David Ben-Zvi
Workshop on TFT's at Northwestern
Notes, errors, and incompleteness by Alex Hoffnung

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DRAFT VERSION ONLY

1 Lecture 4

$$action : Z(S^1) \leftrightarrow EndA : char$$

$A \in Z(\cdot)$
$ch(A) \in Z(S^1)$
$Id_A \in EndA$

- character of reps of Γ
- Chern character $\in (HH_*(Z\cdot)))^{S^1}$ where $HH_*(Z\cdot)) = dimZ(\cdot)$

Borel $B \subset G$ reductive /$\mathbb{C}$

$$\begin{array}{ccc} & G/B & \\ {\scriptstyle \pi}\swarrow & & \searrow{\scriptstyle \delta} \\ G/G & & B\backslash G/B \end{array}$$

where $G/B = (G \times G/B)/G$ and $B\backslash G/B = G\backslash G/B \times G/B$.

$$[g] \leftarrow (g, B) \rightarrow (B', g \cdot B')$$

Simultaneous Grothendieck-Springer resolution

$$\begin{array}{ccccccc} \tilde{G}/G = & & \{g, B' | g \in B'\} & & \subset & & \tilde{G}^{reg}/G \xrightarrow{w} H^{reg}/W \\ & \swarrow & & & & & \\ G/G & & \cup & & & & \end{array}$$

$$\begin{array}{ccc} T^*G/B)/G & & \\ & \swarrow & \\ N/G & & \end{array}$$

Definition 1. *(Lusztig) A* **character sheaf** *on G is a $\mathcal{D}$-module on G/G iin the image of $\pi_*\delta$ on $\mathcal{H} = \mathcal{D}(B\backslash G/B)$.*

The basic object is $\xi = \pi_*\delta^! = \pi_*\mathbb{C}_{\tilde{G}/G}$
•*xi* is Harish-Chandra's system of differential equations satisfied by charactes of G-representations.

Character sheaves are geometric avatars of representations (characters) of $G(\mathbb{F}_q)$.

Theorem 2. *(BZ-Nadler) Character sheaves are characters of $\mathcal{H}$-modules. More precisely, $\mathcal{H}$-modules defines a 2-dimensional (unoriented) TFT and $Z(S^1) = HH_*(\mathcal{H})$ (dim) $= HH^*(\mathcal{H})$ (center)*

$$act : Z(S^1) \leftrightarrow \mathcal{H} : tr$$

E.g. $\xi = char(\mathcal{D}(B\backslash G/B)$ with action of Hecke category.
Local operators:

$$\int \mathcal{O}_x(\varphi) e^{-S(\varphi)} D\varphi$$

E_n structures for $Z(S^{n-1})$ local operators

STUFF HERE
In 2-dimensional gauge theory, there are **disorder operators**
STUFF HERE
4d gauge theory I

The B-model B_G (4D analog of Z_G^Q)

$$B_G(N^3) = R\Gamma(\mathcal{M} + G(N^3), \mathcal{O})$$

$$B_G(\Sigma) = \mathcal{O}_{cohsheaves}(\mathcal{M}_G(\Sigma))$$

This is the main object of study. So we assign to the circle:

$$B_G(S^1) = \mathcal{O}(G/G) - modules$$

$$B_G(\cdot) = (\infty, 2) - \text{categories over } BG.$$

PICTURES

Wilson (loop) operators

TFT interpretation of Geometric Langland's due to (Kapustin-Witten)(Belinson-Drinfeld)

A_G a 4-dimensional gauge theory (closer to $Z_G^{\mathcal{D}}$)

Σ now an algebraic curve/Riemann surface

$Bundles_G\Sigma$ = holomorphic G-bundles on Σ.

$A_G(\Sigma) = \mathcal{D}(Bun_G\Sigma)$

Local operators"' "t'Hooft/Hecke operators. These are examples of disorder operators.

PHYSICS PICTURE

Insert singularity, G is like a monopole alone line in 4-dimensions

$Bun_G(S^2) =$ possible local singularities $\leftrightarrow \hom(T^\vee, \mathbb{C}^*)/W \leftrightarrow$ irreps of $G^\vee$

$$A_G(S^2) \Leftrightarrow \mathcal{H}_{sph}, \text{spherical Hecke category}$$

A LITTLE MORE HERE...

Theorem 3. *(Geometric Satake)* $A_G(S^2) = \mathcal{H}_{sph} \cong \mathcal{O}(\mathcal{M}_{G^\vee}(S^2)) = B_{G^\vee}(S^2)$

Conjecture:

$$\mathcal{D}(Bun_G\Sigma) \cong \mathcal{O}(Loc_{G^\vee}\Sigma)$$

with actions of $\mathcal{H}_{sph}$ and $RepG^\vee$.

Conjecture: (Electric-Magnetic duality)

$$A_G \cong B_{G^\vee}$$

as "TFT's".