with Laughlin's gauge argument

1) Create vortex-antivortex pair
2) Wrap around in y-direction and annhilate
3) There is a phase in the x-direction like

$$e^{2\pi i x/L_x}$$

4) The momentum is $\Delta p_x = \frac{2\pi\hbar\mathcal{N}}{L_x} = 2\pi\hbar n L_y$

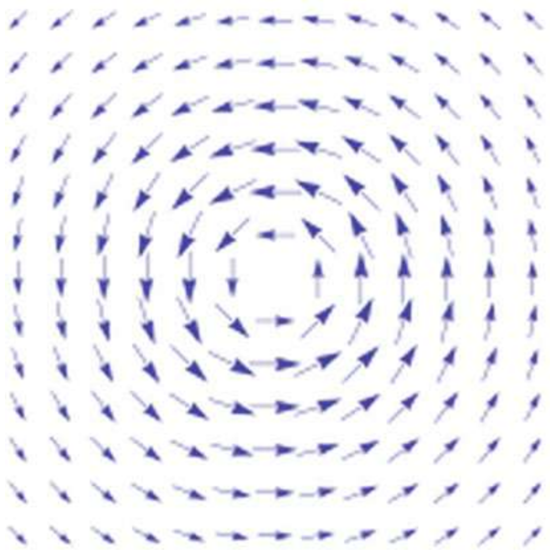
5) Integrating gives same as above

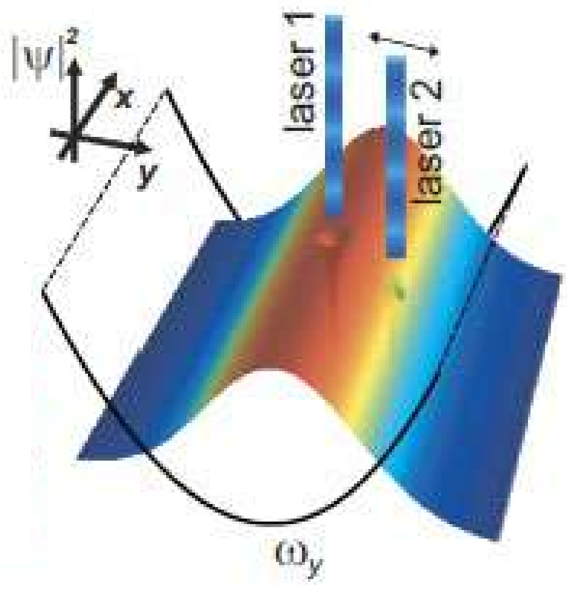
Some numerics

Simulate 2D Gross Pitaevskii equation in a harmonic trap in y-direction

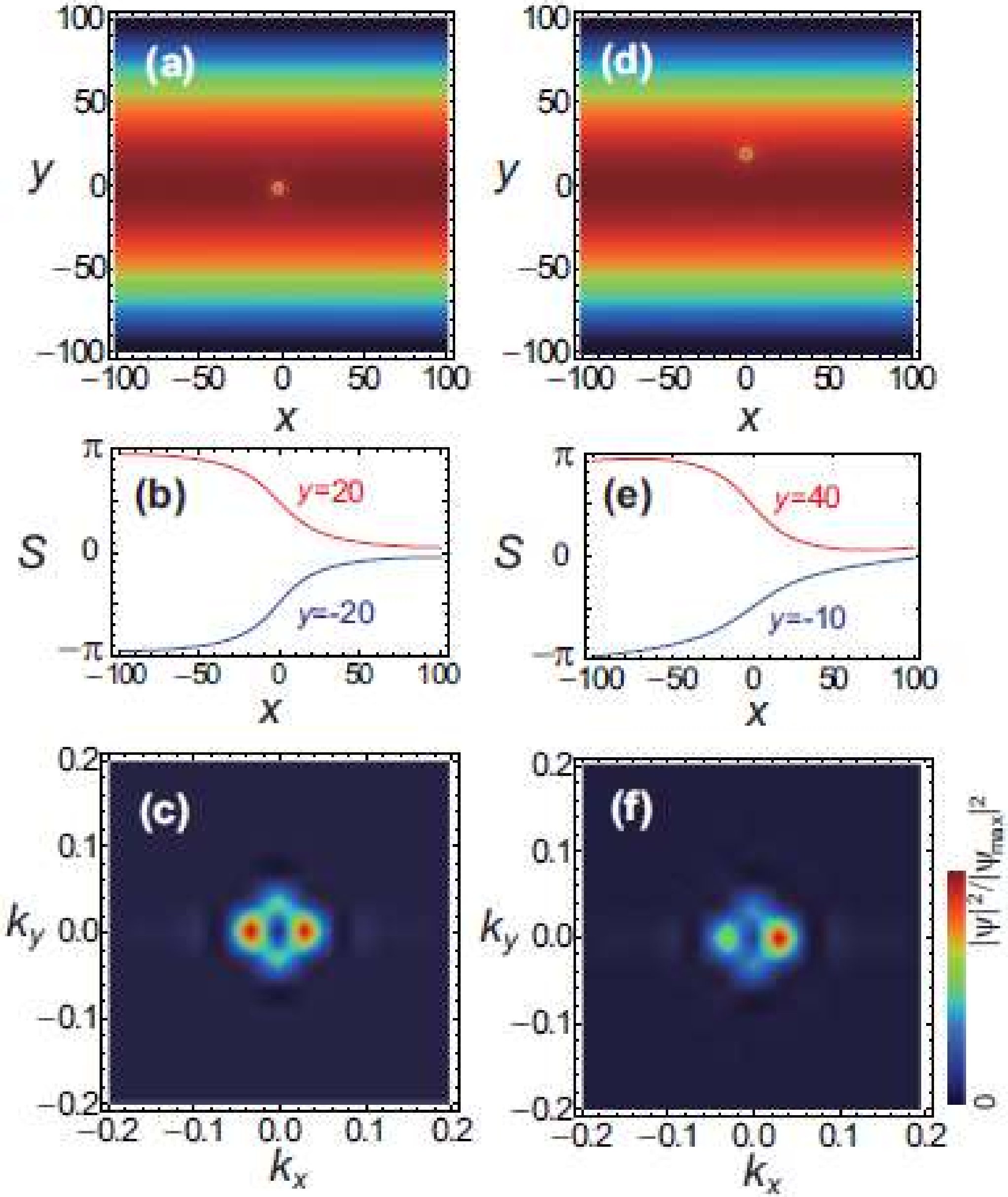
$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\omega_y^2 y^2 + V_0\sum_k \delta(\boldsymbol{r}-\boldsymbol{r}_k) + g|\psi|^2\right]\psi$$

Use open boundaries in y-direction, antiperiodic
Mobius boundary conditions in x-direction





One vortex case

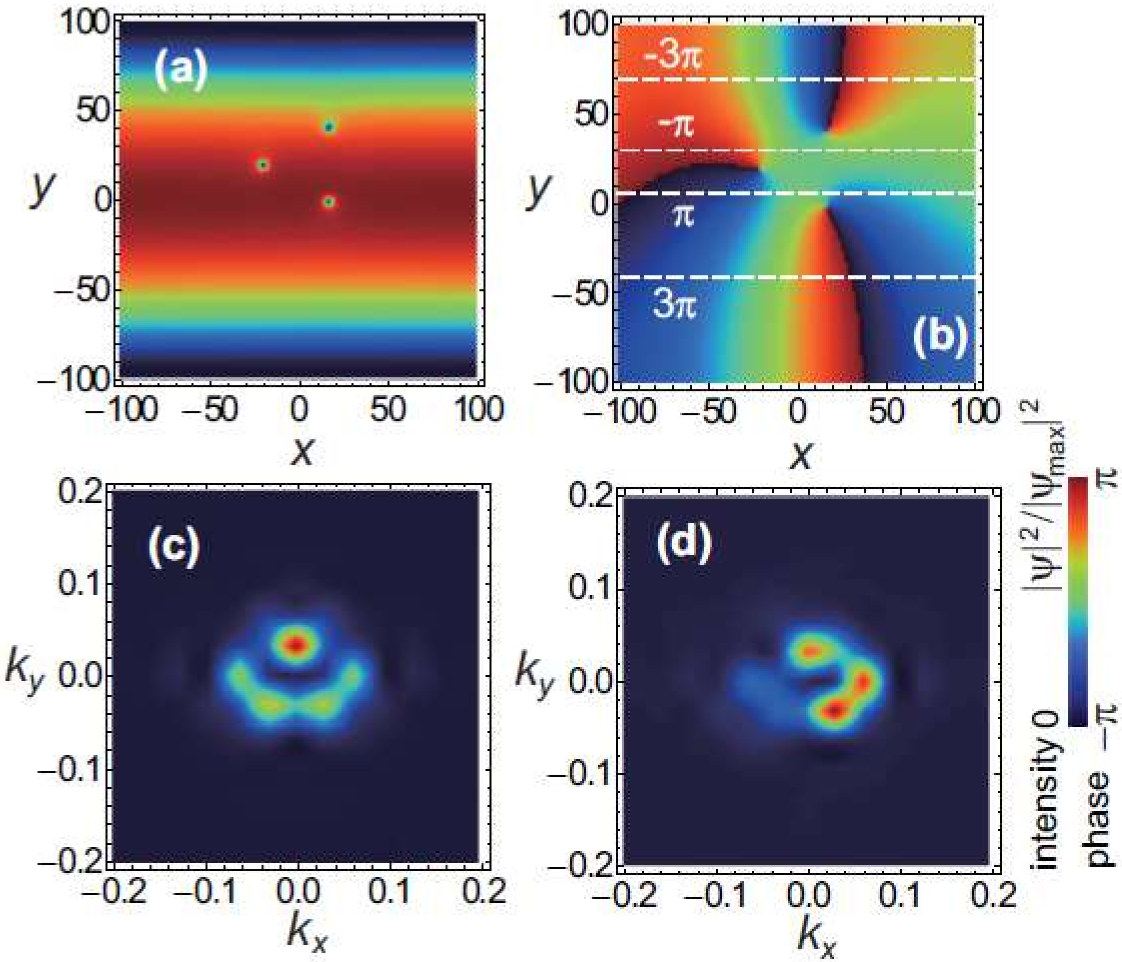


Length scale is healing length

$$\xi = \sqrt{\frac{\hbar^2}{2mgn}}$$

$$\hbar\omega_y/E_0 = 0.02,\ V_0/E_0 = 1,\ k_0 = 0$$

Multi-vortex version



Current displacement relation is simply the sum of contributions from each vortex:

$$J_x = \frac{\hbar k_0 N}{m} + G_{xy} A_y$$

$$A_y = \sum_{l=0}^{N} \frac{2l - N}{N} \int_{y_l}^{y_{l+1}} dy \rho(y)$$

Infinitesimal displacements

$$\delta J_x \approx \frac{h}{m} \delta y \frac{1}{N} \sum_{l=1}^{N} \rho(y_l)$$

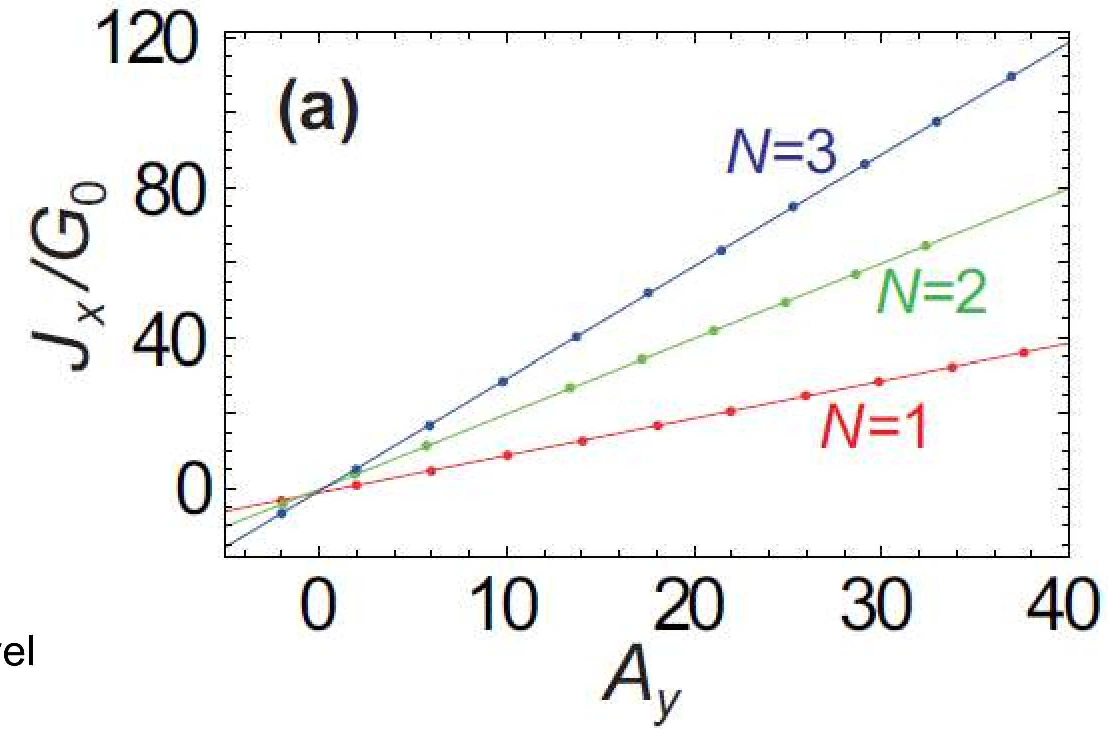
Estimate of h/m

Obtain current-asymmetry relations quantized in G0

$$G_0 = \frac{h}{2m}$$

N=Number of vortices

Gradients are integral to ~1% level



Why only 1%?

- Due to numerical restrictions, only short channels simulated
- Vortex size corrections significant?
- Numerical issues?

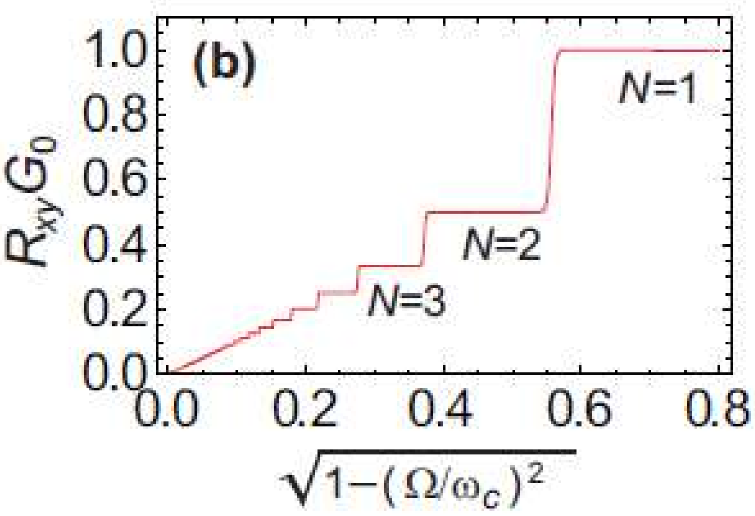
Quantum hall behavior

We can get a very quantum Hall effect like curve by assuming that the number of vortices is generated according to

$$N = \left\lfloor \kappa \frac{\Omega/\omega_c}{\sqrt{1-(\Omega/\omega_c)^2}} \right\rceil$$

A. Kato, Phys. Rev. A **84**, 053623 (2011).

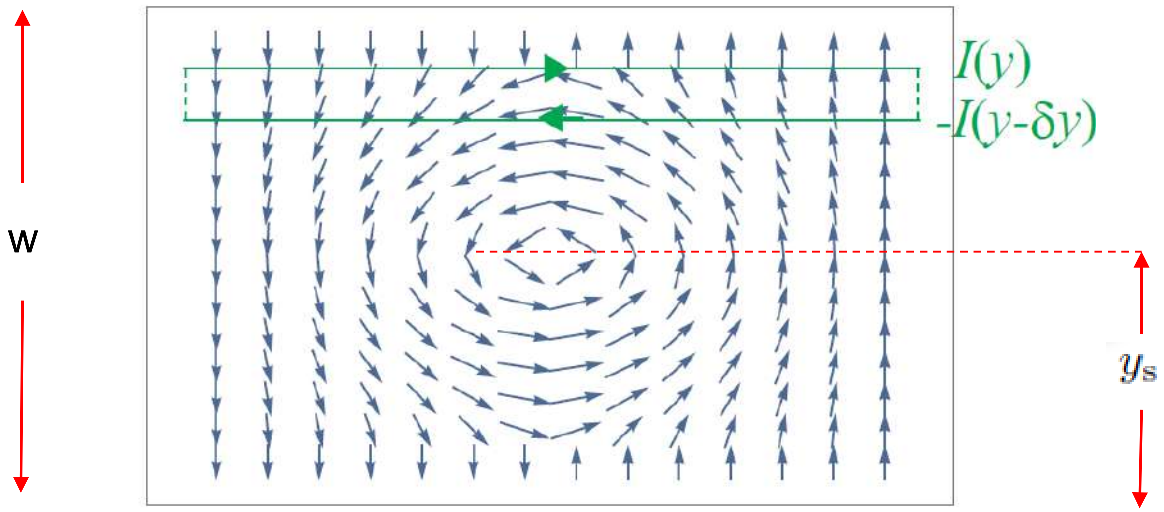
ωc= critical rotation frequency where vortices proliferate
K= proportionality constant



$$G_0 = \frac{h}{2m}$$

Skyrmion quantum spin Hall effect

Using this new quantized spin current, we can define a new kind of QSHE



Defining

$$I(y) = \int_{-\infty}^{\infty} j_Q^x(x)dx$$

$$I(y) = \begin{cases} j_0 & \text{if } y < y_s \\ j_0 + \frac{h}{M}\bar{m} & \text{if } y > y_s \end{cases}$$

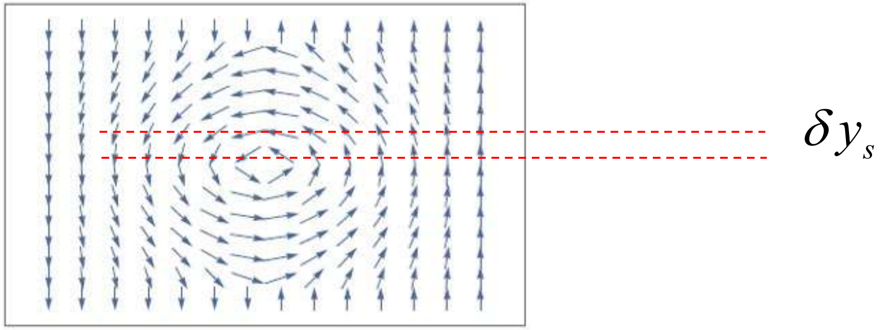
Then the total quantized spin current is

$$J_Q^x \equiv \int_{-\infty}^{\infty} j_Q^x(x)dxdy = \int_{-\infty}^{\infty} I(y)dy = j_0 w + \frac{h}{M}\bar{m}y_s$$

Current-asymmetry relation

For a uniform spin density, the transverse quantity is the vortex displacement

$$\sigma_Q = \frac{dJ_Q^x}{dy_s} = \frac{h}{M}\bar{m}$$



For a non-uniform spin density, we can define an asymmetry parameter as before

$$A_y = \int_{\infty}^{y_s} \rho(y)dy - \int_{y_s}^{\infty} \rho(y)dy$$

$$\sigma_Q = \frac{dJ_Q^x}{dA_y}$$

Chen and TB Phys. Rev. B 99, 184427 (2019)

S=1 BEC Example

Consider a S=1 spin texture, e.g. a Rb87 BEC

$$\boldsymbol{S} = (S_x, S_y, S_z)$$

$$|\psi(\boldsymbol{x})\rangle = e^{i(f(\boldsymbol{x})+g(\boldsymbol{x})\boldsymbol{u}\cdot\boldsymbol{S})}|\psi_0\rangle$$

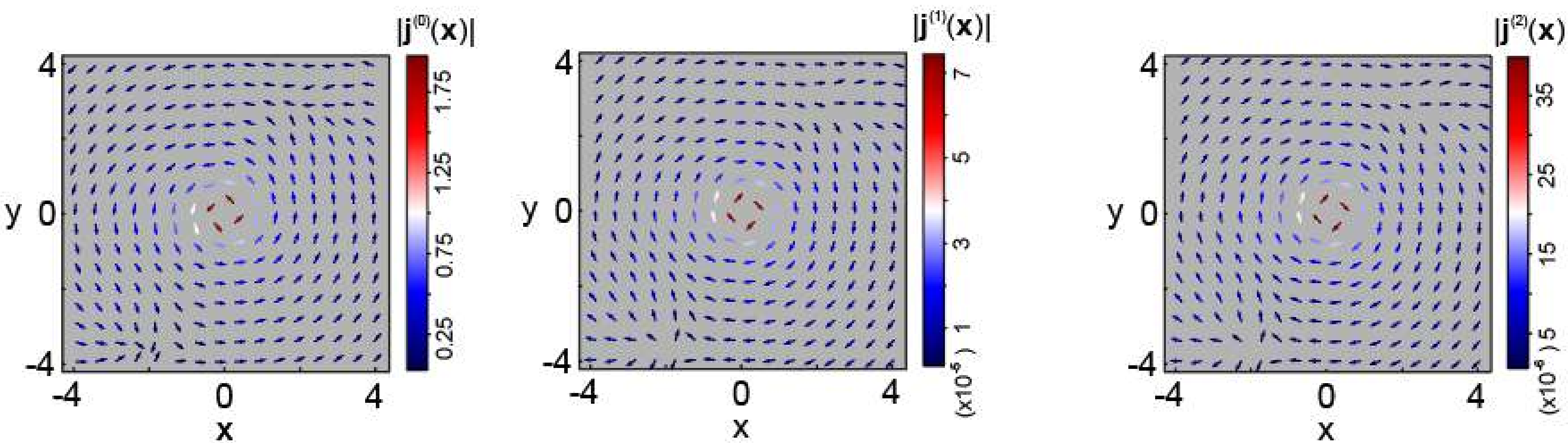
$$u = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$f(\boldsymbol{x}) = m\theta \qquad g(\boldsymbol{x}) = m'\theta n(\boldsymbol{x}) \qquad n(\boldsymbol{x}) = 1+\gamma\sum_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{|\boldsymbol{x}-\boldsymbol{x}_i|^2}{2\sigma^2}}$$

(noise function)

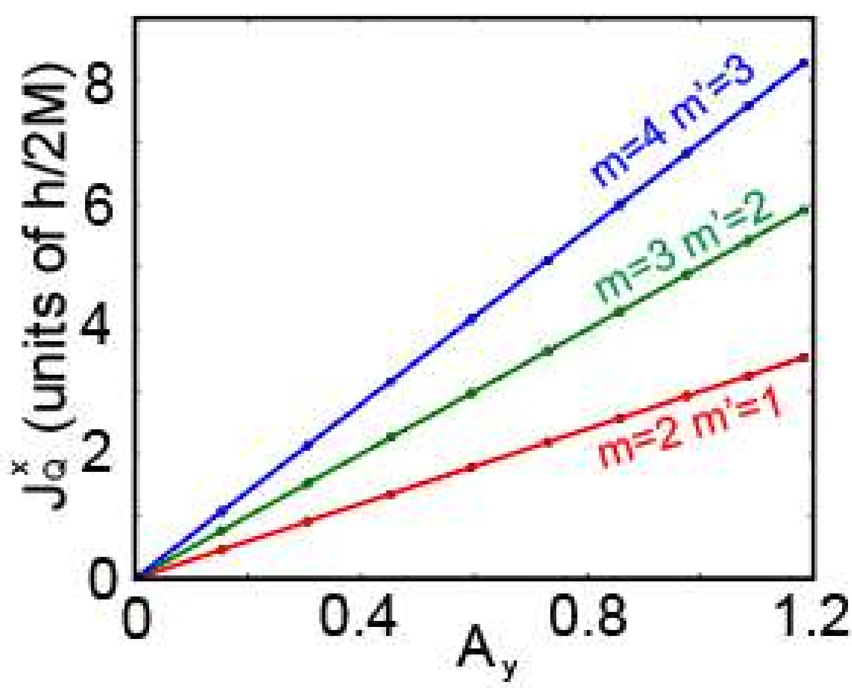
lth order spin currents

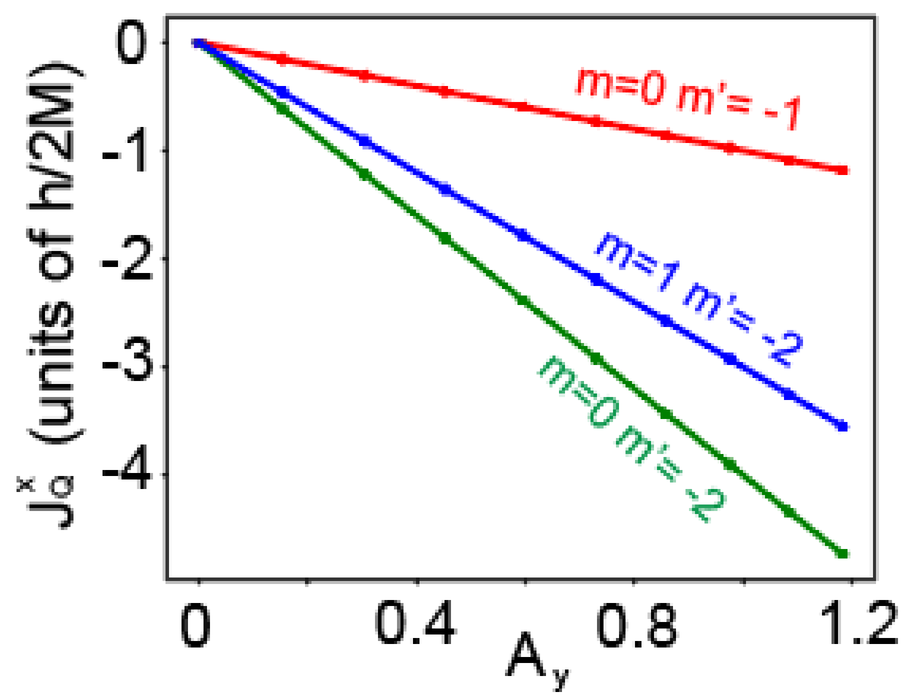
$$\boldsymbol{j}^{(l)}(\boldsymbol{x}) \equiv -\frac{i\hbar}{2M}\langle\psi(\boldsymbol{x})|(\boldsymbol{u}\cdot\boldsymbol{S})^l|\boldsymbol{\nabla}\psi(\boldsymbol{x})\rangle + \mathrm{H.c.}$$

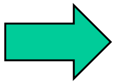


Current-asymmetry relations

For various choices of f(x) and g(x)

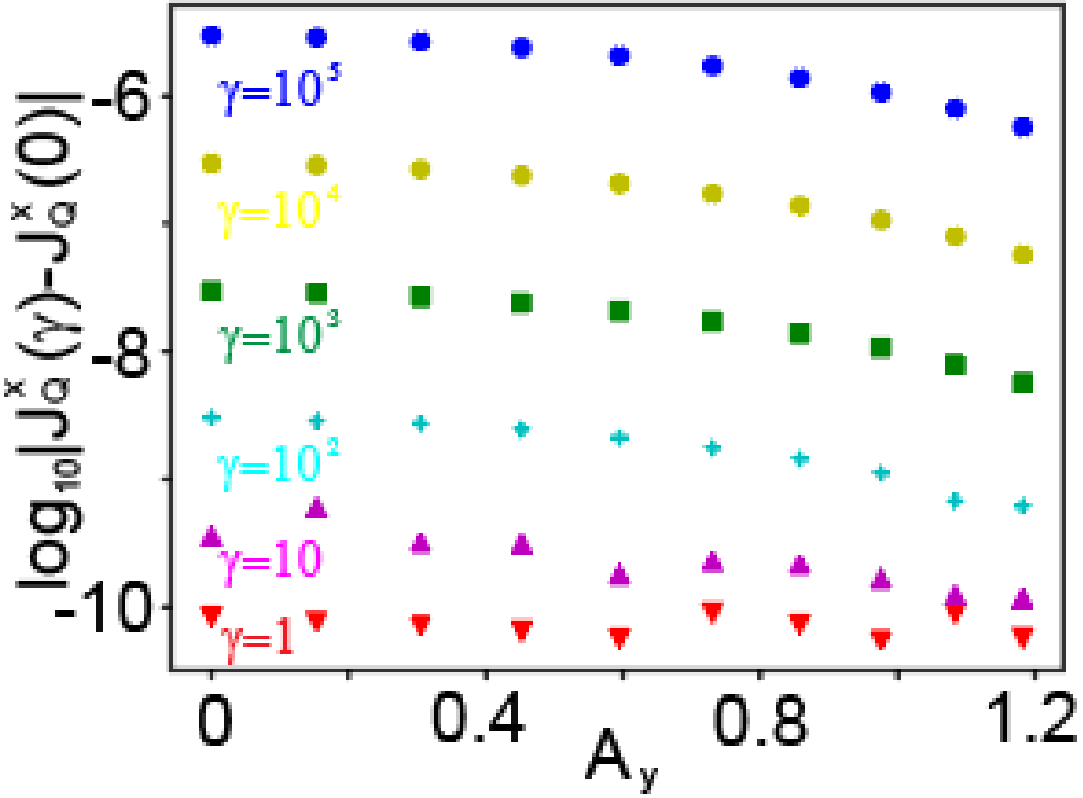




 Perfect linear relations

Deviation from ideal relation

Very accurate quantization, even for large noise



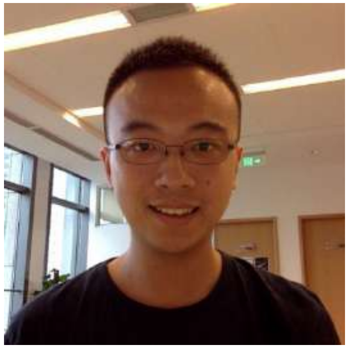
Summary and conclusions

- Topological quantum states offer an exciting route towards quantum metrology and computing
- Kitaev chains can store quantum information robustly using Majorana Zero Modes.
- Experimentally performed quantum simulation of teleportation of anyon encoded states.
- Proposed a simple way to observe quantum Hall physics by manipulating vortices in BECs
- Conductance plateaus seen in units of m/h : a potential method of precisely measuring the mass of the BEC atoms
- Requires a precise measurement of the vortex position in the density distribution. This can be performed using phase contrast imaging

References

Teleportation Majoranas: Huang, Narozniak, TB et al. Phys. Rev. Lett. **126**, 090502 (2021)
QHE in BEC: TB and Dowling Phys. Rev. A **92**, 023629 (2015)
QSHE: Chen and TB Phys. Rev. B 99, 184427 (2019)

Acknowledgments



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