SUPERGRAVITIES IN 5 DIMENSIONS

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Dedicated to Joël Scherk

1 WHY 5 DIMENSIONS ?

In supersymmetry the consideration of theories in dimensions $D > 4$ has been very fruitful. In particular the supersymmetric N=4 Yang-Mills theory has been derived from the N=1 supersymmetric Yang-Mills theory in 10 dimensions (Gliozzi, Olive & Scherk, 1977 ; Brink, Scherk & Schwarz, 1977). More recently, starting from the N=1 supergravity in 11 dimensions (Cremmer, Julia & Scherk, 1978) the N=8 supergravity in 4 dimensions has been derived with its unexpected symmetries E_7 global x SU(8) local (Cremmer & Julia, 1978 and 1979).

We would like today to concentrate on supergravities in 5 dimensions for essentially four reasons :

(i) For extended supergravities (especially N=8) the structure is simpler in 5 dimensions than in 4 dimensions because all the invariances are invariances of the Lagrangian instead of invariances of equations of motion. (This is related to the duality transformations on vector fields for the theories in 4 dimensions). This could therefore lead to a better understanding of these extended supergravities.

(ii) From the theories in 5 dimensions we can obtain spontaneously broken supersymmetric theories in 4 dimensions by a generalized dimensional reduction (Scherk & Schwarz, 1979). In particular, spontaneously broken N=8 supergravity with 4 mass parameters has been constructed in this way (Cremmer, Scherk & Schwarz, 1979).

(iii) The knowledge of the theory on-shell in 5 dimensions allows one to have an off-shell formulation in 4 dimensions modulo some differential constraints on the fields using the dimensional reduction by Legendre transformation (Sohnius, Stelle & West, 1980). In particular, an off-shell formulation of extended N=8 supersymmetry has been derived (Cremmer, Ferrara, Stelle & West, 1980).

(iv) From the "Lagrangian builder" point of view it shows how the conjecture of the bosonic symmetries allows one to construct the complete theory up to a few coefficients.

The plan of my talk will be the following :

(1) I shall give some notation and definitions of symplectic spinors in 5 dimensions.

(2) I shall give the particle contents of all supergravities in 5 dimensions.

(3) I shall briefly recall some facts about global and local symmetries in supergravity.

(4) The main part of the talk will be devoted to the description and construction of the N=8 supergravity in 5 dimensions.

(5) Finally, I shall give the consistent sets of truncation which lead to the N=6, 4 and 2 supergravities in 5 dimensions.

2 SYMPLECTIC SPINORS

The metric of the 5-dimensional spacetime is

$$\eta_{rs} = (+, -, -, -, -)$$

The γ matrices are defined by their anticommutation relations

$$\{\gamma_r , \gamma_s\} = 2\, \eta_{rs}$$

$\gamma_0 , \gamma_1 , \gamma_2 , \gamma_3$ are the same as in 4 dimensions and are pure imaginary. Since $(\gamma_5)^2 = +1$ we must define

$\gamma_4 = i\ \gamma_5$ which is real.
This shows that there are no Majorana spinors in 5 dimensions.
γ_0 and γ_5 are antisymmetric and $\gamma_1 , \gamma_2 , \gamma_3$ are symmetric.
The five γ matrices are related by

$$\gamma_{rstuv} = \varepsilon_{rstuv}$$

where γ_{rstuv} is the totally antisymmetric product of $\gamma_r\ \gamma_s\ \gamma_t\ \gamma_u\ \gamma_v$ and ε_{rstuv} is the usual Levi-Civita symbol with 5 indices ($\varepsilon_{01234} = +1$).

In 5 dimensions the N extended supersymmetry algebra can only be defined for even N and it has a natural isomorphism which is the USp(N) symplectic symmetry (compact)

$$\{\bar{Q}^a_\alpha , Q^b_\beta\} = \Omega^{ab}(\gamma^\mu)_{\beta\alpha}\, P_\mu$$

$(\mu = 0 \ldots 4 \; ; \; \alpha = 1 \ldots 4 \; ; \; a = 1 \ldots N)$

The charges Q^a_α (and consequently the spinor fields) satisfy a generalized Majorana condition

$$Q^a_\alpha = C_5 \bar{Q}^{ta}_\alpha$$

where C_5 satisfies

$$C_5 \gamma_\mu C_5^{-1} = \gamma_\mu^t$$

$$\bar{Q}^a = (Q_a^*)^t \gamma_0$$

Ω^{ab} is the real symplectic metric and is used to raise or lower indices

$$Q_a = \Omega_{ab} Q^b$$

from which we deduce $\bar{Q}_a = \Omega_{ab} \bar{Q}^b = -(Q^a)^{*t} \gamma_0$

We can choose $C_5 = \gamma_0 \gamma_5$. In this case the symplectic spinors are defined by

$$Q^a = \gamma_5 (Q_a)^*$$

From these definitions we deduce the important property of bilinear expressions in Fermi fields :

$$\bar{\psi}^a \gamma_{\mu_1} \ldots \gamma_{\mu_M} \chi^b = \bar{\chi}^b \gamma_{\mu_M} \ldots \gamma_{\mu_1} \psi^a \qquad , \quad \forall M$$

Finally let us give the Fierz transformation in 5 dimensions :

$$\bar{\varepsilon}_1 \varepsilon_2 \bar{\varepsilon}_3 \varepsilon_4 = -\frac{1}{4} \{ \bar{\varepsilon}_1 \varepsilon_4 \bar{\varepsilon}_3 \varepsilon_2 + \bar{\varepsilon}_1 \gamma_r \varepsilon_4 \bar{\varepsilon}_3 \gamma^r \varepsilon_2 - \frac{1}{2} \bar{\varepsilon}_1 \gamma_{rs} \varepsilon_4 \bar{\varepsilon}_3 \gamma^{rs} \varepsilon_2 \}$$

3 SUPERGRAVITIES IN 5 DIMENSIONS

The physical states of 2N extended supersymmetric massless multiplets in 5 dimensions are classified by USp(2N) (compact) as the states of the massive multiplet with central charge in 4 dimensions. It is the 5th dimension which is related to the central charge in 4 dimensions.

In the simplest multiplets, the representations of USp(2N) which appear are the antisymmetric and traceless tensors $R^{abc \ldots m}$ with

$$\Omega_{ab} R^{abc \ldots m} = 0$$

with $m \leqslant N$. For $N < m \leqslant 2N$ an antisymmetric traceless tensor is automatically zero because the Levi-Civita tensor with 2N indices can be written in terms of Ω_{ab}

$$\varepsilon_{a_1 a_2 \ldots . a_{2N-1}\, a_{2N}} \sim \Omega_{[a_1 a_2}\, \Omega_{a_3 a_4} \ldots\, \Omega_{a_{2N-1}\, a_{2N}]}$$

The fields also satisfy the same kind of generalized reality condition as the spinor charges

$$A_\mu{}^{ab} = (A_{\mu\, ab})^*$$
$$\chi^{abc} = \gamma_5 (\chi_{abc})^*$$

The lowest spin supermultiplet for 2N supersymmetry has states from spin s up to s=N (SU(2) is the little group of Lorentz group in 5 dimensions) and has the following content

$s=N$	$s=N-1/2$	$s=N-1$	...	$s=0$
ϕ	ϕ^a	ϕ^{ab}	...	$\phi^{a_1 \ldots a_N}$

where all $\phi^{ab\ldots}$ are antisymmetric and traceless.
Other multiplets can be obtained by combining this multiplet with states of angular momentum J and an arbitrary representation of USp(2N) (Ferrara & Zumino, 1979).

This allows a simple construction of the representations of extended supergravities in 5 dimensions. They are given, in the following table, as well as the lowest spin supermultiplet.

s	2	3/2	1	1/2	0	group
N=8	1	8	27	48	42	USp(8)
N=6	1	6	14+1	14'+6	14	USp(6)
	(J=$\frac{1}{2}$)	⊗[1	6	14	14']	
N=4	1	4	5+1	4	1	USp(4)
	(J=1)	⊗	[1	4	5]	
N=2	1	2	1			USp(2)
			(J=$\frac{3}{2}$)	⊗ [1 ,	2]	

As in 4 dimensions the N=8 supergravity multiplet is also the lowest spin supermultiplet.

4 GLOBAL AND LOCAL SYMMETRIES IN SUPERGRAVITY

The dimensional reduction shows that for maximal extended supergravities in D dimensions obtained from N=1 supergravity in 11 dimensions (Cremmer & Julia, 1979 ; Cremmer, 1980), the theory is invariant under the product of a non-compact global group and a compact local group

$$SL(11 - D,R)\text{global} \times SO(11 - D)\text{local}$$

SO(11 - D)local acts on Fermi fields and scalar fields.
SL(11 - D,R)global acts on tensor fields and scalar fields.
The scalar fields which come from the metric in 11 dimensions are described by the coset GL(11 - D,R)/SO(11 - D) (after a Weyl rescaling). We expect that all scalar fields can be described in this geometric way by a coset G/H (i.e. a matrix of G defined up to a local transformation of H) as the vielbein e_μ^r is described by the coset GL(D,R)/SO(D - 1, 1). If G is non compact, there is no problem of positivity if H is the maximal compact subgroup of G. The symmetries G and H can be conjectured by simple counting arguments if we remember that H is the maximal group linearly realised on all fields. (This H is the diagonal subgroup of $G_{\text{global}} \times H_{\text{local}} \supset H_{\text{global}} \times H_{\text{global}} \supset H$).

We shall give below the content and the symmetries of maximal supergravities in D=9...3 after duality transformations which convert a p tensor field into a (D - 2 - p) tensor field

D=9 $GL(2, R)\text{global} \otimes SO(2)\text{local}$

$1\, e_\mu^r$, $2\, \Psi_\mu$, $1 A_{\mu\nu\rho}$, $2 A_{\mu\nu}$, $3 A_\mu$, $4\, \chi$, 3 scalars

D=8 $E_{3(+3)}=SL(3,R) \times SL(2,R)\text{global} \otimes [SO(3) \times SO(2)]\text{ local}$

$1\, e_\mu^r$, $2\, \Psi_\mu$, $1 \utilde{A_{\mu\nu\rho}}$, $3 A_{\mu\nu}$, $6 A_\mu$, $6\, \chi$, 7 scalars

D=7 $E_{4(+4)}=SL(5,R)\text{global} \otimes SO(5)\text{local}$

$1\, e_\mu^r$, $4\, \Psi_\mu$, $5 A_{\mu\nu}$, $10 A_\mu$, $16\, \chi$, 14 scalars

D=6 $E_{5(+5)}=SO(5, 5)\text{global} \otimes SO(5) \times SO(5)\text{ local}$

$1\, e_\mu^r$, $4\, \Psi_\mu$, $5 \utilde{A_{\mu\nu}}$, $16 A_\mu$, $20\, \chi$, 25 scalars

D=5 $E_{6(+6)}\text{global} \otimes USp(8)\text{local}$

$1\, e_\mu^r$, $8\, \Psi_\mu$, $27 A_\mu$, $48\, \chi$, 42 scalars

D=4 $E_{7(+7)}\text{global} \otimes SU(8)\text{local}$

$1\, e_\mu^r$, $8\, \Psi_\mu$, $\utilde{28 A_\mu}$, $56\, \chi$, 70 scalars

D=3 $E_{8(+8)}\text{global} \otimes SO(16)\text{local}$

$1\, e_\mu^r$, $16\Psi_\mu$, $128\, \chi$, 128 scalars

Let us note that in 3 dimensions there is no degree of freedom for the graviton and the gravitinos. The underlined tensor fields need duality transformations to form a representation of the global group. The global symmetry will not be a symmetry of the Lagrangian but only of the equations of motion : the symmetry will exchange the Bianchi identity for the field strength of the tensor with its equation of motion.

It has been seen that in 4 dimensions, for all extended supergravities, the scalar fields are described by a coset, the local symmetry being U(N). In the same way, we can conjecture that all extended supergravities in 5 dimensions have a global symmetry G and a local symmetry USp(2N), the scalar fields being described by G/USp(2N). This gives the following table

N=8 $E_{6(+6)}$ global $\otimes$ USp(8) local

N=6 SU*(6) global $\otimes$ USp(6) local

N=4 USp(4) x R global $\otimes$ USp(4) local

N=2 USp(2) global $\otimes$ USp(2) local

5 N=8 SUPERGRAVITY IN 5 DIMENSIONS

As we have seen, the free particle spectrum is described by the fields $h_{\mu\nu}$, $\Psi_\mu{}^a$, $A_\mu{}^{ab}$, χ^{abc} and ϕ^{abcd} where $a = 1 \ldots 8$ and these fields are pseudoreal in the sense previously defined, completely antisymmetric and traceless in the internal USp(8) indices.

We have seen that we expect the theory to have a global symmetry E_6 and a local symmetry USp(8). Let us first briefly describe E_6. It has 78 generators and the fundamental representation has dimension 27. We are interested in the non-compact form which has 42 non-compact generators and 36 compact ones which generate the maximal subgroup USp(8). The 27 representation acts in the vector space spanned by $Z^{\alpha\beta}$ (α , $\beta = 1 \ldots 8$) such that

$$Z^{\alpha\beta} = - Z^{\beta\alpha} = (Z_{\alpha\beta})^*$$

$$\Omega_{\alpha\beta} Z^{\alpha\beta} = 0 \quad , \quad Z_{\alpha\beta} = \Omega_{\alpha\gamma} \Omega_{\beta\delta} Z^{\gamma\delta}$$

and the infinitesimal transformations of E_6 are given by

$$\delta Z^{\alpha\beta} = \Lambda^\alpha{}_\gamma Z^{\gamma\beta} + \Lambda^\beta{}_\gamma Z^{\alpha\gamma} + \Sigma^{\alpha\beta\gamma\delta} Z_{\gamma\delta}$$

where $\Lambda^\alpha{}_\gamma$ is an antihermitian matrix such that $\Lambda_{\alpha\gamma}$ is symmetric and $\Sigma^{\alpha\beta\gamma\delta}$ is totally antisymmetric, traceless and pseudoreal

$$\Sigma^{\alpha\beta\gamma\delta} = (\Sigma_{\alpha\beta\gamma\delta})^*$$

There is no quadratic invariant for E_6 : 27 x 27 $\not\supset$ 1 in particular $Z^{\alpha\beta} Z_{\alpha\beta}$ is not invariant for E_6. We can form an invariant from 27 x $\overline{27}$ where $\overline{27}$ is spanned by $\tilde{Z}^{\alpha\beta}$

$$\tilde{Z}^{\alpha\beta} = - \tilde{Z}^{\alpha\beta} = - (\tilde{Z}_{\alpha\beta})^{*} \quad ; \quad \Omega_{\alpha\beta}\, \tilde{Z}^{\alpha\beta} = 0$$

which transforms under E_6 by

$$\delta\tilde{Z}^{\alpha\beta} = \Lambda^{\alpha}{}_{\gamma}\, \tilde{Z}^{\gamma\beta} + \Lambda^{\beta}{}_{\gamma}\, \tilde{Z}^{\alpha\gamma} - \Sigma^{\alpha\beta\gamma\delta}\, \tilde{Z}_{\gamma\delta}$$

$Z_{\alpha\beta}\, \tilde{Z}^{\alpha\beta}$ is an invariant under E_6.
Both $Z_{\alpha\beta}\, Z^{\alpha\beta}$ and $Z_{\alpha\beta}\, \tilde{Z}^{\alpha\beta}$ are invariant under the subgroup USp(8). There exists a trilinear invariant for E_6 : 27 x 27 x 27 =1+

$$J = Z^{\alpha\beta}\, \Omega_{\beta\gamma}\, Z^{\gamma\delta}\, \Omega_{\delta\varepsilon}\, Z^{\varepsilon\lambda}\, \Omega_{\lambda\alpha}$$

These properties of E_6 are all we need to obtain the general structure of the theory.

The fields of the N=8 supergravity are :

- the graviton $e_\mu^{\ r}$, an element of GL(5,R)/SO(4, 1)
- the 8 gravitinos Ψ_μ^a which are in the representation 8 of USp(8) and singlets for E_6
- the 27 vector fields $A_\mu^{\alpha\beta}$ which are singlets for USp(8) and in the 27 representation of E_6
- the 48 spin 1/2 fields χ^{abc} which are in the representation 48 of USp(8) and singlets for E_6
- the 42 scalar fields will be described by an element $\mathcal{V}_{\alpha\beta}{}^{ab}$ of the coset $E_{6(+6)}$/USp(8) (78 - 36 = 42). It transforms as $\overline{27}$ under E_6 and 27 under USp(8). The indices α, β =1...8 are the 'curved' indices of E_6 and a , b =1...8 are the flat indices of USp(8) and $\mathcal{V}_{\alpha\beta}{}^{ab}$ is a 27-bein connecting these two groups.

The self-interaction of the scalar fields is described by a non linear σ -model associated to the coset E_6/USp(8) and therefore by the Lagrangian

$$\mathcal{L}_s \sim D_\mu \mathcal{V}_{\alpha\beta}{}^{ab}\, D^\mu \tilde{\mathcal{V}}_{ab}{}^{\alpha\beta} = - \mathrm{Tr}\, (\tilde{\mathcal{V}} D_\mu \mathcal{V})^2$$

where $\tilde{\mathcal{V}}$ is the inverse of $\mathcal{V}$

$$\tilde{\mathcal{V}}_{ab}{}^{\alpha\beta}\, \mathcal{V}_{\alpha\beta}{}^{cd} = \frac{1}{2}\, (\delta_a{}^c\, \delta_b^d - \delta_a{}^d\, \delta_b^c) + \frac{1}{8}\, \Omega_{ab}\, \Omega^{cd}$$

D_μ is the covariant derivative with respect to USp(8) using the associated connexion $\Omega_{\mu\, a}{}^b$. Since there is no kinetic term for $\Omega_{\mu\, a}{}^b$ we can solve its equations of motion. Since $\mathcal{V}$ is an element of E_6 we have the following decomposition for $\mathcal{V}^{-1}\partial_\mu \mathcal{V}$ which is in the Lie algebra of E_6

$$\tilde{\mathcal{V}}_{cd}{}^{\alpha\beta}\, \partial_\mu\, \mathcal{V}_{\alpha\beta}{}^{ab} = 2\, Q_{\mu[c}{}^{[a}\, \delta_{d]}{}^{b]} + P_\mu{}^{ab}{}_{cd}$$

where $Q_{\mu\ c}^{\ a}$ belongs to the Lie algebra of USp(8) and $P_{\mu\ abcd}$ is in the orthogonal part to USp(8) (with respect to the Killing metric). For Ω_μ we get

$$\Omega_{\mu\ a}^{\ b} = Q_{\mu\ a}^{\ b}$$

The Lagrangian then becomes

$$\mathcal{L}_S \sim |P_{\mu\ abcd}|^2$$

$Q_{\mu a}^{\ \ b}$ and $P_{\mu abcd}$ are obviously invariant under E_6. If we restrict ourselves to the scalar fields, as in the case of general relativity, we can describe them by a metric $G_{\alpha\beta,\gamma\delta}$ instead of the 27-bein $\mathcal{V}_{\alpha\beta}^{\ \ ab}$ (to be compared to $g_{\mu\nu}$ and $e_\mu^{\ r}$) The metric is invariant under the local group USp(8) and covariant under E_6. It is given by

$$G_{\alpha\beta,\gamma\delta} = \mathcal{V}_{\alpha\beta}^{\ \ ab}\,\Omega_{ac}\,\Omega_{bd}\,\mathcal{V}_{\gamma\delta}^{\ \ cd}$$

and is characterized by the property :

$$G_{\alpha\beta,\gamma\delta} = G_{\gamma\delta,\alpha\beta}$$

The Lagrangian is then written as

$$\mathcal{L}_S \sim \partial_\mu G_{\alpha\beta,\gamma\delta}\,\partial^\mu (G^{-1})^{\alpha\beta,\gamma\delta}$$

This metric must also be used to describe the interaction of the vector fields since there is no quadratic invariant for E_7. The generalized "kinetic" term for the vectors is then given by

$$\mathcal{L}_{V^2} \sim G_{\alpha\beta,\gamma\delta}\,F_{\mu\nu}^{\ \ \alpha\beta}\,F_{\rho\sigma}^{\ \ \gamma\delta}\,g^{\mu\rho}\,g^{\nu\sigma}$$

As in 11 dimensions there also exists a trilinear gauge invariant coupling (up to a total derivative) of the vectors which is required by supersymmetry. Since there is a trilinear E_6 invariant J, we do not need the scalar metric (nor the metric tensor $g_{\mu\nu}$)

$$\mathcal{L}_{V^3} \sim \varepsilon^{\mu\nu\rho\sigma\lambda}\,\Omega_{\alpha\beta}\,F_{\mu\nu}^{\ \ \beta\gamma}\,\Omega_{\gamma\delta}\,F_{\rho\sigma}^{\ \ \delta\varepsilon}\,\Omega_{\varepsilon\eta}\,A_\lambda^{\eta\alpha}$$

The couplings to the fermions can no longer be described by the metric, but require the 27-bein $\mathcal{V}$. The "kinetic" terms for the fermionic fields $\Psi_\mu^{\ a}$ and χ^{abc} will be covariant with respect to the local Lorentz group SO(4, 1) with the connexion $\omega_{\mu\ rs}$ and the local group USp(8) with the connexion $Q_{\mu\ a}^{\ b}$

$$D_\mu \Psi_\rho^{\ a} = (\partial_\mu\,\delta^a_b - Q_{\mu\ b}^{\ a} + \frac{1}{4}\,\omega_{\mu rs}\,\gamma^{rs}\,\delta^a_b)\,\Psi_\rho^{\ b}$$

$$D_\mu \chi^{abc} = (\partial_\mu\,\delta^{[a}_{\ d} - 3Q_{\mu\ d}^{\ [a} + \frac{1}{4}\,\omega_{\mu rs}\,\gamma^{rs}\,\delta^{[a}_{\ d})\,\chi^{bc]\,d}$$

As usual there exists a Noether-type coupling required by supersymmetry

$$P_{\rho}{}^{abcd}\, \bar{\Psi}_{\mu\, a}\, \gamma^{\rho}\, \gamma^{\mu}\, \chi_{bcd}$$

The coupling of fermions to $F_{\mu\nu}{}^{\alpha\beta}$ must occur only through the E_6 invariant (and scalar under the general coordinate transformation)

$$F_{rs}{}^{ab} = \mathcal{V}_{\alpha\beta}{}^{ab}\, F_{\mu\nu}{}^{\alpha\beta}\, e^{\mu}{}_{r}\, e^{\nu}{}_{s}$$

Let us note that $\mathscr{L}_{V^2}$ can be written as

$$\mathscr{L}_{V^2} \sim (F_{rs}{}^{ab})^2$$

but $F_{rs}{}^{ab}$ in no longer a curl.

The supersymmetry transformation laws $\delta\phi$ are conjectured to be covariant with respect to USp(8) and E_6. Therefore $\mathscr{L}$ and $\delta\phi$ are now defined up to numerical coefficients, quartic fermionic terms for $\mathscr{L}$ and trilinear fermionic terms for $\delta\, \Psi_{\mu\, a}$ and $\delta\, \chi_{abc}$. In particular all the non-polynomial structure in the scalar fields is fixed. Supersymmetry is used to get rid of the remaining arbitrariness.

(i) Numerical coefficients (and Lorentz structure) in $\delta\,\Psi$ and $\delta\,\chi$ are determined by checking the supersymmetry invariance of $\mathscr{L}$ in the terms of the type $\bar{\varepsilon}\, \Psi$, $\bar{\varepsilon}\, \chi$

(ii) Quartic terms in $\mathscr{L}$ and trilinear terms in $\delta\,\Psi$ and $\delta\,\chi$ are determined in two independent ways :

- we require supercovariant equations of motion for fermionic fields
- we require the closure of the supersymmetry algebra on the bosonic fields

$$[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}] = \delta_G + \delta_{\varepsilon'} + \delta_L + \delta_{USp(8)} + \delta_{U(1)}$$

where δ_G is the general coordinate transformation, $\delta_{\varepsilon'}$ a new supersymmetry transformation, δ_L a local Lorentz transformation, $\delta_{USp(8)}$ a local USp(8) transformation and $\delta_{U(1)}$ an Abelian gauge transformation on the vector fields. At this stage only the χ^4 terms in $\mathscr{L}$ are still undetermined. They are determined by checking $\delta\mathscr{L}$ in the terms of the type $\varepsilon\, \chi^3$ or by looking at the closure on fermionic fields which requires the fermionic equations of motion.

The Lagrangian is then written, (we have put K=1)

$$e^{-1}\mathscr{L} = -\frac{1}{4} R(\omega) - \frac{i}{2} \bar{\Psi}_{\mu}{}^{a}\, \gamma^{\mu\nu\rho}\, D_{\nu}\, \Psi_{\rho\, a} - \frac{1}{8} g^{\mu\rho} g^{\nu\sigma} G_{\alpha\beta,\gamma\delta} F_{\mu\nu}{}^{\alpha\beta} F_{\rho\sigma}{}^{\gamma\delta}$$

$$+ \frac{i}{12} \bar{\chi}^{abc} \gamma^{\mu} D_{\mu} \chi_{abc} + \frac{1}{24} g^{\mu\nu} P_{\mu\, abcd} P_{\nu}{}^{abcd}$$

$$- \frac{e^{-1}}{12} \varepsilon^{\mu\nu\rho\sigma\lambda} (F_{\mu\nu})^{\alpha}{}_{\beta} (F_{\rho\sigma})^{\beta}{}_{\gamma} (A_{\lambda})^{\gamma}{}_{\alpha} + \frac{i}{3\sqrt{2}} P_{\rho\, abcd} \bar{\Psi}_{\mu}{}^{a} \gamma^{\rho} \gamma^{\mu} \chi^{bcd}$$

$$+ \frac{i}{4} \mathcal{V}_{\alpha\beta}{}^{ab} F_{\mu\nu}{}^{\alpha\beta} \{\bar{\Psi}^{\rho}_{a} \gamma_{[\rho} \gamma^{\mu\nu} \gamma_{\sigma]} \Psi^{\sigma}_{b} + \frac{1}{\sqrt{2}} \bar{\Psi}^{c}_{\rho} \gamma^{\mu\nu} \gamma^{\rho} \chi_{abc} + \frac{1}{2} \bar{\chi}_{acd} \gamma^{\mu\nu} \chi_{b}{}^{cd}\}$$

$$+ e^{-1} \mathcal{L}_{4}$$

$\mathcal{L}_4$ represents the quartic fermionic terms. Except for the Ψ^4 terms it is not enough to replace $\omega_{\mu\, rs}$, $P_{\rho\, abcd}$ and F_{ab} by $(\omega + \hat{\omega})/2$, $(P + \hat{P})/2$, $(F + \hat{F})/2$ (^ means supercovariant extension) to reabsorb all the quartic terms. For completeness we give $\mathcal{L}_4$ below :

$$e^{-1} \mathcal{L}_{4} = - \frac{1}{16} [\bar{\Psi}^{\rho a} \gamma^{rs} \Psi_{\rho a} \bar{\Psi}^{\sigma b} \gamma_{rs} \Psi_{\sigma b} - \bar{\Psi}^{\rho a} \gamma^{rs} \Psi^{\sigma}{}_{a} \bar{\Psi}_{\rho}{}^{b} \gamma_{rs} \Psi_{\sigma b}]$$

$$+ \frac{e^{-1}}{8} \varepsilon^{\mu\nu\rho\sigma\lambda} \bar{\Psi}_{\rho}{}^{a} \Psi_{\sigma a} \bar{\Psi}_{\lambda}{}^{b} \gamma_{\nu} \Psi_{\mu b} - \frac{1}{4} \bar{\Psi}^{\rho a} \Psi^{\sigma}{}_{a} \bar{\Psi}^{b}{}_{\rho} \Psi_{\sigma b}$$

$$+ \frac{1}{8} [\bar{\Psi}^{\rho a} \gamma^{\lambda} \Psi_{\sigma a} \bar{\Psi}^{b}{}_{\lambda} \gamma^{\sigma} \Psi_{\rho b} - 2 \bar{\Psi}^{\rho a} \gamma^{\sigma} \Psi_{\sigma a} \bar{\Psi}^{\lambda b} \gamma_{\lambda} \Psi_{\rho b}]$$

$$+ \frac{1}{2\sqrt{2}} \bar{\chi}^{abc} \gamma^{\mu} \gamma^{\rho\sigma} \Psi_{\mu c} \bar{\Psi}_{\rho a} \Psi_{\sigma b}$$

$$+ \frac{1}{96} [\bar{\chi}^{abc} \gamma^{\rho\sigma} \chi_{abc} \bar{\Psi}^{\mu d} \gamma_{\rho\sigma} \Psi_{\mu d} - \bar{\chi}^{abc} \gamma^{\rho\sigma} \chi_{abc} \bar{\Psi}_{\rho}{}^{d} \Psi_{\sigma d}$$

$$+ \bar{\chi}^{abc} \gamma^{\mu\rho\sigma} \chi_{abc} \bar{\Psi}_{\rho}{}^{d} \gamma_{\mu} \Psi_{\sigma d}]$$

$$+ \frac{1}{32} [\bar{\chi}^{abc} (\gamma^{\rho\sigma} - 3 g^{\rho\sigma}) \chi^{d}{}_{ab} \bar{\Psi}_{\rho c} \Psi_{\sigma d}$$

$$+ \bar{\chi}^{abc} (-\gamma^{\rho\sigma\lambda} + \gamma^{\sigma} g^{\rho\lambda} + \gamma^{\rho} g^{\sigma\lambda} + 3\gamma^{\lambda} g^{\rho\sigma}) \chi^{d}{}_{ab} \bar{\Psi}_{\rho c} \gamma_{\lambda} \Psi_{\sigma d}$$

$$+ \frac{1}{2} \bar{\chi}^{abc} (\gamma^{\rho\sigma\mu\nu} - 3 g^{\rho\sigma} \gamma^{\mu\nu} - 2 g^{\rho\mu} \gamma^{\sigma\nu} - 2 g^{\sigma\mu} \gamma^{\rho\nu} - 2 g^{\sigma\mu} g^{\rho\nu}) \chi^{d}{}_{ab} \bar{\Psi}_{\rho c} \gamma_{\mu\nu} \Psi_{\sigma d}]$$

$$- \frac{1}{8\sqrt{2}} \bar{\chi}^{abc} \gamma_{\rho\sigma} \chi^{d}{}_{bc} \bar{\chi}_{ade} (g^{\mu\rho} \gamma^{\sigma} + \frac{1}{6} \gamma^{\mu\rho\sigma}) \Psi_{\mu}{}^{e}$$

$$+ \frac{1}{80} [\bar{\chi}_{bcd} \gamma^{r} \chi^{bc}{}_{e} \bar{\chi}^{d}{}_{fg} \gamma_{r} \chi^{efg}$$

$$- \bar{\chi}_{bcd} \gamma^{rs} \chi^{bc}{}_{e} \bar{\chi}^{d}{}_{fg} \gamma_{rs} \chi^{efg}$$

$$+ \frac{7}{48} \bar{\chi}_{abc} \gamma^{rs} \chi^{abc} \bar{\chi}_{efg} \gamma_{rs} \chi^{efg}]$$

The supersymmetry transformation laws are given by

$$\delta e_{\mu}{}^{r} = -i\,\bar{\varepsilon}^{a}\,\gamma^{r}\,\Psi_{\mu a}$$

$$\tilde{\mathcal{V}}^{\alpha\beta}{}_{cd}\,\delta\mathcal{V}_{\alpha\beta,ab} = -2i\sqrt{2}\left(\bar{\varepsilon}_{[a}\,\chi_{bcd]} + \frac{3}{4}\,\Omega_{[ab}\,\bar{\varepsilon}_{e}\,\chi^{e}{}_{cd]}\right)$$

$$\delta A_{\mu}{}^{\alpha\beta} = 2i\,\tilde{\mathcal{V}}^{\alpha\beta}{}_{ab}\left(\bar{\varepsilon}^{a}\,\Psi_{\mu}{}^{b} + \frac{1}{2\sqrt{2}}\,\bar{\varepsilon}_{c}\,\gamma_{\mu}\,\chi^{abc}\right)$$

$$\begin{aligned}\delta\Psi_{\mu a} &= (D_{\mu}(\hat{\omega})\delta_{a}{}^{b} + Q_{\mu a}{}^{b})\varepsilon_{b} - \tfrac{1}{6}\hat{F}_{rsab}(\gamma^{rs}\gamma_{\mu} + 2\gamma^{r}e_{\mu}{}^{s})\varepsilon^{b}\\ &\quad + \frac{i\sqrt{2}}{4}\left(3\,\bar{\varepsilon}^{b}\,\Psi_{\mu}{}^{c}\,\chi_{abc} - \bar{\varepsilon}^{b}\,\gamma^{r}\,\Psi_{\mu}{}^{c}\,\gamma_{r}\,\chi_{abc}\right)\\ &\quad - \frac{i}{12}(\gamma_{\rho\mu} + 2g_{\rho\mu})\varepsilon_{d}\,\bar{\chi}_{abc}\gamma^{\rho}\chi^{bcd} - \frac{i}{12}(\gamma_{\rho\sigma\mu} + \gamma_{\rho}g_{\sigma\mu})\varepsilon_{d}\,\bar{\chi}_{abc}\gamma^{\rho\sigma}\,\chi^{bcd}\end{aligned}$$

$$\begin{aligned}\delta\chi_{abc} &= \sqrt{2}\,\hat{P}_{\mu\ abcd}\varepsilon^{d} - \frac{3}{2\sqrt{2}}\,\gamma^{rs}\left(\hat{F}_{rs[ab}\,\varepsilon_{c]} + \tfrac{1}{3}\Omega_{[ab}\hat{F}_{rsc]d}\,\varepsilon^{d}\right)\\ &\quad + \frac{3i\sqrt{2}}{8}\Big[3\,\varepsilon_{g}\,\bar{\chi}^{gf}{}_{[a}\,\chi_{bc]f} - \varepsilon_{g}\,\bar{\chi}^{gfd}\,\chi_{fd[a}\,\Omega_{bc]}\\ &\quad - \gamma_{r}\,\varepsilon_{g}\,\bar{\chi}^{gf}{}_{[a}\,\gamma^{r}\,\chi_{bc]f} + \frac{\gamma_{r}}{3}\,\varepsilon_{g}\,\bar{\chi}^{gfd}\,\gamma^{r}\,\chi_{fd[a}\,\Omega_{bc]}\\ &\quad + \frac{1}{2}\,\gamma_{rs}\,\varepsilon_{g}\,\bar{\chi}^{gf}{}_{[a}\,\gamma^{rs}\,\chi_{bc]f} - \frac{\gamma_{rs}}{6}\,\varepsilon_{g}\,\bar{\chi}^{gfd}\,\gamma^{rs}\,\chi_{fd[a}\,\Omega_{bc]}\Big]\end{aligned}$$

with :

$$\hat{\omega}_{\mu rs} = \omega^{\circ}_{\mu rs}(e) + \tfrac{i}{2}[\bar{\Psi}_{\mu}{}^{a}\gamma_{s}\Psi_{ra} - \bar{\Psi}^{a}{}_{r}\gamma_{\mu}\Psi_{sa} + \bar{\Psi}^{a}{}_{s}\gamma_{r}\Psi_{\mu a}] - \frac{i}{24}\,\bar{\chi}^{abc}\,\gamma_{\mu rs}\,\chi_{abc}$$

$$\hat{P}_{\rho\ abcd} = P_{\rho\ abcd} + 2i\sqrt{2}\left(\bar{\Psi}_{\rho[a}\,\chi_{bcd]} + \frac{3}{4}\,\Omega_{[ab}\,\bar{\Psi}_{\rho e}\,\chi^{e}{}_{cd]}\right)$$

$$\hat{F}_{\rho\sigma ab} = F_{\rho\sigma ab} - 2i\left[\bar{\Psi}_{\rho[a}\,\Psi_{\sigma b]} + \frac{1}{8}\,\Omega_{ab}\,\bar{\Psi}_{\rho}{}^{c}\,\Psi_{\sigma c} - \frac{1}{\sqrt{2}}\,\bar{\Psi}_{[\rho}{}^{c}\,\gamma_{\sigma]}\,\chi_{abc}\right]$$

Let us note that since $\mathcal{V}^{-1}\,\delta\mathcal{V}$ has no component in USp(8), $Q_{\mu\ a}{}^{b}$ is supercovariant by itself but $P_{\mu\ abcd}$ is not.

The fermionic equations of motion are :

$$\begin{aligned}&-i\,\gamma^{\mu\nu\rho}\,\hat{\mathcal{D}}_{\mu}\,\Psi_{\nu a} + \frac{i}{3\sqrt{2}}\,\hat{P}_{\mu\ abcd}\,\gamma^{\mu}\gamma^{\rho}\,\chi^{bcd} + \frac{i}{4\sqrt{2}}\,\gamma^{\mu\nu}\gamma^{\rho}\,\chi_{abc}\,\hat{F}_{\mu\nu}{}^{bc}\\ &- \frac{1}{8\sqrt{2}}\left(\frac{1}{6}\,\gamma^{\rho\mu\nu} - g^{\rho\mu}\,\gamma^{\nu}\right)\chi_{abc}\,\bar{\chi}^{bde}\,\gamma_{\mu\nu}\,\chi^{c}{}_{de} = 0\end{aligned}$$

$$
\begin{aligned}
&\tfrac{i}{6}\,\gamma^{\mu}\hat{\mathscr{D}}_{\mu}\,\chi_{abc} + \tfrac{i}{4}\,\gamma^{\mu\nu}(\chi^{d}{}_{[ab}\,\hat{F}_{\mu\nu\;c]d} - \text{Trace } abc) \\
&\quad - \tfrac{1}{20}[\gamma^{\mu}\,\chi_{e[ab}\,\bar{\chi}_{c]fg}\,\gamma_{\mu}\,\chi^{efg} - \gamma^{\rho\sigma}\,\chi_{e[ab}\,\bar{\chi}_{c]fg}\,\gamma_{\rho\sigma}\,\chi^{efg} \\
&\quad + \tfrac{1}{24}\,\gamma^{\rho\sigma}\,\chi_{abc}\,\bar{\chi}_{efg}\,\gamma_{\rho\sigma}\,\chi^{efg} - \text{Trace } abc] \;=\; 0
\end{aligned}
$$

$\hat{\mathscr{D}}_{[\mu}\Psi_{\nu]a}$ and $\hat{\mathscr{D}}_{\mu}\,\chi_{abc}$ are supercovariant extensions of $D_{[\mu}\,\Psi_{\nu]a}$ and $D_{\mu}\,\chi_{abc}$ defined such that their variation by supersymmetry has no derivative of ε.

The algebra of supersymmetry is given by

$$
[\delta_{\varepsilon_2},\delta_{\varepsilon_1}] = \delta_G(\xi_{\mu}) + \delta_S(\varepsilon') + \delta_L(\Sigma^{rs}) + \delta_{USp(8)}(\Lambda_a{}^{b}) + \delta_{U(1)}(U^{\alpha\beta})
$$

with

$$
\begin{aligned}
\xi_{\mu} &= -i\,\bar{\varepsilon}_1{}^{c}\,\gamma_{\mu}\,\varepsilon_{2c} \\
\bar{\varepsilon}'^{a} &= -\xi^{t}\,\bar{\Psi}^{a}{}_{t} + \tfrac{i\sqrt{2}}{4}(3\,\bar{\varepsilon}_{1b}\,\varepsilon_{2c}\,\bar{\chi}^{abc} - \bar{\varepsilon}_{1b}\,\gamma^{\rho}\,\varepsilon_{2c}\,\bar{\chi}^{abc}\,\gamma_{\rho}) \\
\Sigma_{rs} &= \xi^{t}\,\hat{\omega}_{t,rs} + \tfrac{i}{3}\,\hat{F}_{uv,ab}\,\bar{\varepsilon}_1^{a}(\gamma_{rs}{}^{uv} + 4\delta^{u}_{r}\,\delta^{v}_{s})\,\varepsilon_2^{b} \\
&\quad - \tfrac{1}{6}\,\bar{\varepsilon}_{1a}\,\gamma_{rst}\,\varepsilon_{2b}\,\bar{\chi}^{a}{}_{cd}\,\gamma^{t}\,\chi^{bcd} \\
&\quad + \tfrac{1}{6}\,\bar{\varepsilon}_{1a}(\gamma_{rstu} + \eta_{rt}\,\eta_{su})\,\varepsilon_{2b}\,\bar{\chi}^{a}{}_{cd}\,\gamma^{tu}\,\chi^{bcd} \\
\Lambda_a{}^{b} &= \xi^{t}\,Q_{ta}{}^{b} - \tfrac{8}{3}[(\bar{\varepsilon}_2{}^{[b}\,\chi^{cde]} - \tfrac{3}{4}\,\Omega^{[bc}\,\bar{\varepsilon}_{2f}\,\chi^{de]f}) \\
&\qquad\qquad \times(\bar{\varepsilon}_{1[a}\,\chi_{cde]} + \tfrac{3}{4}\,\Omega_{[ac}\,\bar{\varepsilon}_{1f}\,\chi_{de]}{}^{f}) - \varepsilon_1 \leftrightarrow \varepsilon_2] \\
U^{\alpha\beta} &= -\xi^{\rho}\,A_{\rho}{}^{\alpha\beta} + 2i\,\tilde{\mathcal{V}}^{\alpha\beta}{}_{ab}\,\bar{\varepsilon}_1^{a}\,\varepsilon_2^{b}
\end{aligned}
$$

We have seen that the conjectured E_6 global ⊗ USp(8) local was the clue to construct the N=8 supergravity in 5 dimensions. There still remain some complications in the quartic fermionic terms. This could be a sign that there is still some structure to be discovered. We can hope that it would be easier to discover it in 5 than in 4 dimensions. Another problem could also be more easily solved in 5 dimensions : the construction of the multiplet containing the connexion of USp(8), $Q_{\mu\, a}^{\ \ b}$. This is of crucial importance in 4 dimensions where we conjectured that the local SU(8) could become dynamical at the quantum level (Cremmer & Julia, 1979, 1980). Some conjectures have been made on this multiplet which could lead to a grand unified model based on SU(5) with 3 families (Ellis, Gaillard, Maiani & Zumino, 1980 ; Zumino, 1980).

6 SUPERGRAVITIES N=6, 4, 2

In order to derive supergravities N=6, 4, 2 from N=8 by consistent truncations, it is useful to choose a particular representation for Ω_{ab} namely

$$\Omega_{ab} = \begin{pmatrix} \begin{matrix}0&1\\-1&0\end{matrix} & & & \\ & \begin{matrix}0&1\\-1&0\end{matrix} & & \bigcirc \\ & & \begin{matrix}0&1\\-1&0\end{matrix} & \\ \bigcirc & & & \begin{matrix}0&1\\-1&0\end{matrix} \end{pmatrix}$$

We shall describe the consistent truncations for N=6, 4, 2 and give the complete results only for N=2.

6.1 N=6 Supergravity

The invariance of the theory is SU*(6)global x USp(6). We note by a =1...6 the indices of USp(6) and by α =1...6 the indices of SU*(6). We keep the fields $\Psi_{\mu a}$, χ_{abc} $\chi_{a\,78}$, $A_\mu^{\ \alpha\beta}$, $A_\mu^{\ 78}$, $\mathcal{V}_{\alpha\beta}^{\ \ ab}$, $\mathcal{V}^{ab}_{\ \ 78}$, $\mathcal{V}_{\alpha\beta}^{\ \ 78}$ and $\mathcal{V}_{78}^{\ \ 78}$. (For $\mathcal{V}$ we have also to use the local USp(8) invariance before making the truncation). We must take the traceless condition for N=8 into account

$$2\,\Omega_{78}\,A_\mu^{\ 78} + \Omega_{\alpha\beta}\,A_\mu^{\ \alpha\beta} = 0$$

this implies that for $\mathcal{V}^{-1}\,\partial_\mu\,\mathcal{V}$ (N=8) only the following components Q and P remain

$$Q_{\mu\, a}^{\ \ b}\ ,\ P_{\mu\, abcd}\ ,\ P_{\mu\, ab78}\ (P_{\mu\, 78ab})$$

with the trace conditions

$$\Omega^{ab}\,\Omega^{cd}\,P_{\mu\,abcd} = 0\,,\quad \Omega^{ab}\,P_{\mu\,ab78} = 0$$

$$\Omega^{ab}\,P_{\mu\,abcd} + 2\,\Omega^{78}\,P_{\mu\,78cd} = 0$$

and $P_{\mu\,abcd}$ totally antisymmetric : this is equivalent to an antisymmetric $P_{\mu\,ab}$ traceless which can be identified as $P_{\mu\,ab\,78}$ corresponding to the 14 scalar fields ($P_{\mu\,abcd} \sim \varepsilon_{abcdef}\,P_\mu{}^{ef}$). The theory is invariant under the N=6 supersymmetry obtained by restricting ε to $\varepsilon^a{}_r$ $(\varepsilon^7=\varepsilon^8=0)$. The content of the theory is as expected 1 graviton $e_\mu{}^a$, 6 gravitinos $\Psi_\mu{}^a$, 14+1 vector fields $A_\mu{}^{\alpha\beta}$, 14+6 spinor fields and 14 scalar fields.

6.2 N=4 Supergravity

The symmetries are (USp(4) x R)global ⊗ USp(4)local. We keep $\Psi_\mu{}^a$ (a=1...4); $A_\mu{}^{\alpha\beta}$ (α =1...4) $A_\mu{}^{56}$ with the condition

$$\Omega_{\alpha\beta}\,A_\mu{}^{\alpha\beta} + 4\,\Omega_{56}\,A_\mu{}^{56} = 0$$

as well as $A_\mu{}^{78}$ defined for N=6 in terms of $A_\mu{}^{\alpha\beta}$ and $A_\mu{}^{56}$. In the same way we keep χ_{abc} and χ_{56c} ($\chi_{a\,78}$ being a function of the previous ones) with the condition

$$\Omega^{ab}\,\chi_{abc} + 4\,\Omega^{56}\,\chi_{56\,c} = 0$$

This corresponds to 4 spin 1/2 since $\chi_{abc} \sim \varepsilon_{abcd}\,\chi^d$ from which we deduce $\chi_{56c} \sim \chi_c$. On $\mathcal{V}^{\alpha\beta}{}_{ab}$ we make the same truncation as on $A_\mu{}^{\alpha\beta}$. This implies, for $P_{\mu\,abcd}$ and $P_{\mu\,56\,cd}$, the relation

$$\Omega^{ab}\,P_{\mu\,abcd} + 4\,\Omega^{56}\,P_{\mu\,56\,cd} = 0$$

This corresponds to 1 scalar field : $P_{\mu\,abcd} \sim \varepsilon_{abcd}\,\phi$

$P_{\mu\,ab\,56} \sim \Omega_{ab}\,\phi$

The remaining $Q_{\mu a}{}^b$ has the form $\mathcal{V}^{-1}\,\partial_\mu \mathcal{V}$ where $\mathcal{V}$ is an element of USp(4) and therefore being a pure gauge it can be reabsorbed by redefining the fermionic field with the USp(4) transformation . The theory is invariant under N=4 supersymmetry with parameter ε^a . The content is 1 graviton, 4 gravitinos, 5+1 vector fields, 4 spin 1/2 fields and 1 scalar field.

6.3 N=2 Supergravity

From N=4 we keep $\Psi_{\mu\,a}$ (a =1, 2) $A_{\mu\,ab}$ with the relation

$$\Omega^{ab}\,A_{\mu\,ab} + 6\,\Omega^{34}\,A_{\mu\,34} = 0$$

There is no X_{abc} left and we can replace $\mathcal{V}^{ab}{}_{\alpha\beta}$ by "1". The truncation can also be directly made from N=8, keeping $\Psi_{\mu a}$, $A_{\mu\,12}$, $A_{\mu\,34}$, $A_{\mu\,56}$ and $A_{\mu\,78}$ with the relations

$$A_{\mu\,78} = A_{\mu\,56} = A_{\mu\,34} = -\frac{1}{3} A_{\mu\,12}$$

After renormalization of the vector fields we get the Lagrangian for N=2 supergravity with field $e_\mu{}^r$, $\Psi_\mu{}^a$ (a =1, 2) and A_μ .

$$e^{-1}\mathscr{L} = -\frac{1}{4}R(\omega) - \frac{i}{2}\bar{\Psi}_\mu{}^a\gamma^{\mu\nu\rho} D_\nu\left(\frac{\omega+\hat{\omega}}{2}\right) \Psi_{\rho a} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}\, g^{\mu\rho} g^{\nu\sigma}$$

$$+ \frac{e^{-1}}{6\sqrt{3}} \varepsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}F_{\rho\sigma}A_\lambda - \frac{i\sqrt{3}}{16}(F_{\mu\nu}+\hat{F}_{\mu\nu})\bar{\Psi}^{\rho c}\gamma_{[\rho}\, \gamma^{\mu\nu}\, \gamma_{\sigma]}\, \Psi^\sigma{}_c$$

with

$$\hat{F}_{\mu\nu} = F_{\mu\nu} + \frac{\sqrt{3}}{4}\bar{\Psi}_\mu{}^c\, \Psi_{\nu c}$$

and ω is given by the 1st order formalism if $\hat{\omega}$ is defined as

$$\hat{\omega}_{\mu rs} = \omega_{\mu rs} + \frac{i}{4}\bar{\Psi}^{\rho a}\, \gamma_{\mu\, rs\, \rho\sigma}\, \Psi^\sigma{}_a$$

After solving the equations of motion for ω we get, as usual

$$\hat{\omega}_{\mu rs} = \omega^0{}_{\mu rs}(e) + \frac{i}{2}(\bar{\Psi}_\mu{}^a\, \gamma_s\, \Psi_{ra} - \bar{\Psi}^a{}_r\, \gamma_\mu\, \Psi_{sa} + \bar{\Psi}_s{}^a\, \gamma_r\, \Psi_{\mu a})$$

is invariant under the following N=2 supersymmetry transformation

$$\delta\, e_\mu{}^r = -i\, \bar{\varepsilon}^a\, \gamma^r\, \Psi_{\mu a}$$

$$\delta\, \Psi_{\mu a} = [D_\mu(\hat{\omega}) + \frac{1}{4\sqrt{3}}\hat{F}_{\rho\sigma}(\gamma^{\rho\sigma}\, \gamma_\mu + 2\gamma^\rho\, \delta_\mu{}^\sigma)]\, \varepsilon_a$$

$$\delta\, A_\mu = -\frac{\sqrt{3}}{4}\bar{\varepsilon}^a\, \Psi_{\mu a}$$

All quartic terms are contained in the replacement of ω and F by $(\omega+\hat{\omega})/2$ and $(F+\hat{F})/2$ in the bilinear fermionic terms. We note that N=2 supergravity in 5 dimensions has exactly the same structure as the N=1 supergravity in 11 dimensions where everything comes from. This should be compared with the partial purely geometric results obtained by D'Auria & Fré, 1980 ; D'Auria, Fré & Regge, 1980 for this N=2 supergravity in 5 dimensions.

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