## CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Preface</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td><strong>PART ONE. GRAVITY AND DIFFERENTIAL GEOMETRY</strong></td>
<td></td>
</tr>
<tr>
<td>I.0</td>
<td>Introduction</td>
<td>3</td>
</tr>
<tr>
<td>I.1</td>
<td>Exterior Calculus</td>
<td>9</td>
</tr>
<tr>
<td>L.1.1</td>
<td>Exterior forms on vector spaces</td>
<td>10</td>
</tr>
<tr>
<td>L.1.2</td>
<td>Mappings and operations on forms</td>
<td>23</td>
</tr>
<tr>
<td>L.1.3</td>
<td>Differentiable manifolds, vector fields and differential forms</td>
<td>28</td>
</tr>
<tr>
<td>L.1.4</td>
<td>Functions, vector fields and differential forms</td>
<td>37</td>
</tr>
<tr>
<td>L.1.5</td>
<td>Exterior differentiation and behaviour under mappings</td>
<td>47</td>
</tr>
<tr>
<td>L.1.6</td>
<td>The vielbein basis</td>
<td>53</td>
</tr>
<tr>
<td>L.1.7</td>
<td>Lie derivative, coordinate transformations and invariance</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>Appendix: The δ Operator and the Hodge Decomposition</td>
<td>72</td>
</tr>
<tr>
<td>I.2</td>
<td>Riemannian Manifolds</td>
<td>75</td>
</tr>
<tr>
<td>L.2.1</td>
<td>Introduction</td>
<td>75</td>
</tr>
<tr>
<td>L.2.2</td>
<td>Geometry of the linear spaces</td>
<td>76</td>
</tr>
<tr>
<td>L.2.3</td>
<td>The geometry of general Riemannian manifolds in the vielbein basis</td>
<td>80</td>
</tr>
<tr>
<td>L.2.4</td>
<td>Relation with the standard world-tensor formalism</td>
<td>91</td>
</tr>
<tr>
<td>I.3</td>
<td>Group Manifolds and Maurer-Cartan Equations</td>
<td>97</td>
</tr>
<tr>
<td>L.3.1</td>
<td>Introduction</td>
<td>97</td>
</tr>
<tr>
<td>L.3.2</td>
<td>Lie groups as manifolds: left and right invariant vector fields</td>
<td>98</td>
</tr>
<tr>
<td>L.3.3</td>
<td>Maurer-Cartan equations</td>
<td>104</td>
</tr>
<tr>
<td>L.3.4</td>
<td>Adjoint representation and Killing metric</td>
<td>107</td>
</tr>
<tr>
<td>L.3.5</td>
<td>Killing metric</td>
<td>113</td>
</tr>
<tr>
<td>L.3.6</td>
<td>Riemannian geometry of semisimple groups</td>
<td>116</td>
</tr>
<tr>
<td>L.3.7</td>
<td>Soft group manifolds</td>
<td>119</td>
</tr>
<tr>
<td>L.3.8</td>
<td>The example of Poincaré and anti de Sitter soft group manifold</td>
<td>131</td>
</tr>
<tr>
<td>I.4</td>
<td>Poincaré Gravity</td>
<td>141</td>
</tr>
<tr>
<td>L.4.1</td>
<td>Poincaré gravity</td>
<td>141</td>
</tr>
<tr>
<td>L.4.2</td>
<td>Extension to the soft group manifold</td>
<td>152</td>
</tr>
<tr>
<td>L.4.3</td>
<td>Building rules for the gravity Lagrangians</td>
<td>157</td>
</tr>
<tr>
<td>L.4.4</td>
<td>Gravity in de Sitter and anti de Sitter space</td>
<td>166</td>
</tr>
<tr>
<td>I.5</td>
<td>Coupling of Gravity to Matter Fields</td>
<td>170</td>
</tr>
<tr>
<td>L.5.1</td>
<td>Geometrical Lagrangian for scalar fields on a rigid background</td>
<td>170</td>
</tr>
<tr>
<td>L.5.2</td>
<td>Extension to the Poincaré group manifold and interpretation of the Lorentz transformation rules as variational equations</td>
<td>174</td>
</tr>
</tbody>
</table>
I.5.3. The interaction of the scalar fields with gravity and the effective cosmological constant 176
I.5.4. The field equation of a massless scalar field in anti de Sitter space (in general in a curved space) 181
I.5.5. Geometrical Lagrangians for spin 1 fields 185
I.5.6. Geometrical Lagrangian for spin 1/2 fields 188

Chapter I.6. Differential Geometry of Coset Manifolds 190
I.6.1. Introduction 190
I.6.2. Classification of coset manifolds 195
I.6.3. Coordinates on G/H and finite G-transformations 197
I.6.4. Finite transformations on G/H 204
I.6.5. Infinitesimal transformations and Killing vectors 210
I.6.6. vielbeins and metric on G/H 212
I.6.7. Covariant Lie derivative 219
I.6.8. Geodesics 222
I.6.9. Invariant measure 225
I.6.10. Connection and curvature 226
I.6.11. Rescalings 231
I.6.12. A Note on the isometries of G/H 235
I.6.13. Some examples 240
I.6.15. Homotopy and (co)homology of coset spaces 262

Chapter I.7. Applications of the Formalism and Miscellaneous Examples 272
I.7.1. The Brans-Dicke theory 272
I.7.2. Minimal coupling of pseudoscalars through a torsion mechanism 278
I.7.3. The Schwarzschild solution 283

Bibliography 296

PART TWO. THE ALGEBRAIC BASIS OF SUPERSYMMETRY

Chapter II.1. Introduction 301

Chapter II.2. Super Lie algebras, Supermanifolds and Supergroups
II.2.1. The definition of superalgebras and the example of N-extended super Poincaré algebra 310
II.2.2. Classification of the simple superalgebras whose Lie algebra is reductive 323
II.2.3. Grassmann algebras 333
II.2.4. Supermanifolds 338
II.2.5. Supergroups and graded matrices 343
II.2.6. Osp(4/N) as the N-extended supersymmetry algebra in anti de Sitter space 352

Chapter II.3. Super Maurer-Cartan Equations and the Geometry of Superspace
II.3.1. Maurer-Cartan equations of supergroups on supergroup manifolds 360
II.3.2. Maurer-Cartan equations of Osp(4/N) and Osp(4/N) 364
II.3.3. Osp(4/N) Maurer-Cartan equations as the structural equations of rigid superspace 370
II.3.4. Killing vectors on superspace, that is the generators of the supersymmetry algebra of supersimetrics 380

Chapter II.4. Poincaré Supermultiplets
II.4.1. How to construct the unitary irreducible representations of the N-extended Poincaré superalgebra 390
II.4.2. Massive multiplets without central charges 395
II.4.3. Massive multiplets with central charges 411
II.4.4. Massless multiplets 416

Chapter II.5. Supermultiplets in Anti de Sitter Space
II.5.1. Free field equations and the concept of mass in anti de Sitter space 425
II.5.2. Unitary irreducible representations of SO(2,3) 435
II.5.3. Unitary irreducible representations of Osp(4/N) 448
II.5.4. Osp(4/1) supermultiplets 454
II.5.5. Remarks about the N-extended case and the example of the Osp(4/2) multiplets 464

Chapter II.6. Supersymmetric Field Theories: The Example of the Wess-Zumino Multiplet
II.6.1. Supersymmetric field-theories corresponding to an irreducible representation of the supersymmetry algebra 473
II.6.2. The Wess-Zumino model: the simplest example of a supersymmetric field theory 477
II.6.3. Superfield interpretation of the Wess-Zumino model and rheonomy 489
II.6.4. The integrability of the rheonomic conditions and the Bianchi identities 500
II.6.5. The rheonomic action principle 503

Chapter II.7. T-matrix Algebra and Spinors in 4 ≤ D ≤ 11
II.7.1. The construction of T-matrices 519
II.7.2. The charge conjugation matrix 523
II.7.3. Majorana, Weyl and Majorana-Weyl spinors 526
II.7.4. Useful formulas in T-matrix algebra 530

Chapter II.8. Fierz Identities and Group Theory
II.8.1. Introduction 535
II.8.2. The structure of forms on N-extended D = 4 superspace 537
II.8.3. Fierz decompositions in the N = 1, D = 4 superspace 545
II.8.4. The N = 2, D = 4 case 547
II.8.5. The N = 3, D = 4 case 551
II.8.6. The N = 2, D = 5 case 555
II.8.7. Systematics of Fierz identities in eleven dimensions 553
II.8.8. Irreducible representations of SO(1,9) and the irreducible basis of the D = 10 superspace 567
Chapter II.9. Super Yang-Mills Theories
   II.9.1. Introduction
   II.9.2. Super Yang-Mills theories in $D = 4$
   II.9.3. The action principle for $N = 1$, $D = 10$ super Yang-Mills theory

Historical Remarks and References

Volume 2

PART THREE. SUPERGRAVITY IN THE RHEONOMY FRAMEWORK

Chapter III.1. Introduction

Chapter III.2. Supergravity in the Standard Component Approach
   III.2.1. Local supersymmetry and gravity
   III.2.2. Space-time Lagrangian of $D = 4$, $N = 1$ supergravity
   III.2.3. The equations of motion of $D = 4$, $N = 1$ supergravity
   III.2.4. Supersymmetry transformations and action invariance
   III.2.5. On-shell supersymmetry invariance
   III.2.6. The linearized theory of supergravity
   Appendix III.2.A. Commutator of Two Supersymmetries on the
                    Gravitino Field

Chapter III.3. Supergravity in Superspace and the Rheonomy Principle
   III.3.1. From space-time to superspace
   III.3.2. Geometry of superspace
   III.3.3. The rheonomy principle
   III.3.4. An extended action principle
   III.3.5. $D = 4$, $N = 1$ supergravity and rheonomy
   III.3.6. Rheonomic constraints and Bianchi identities
   III.3.7. On-shell supersymmetry
   III.3.8. Action invariance and off-shell supersymmetry
   III.3.9. Building rules for supergravity Lagrangians
   III.3.10. Retrieving $N = 1$, $D = 4$ supergravity from the building rules
   III.3.11. Extension of and de Sitter supergravity
   III.3.12. Building rules for supergravity theories using rheonomy and
             Bianchi identities

Chapter III.4. $D = 4$, $N = 2$ Simple Supergravity
   III.4.1. Introduction
   III.4.2. Rheonomic solution of the $N = 2$, $D = 4$ Bianchi identities
   III.4.3. The Lagrangian of $N = 2$, $D = 4$ supergravity

Chapter III.5. The $D = 5$, $N = 2$ Supergravity Theory
   III.5.1. Introduction
   III.5.2. Identification of the supergroup and construction of its curvatures
   III.5.3. Construction of the Lagrangian
   III.5.4. Superspace equations of motion and on-shell supersymmetry
   III.5.5. The second order formulation and the contracted version of the theory

Chapter III.6. The Theory of Free Differential Algebras and
               Some Applications
   III.6.1. Introduction
   III.6.2. The concept of free differential algebra
   III.6.3. The structure of free differential algebras and some theorems
Volume 3

PART SIX. HETEROTIC SUPERSTRINGS AND SUPERGRAVITY

Chapter VI.1. Introduction

Chapter VI.2. Elements of Two-dimensional Differential Geometry and of Riemann Surface Theory

Chapter VI.3. The Classical Action of the Heterotic Superstrings and Their Canonical Quantization

Chapter VI.4. The BRST Charge and the Ghost Fields

Chapter VI.5. Quantum Determination of the Target Manifold and Kac-Moody Algebras

Chapter VI.6. The Polyakov Path Integral and the Partition Function of String Models

Bibliographical Note