Uncharted Territories in Geometric Engineering

Strings and Geometry 2023 - Philadelphia

Michele Del Zotto - Uppsala University

European Research Council
Established by the European Commission

Simons Collaboration on Global Categorical Symmetries

BASED ON:

[1] 2209.10551 w/ LIU, OEHLMANN

[2] 2212.05331 w/ " , "

[3] 2303.XXXXX w/ " , "

[4] 2303.XXXXX w/ ACHARYA, HECKMAN, HÜBNER, TORRES

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MOSTLY THESE TWO

BASED ON: → SEE MUYANG'S TALK

[1] 2209.10551 w/ LIU, OEHLMANN

[2] 2212.05331 w/ " , "

[3] 2303.XXXXX w/ " , "

[4] 2303.XXXXX w/ ACHARYA, HECKMAN, HÜBNER, TORRES

MOSTLY THESE TWO

COMMERCIALS

COMMERCIALS

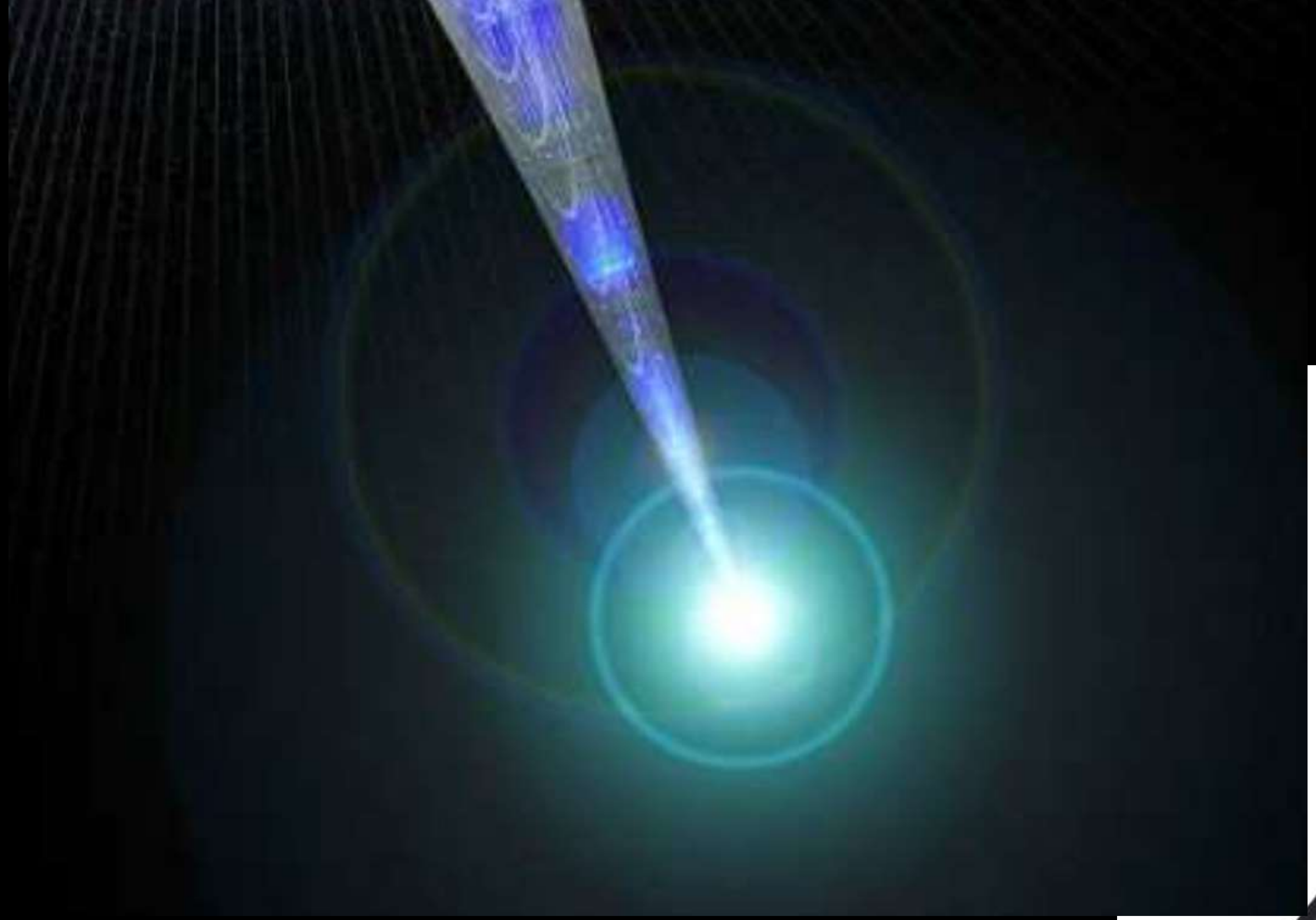
- NORDITA CATEGORICAL ASPECTS OF SYMMETRIES PROGRAM
  CONFERENCE AUG 22-25, 2023
- GCS CONFERENCE + GRADUATE SCHOOL
  LES DIABLERET AUG 27 - SEPT 8, 2023

DETAILS ON: SCGCS.BERKELEY.EDU

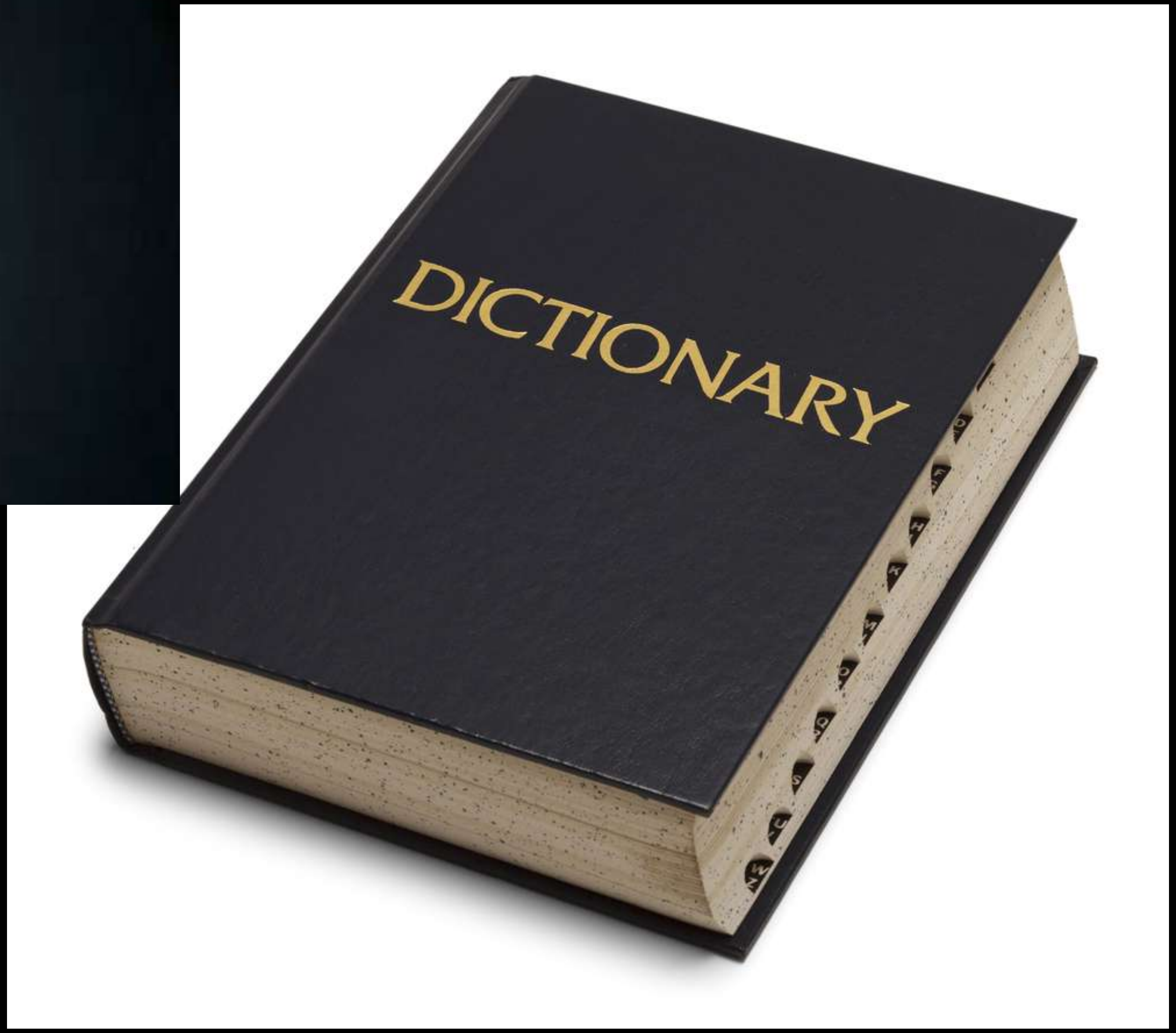
GEOMETRIC ENGINEERING

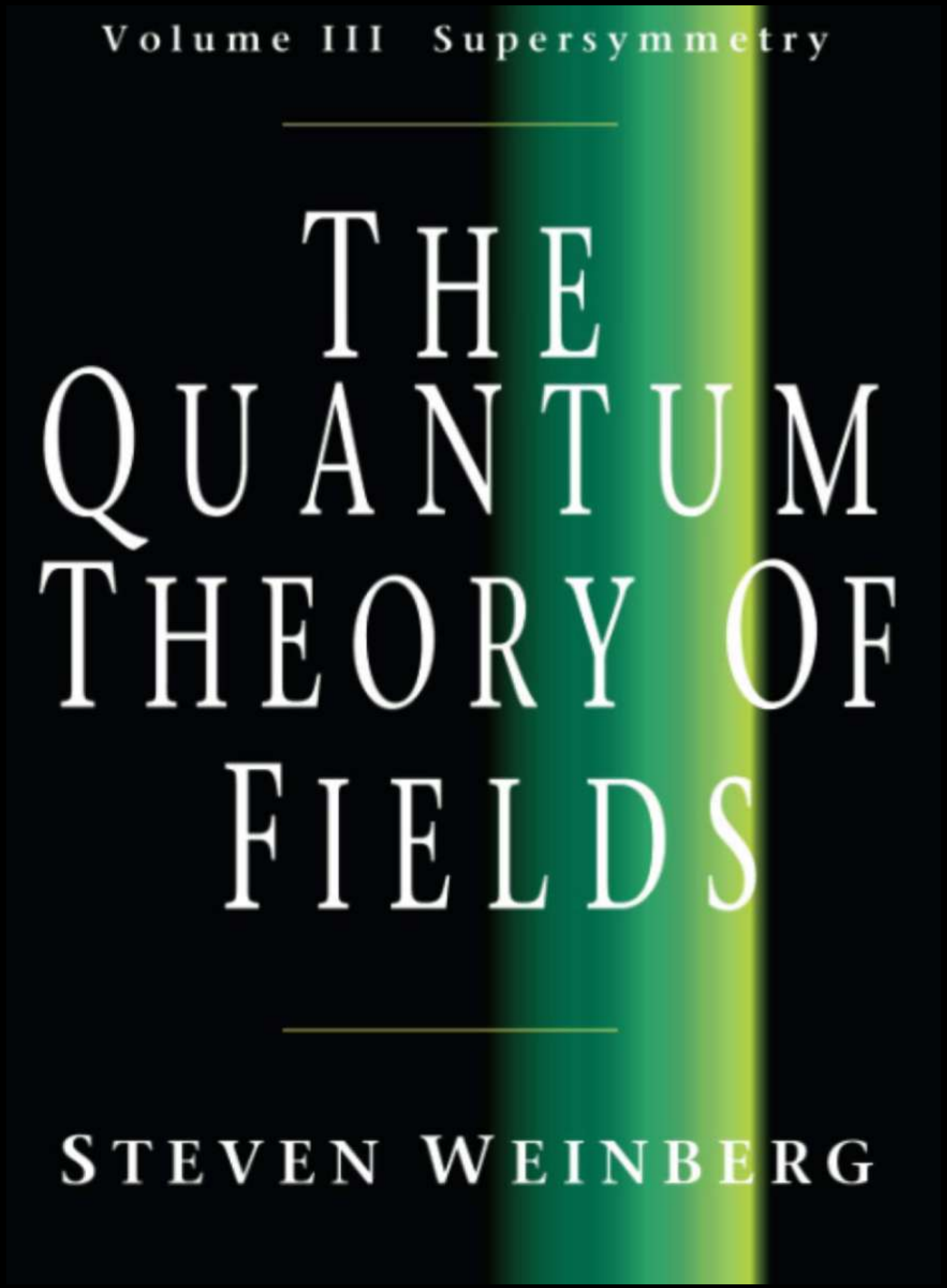
[KATZ, KLEMM, VAFA 96]

[LEUNG, VAFA 97]

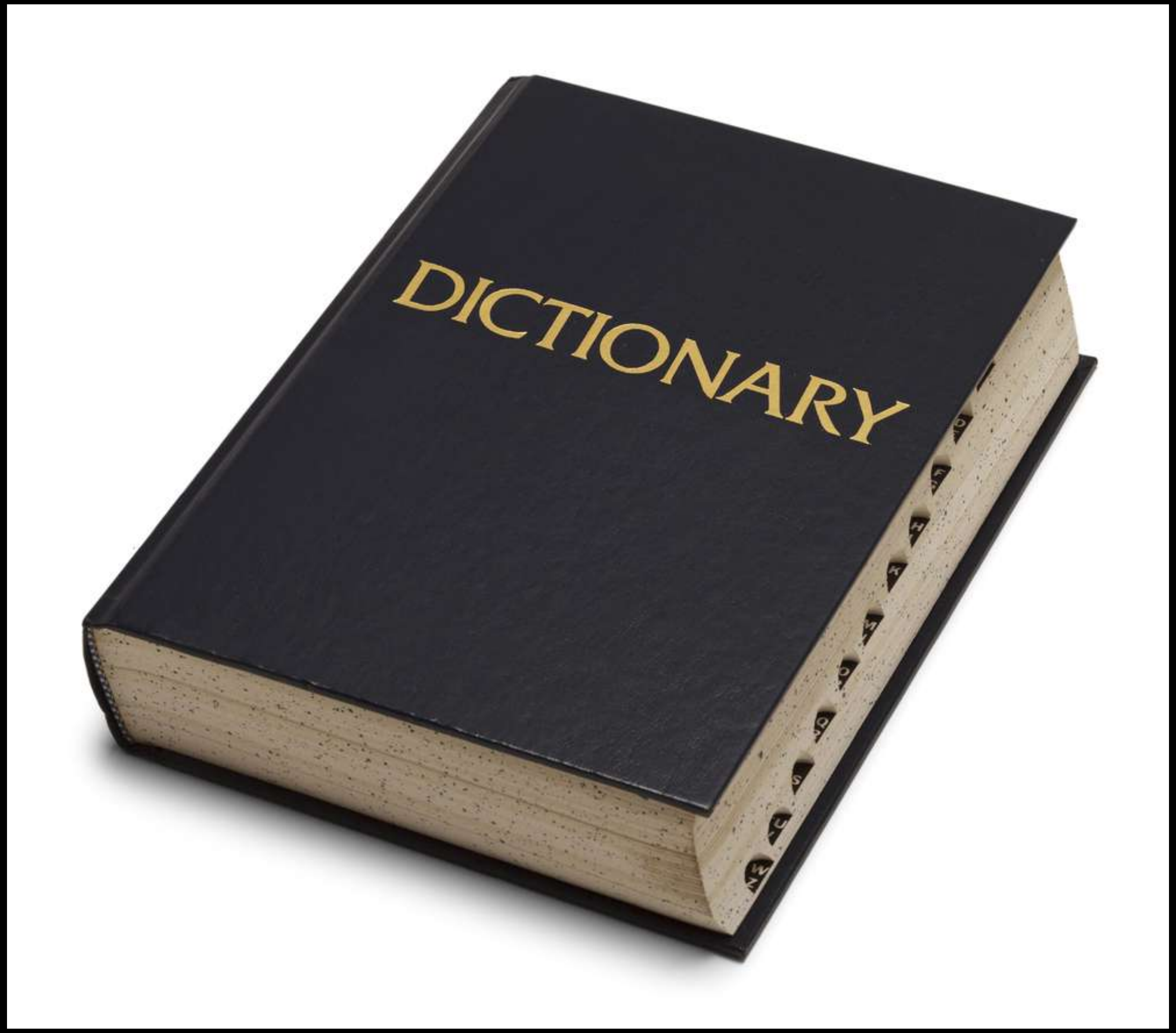


GEOMETRIC SINGULARITIES





IN TODAY'S TALK WE WILL EXPLORE TWO OPEN GEOMETRIC ENGINEERING QUESTIONS

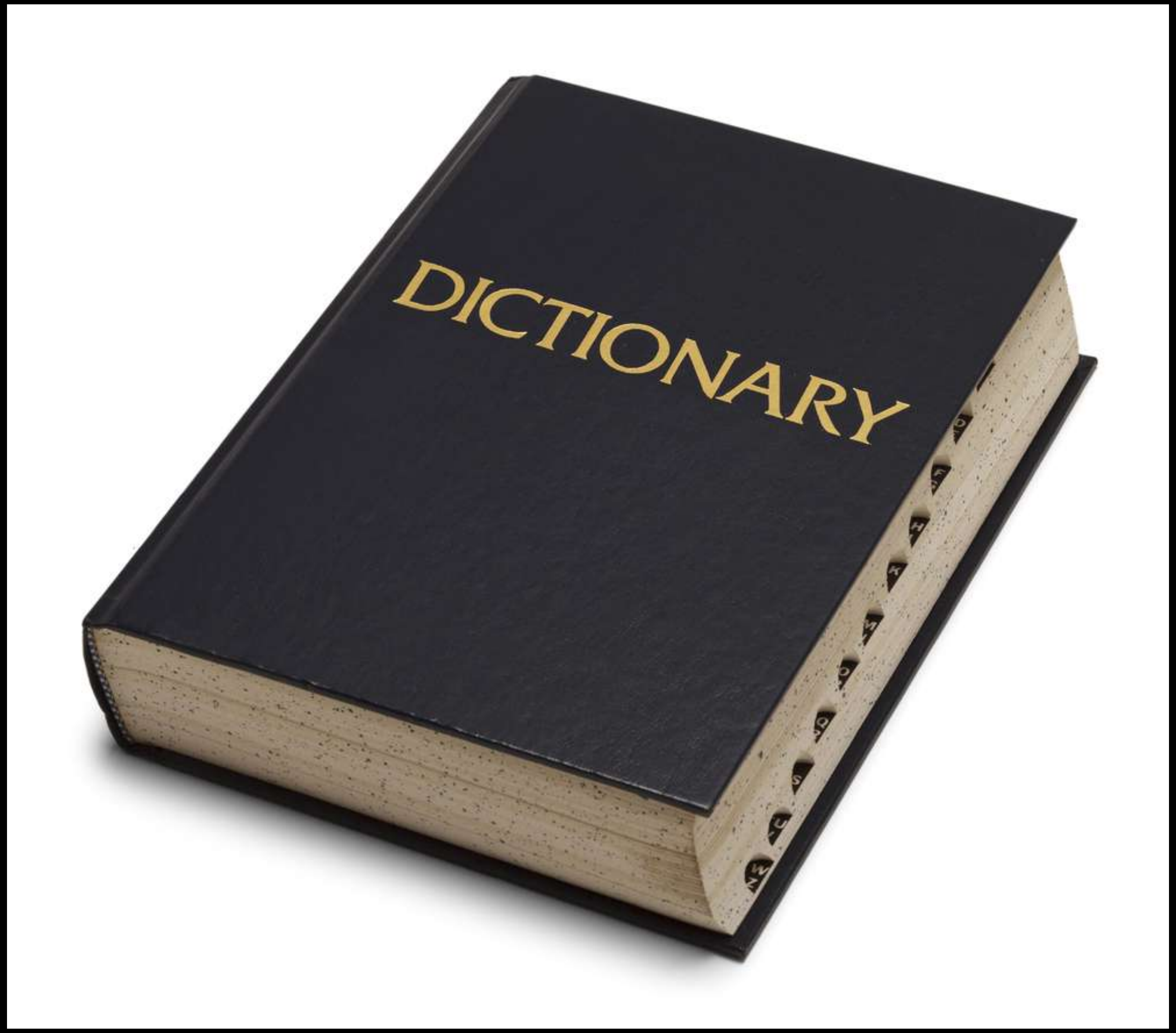


IN TODAY'S TALK WE WILL EXPLORE TWO OPEN GEOMETRIC ENGINEERING QUESTIONS

1

GEOMETRIC ENGINEERING LIMIT OF HETEROTIC STRINGS

[1-3]



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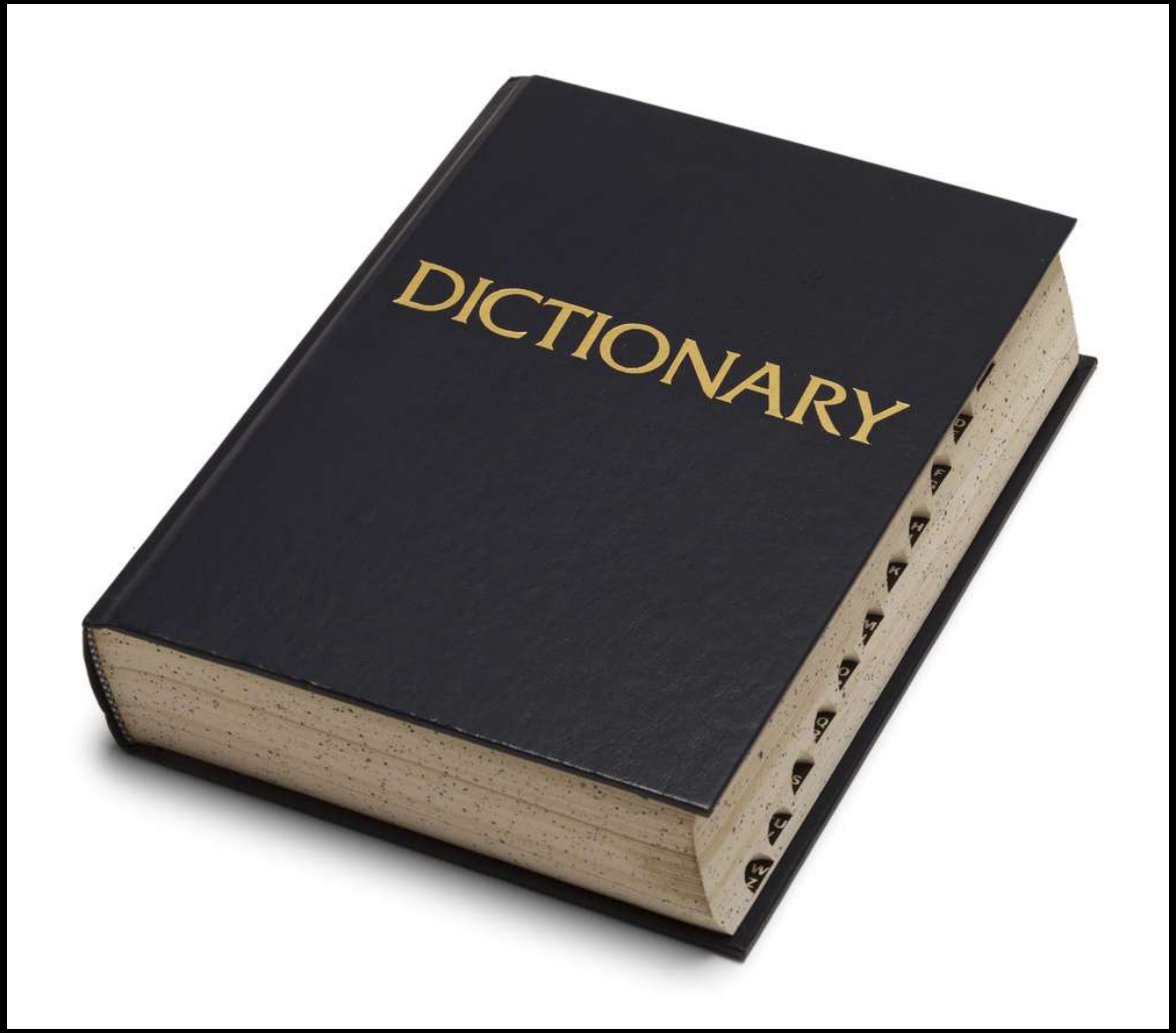


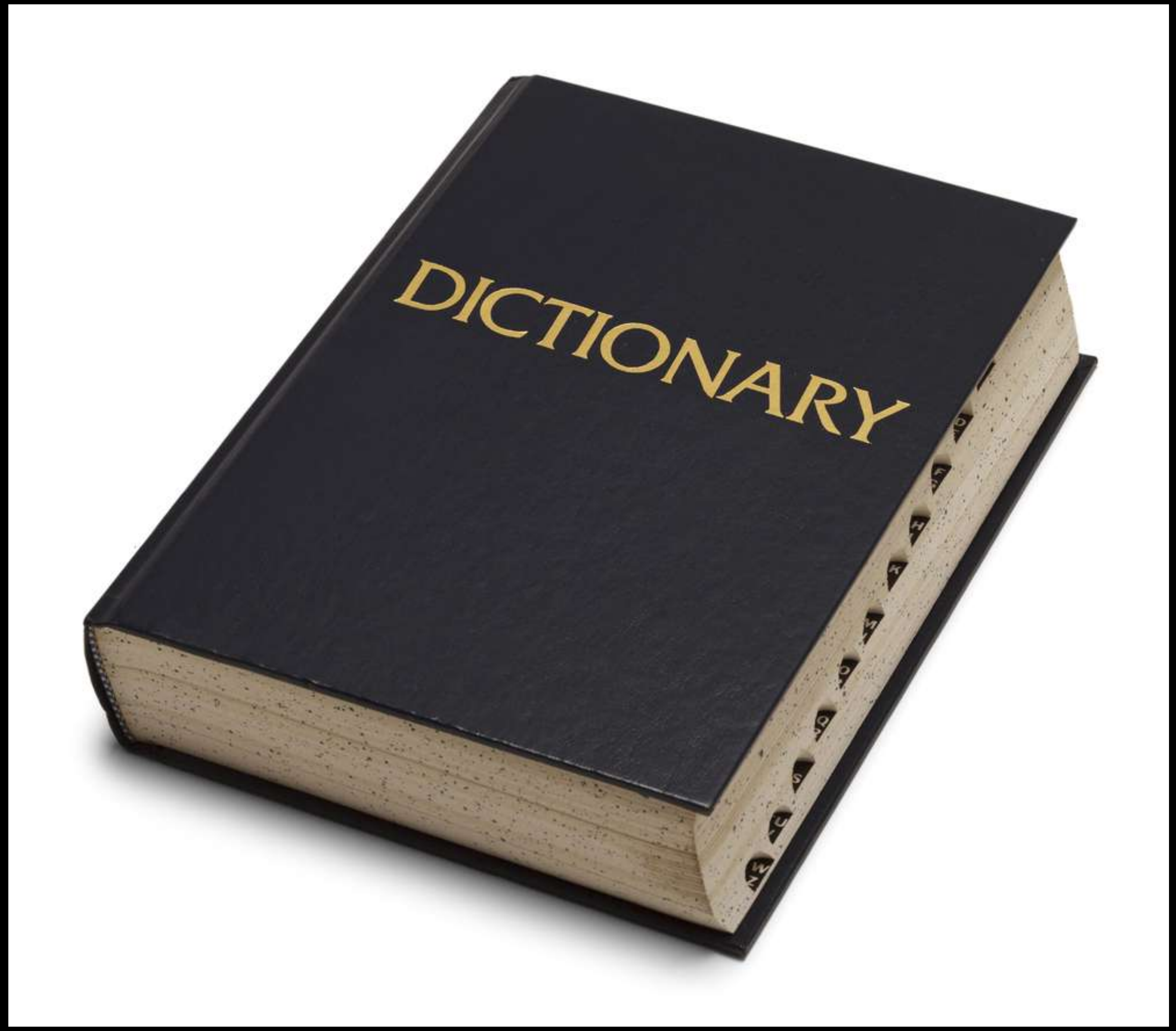
2

GEOMETRIC ENGINEERING OF M-THEORY ON SINGULAR G_2 HOLONOMY CONES

[4]

IN TODAY'S TALK WE WILL EXPLORE TWO OPEN GEOMETRIC ENGINEERING QUESTIONS:

1. GEOMETRIC ENGINEERING LIMIT OF HETEROTIC STRINGS [1-3]

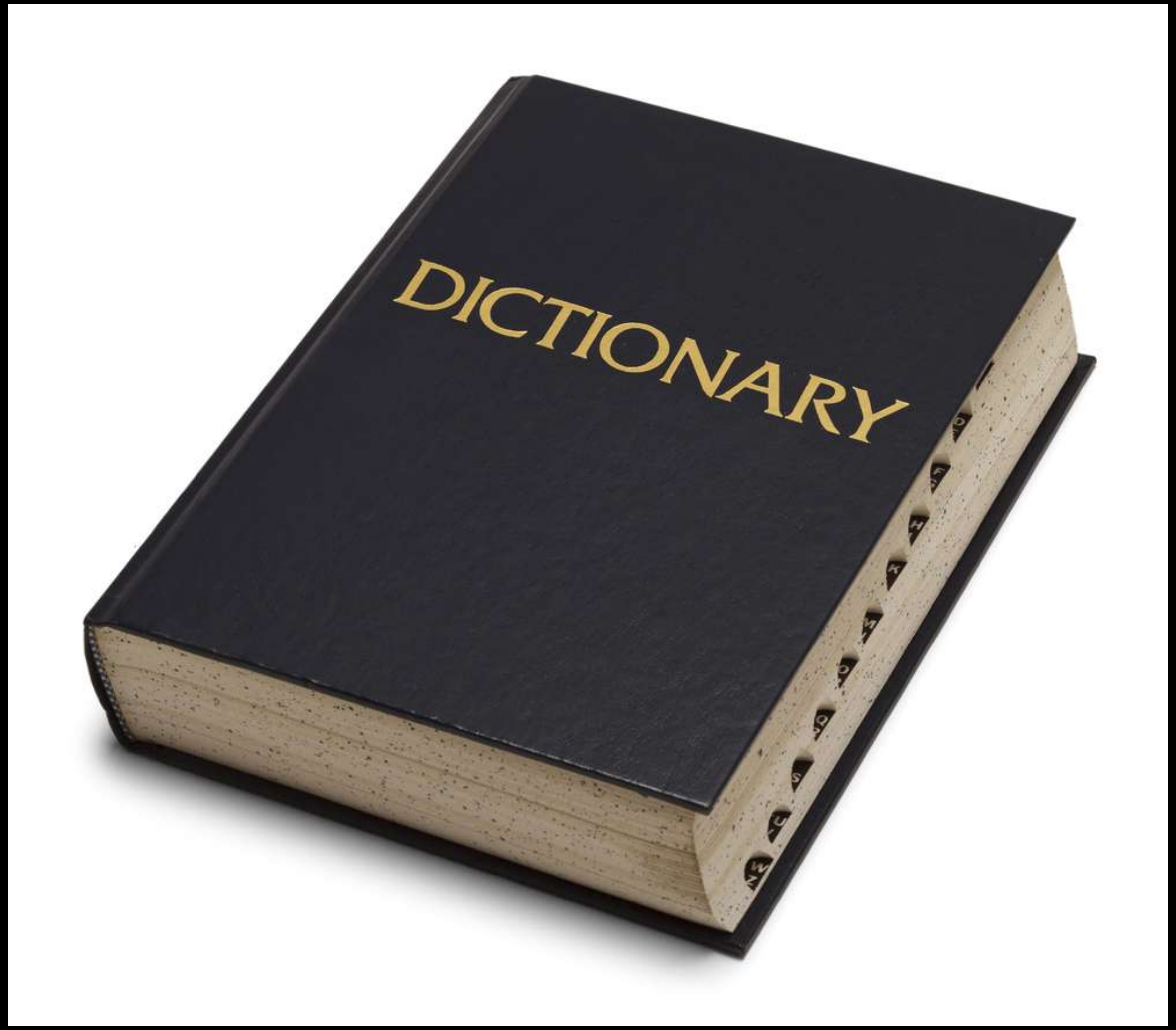


2. GEOMETRIC ENGINEERING OF M-THEORY ON SINGULAR G_2 HOLONOMY CONES [4]

IN BOTH CASES PROGRESS THANKS TO RECENT ADVANCES IN SCFTS AND SYMMETRIES

IN TODAY'S TALK WE WILL EXPLORE TWO OPEN GEOMETRIC ENGINEERING QUESTIONS:

① GEOMETRIC ENGINEERING LIMIT OF HETEROTIC STRINGS [1-3]



② GEOMETRIC ENGINEERING OF M-THEORY ON SINGULAR G_2 HOLONOMY CONES [4]

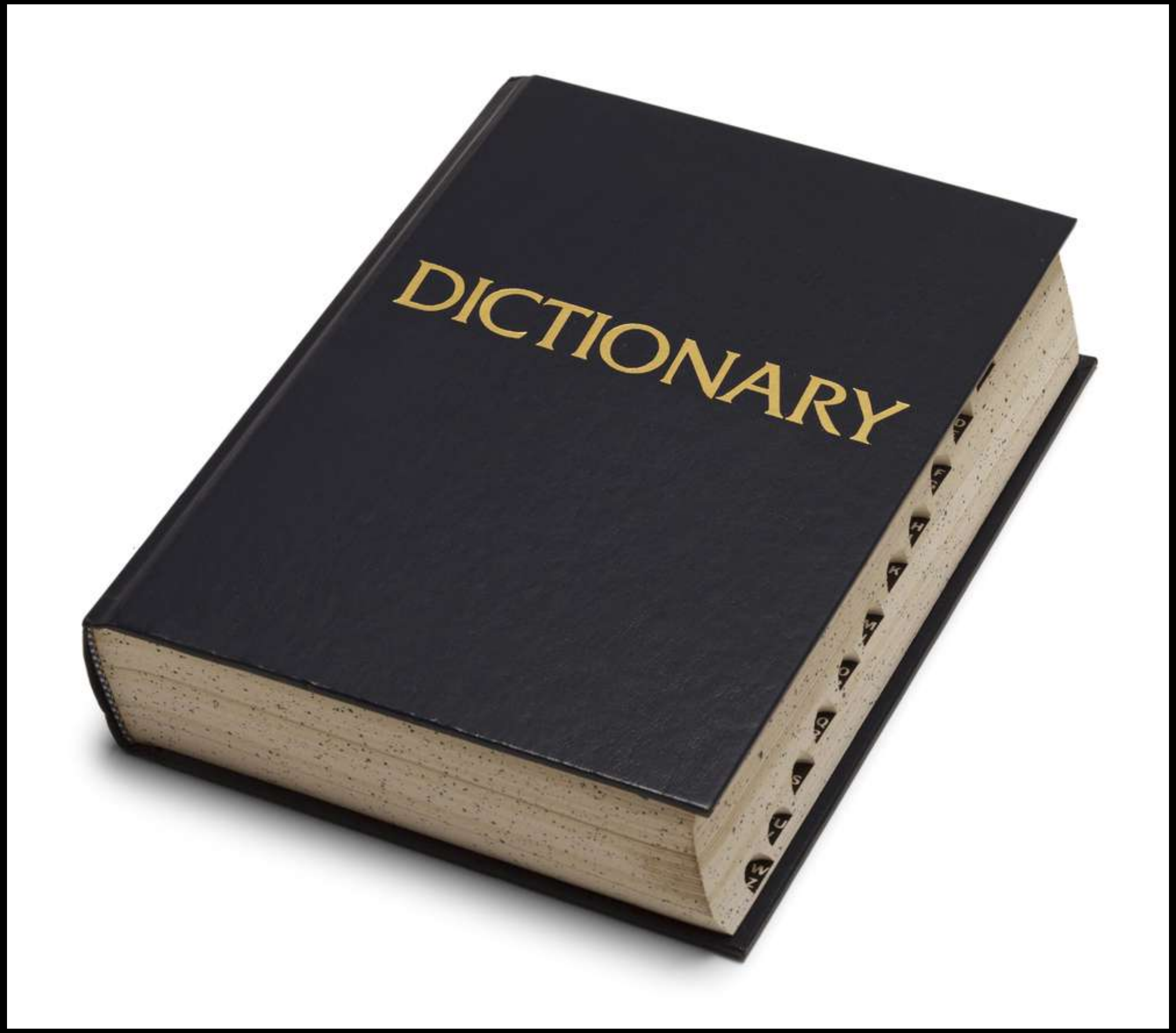
IN BOTH CASES PROGRESS THANKS TO RECENT ADVANCES IN SCFTS AND SYMMETRIES.

NAIVELY DISTINCT BUT $\exists$ INTERPLAY VIA DUALITY

1

GEOMETRIC ENGINEERING LIMIT OF HETEROTIC STRINGS

[1-3]



SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_{\mathfrak{g}} = \mathbb{C}^2 / \Gamma_{\mathfrak{g}}$$

$$\Gamma_{\mathfrak{g}} \subseteq SU(2)$$

$\mathfrak{g} \in ADE$

MACKAY

CORRESPONDENCE

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WELL-KNOWN BUILDING BLOCKS IN GEOMETRIC ENGINEERING DICTIONARIES

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WELL-KNOWN BUILDING BLOCKS IN GEOMETRIC ENGINEERING DICTIONARIES:

$$M/X^4_{\mathfrak{g}} \equiv 7D\ \mathfrak{g}\text{-}SYM$$

[SEN 96]

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[SEN 96]

Γ_{e_6}

WRAPPED M2S $\equiv$ W-BOSON

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[ALBERTINI, DZ, GARCIA ETXEBARRIA, HOSSEINI (20); MORRISON, SCHÄFER-NAMEKI, WILLET (20); APRUZZI, BONETTI, SCHÄFER NAMEKI, GARCIA ETXEBARRIA, HOSSEINI (21)]

ALL GLOBAL FORMS ARE REALIZED $M/\partial X^4_{\mathfrak{g}}$ SYMMETRY TFT

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_{\mathfrak{g}} = \mathbb{C}^2 / \Gamma_{\mathfrak{g}}$$

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- $IIA/X^4_{\mathfrak{g}}$ = 6D $\mathfrak{g}$-SYM (1,1) [SEN 97]

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RMK : $IIA/X = D_{S^1}(M/X)$

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- $IIB/X^4_{\mathfrak{g}}$ = 6D $\mathfrak{g}$-TYPE $(2,0)$ [WITTEN 95]

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RMK : $D_{S^1}(IIA/X^4_{\mathfrak{g}}) \xleftrightarrow{T} D_{S^1}(IIB/X^4_{\mathfrak{g}})$

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LITTLE MEMBRANE THEORY [NEKRASOV, MOORE SHATASHVILI 98]

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- $\mathrm{II}B/X^4_{\mathfrak{g}}$ = 6D $\mathfrak{g}$-TYPE $(2,0)$

LITTLE STRING THEORIES [SEIBERG 97]

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$\boxed{\text{WHAT ABOUT THE HETEROTIC ?}}$

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NAIVELY:

- $\mathrm{Het}_{E_8\times E_8}/X^4_g \equiv$ 6D (1,0)

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LSTs

E

6D LST

M_{LS}

6D SCFT

RMK: $D_{S^1}\left(\mathrm{Het}_{E_8\times E_8}/X^4_{\mathfrak{g}}\right) \xleftrightarrow{T} D_{S^1}\left(\mathrm{Het}_{D_{16}}/X^4_{\mathfrak{g}}\right)$

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BUT HETEROTIC HAS GAUGE BUNDLES !

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BUT HETEROTIC HAS GAUGE BUNDLES !

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_{\mathfrak{g}} = \mathbb{C}^2 / \Gamma_{\mathfrak{g}}$$

HETEROTIC HAS GAUGE BUNDLES $\Rightarrow$ NEED CHOICE OF FLAT CONNECTION

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$$\partial X^4_{\mathfrak{g}} = S^3/\Gamma_{\mathfrak{g}} \qquad \pi_1(S^3/\Gamma_{\mathfrak{g}}) \simeq \Gamma_{\mathfrak{g}}$$

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FOR Het_H PARAMETRIZED BY

$$\xi \in \mathrm{Hom}(\Gamma_{\mathfrak{g}}, H)$$

SIMPLEST SUSY SINGULARITIES: DU VAL

$$X_g^4 = \mathbb{C}^2 / \Gamma_g$$

FOR Het_H PARAMETRIZED BY

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SIMPLEST SUSY SINGULARITIES: DU VAL

$$X^4_{\mathfrak{g}} = \mathbb{C}^2/\Gamma_{\mathfrak{g}}$$

$$\xi \in \mathrm{Hom}(\Gamma_{\mathfrak{g}}, H)$$

$$\mathrm{Het}_H / (X^4_{\mathfrak{g}}, \xi) \in 6D\ (1,0)\ \mathrm{LST}$$

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OUR TASK NOW IS TO DETERMINE THIS LST

$$\mathrm{Het}_H / (X_g^4, \xi) \in 6D\,(1,0)\ \mathrm{LST}$$

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OUR TASK NOW IS TO DETERMINE THIS LST

STRATEGY: [3]

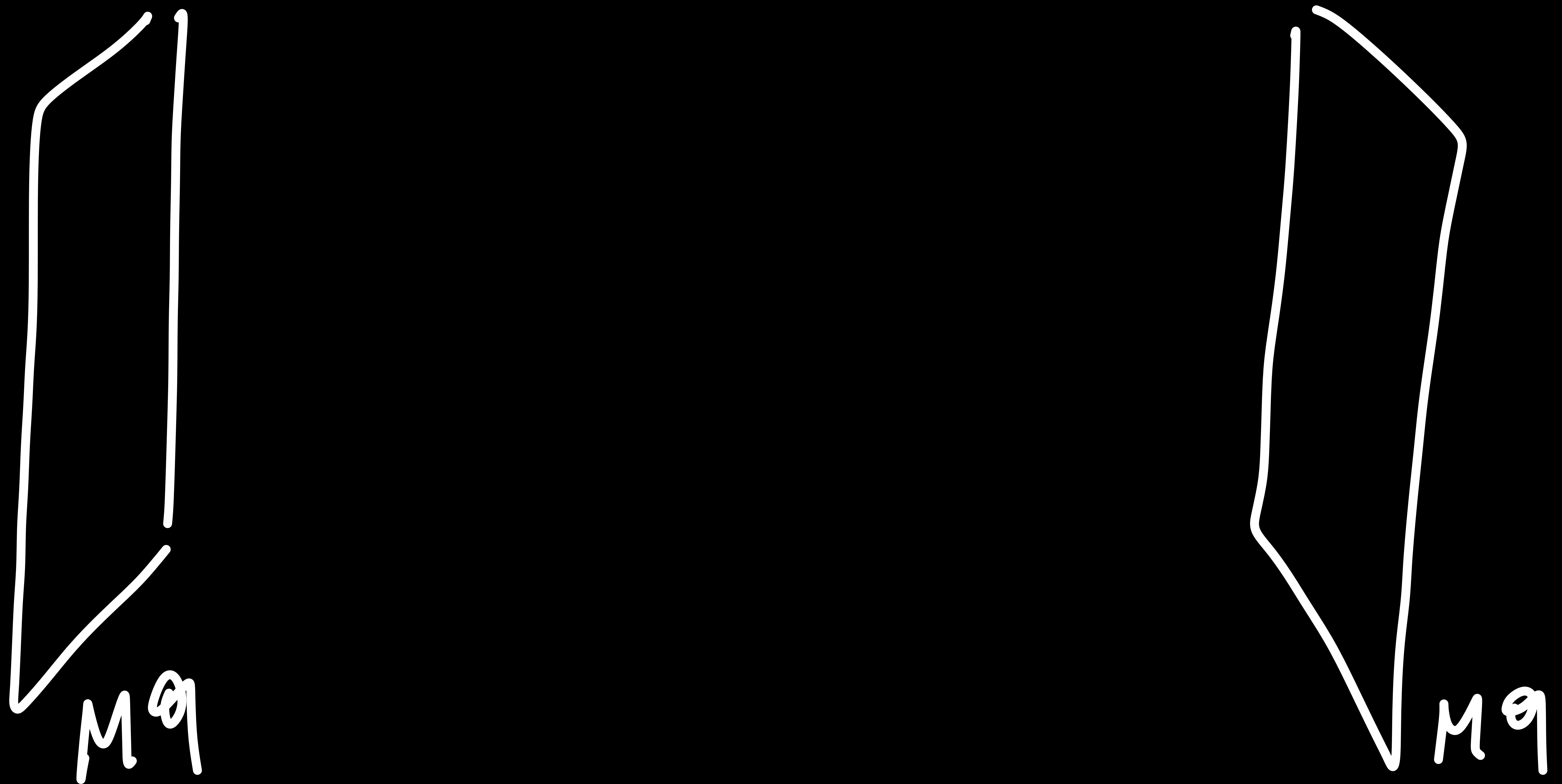
$\mathrm{Het}_{E_8 \times E_8}$ ⟶ HOŘAVA-WITTEN PICTURE

⟶ FRACTIONALIZATION OF M9 FROM ORBI-INSTANTONS

[FREY, RUDELIUS ...
MEKKAREEYA, OHMORI
LAWRIE, LIN,
FAZZI, GIRI,
GIACOMELLI ...]

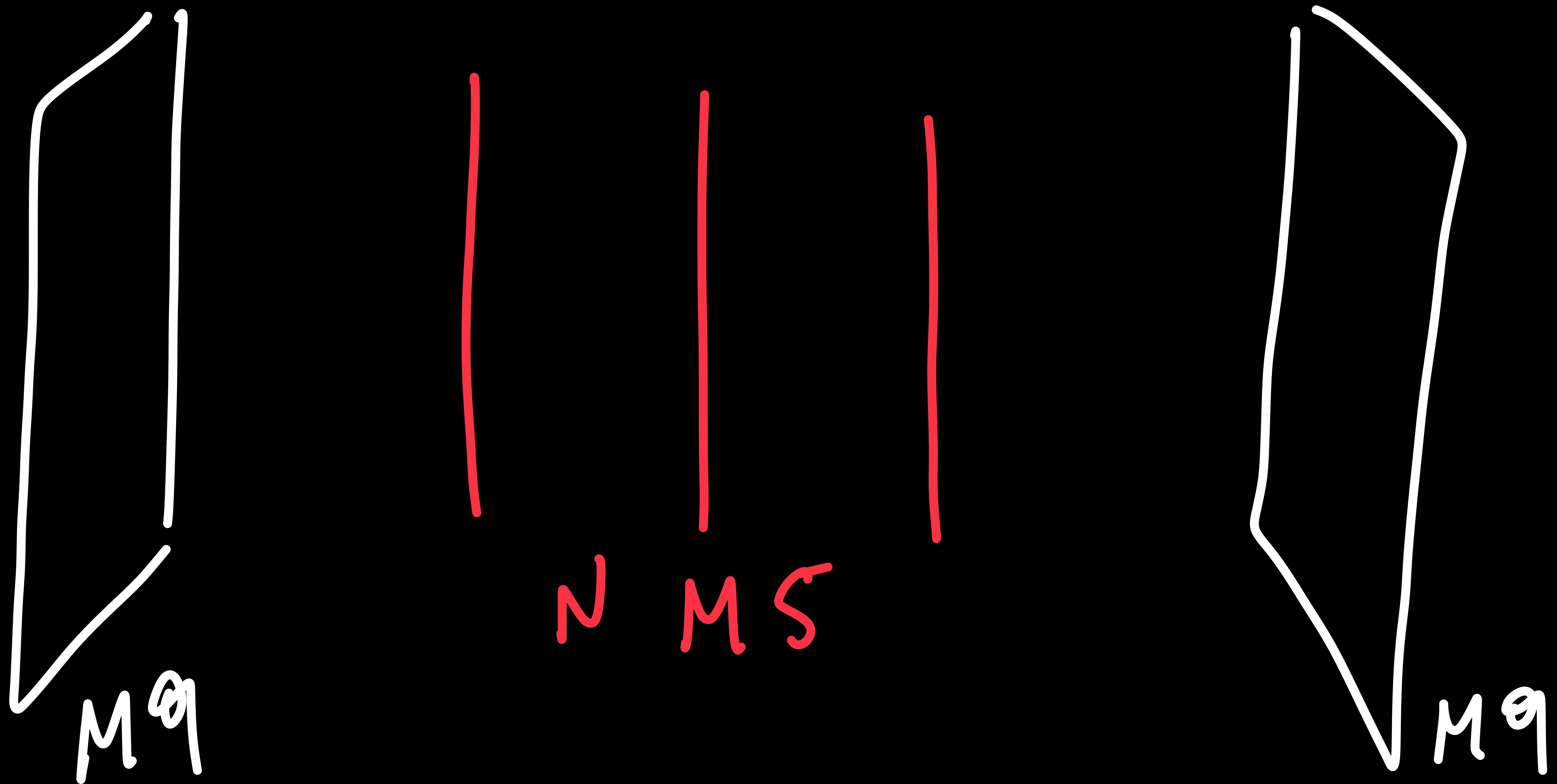
⟶ EMBED IN F-THEORY & GEOMETRIZE T-DUALITY [BHARDWAJ, DZ, HECKMAN, MORRISON, RUDELIUS, VAFA '15]

⟶ Het_{DIC}



HOŘAVA
WITTEN
95-96

$$M/(S^1/\mathbb{Z}_2) = \mathrm{Het}_{E_8\times E_8}$$



Hořava
Witten
95-96

$$M/(S^1/\mathbb{Z}_2)$$
$$= \mathrm{Het}_{E_8\times E}$$

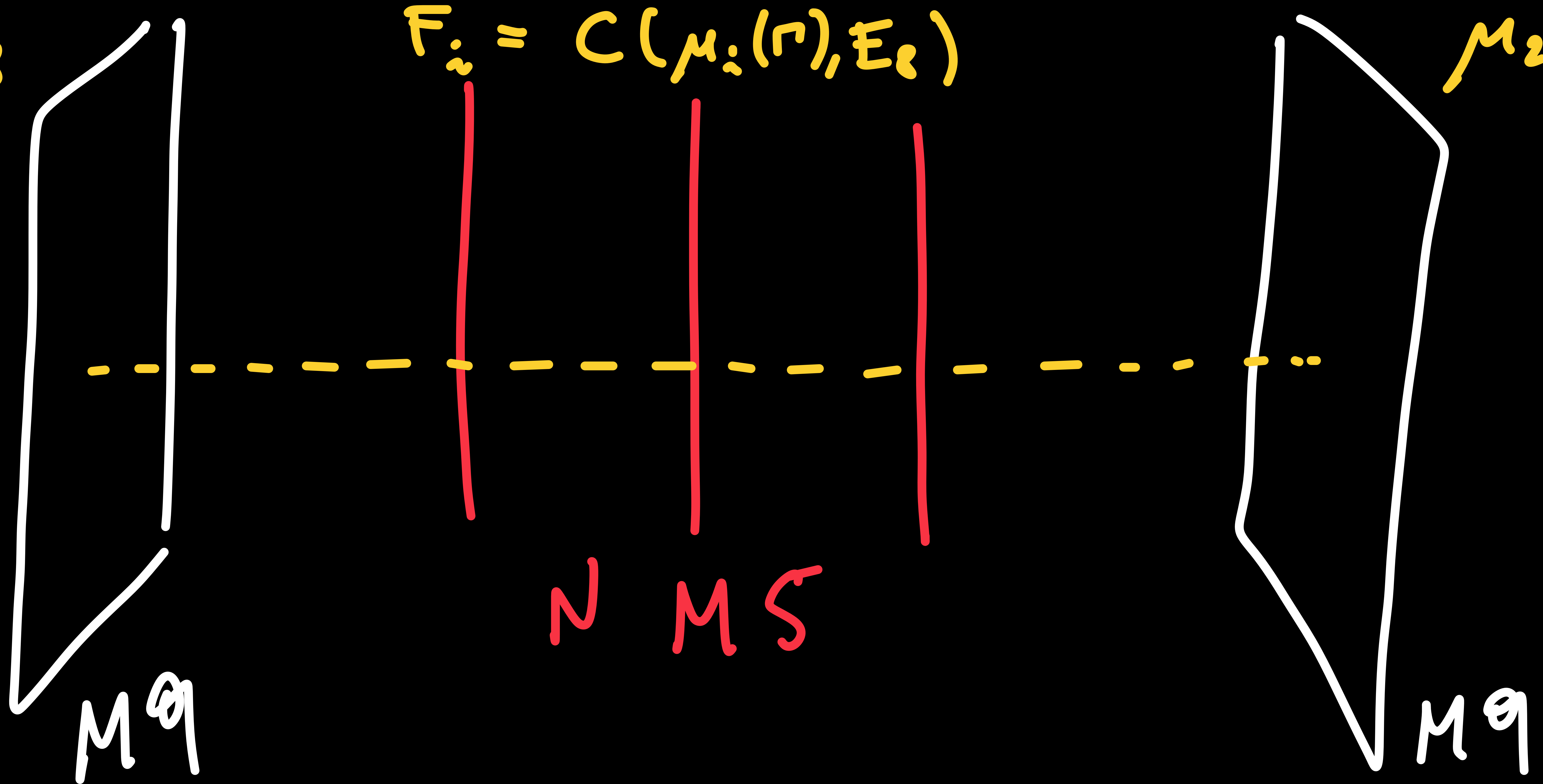
$\mu_1 : \Gamma \to E_8$

$F_i = C(\mu_i(\Gamma), E_8)$

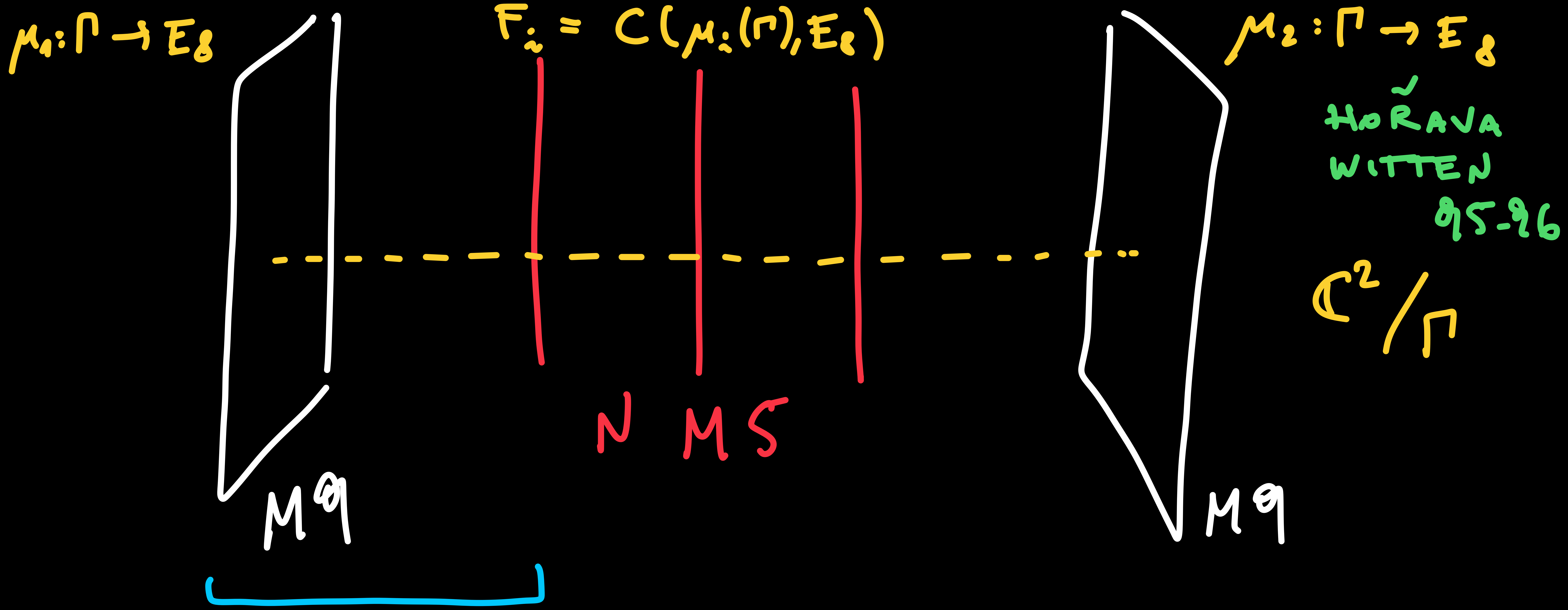
$\mu_2 : \Gamma \to E_8$

HORAVA WITTEN 95-96

$\mathbb{C}^2/\Gamma$



[ASPINWALL-MORRISON 98 ... LOTS OF RESULTS... FONT, GARCIA ETXEBARRIA, LÜST, MEYRHOFER 17 ... [1, 2] SYSTEMATIC UNDERSTANDING]



$\equiv T(\mu_1, \Gamma)$ RUDELIUS FREY 17 : 1 ORBI-INSTANTON

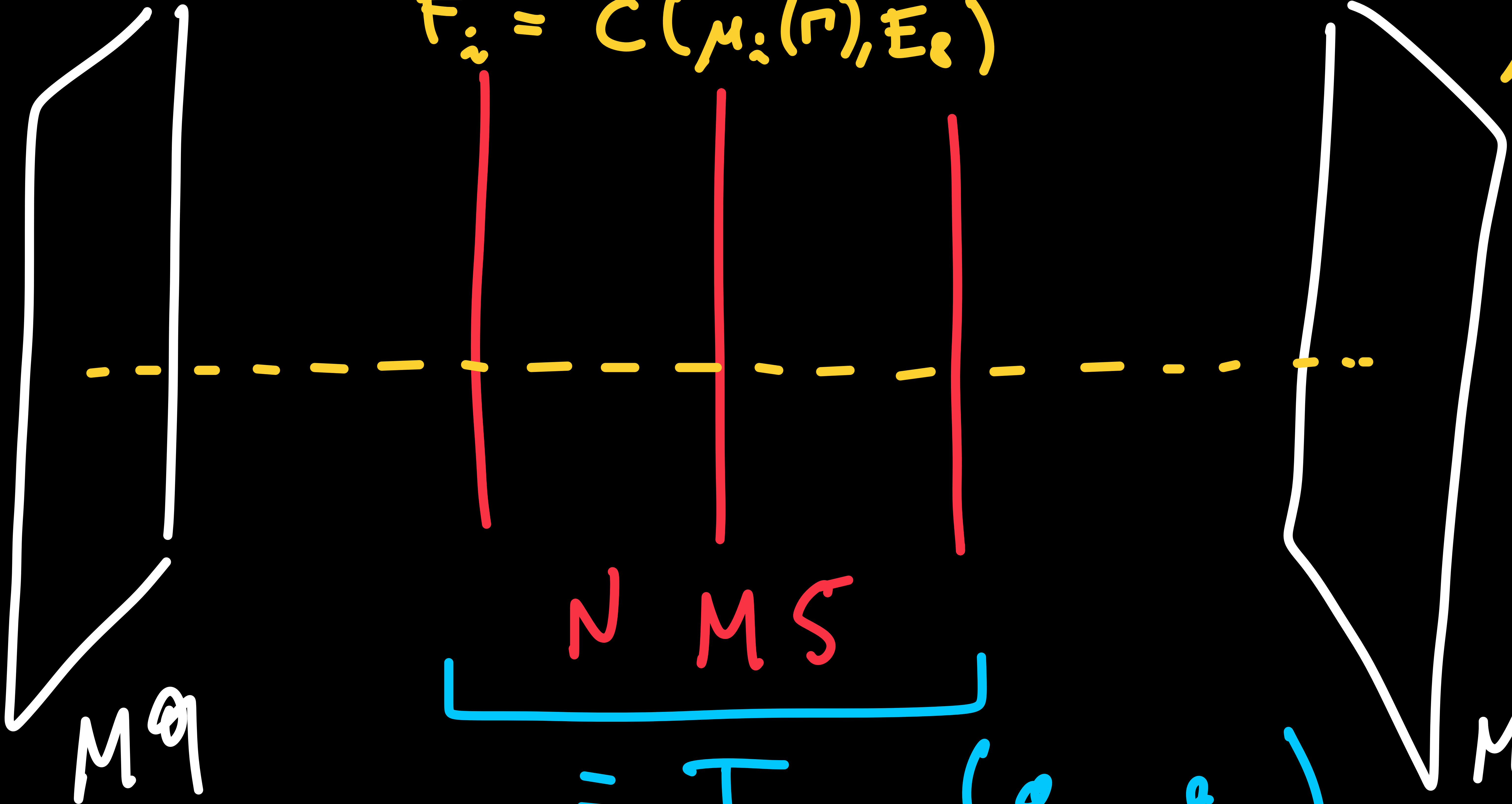
$\mu_1 : \Gamma \to E_8$

$F_i = C(\mu_i(\Gamma), E_8)$

$\mu_2 : \Gamma \to E_8$

HOŘAVA WITTEN 95-96

$\mathbb{C}^2/\Gamma$



$\equiv T_{N-2}(g_\Gamma, g_\Gamma)$

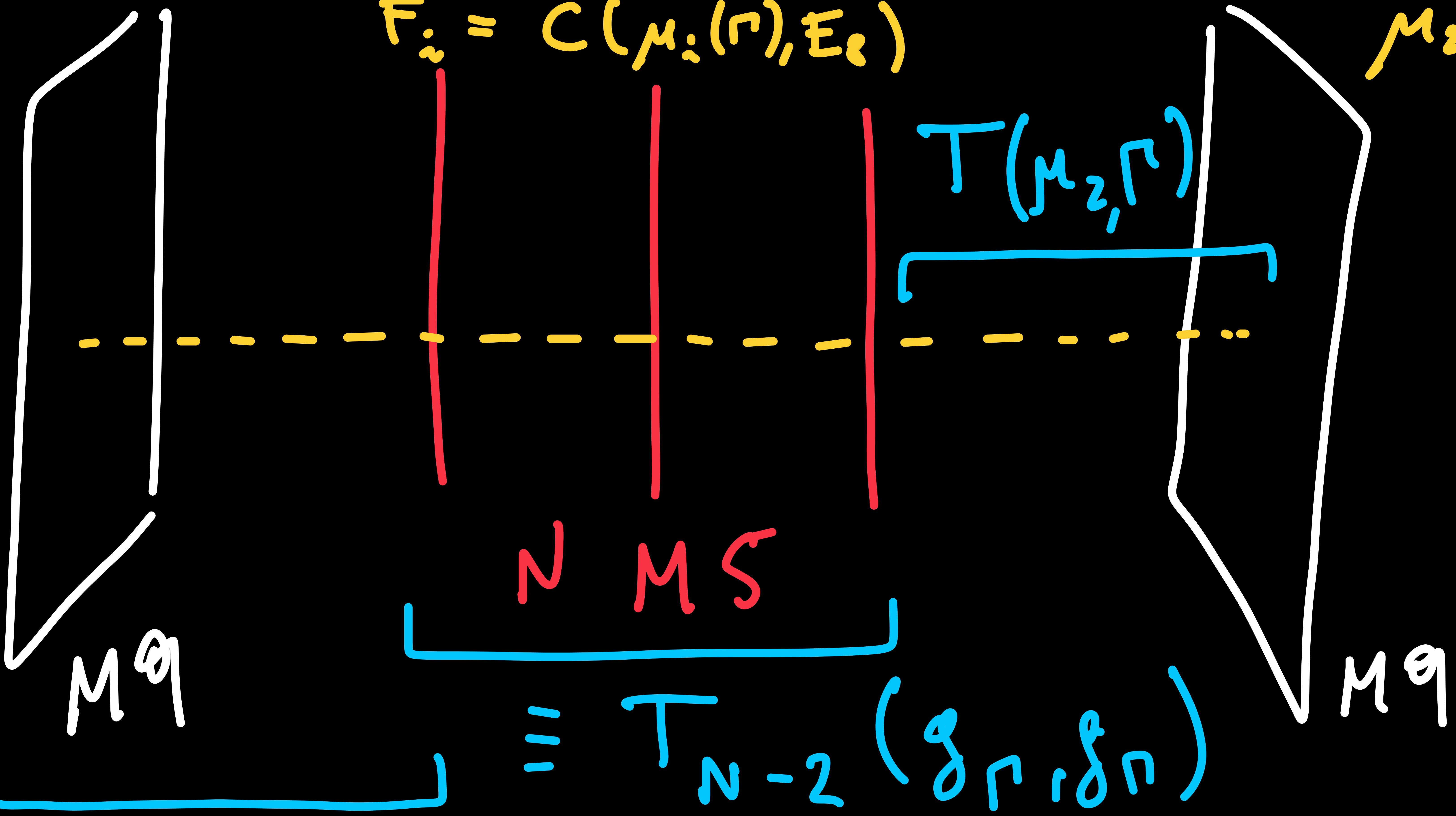
$\equiv T(\mu_1, \Gamma)$

RUDELIUS FREY 17

CONFORMAL MATTER

[DZ, HECKMAN, TOMASIELLO, VAFA 14]

$\mu_1 : \Gamma \to E_8$ $\quad F_i = C(\mu_i(\Gamma), E_8)$ $\quad \mu_2 : \Gamma \to E_8$



$T(\mu_2, \Gamma)$

HOŘAVA WITTEN 95-96

$\mathbb{C}^2/\Gamma$

$\equiv T_{N-2}(g_\Gamma, g_\Gamma)$

$\equiv T(\mu_1, \Gamma)$

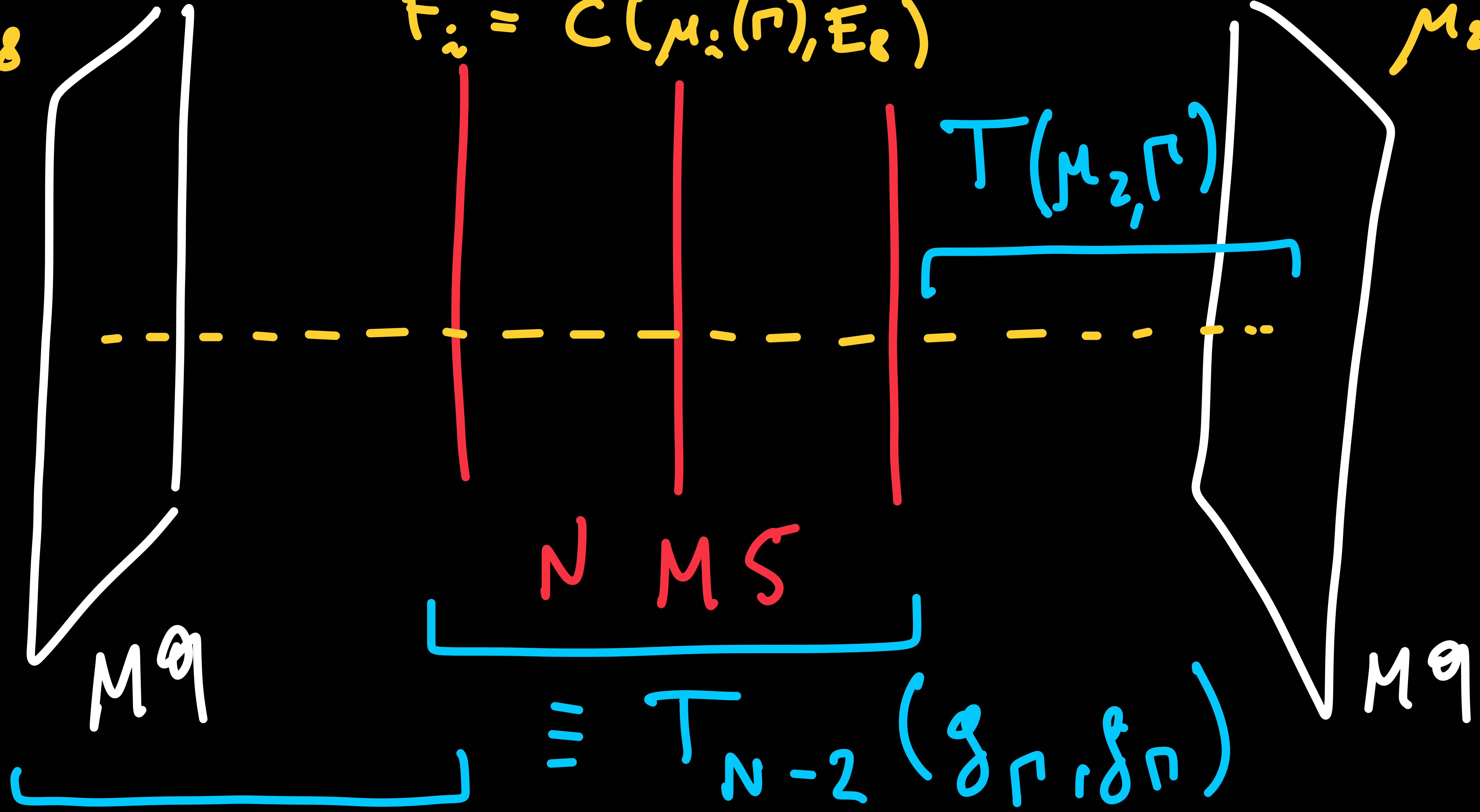
RUDELIUS FREY 17

CONFORMAL MATTER

$\mu_1 : \Gamma \to E_8$ $\qquad F_i = C(\mu_i(\Gamma), E_8)$ $\qquad \mu_2 : \Gamma \to E_8$

HOŘAVA WITTEN 95-96

$\mathbb{C}^2/\Gamma$



$\equiv T_{N-2}(g_\Gamma, g_\Gamma)$

$\equiv T(\mu_1, \Gamma)$

RUDELIUS FREY 17

CONFORMAL MATTER

ALL KNOWN 6D SCFTs!

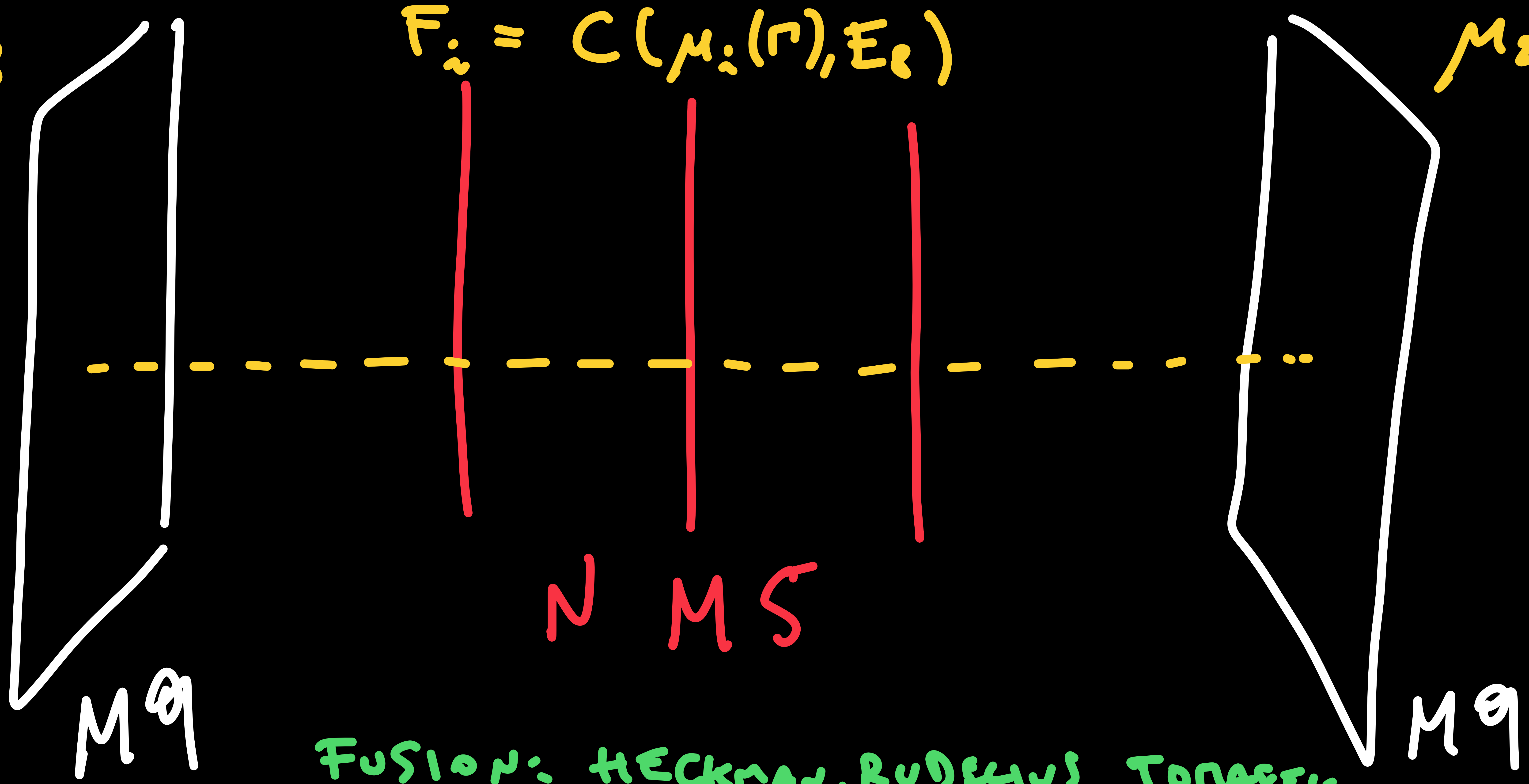
$\mu_1 : \Gamma \to E_8$

$F_i = C(\mu_i(\Gamma), E_8)$

$\mu_2 : \Gamma \to E_8$

HORAVA WITTEN 95-96

$\mathbb{C}^2/\Gamma$



GENERALIZED QUIVER!

FUSION: HECKMAN, RUDELIUS, TOMASIELLO 18
DZ, LOCKHART 18

$$\equiv T(\mu_1, \Gamma) \overset{\mathfrak{g}_\Gamma}{\longrightarrow} T_{N-2}(\mathfrak{g}_\Gamma, \mathfrak{g}_\Gamma) \overset{\mathfrak{g}_\Gamma}{\longrightarrow} T(\mu_2, \Gamma)$$

EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$

e_6 ... e_6 e_6 ... e_6 ...

E_6] 1 3 1 5 1 3 1 6 1 3 1 6 1 3 1 ... 6 1 3 1 6 1 3 1 5 1 3 2 1 [E_7

$T(E_6, \Gamma_g = E_6)$

$T(E_6 E_6)$

$N-2$

$T(E_7, \Gamma_g = E_6)$

$N+1$

IS T-DUAL TO [MUYANG'S TALK]

Sp_{7+N} SO_{22+4N} Sp_{6+3N} SU_{8+4N} SU_{4+2N}

$[SO_{22}]$ 1 4 1 2 2

$[Sp_1]$ $[SO_2]$

$\hat{k}_{\rho} = 2$ $\hat{\kappa}_R = 69 + 24N$ 2-GROUP STRUCTURE

[INTRILIGATOR OCUM 97]

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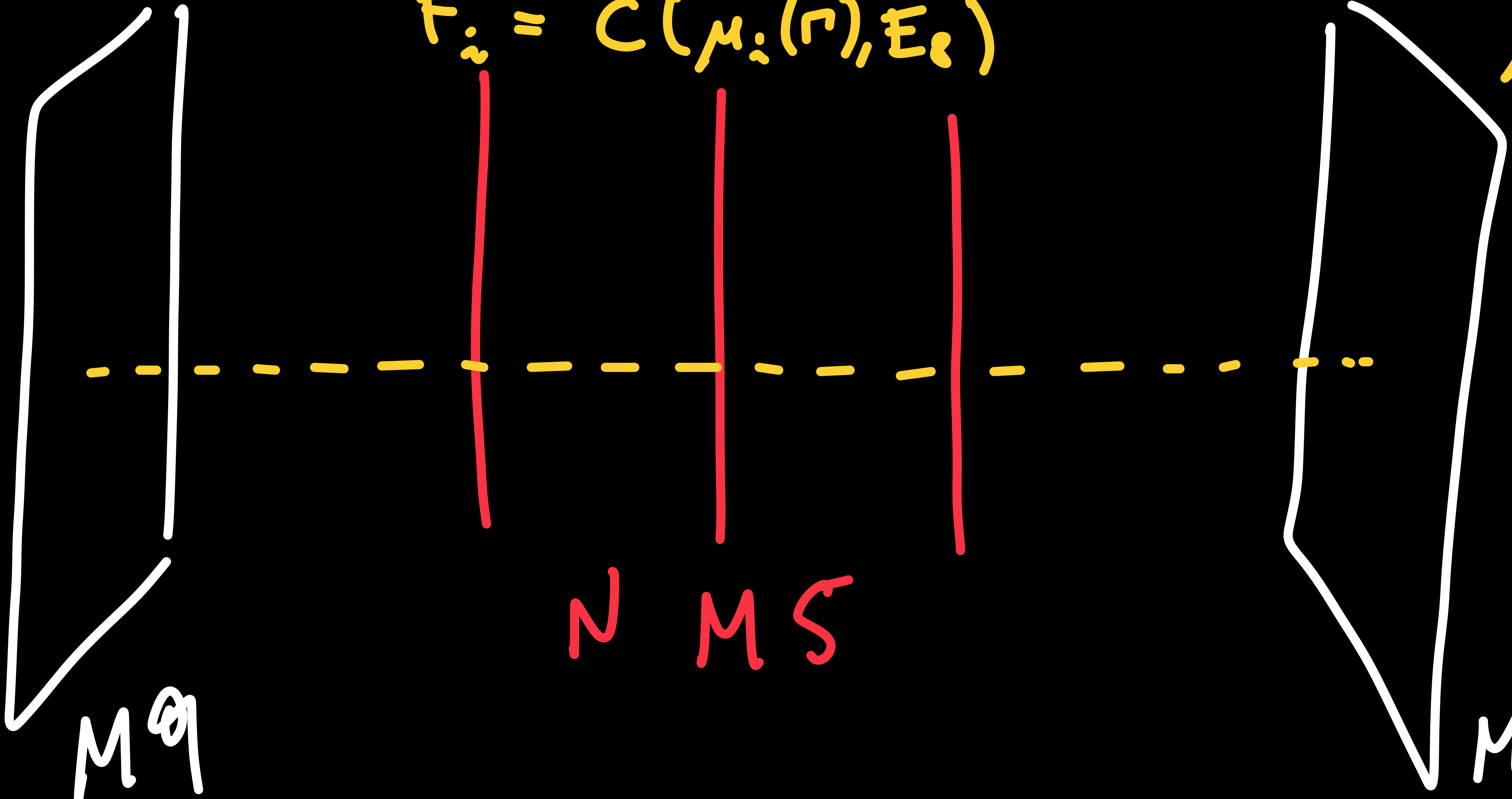

HOŘAVA
WITTEN
95-96

$\mathbb{C}^2/\Gamma$

$$\equiv T(\mu_1, \Gamma) \xrightarrow{\;\; g_\Gamma \;\;} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{\;\; g_\Gamma \;\;} T(\mu_2, \Gamma)$$

$\mu_1 : \Gamma \to E_8$ $\qquad F_i = C(\mu_i(\Gamma), E_8)$ $\qquad \mu_2 : \Gamma \to E_8$

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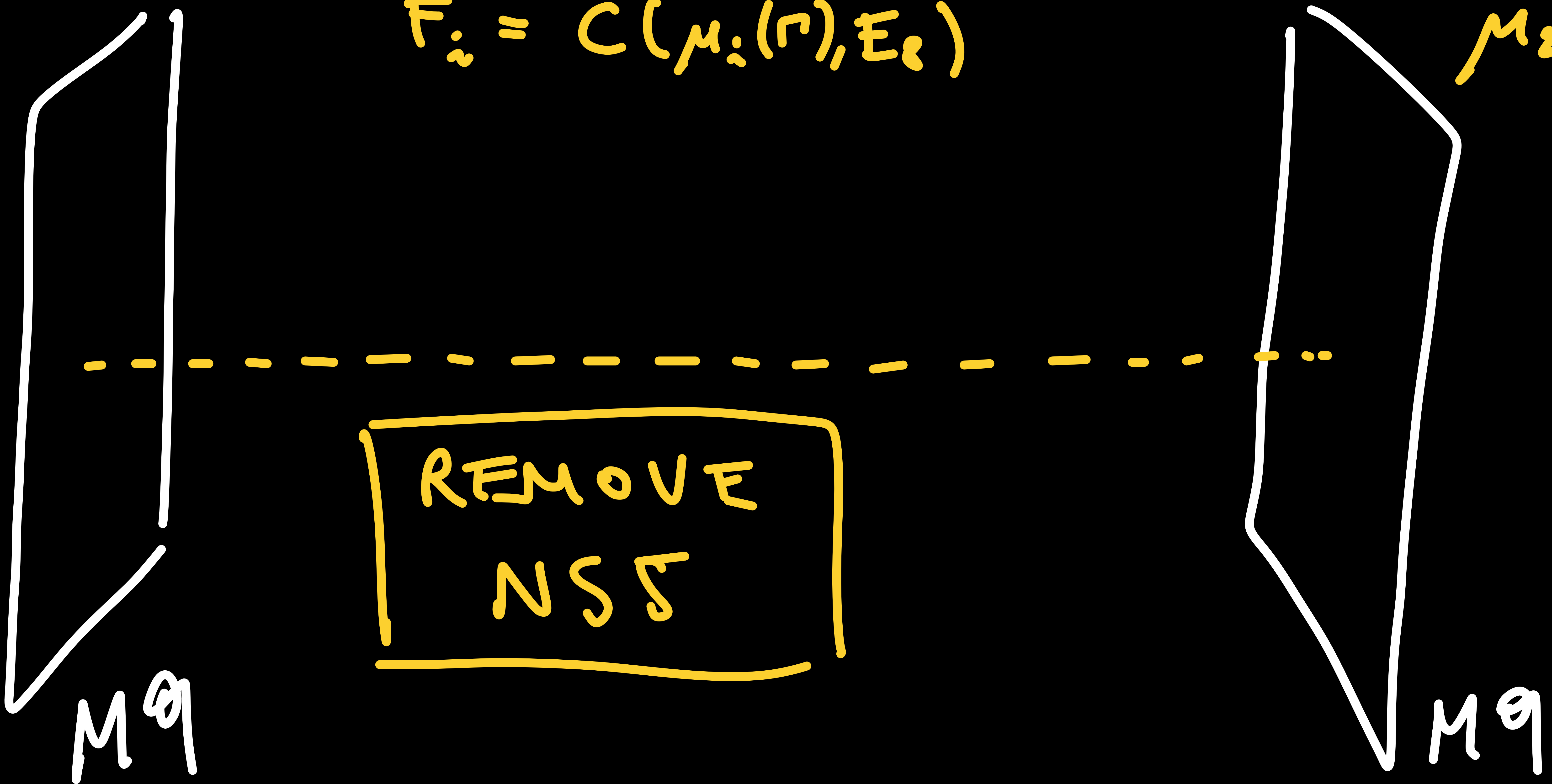
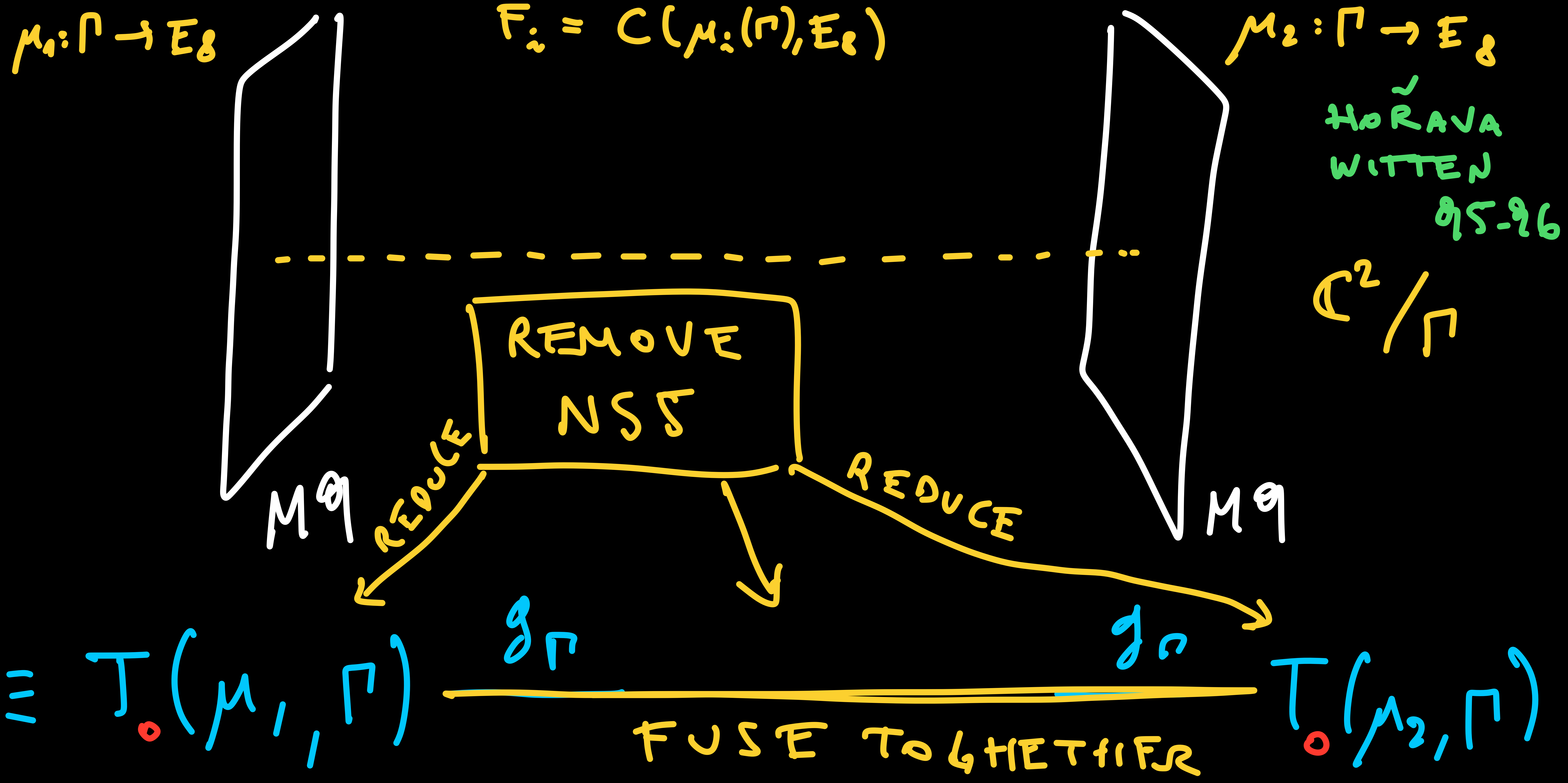


$\mathbb{C}^2/\Gamma$

$$\equiv T(\mu_1, \Gamma) \xrightarrow{\; g_\Gamma \;} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{\; g_\Gamma \;} T(\mu_2, \Gamma)$$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WITTEN
95-96
$\mathbb{C}^2/\Gamma$
REMOVE
NS5
REDUCE
REDUCE
M9
M9
g_Γ
g_Γ
$= T_0(\mu_1, \Gamma)$
$T_0(\mu_2, \Gamma)$
FUSE TOGHETHER

EXAMPLE: $\mathfrak{g} = e_6$, $F_1 = E_6$, $F_2 = E_7$

$$\begin{array}{c} \ldots \quad e_6 \quad \ldots \quad e_6 \qquad e_6 \quad \ldots \quad e_6 \quad \ldots \\ E_6 \,]\; 1\;3\;1\;5\;1\;3\;1\;6\;1\;3\;1\;\;6\;1\;3\;1 \cdots 6\;1\;3\;1\;6\;1\;3\;1\;5\;1\;3\;2\;1\;[\, E_7 \end{array}$$

$T(E_6, \Gamma_{\mathfrak{g}} = E_6)$

$T(E_6 E_6)$

$N-2$

$T(E_7, \Gamma_{\mathfrak{g}} = E_6)$

$N+1$

EXAMPLE: $g = e_6$, $F_1 = E_6$, $F_2 = E_7$

$$E_6 \,]\; \overbrace{1\,3\,1\,5\,1\,3\,1}^{\cdots}\; \overset{e_6}{6}\; \overbrace{1\,3\,1}^{\cdots}\; \overset{e_6}{6}\; 1\,3\,1 \cdots \overset{e_6}{6}\; \overbrace{1\,3\,1}^{\cdots}\; \overset{e_6}{6}\; \overbrace{1\,3\,1\,5\,1\,3\,2\,1}^{\cdots}\; [\, E_7$$

$T(E_6, \Gamma_g = E_6)$

$T(E_6 F_6)$

$N-2$

$T(E_7, \Gamma_g = E_6)$

$N+1$

DISAPPEARS IN THE $N \to 0$ LIMIT!

EXAMPLE: $\mathfrak{g} = \mathfrak{e}_6$, $F_1 = E_6$, $F_2 = E_7$

$$\underbrace{[E_6]\; 1\; 3\; 1\; \overset{\cdots}{5}\; 1\; 3\; 1}_{T_0(E_6,\, \Gamma_{\mathfrak{g}} = \mathfrak{e}_6)} \qquad\qquad \begin{matrix}\mathfrak{e}_6 \\ 6\end{matrix} \qquad\qquad \underbrace{1\; 3\; 1\; \overset{\cdots}{5}\; 1\; 3\; 2\; 1\; [E_7]}_{T_0(E_7,\, \Gamma_{\mathfrak{g}} = \mathfrak{e}_6)}$$

REDUCED

REDUCED

IN THE

$N \to 0$ LIMIT!

EXAMPLE: $\mathfrak{g} = \mathfrak{e}_6, \quad F_1 = E_6, \quad F_2 = E_7$

$$\underbrace{[E_6]\; 1\; 3\; \overset{\cdots}{1\; 5\; 1}\; 3\; 1}_{\substack{T_0(E_6,\, \Gamma_{\mathfrak{g}} = \mathfrak{e}_6) \\ \text{REDUCED}}}\;\; \overset{\mathfrak{e}_6}{6}\;\; \underbrace{1\; 3\; \overset{\cdots}{1\; 5\; 1}\; 3\; 2\; 1\; [E_7]}_{\substack{T_0(E_7,\, \Gamma_{\mathfrak{g}} = \mathfrak{e}_6) \\ \text{REDUCED}}}$$

IS THE 6D LST FROM $\mathrm{Het}_{E_8 \times E_8} / X^4_{\mathfrak{e}_6}, \mu_1, \mu_2$

IS T-DUAL TO N→0 LIMIT OF:

$$
\begin{array}{cccccc}
 & Sp_{7+N} & SO_{22+4N} & Sp_{6+3N} & SU_{8+4N} & SU_{4+2N} \\
[SO_{22}] & 1 & 4 & 1 & 2 & 2 \\
 & & [Sp_1] & [SO_2] & &
\end{array}
$$

$$\boxed{\hat{\kappa}_P = 2 \qquad \hat{\kappa}_R = 69 + 24N}$$

2-GROUP STRUCTURE

[INTRILIGATOR OLUM 97]

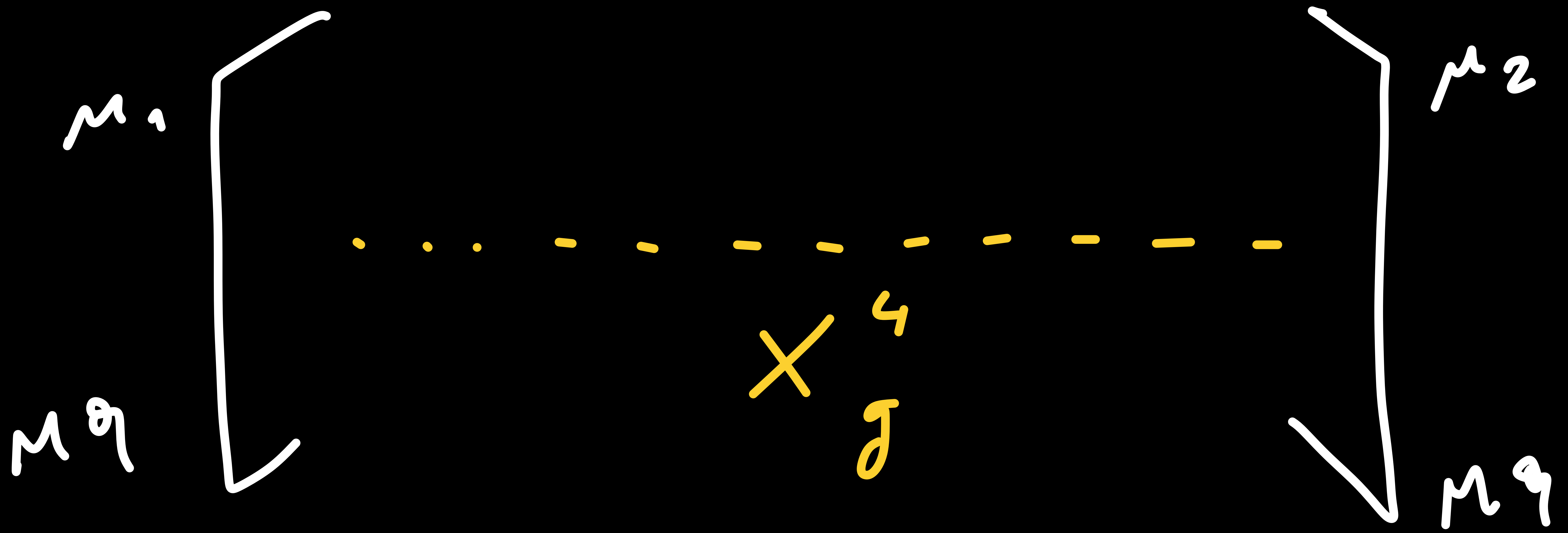
IS T-DUAL TO

$$
\begin{array}{cccccc}
 & \mathfrak{sp}_7 & \mathfrak{so}_{22} & \mathfrak{sp}_6 & \mathfrak{su}_8 & \mathfrak{su}_4 \\
[\mathfrak{so}_{22}] & 1 & 4 & 1 & 2 & 2 \\
 & & [\mathfrak{sp}_1] & [\mathfrak{so}_2] & &
\end{array}
$$

$$\boxed{\hat{\kappa}_P = 2 \qquad \hat{\kappa}_R = 69}$$ 2-GROUP STRUCTURE

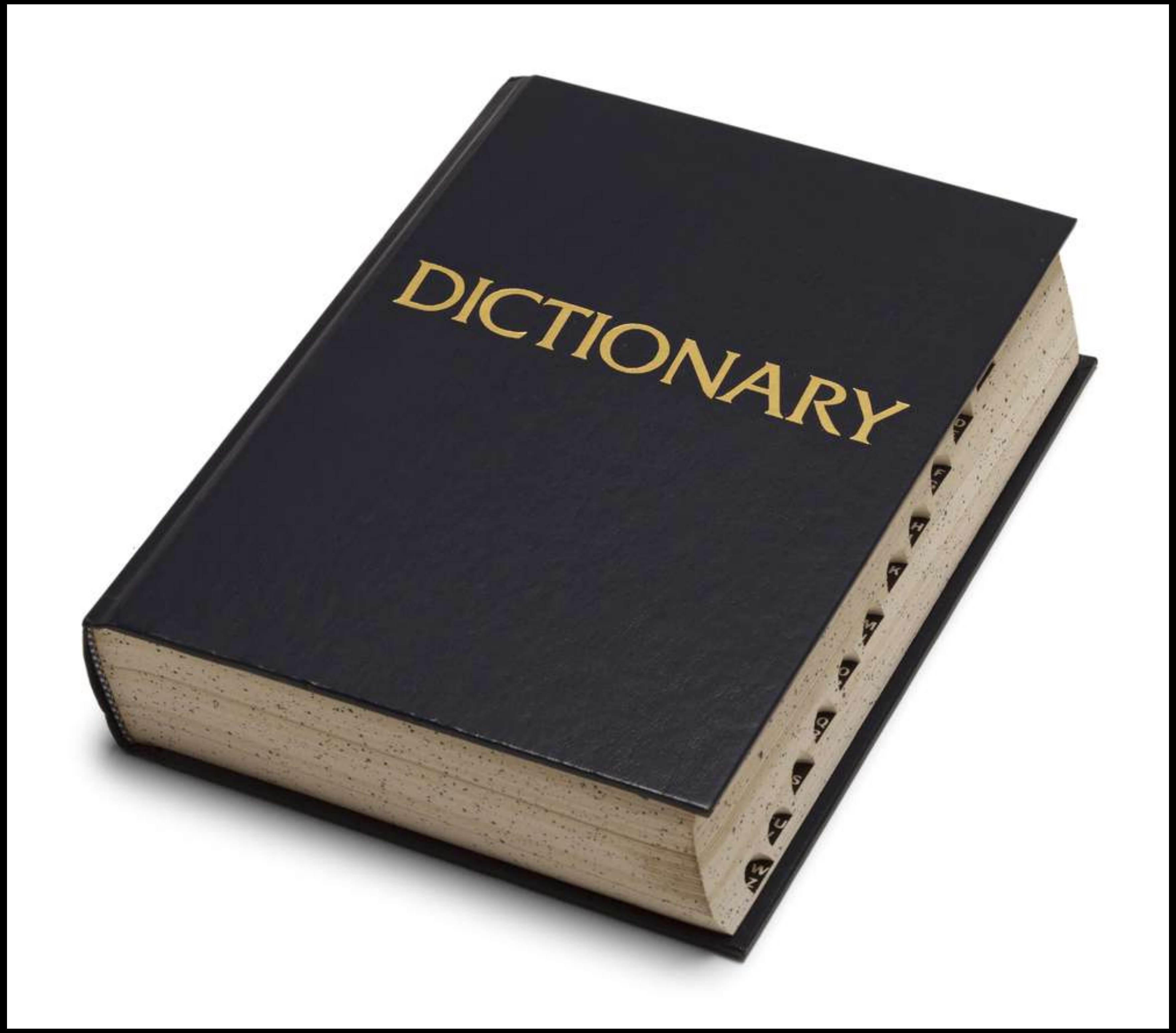
IS THE 6D LST : $\mathrm{Het}_{Spin(32)/\mathbb{Z}_2} / (X^4_{e_6}, \lambda)$

IN THE PAPER: GENERAL PRESCRIPTION TO CAPTURE GEOMETRIC ENGINEERING LIMIT $\forall$



$$(X^4_g, \mu_1, \mu_2) \overset{T}{\updownarrow} (X^4_g, \lambda)$$

ALSO COMMENT ON TURNING ON TORSIONAL C_3 FLUX ON S^3/Γ_g $\longrightarrow$ NON SIMPLY LACED VERSIONS. [TRIPLES FLUXES & STRINGS]



2

GEOMETRIC ENGINEERING OF M-THEORY ON SINGULAR G_2 HOLONOMY CONES

M / X^7 : G_2 HOLONOMY $\longrightarrow$ $\boxed{\text{4D } N=1 \text{ SQFT}}$

- RICH DYNAMICS
  SYMMETRY BREAKING...
  CONFINEMENT...
- NON-PERTURBATIVE CORRECTIONS
  EUCLIDEAN M2 BRANES ON CALIBRATED ASSOCIATIVE 3-CYCLES

$\Rightarrow$ GEOMETRIC ENGINEERING IS HARDER: QUANTUM GEOMETRY

THERE ARE 3 EXAMPLES OF LOCAL G_2'S

- $X^7_1 = \text{CONE}(S^3 \times S^3)$ [BRYANT SALAMON 89]
- $X^7_2 = \text{CONE}(CP^3)$ $\quad ds^2_{X^7} = dr^2 + r^2 ds^2_{X^6}$
- $X^7_3 = \text{CONE}(SU(3)/U(1)\times U(1))$

PHYSICS: [ATIYAH, MALDACENA, VAFA 00; ACHARYA 00; ATIYAH-WITTEN 01; ACHARYA-WITTEN 01; CVETIC, GIBBONS, LÜ, POPE 01]

THERE ARE 3 EXAMPLES OF LOCAL G_2'S

- $X_1^7 = \text{CONE}(S^3 \times S^3) = \mathcal{S}(S^3)$
- $X_2^7 = \text{CONE}(\mathbb{CP}^3) = \Lambda^2_-(S^4)$
- $X_3^7 = \text{CONE}(SU(3)/U(1)\times U(1)) = \Lambda^2_-(\mathbb{CP}^2)$

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- $X_1^7 = \text{CONE}(S^3 \times S^3) = \mathcal{S}(S^3)$
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WE WANT SCFTS

THERE ARE 3 EXAMPLES OF LOCAL G_2'S

- $X_1^7 = \text{CONE}(S^3 \times S^3) = \mathcal{S}(S^3)$ ⚠ QUANTUM GEOMETRY
- $X_2^7 = \text{CONE}(\mathbb{CP}^3) = \Lambda^2_-(S^4)$
- $X_3^7 = \text{CONE}(SU(3)/U(1)\times U(1)) = \Lambda^2_-(\mathbb{CP}^2)$

VERY INTERESTING BUT ≠ QUALITATIVE FEATURES

WE WANT SCFTS THAT GENERALIZE MATTER FOR 4D $N=1$ G_2 EXAMPLES

THERE ARE 3 EXAMPLES OF LOCAL G_2'S

- $X_1^7 = \text{CONE}(S^3 \times S^3) = \mathcal{S}(S^3)$ ⚠ QUANTUM GEOMETRY
- $X_2^7 = \text{CONE}(\mathbb{CP}^3) = \Lambda^2_-(S^4)$
- $X_3^7 = \text{CONE}(SU(3)/U(1)\times U(1)) = \Lambda^2_-(\mathbb{CP}^2)$

WE WANT SCFTS THAT GENERALIZE MATTER FOR 4D N=1 G_2 EXAMPLES

EM2

VERY INTERESTING BUT ≠ QUALITATIVE FEATURES

THERE ARE 3 EXAMPLES OF LOCAL G_2'S

- $X^7_1 = \text{CONE}(S^3 \times S^3) = \mathbb{S}(S^3)$ ⚠ QUANTUM GEOMETRY
- $X^7_2 = \text{CONE}(\mathbb{CP}^3) = \Lambda^2_-(S^4)$
- $X^7_3 = \text{CONE}(SU(3)/U(1)\times U(1)) = \Lambda^2_-(\mathbb{CP}^2)$

EM2

VERY INTERESTING BUT ≠ QUALITATIVE FEATURES

WE WANT SCFTS

FOCUS ON THE OTHER TWO EXAMPLES AND TAKE ORBIFOLDS!

$$\boxed{\begin{aligned} &\cdot\ X_2^7 = \text{CONE}\,(\mathbb{CP}^3) = \Lambda^2_-(S^4) \\ &\cdot\ X_3^7 = \text{CONE}\,\big(SU(3)/U(1)\times U(1)\big) = \Lambda^2_-(\mathbb{CP}^2) \end{aligned}}$$

WE WANT SCFTS

FOCUS ON THE OTHER TWO EXAMPLES AND TAKE ORBIFOLDS!

FINITE SUBGROUPS OF SO(5) ISOMETRY

FINITE SUBGROUPS OF PSU(3) ISOMETRY

LOTS OF UNEXPLORED EXAMPLES ! BUT BECAUSE OF G_2 HOLONOMY : GENERIC NON-ISOLATED SINGULARITIES

$$\boxed{\begin{array}{l} \cdot\ X_2^7 = \text{Cone}\left(\mathbb{CP}^3\right) = \Lambda^2_-\left(S^4\right) \\ \cdot\ X_3^7 = \text{Cone}\left(SU(3)/U(1)\times U(1)\right) = \Lambda^2_-\left(\mathbb{CP}^2\right) \end{array}}$$

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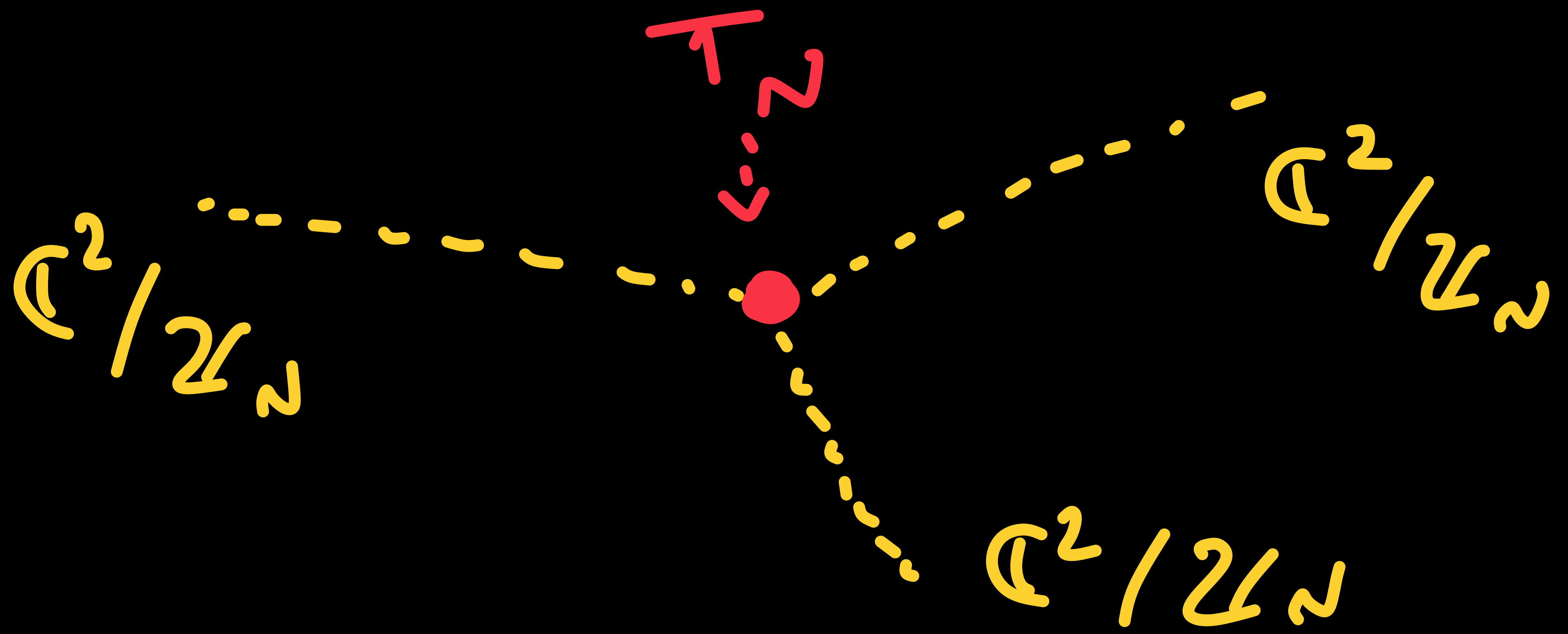
IS THIS A PROBLEM?

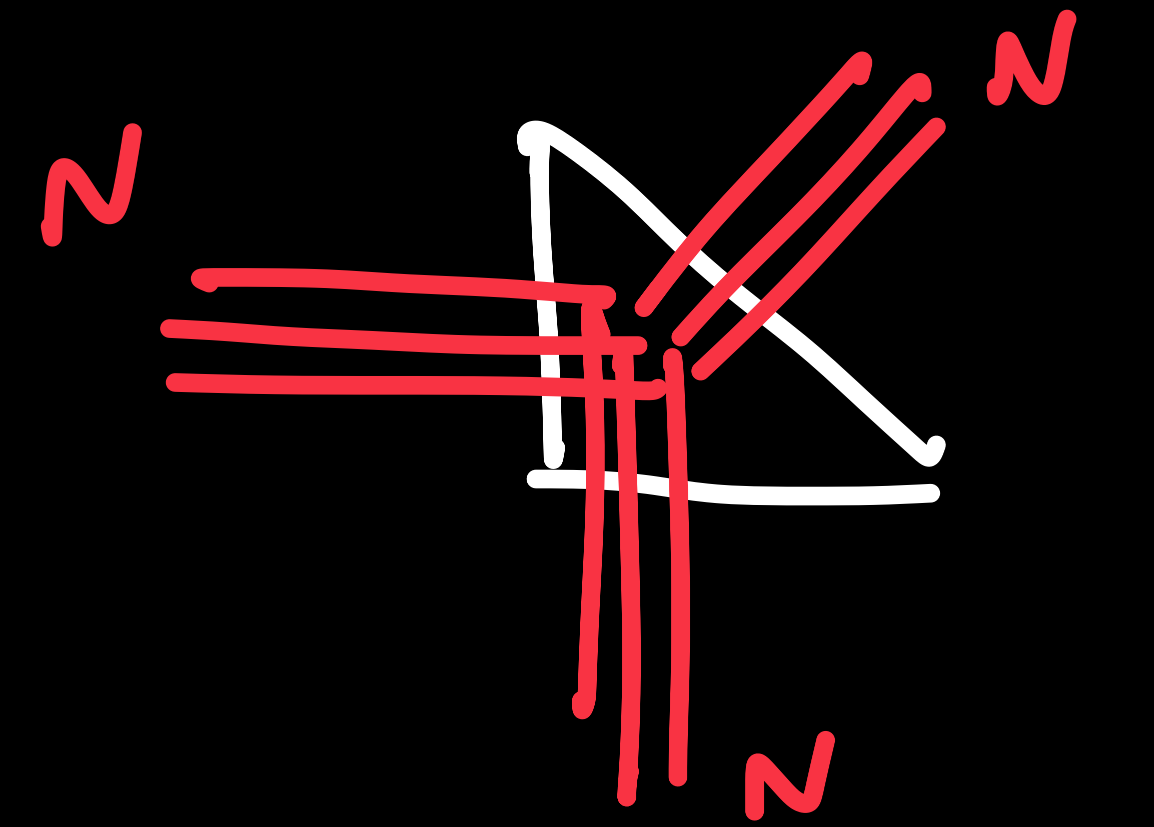
LOTS OF UNEXPLORED EXAMPLES! BUT BECAUSE OF G_2 HOLONOMY $\boxed{\text{GENERIC NON-ISOLATED SINGULARITIES}}$

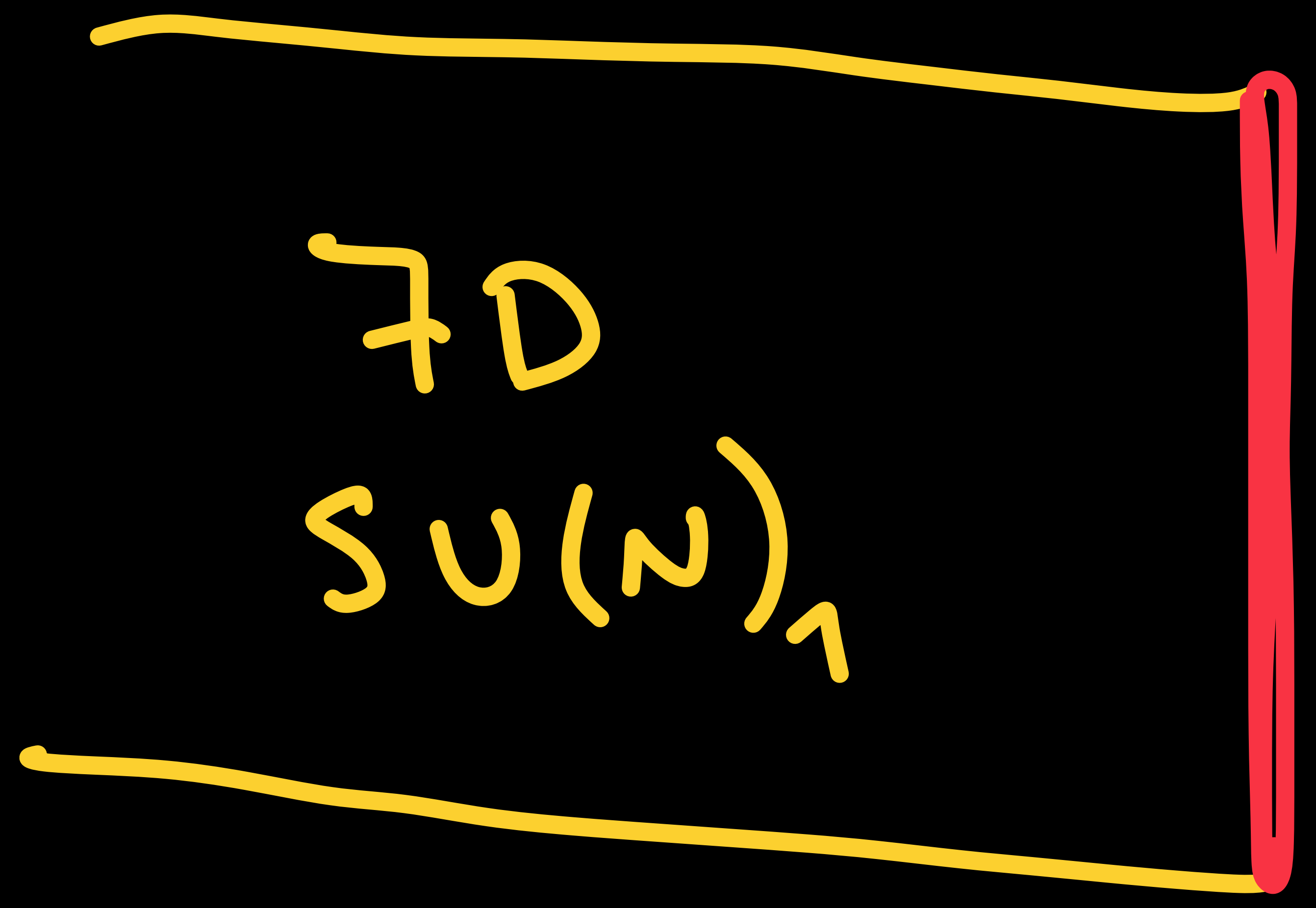
IS THIS A PROBLEM? NOT REALLY, WE ARE QUITE FAMILIAR WITH THESE: [BENINI, BENVENUTI, TACHIKAWA 09]

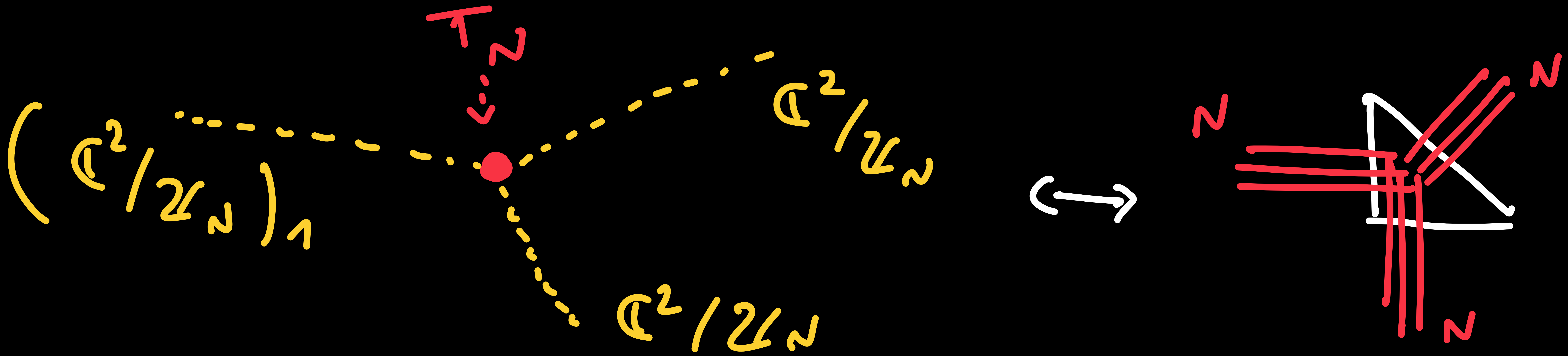
$$M \,/\, \mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N \equiv T_N \quad \text{5D SCFT}$$

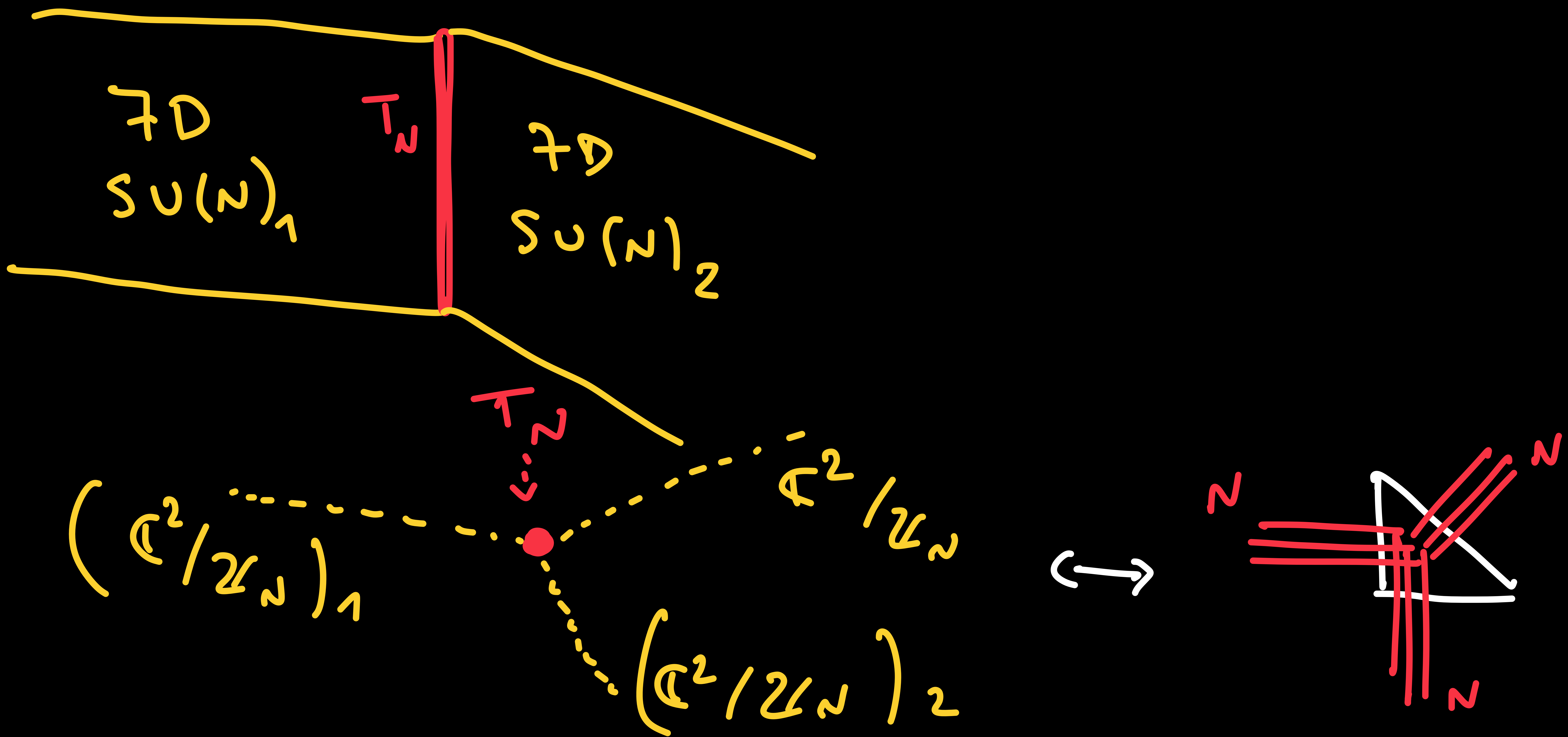


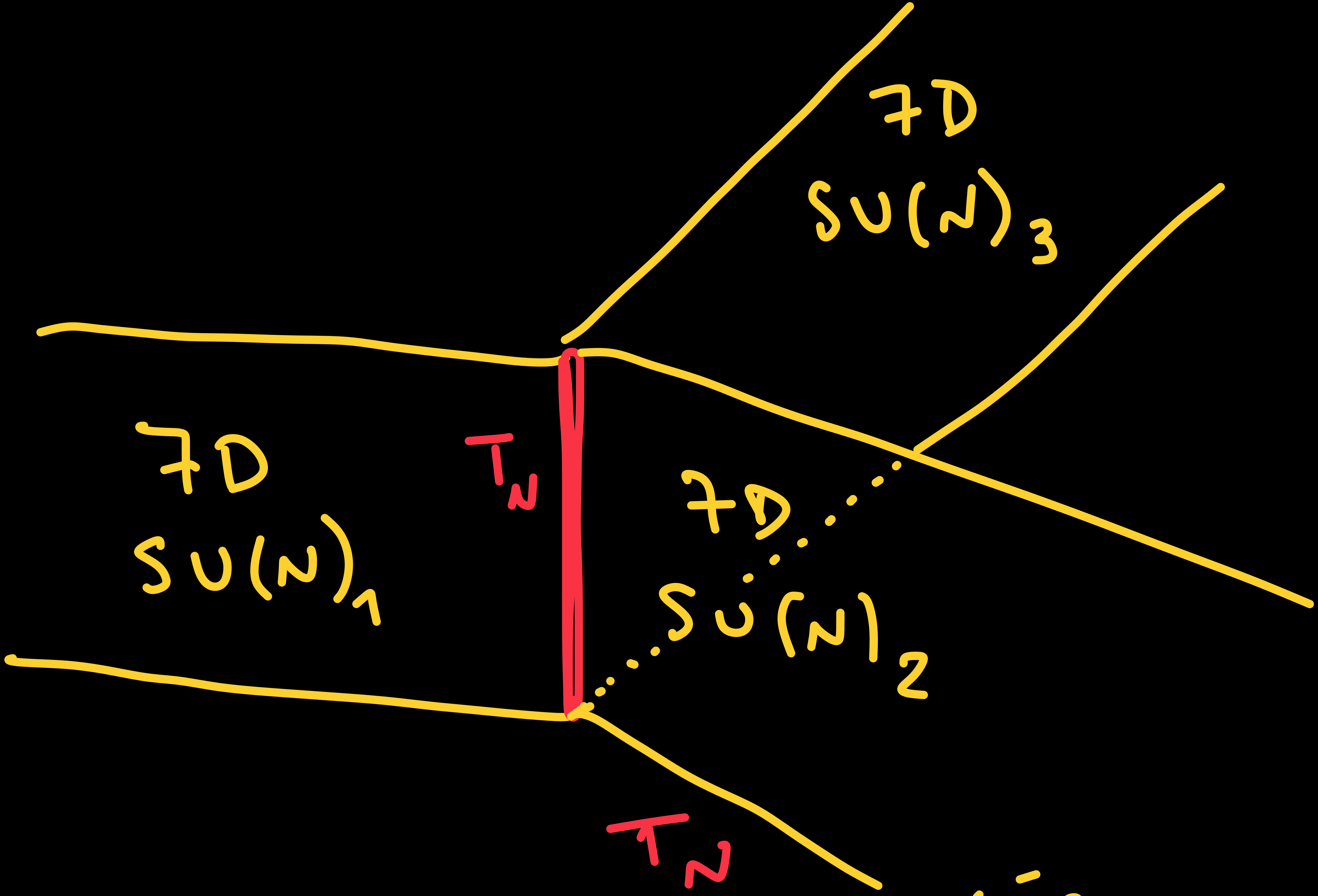
$\longleftrightarrow$



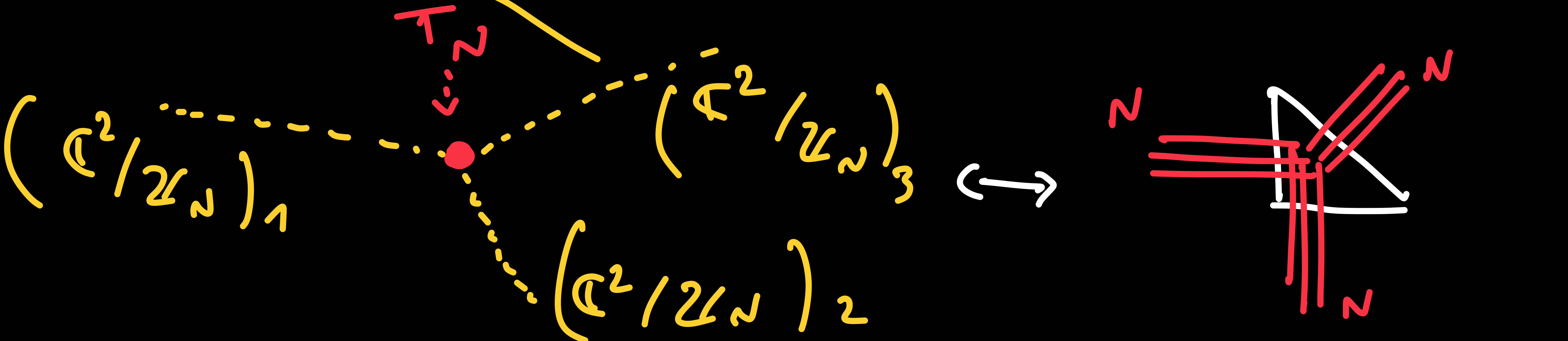

7D
$SU(N)_1$

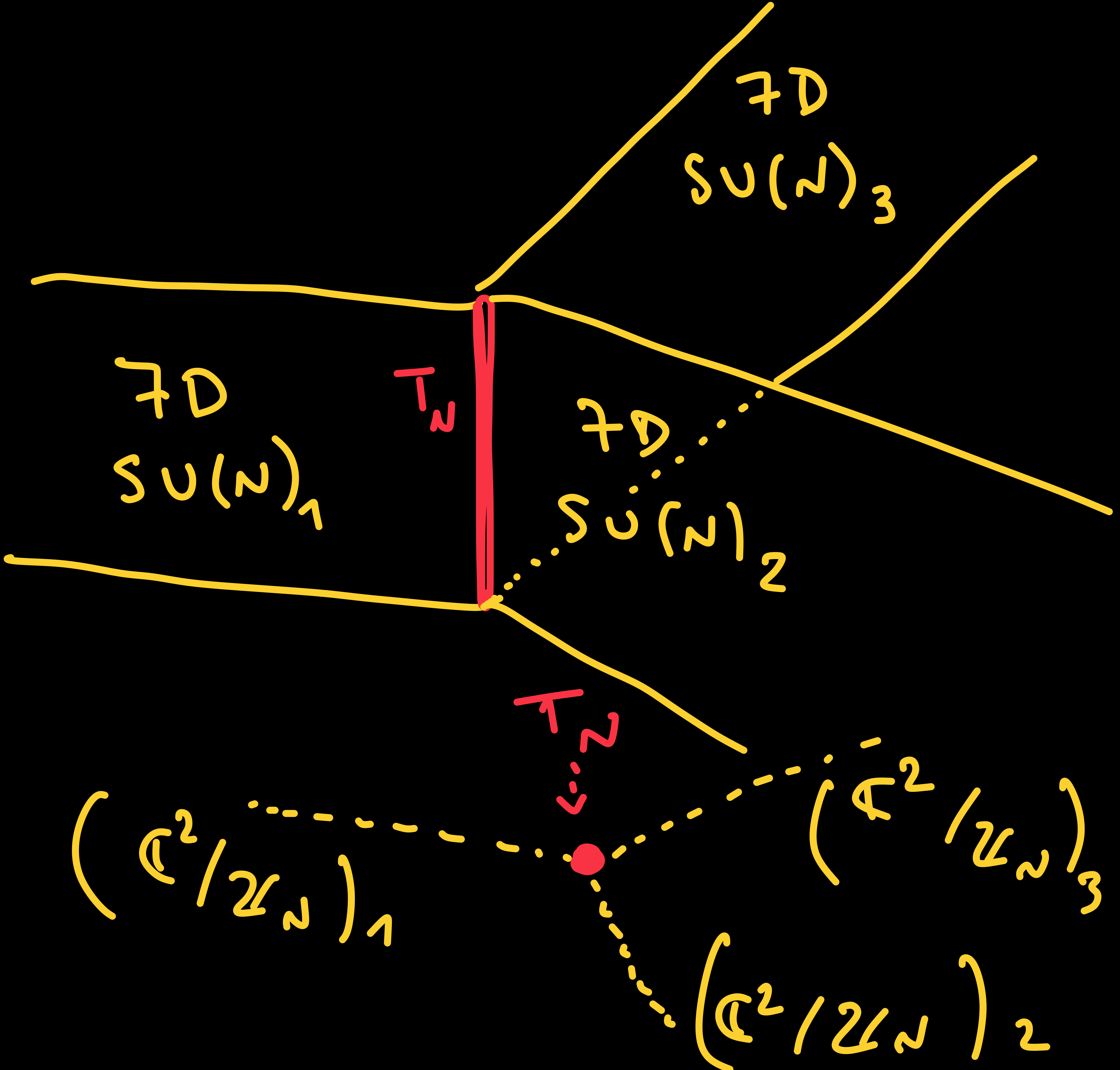

T_N
$(\mathbb{C}^2/\mathbb{Z}_N)_1$
$\mathbb{C}^2/\mathbb{Z}_N$
$\mathbb{C}^2/\mathbb{Z}_N$
$\longleftrightarrow$
N
N
N


7D
SU(N)$_1$
T_N
7D
SU(N)$_2$
T_N
$(\mathbb{C}^2/\mathbb{Z}_N)_1$
$\mathbb{C}^2/\mathbb{Z}_N$
$(\mathbb{C}^2/\mathbb{Z}_N)_2$
$\longleftrightarrow$
N
N
N



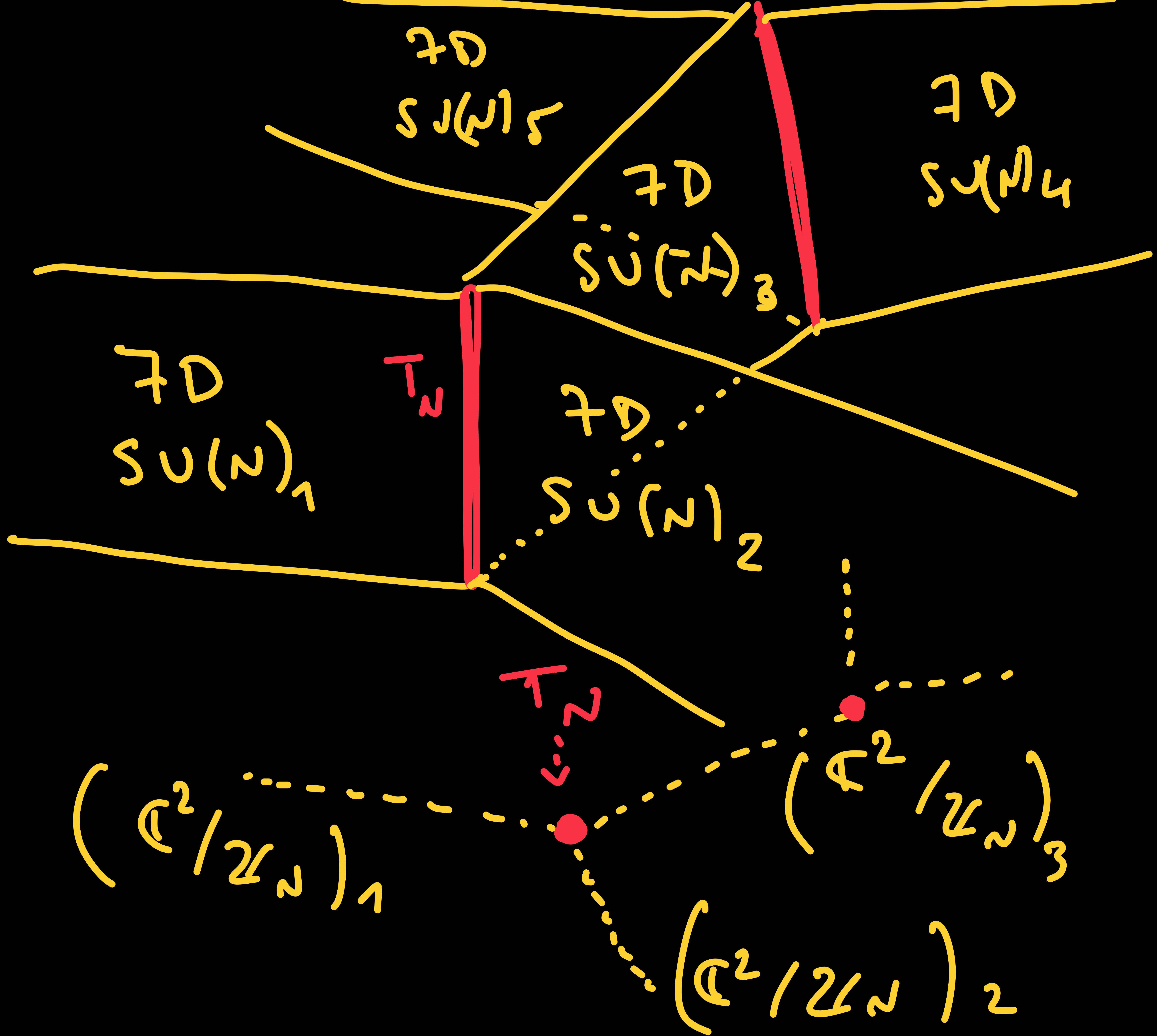
THE 5D T_N THEORY IS A CONFORMAL INTERFACE BETWEEN 7D THEORIES





THE 5D T_N THEORY IS A CONFORMAL INTERFACE BETWEEN 7D THEORIES

THE 7D DOFs DECOUPLE AND ENCODE GLOBAL SYMMETRY



GAUGING = GLUEING INTERFACES [HAYASHI, JEFFERSON, KIM, OHMORI, VAFA] '19

THE 5D T_N THEORY [APRUZZI, SCHÄFER NAMEKI, WANG '19]

CONFORMAL INTERFACE BETWEEN 7D THEORIES

THE 7D DOFs DECOUPLE AND ENCODE GLOBAL SYMMETRY

FOR G_2 - ORBIFOLDS OF X_2^7 AND X_3^7

FIXED LOCUS CAN HAVE:

- CODIMENSION 4 $\longleftrightarrow$ 7D THEORY M / X_7^4

FOR G_2-ORBIFOLDS OF X^7_2 AND X^7_3

FIXED LOCUS CAN HAVE:

- CODIMENSION 4 $\longleftrightarrow$ 7D THEORY M / X^4_7
- CODIMENSION 6 $\longleftrightarrow$ 5D THEORY M / X^6_n

e.g. $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$

FOR G_2 - ORBIFOLDS OF X^7_2 AND X^7_3

FIXED LOCUS CAN HAVE:

- CODIMENSION 4 $\longleftrightarrow$ 7D THEORY M / X^4_7

- CODIMENSION 6 $\longleftrightarrow$ 5D THEORY M / X^6_n

4D THEORY ARISES AS EDGE MODE OF INTRICATED NESTED 5D / 7D INTERFACES

e.g. $\mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N$


7A
7B
7D
JUNCTION
7D
7C


7A
7B
7D
JUNCTION
7D
T^{5D}_{ADS}
7C

7A
7B
7D
JUNCTION
7D
7C
T^{5D}_{ABD}
T^{5D}_{ADC}

7A
7B
7D
7D
JUNCTION
7C
T^{5D}_{ABD}
T^{5D}_{ADC}
T^{5D}_{ABC}

7A
7B
7D
JUNCTION
7D
7C
T^{5D}_{ABD}
T^{5D}_{BCD}
T^{5D}_{ADC}
T^{5D}_{ABC}

7A
7B
7D
JUNCTION
7D
7C
T^{5D}_{ABD}
T^{5D}_{BCD}
T^{5D}_{ADC}
T^{5D}_{ABC}
T^{4D}_{ABCD}

CRUCIAL QUALITATIVE DIFFERENCE :

CODIMENSION 6 : 5D SCFTs, THAT DO NOT DECOUPLE IN THE IR

WE OBTAIN $\boxed{\text{4D - 5D SYSTEMS}}$

CRUCIAL QUALITATIVE DIFFERENCE:

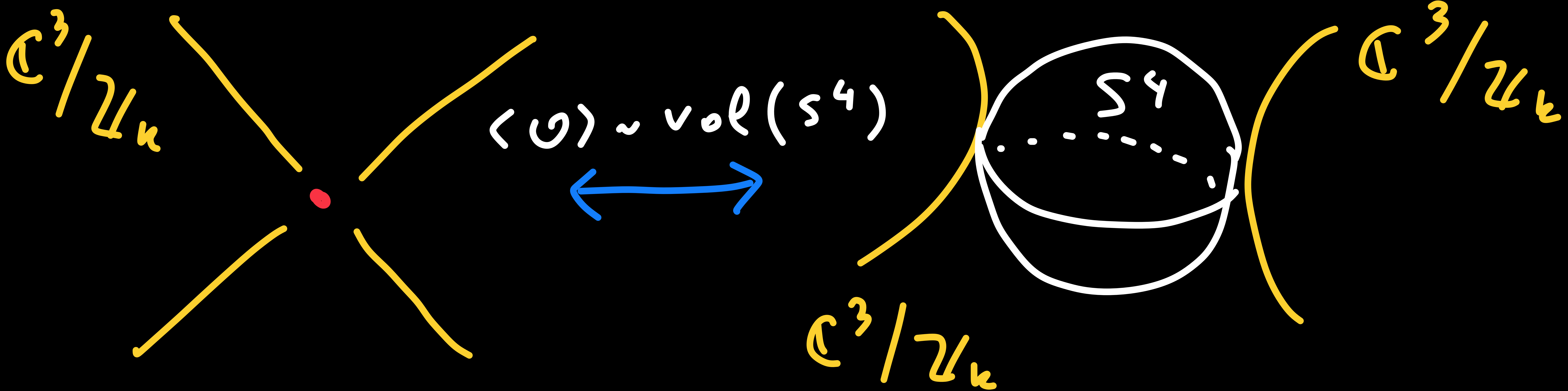
CODIMENSION 6 : 5D SCFTS, THAT DO NOT DECOUPLE IN THE IR

THIS ALLOWS — WE OBTAIN $\boxed{\text{4D - 5D SYSTEMS}}$

QUALITATIVELY NEW FEATURES:

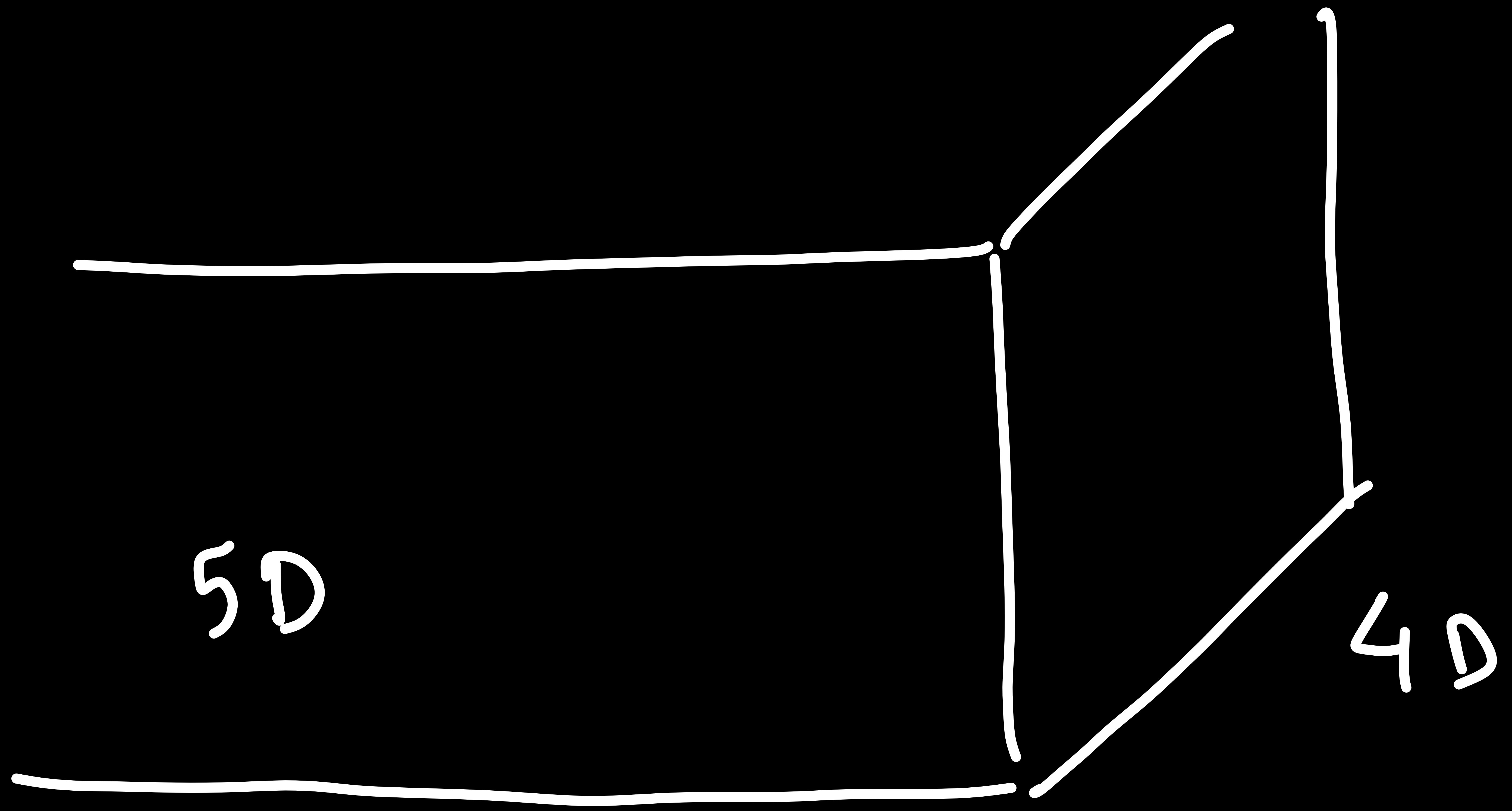
- SPONTANEOUS SYMMETRY BREAKING
- HIGHER SYMMETRY INHERITANCE
- 4D QUASI-SCFTS: NECESSARILY INTERACTING!

CRUCIAL QUALITATIVE DIFFERENCE:



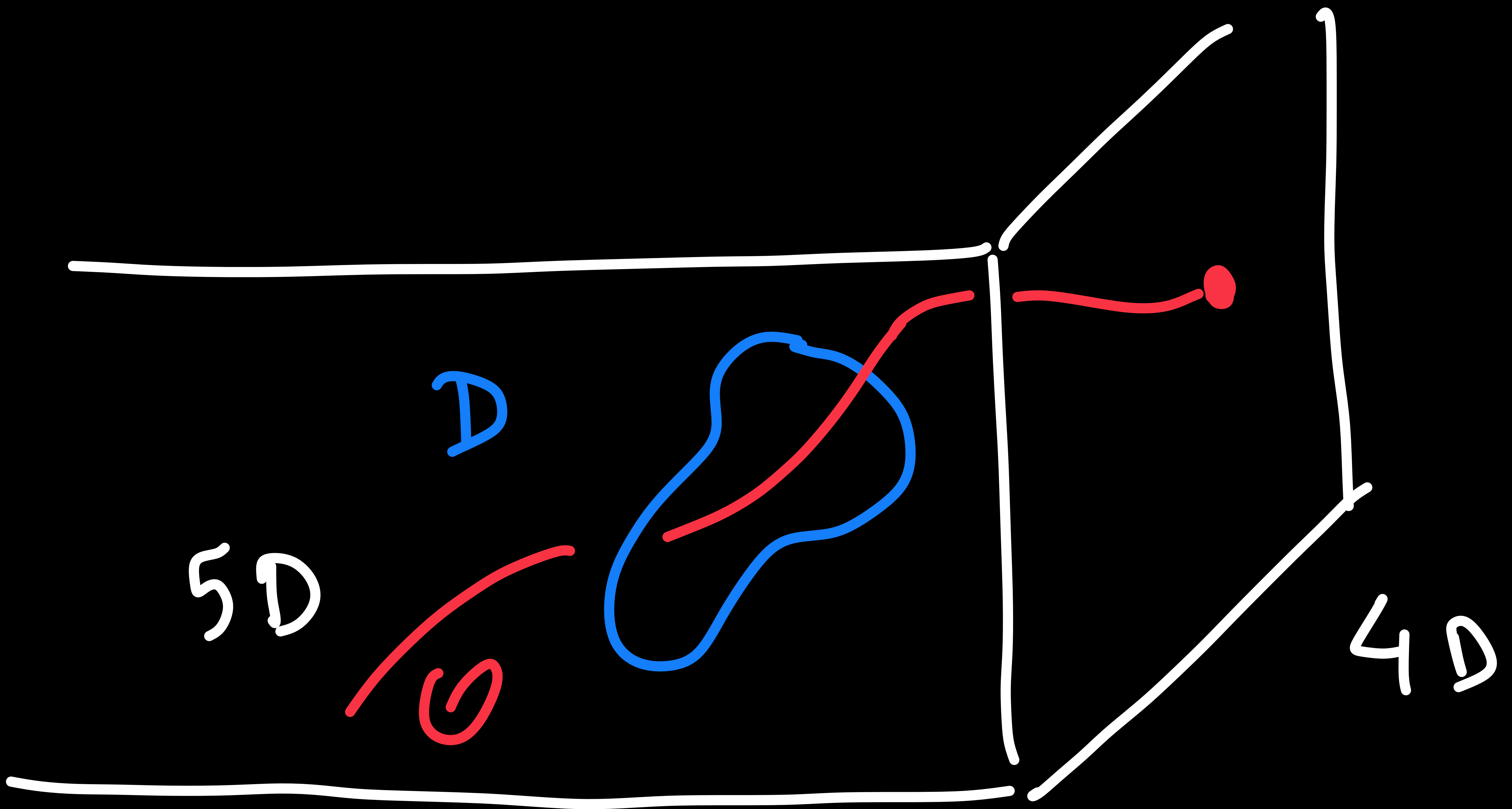
- SPONTANEOUS SYMMETRY BREAKING

CRUCIAL QUALITATIVE DIFFERENCE:



- HIGHER SYMMETRY INHERITANCE

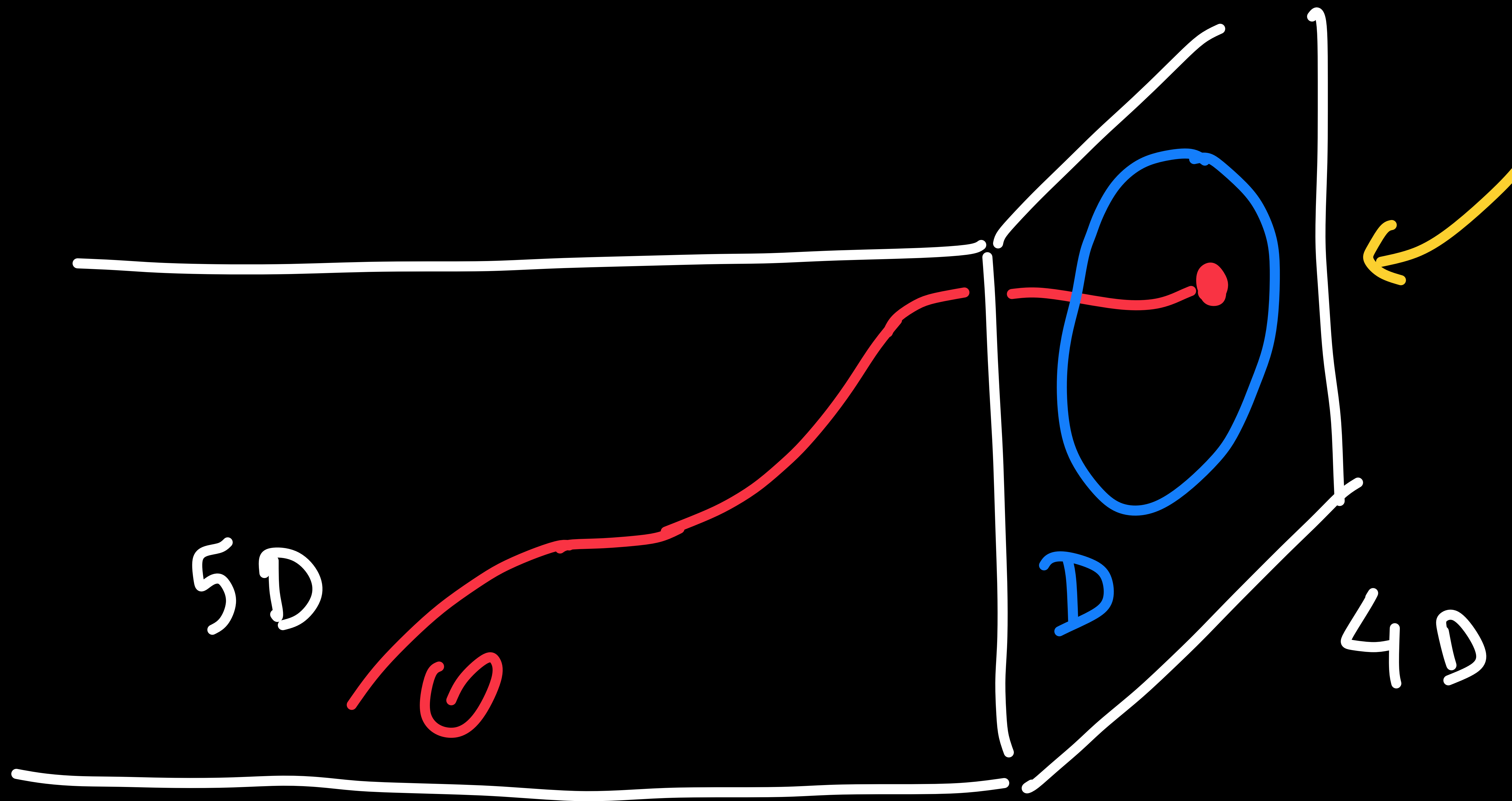
CRUCIAL QUALITATIVE DIFFERENCE:



- HIGHER SYMMETRY INHERITANCE

  5D 1-FORM
  SYMMETRY

CRUCIAL QUALITATIVE DIFFERENCE:

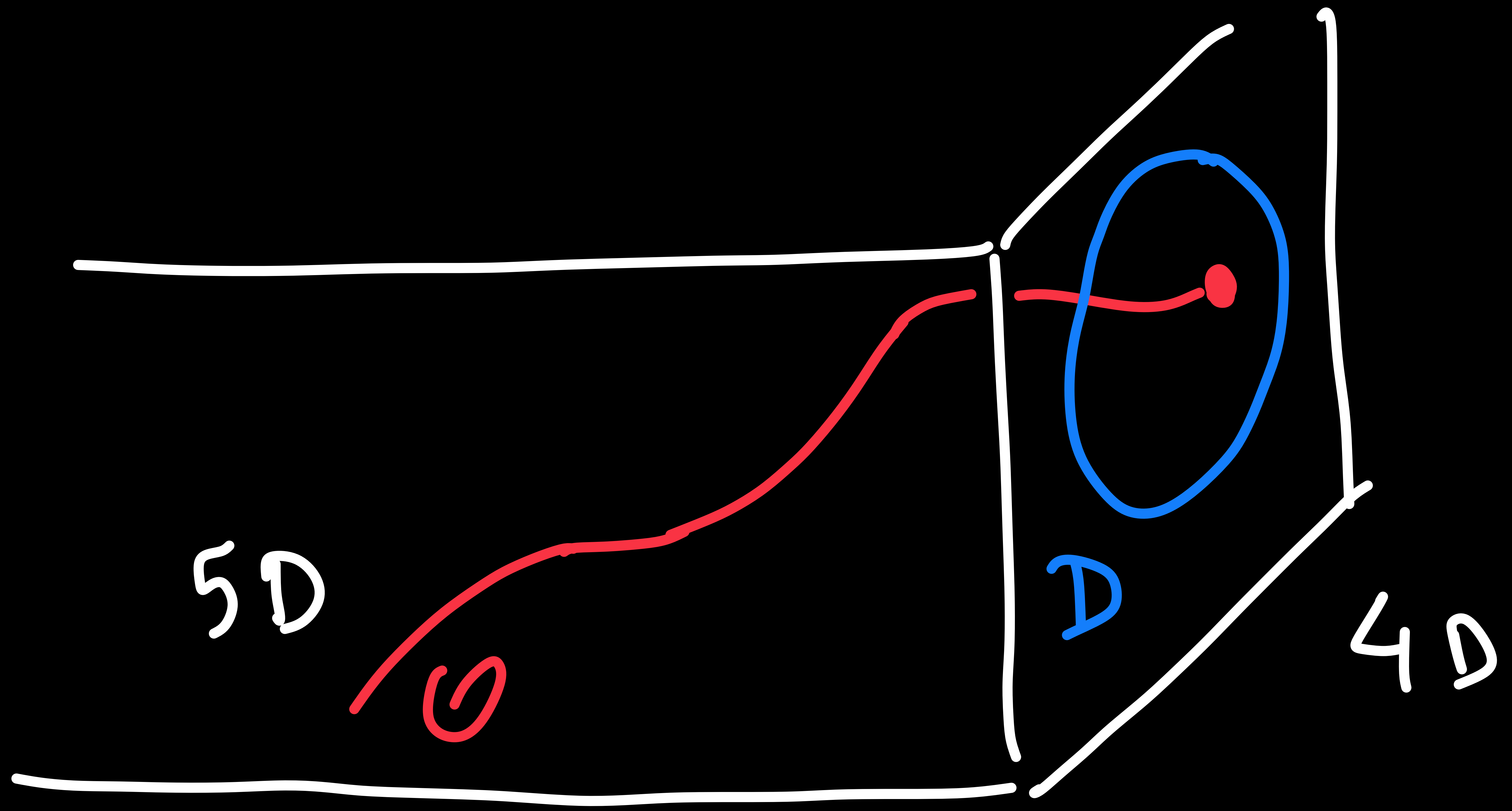


PROVIDED THIS BOUNDARY CONDITION EXISTS

- HIGHER SYMMETRY INHERITANCE

  5D 1-FORM SYMMETRY → 4D 0-FORM SYMMETRY

CRUCIAL QUALITATIVE DIFFERENCE:



EXPLICITLY DETERMINED FROM DEFECT GROUP

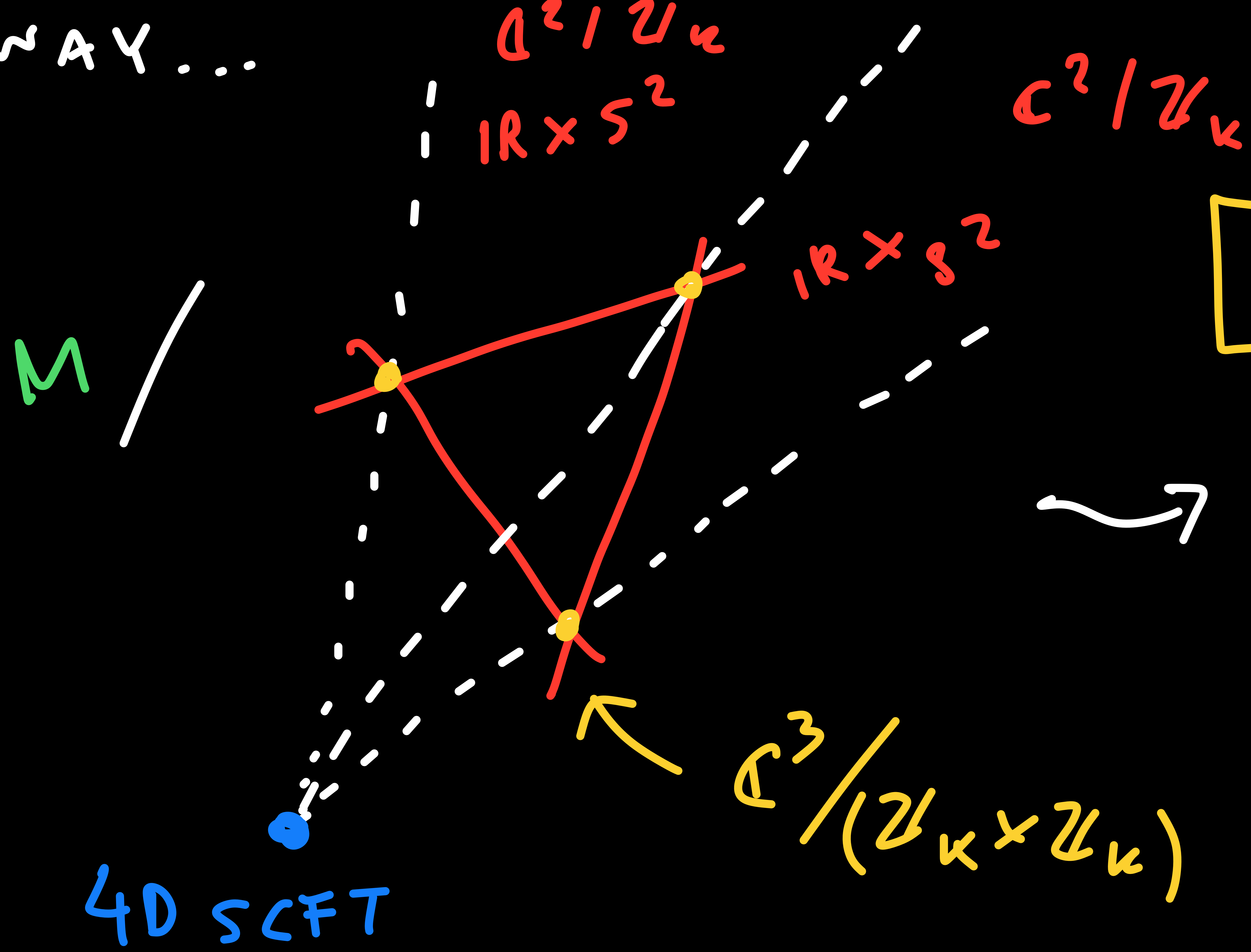
$$\mathbb{D}^{(n)} = \bigoplus_{\substack{p-k+1 \\ =n}} \frac{H_k(X,\partial X)}{H_k(X)}$$

p-BRANES

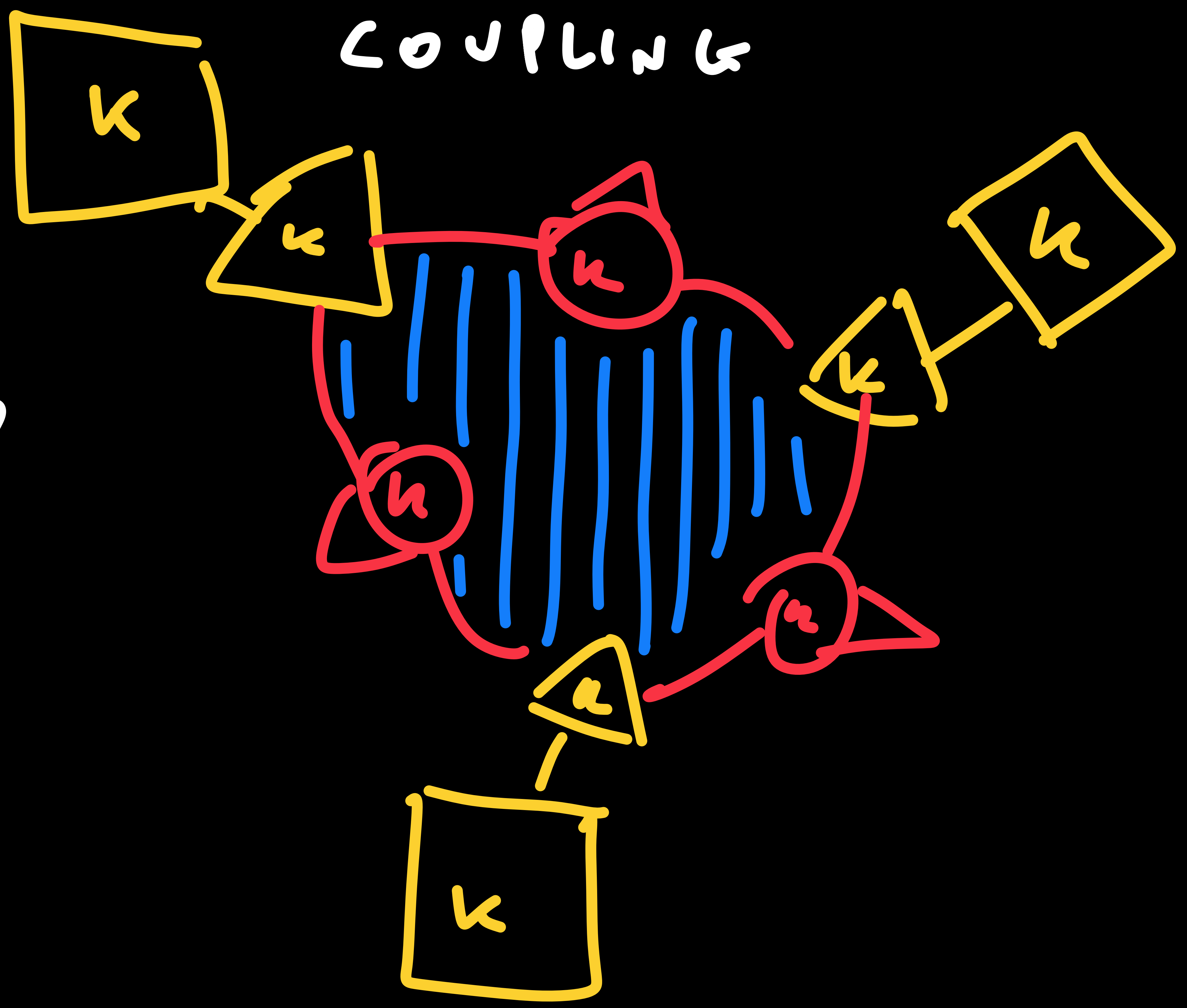
- HIGHER SYMMETRY INHERITANCE

  5D 1-FORM SYMMETRY $\longrightarrow$ 4D 0-FORM SYMMETRY

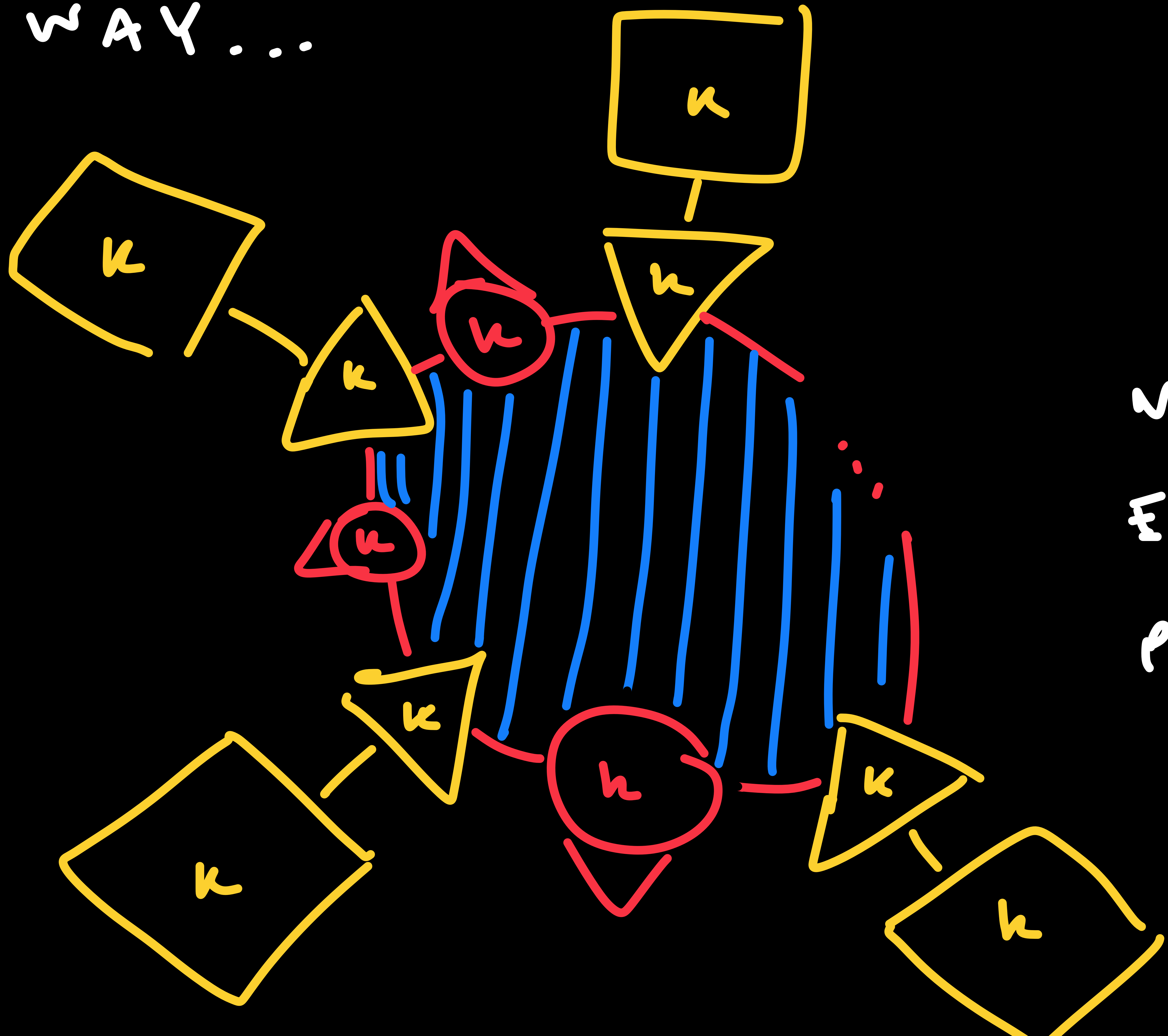
MANY NEW EXAMPLES OF 4D QUASI SCFT ARE GEOMETRICALLY ENGINEERED IN THIS WAY...



5D GAUGING W/ POSITION DEPENDENT COUPLING



MANY NEW EXAMPLES OF 4D QUASI SCFT ARE GEOMETRICALLY ENGINEERED IN THIS WAY...

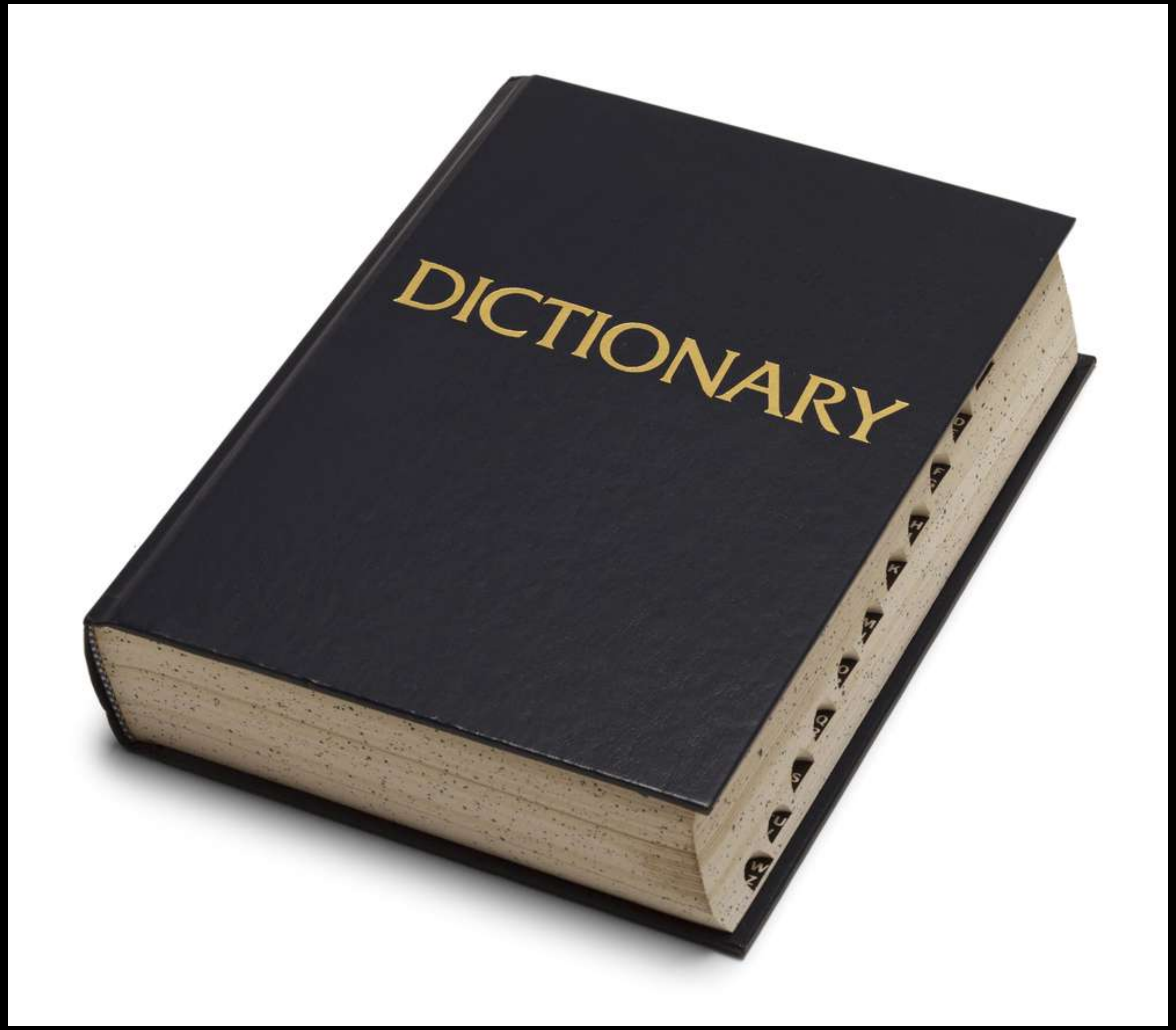


5D GAUGING w/ POSITION DEPENDENT COUPLING

MANY MORE DETAILED EXAMPLES IN THE PAPER [4]

① GEOMETRIC ENGINEERING LIMIT OF HETEROTIC STRINGS

[1-3]



② GEOMETRIC ENGINEERING OF M-THEORY ON SINGULAR G_2 HOLONOMY CONES

[4]

NAIVELY DISTINCT BUT $\exists$ INTERPLAY VIA DUALITY

IN THE FIRST PART OF THE SEMINAR:

$$\mathrm{Het}_H \;/\; \mathbb{C}^2/\Gamma_g$$

BUT HOW ABOUT

$$\mathrm{Het}_H \;/\; \mathbb{C}^3/\Gamma \qquad \Gamma \subseteq SU(3) \quad ?$$

IN THE FIRST PART OF THE SEMINAR:

$$\mathrm{Het}_H \,/\, \mathbb{C}^2/\Gamma_g$$

BUT HOW ABOUT

$$\mathrm{Het}_H \,/\, \mathbb{C}^3/\Gamma \qquad \Gamma \subseteq SU(3) \quad ?$$

T-DUALITY STILL HOLDS:

$$D_{S^1}\,\mathrm{Het}_{E_8\times E_8}/\mathbb{C}^3/\Gamma \xleftrightarrow{T} D_{S^1}\,\mathrm{Het}_{D_{16}}/\mathbb{C}^3/\Gamma$$

$\Rightarrow$ 4D LSTs [GIDEON, KUTASOV], BELOW LS SCALE 3D DUALITY [CLOSSET, SCHÄFER-NAMEKI, WANG]

IN THE FIRST PART OF THE SEMINAR:

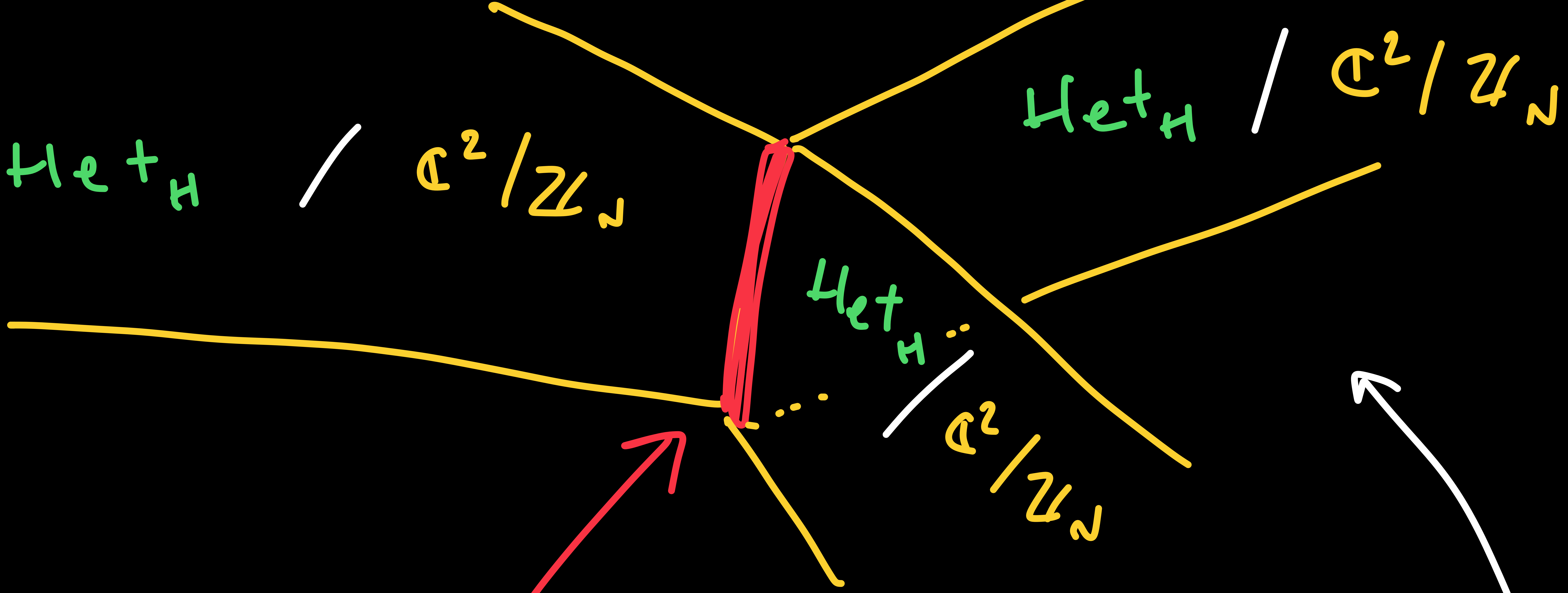
$$\mathrm{Het}_H \;/\; \mathbb{C}^2/\Gamma_g$$

BUT HOW ABOUT

$$\mathrm{Het}_H \;/\; \mathbb{C}^3/\Gamma \qquad \Gamma \subseteq SU(3) \quad ?$$

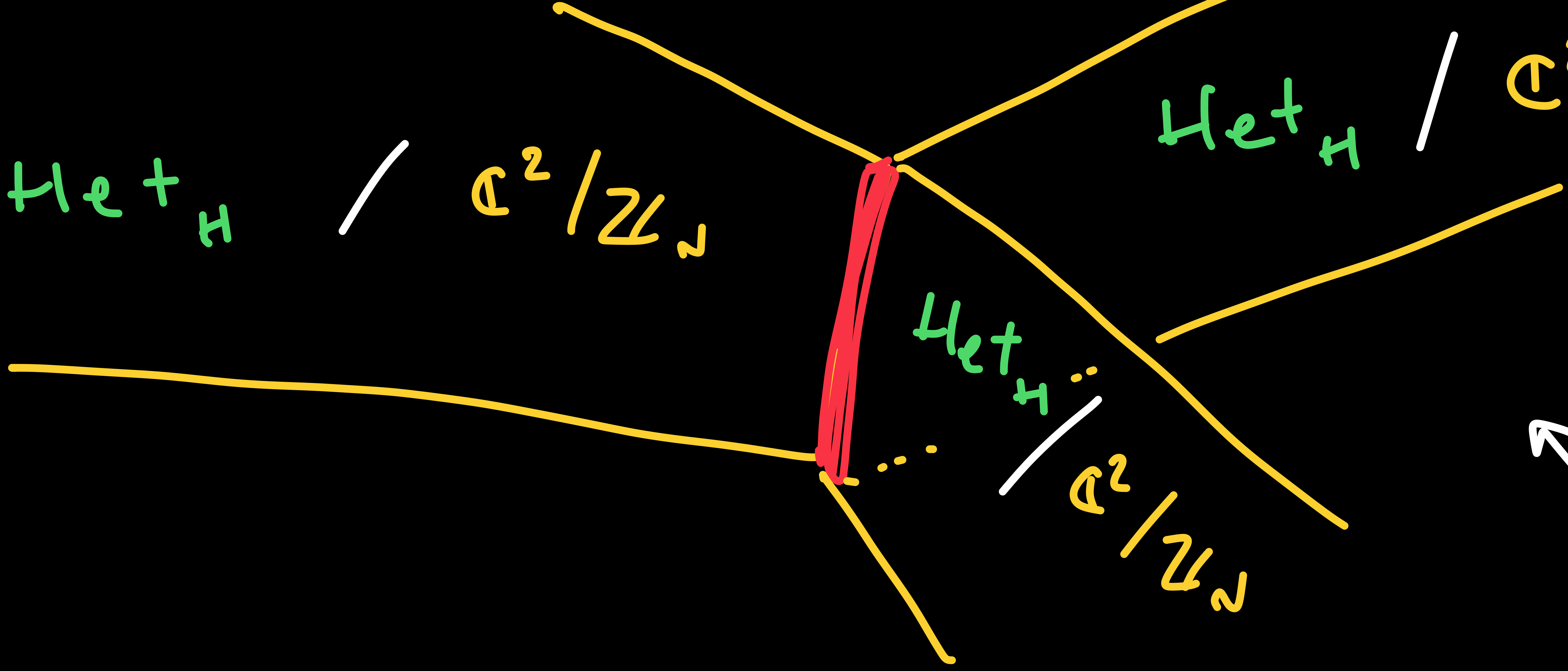
WHENEVER THE SINGULARITIES ARE NOT ISOLATED: THE SAME IDEA APPLIES

$$\mathrm{Het}_H \;/\; \mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_N) \quad \longleftrightarrow$$



WHENEVER THE SINGULARITIES ARE NOT ISOLATED: THE SAME IDEA APPLIES

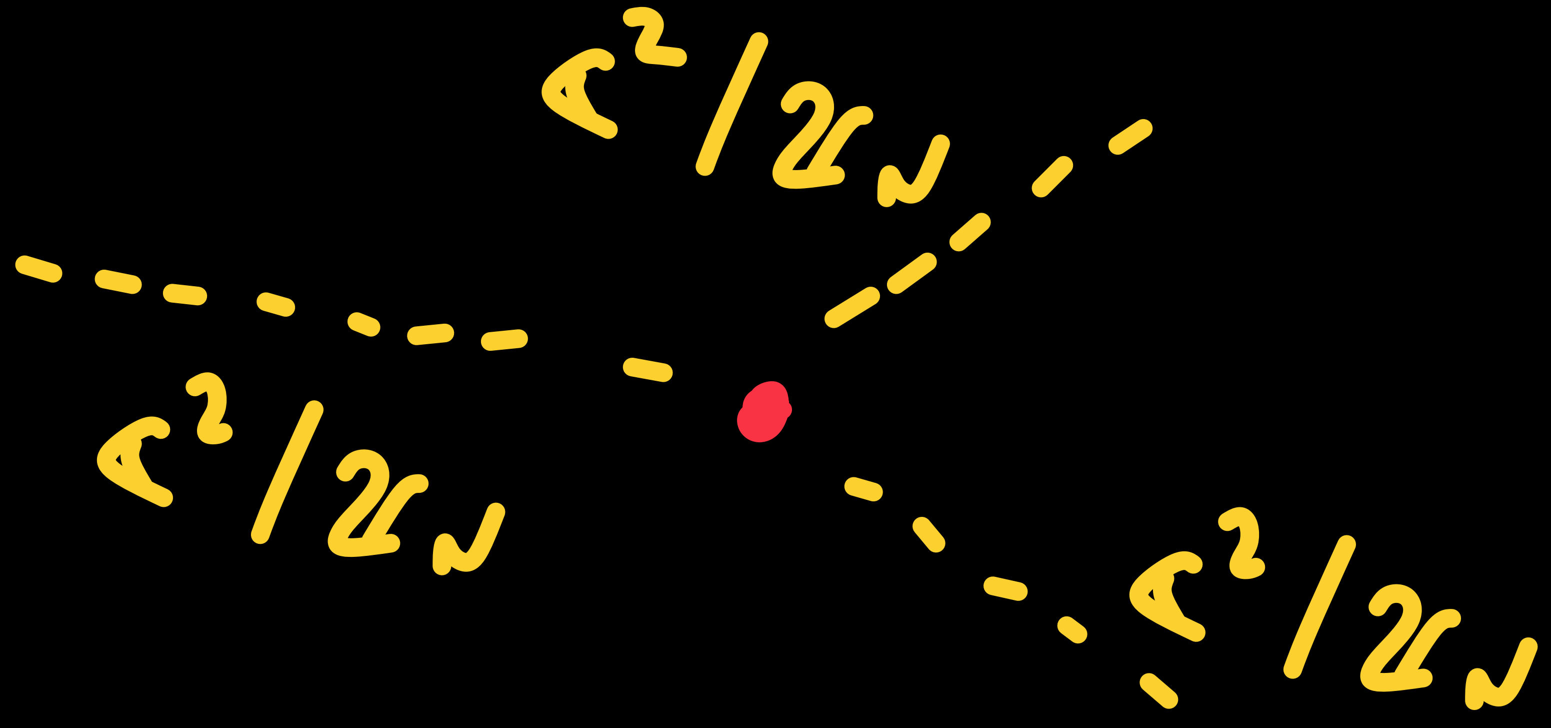
Het$_H$ / $\mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_N)$ $\longleftrightarrow$

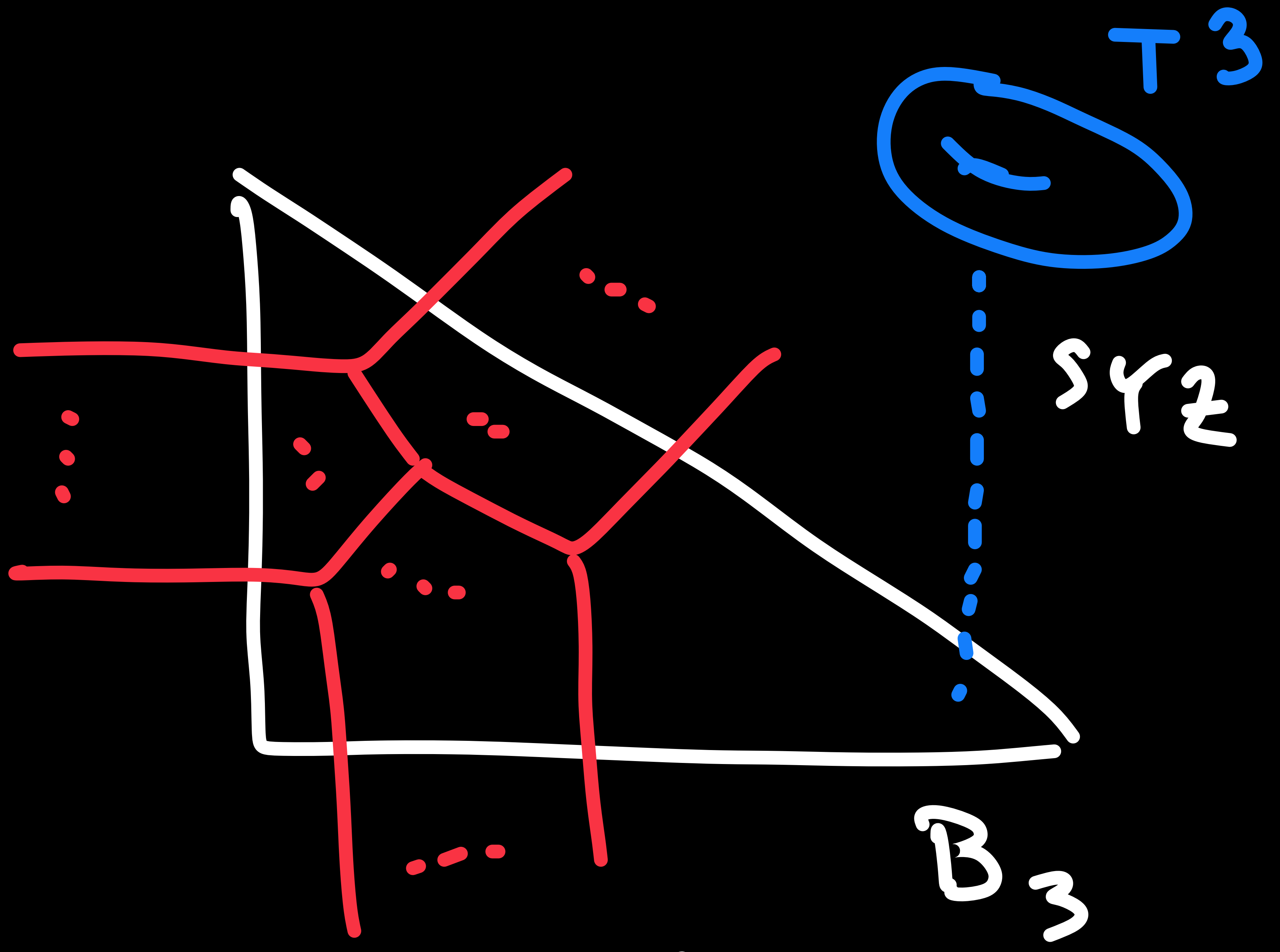


WE CAN ALSO DUALIZE

[AGANAGIC, VAFA, KLEMM]

$$\mathrm{Het}_H / \mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_N)$$

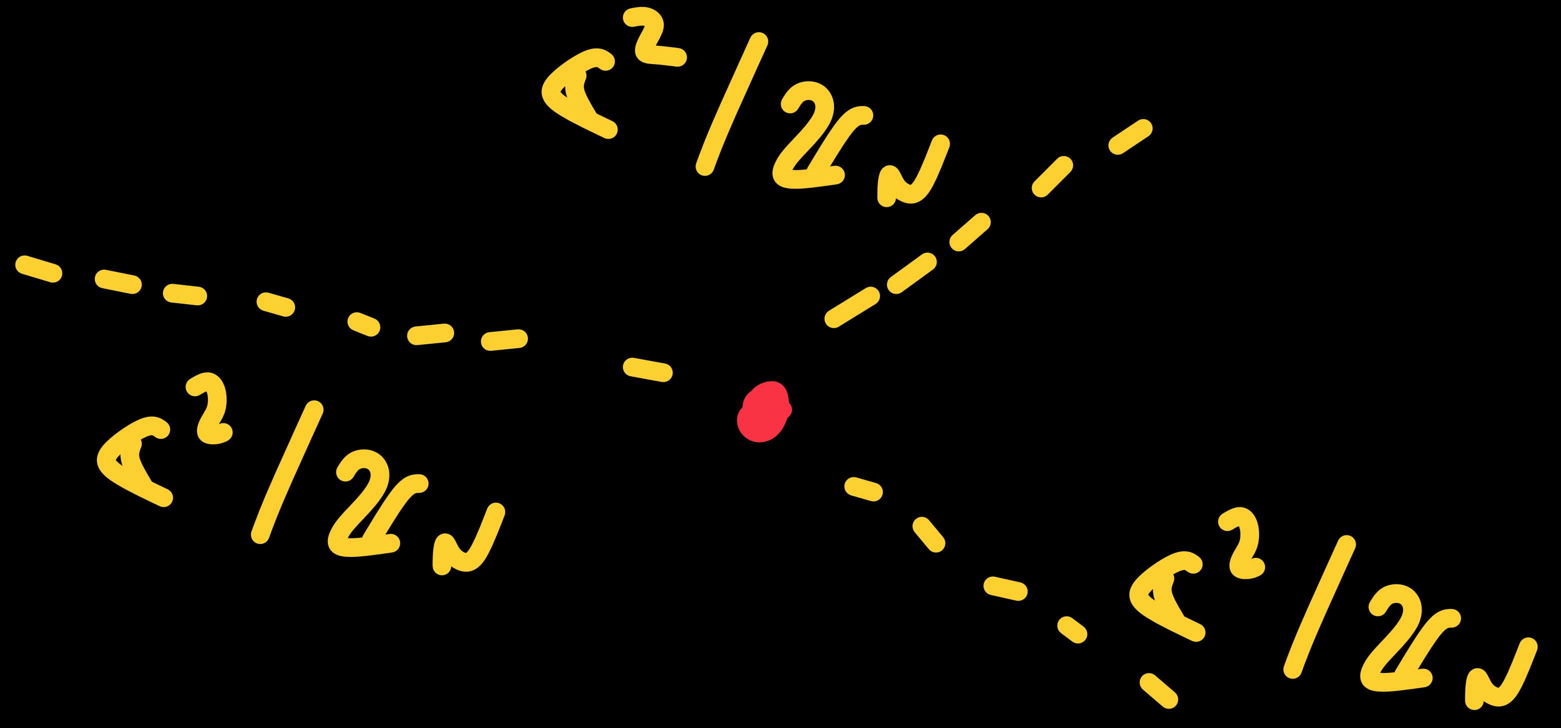




WE CAN ALSO DUALIZE

[AGANAGIC, VAFA, KLEMM]

Het_H / $\mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_N)$



M / G_2 ← K3

$\downarrow$

B_3

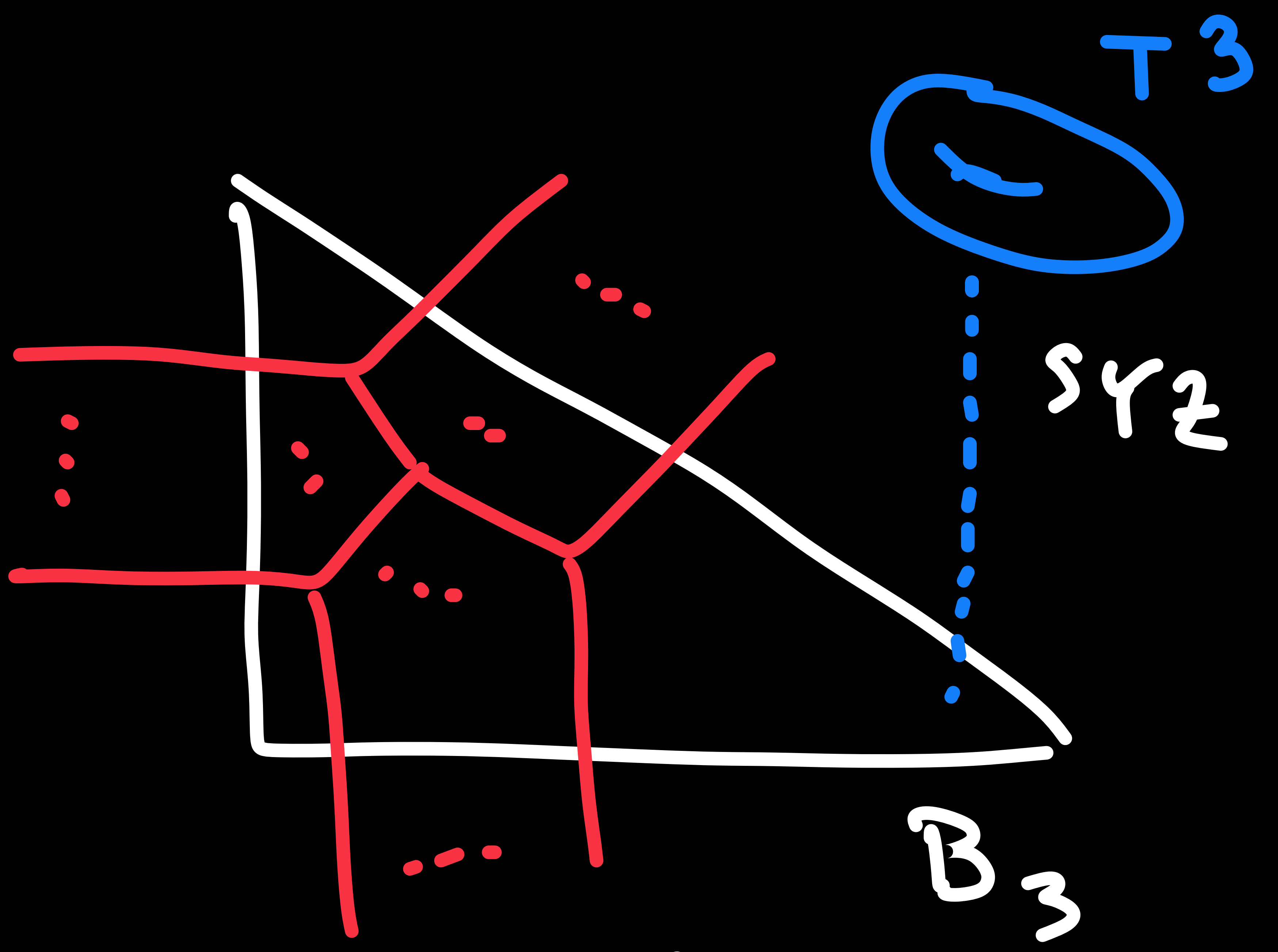
... local models of "toric" G_2's?

FIBERWISE

$\longleftrightarrow$

Het_F / T^3

$\equiv M / K3$

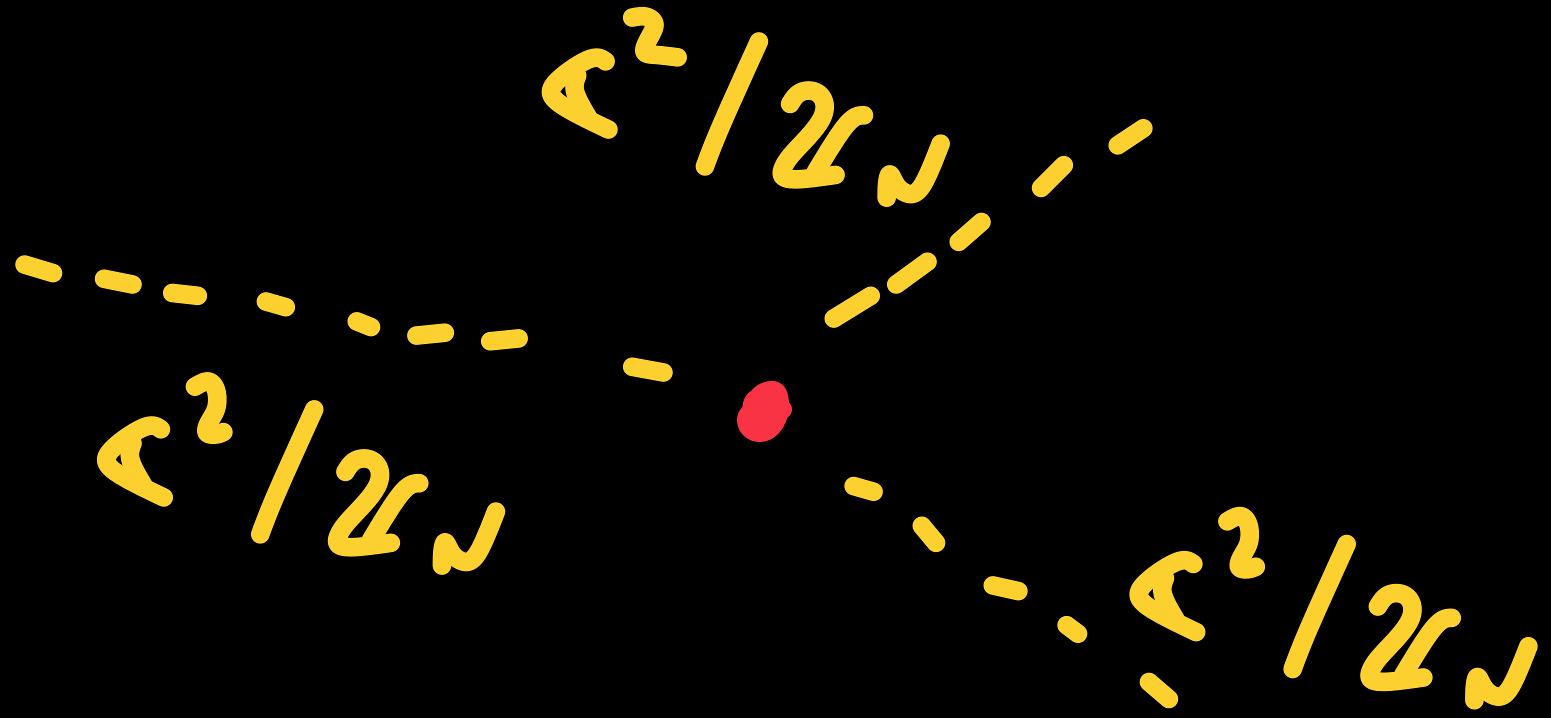


$\updownarrow$

WE CAN ALSO DUALIZE

[AGANAGIC, VAFA, KLEMM]

$Het_H / \mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_N) \longleftrightarrow$



IN SUMMARY: MANY UNCHARTED CORNERS OF GEOMETRIC ENGINEERING LANDSCAPE!

SCFT METHODS $\longleftrightarrow$ GENERALIZED SYMMETRIES

INTERPLAY

ADVANCES THE EXPLORATION

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THANK YOU FOR YOUR ATTENTION!

BACK UP

GENERALIZATION OF SYMMETRIES

KAPUSTIN THORNGREN 13

MANTRA: GAIOTTO KAPUSTIN SEIBERG WILLET 14

SYMMETRIES IN QFT ARE ENCODED BY SUBSECTOR OF (QUASI) TOPOLOGICAL OPERATORS WITH SUPPORTS OF VARIOUS CODIMENSIONS

GENERALIZATION OF SYMMETRIES

MANTRA #2:

FUSION $\longleftrightarrow$ GENERALIZES GROUP LAW

CAN INVOLVE TOPOLOGICAL OPERATORS OF DIFFERENT CODIMENSION

GENERALIZATION OF SYMMETRIES

ANOMALY TFT_{D+1} | QFT_D

$\searrow$ RG

ANOMALY TFT_{D+1} | QFT_D

CONSTRAINTS ON DYNAMICS

GENERALIZATION OF SYMMETRIES

ANOMALY TFT_{D+1} | SYMMETRY TFT_{D+1} | QFT_D

APRUZZI, BONETTI, GARCÍA ETXEBARRIA, HOSSEINI, SCHÄFER-NAMEKI 21

$\xrightarrow{\text{RG}}$

CONSTRAINTS ON DYNAMICS

ANOMALY TFT_{D+1} | SYMMETRY TFT_{D+1} | QFT_D

GAIOTTO, KULP 20 ; DZ, GARCÍA ETXEBARRIA 22

GENERALIZATION OF SYMMETRIES

ANOMALY TFT_{D+1} | QFT_D

DUAL

ANOMALY TFT_{D+1} | $\widetilde{\mathrm{QFT}}_D$

CONSTRAINTS ON DYNAMICS

GENERALIZATION OF SYMMETRIES

ANOMALY TFT_{D+1} | SYMMETRY TFT_{D+1} | QFT_D

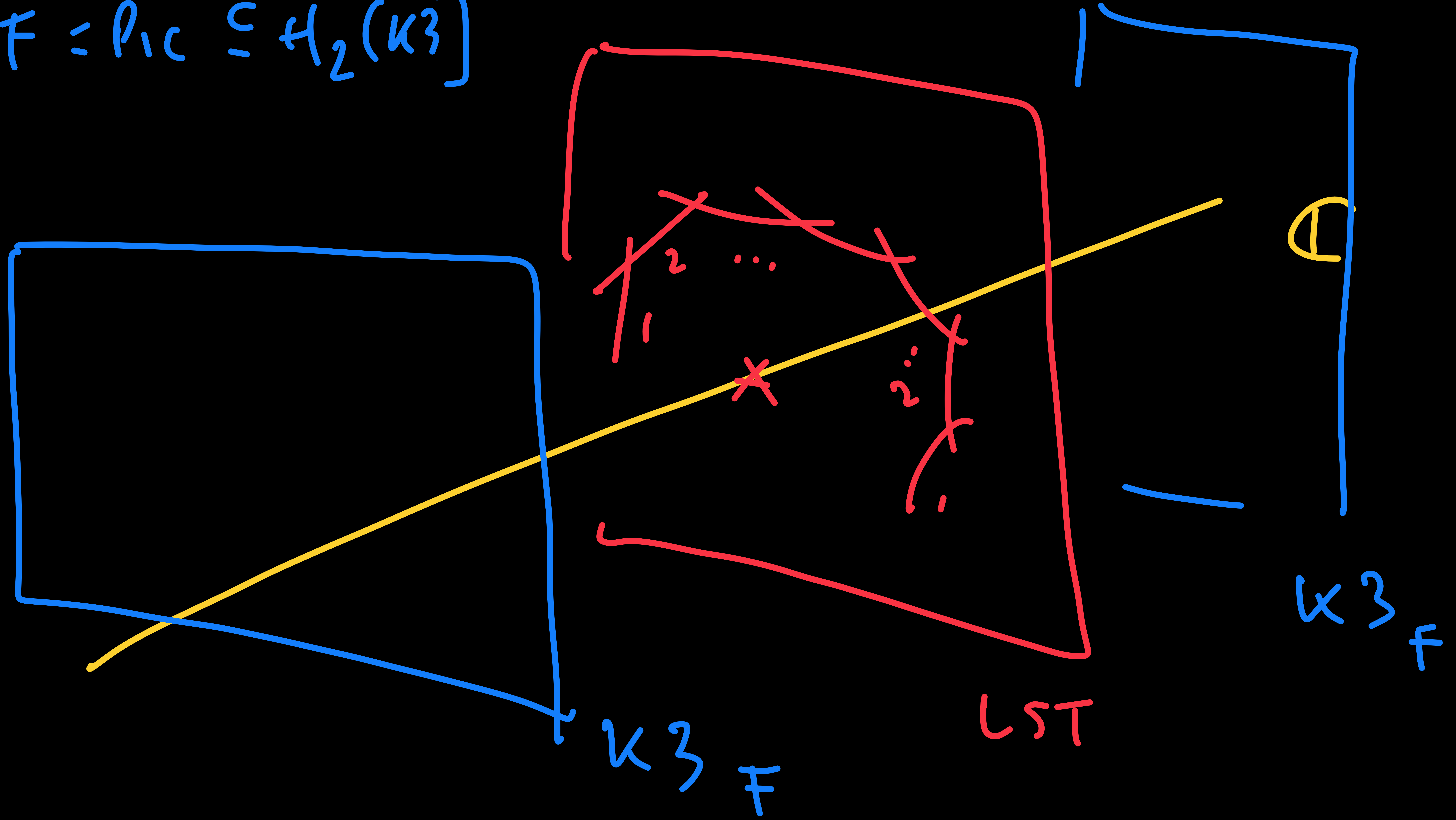
DUAL

CONSTRAINTS ON DYNAMICS

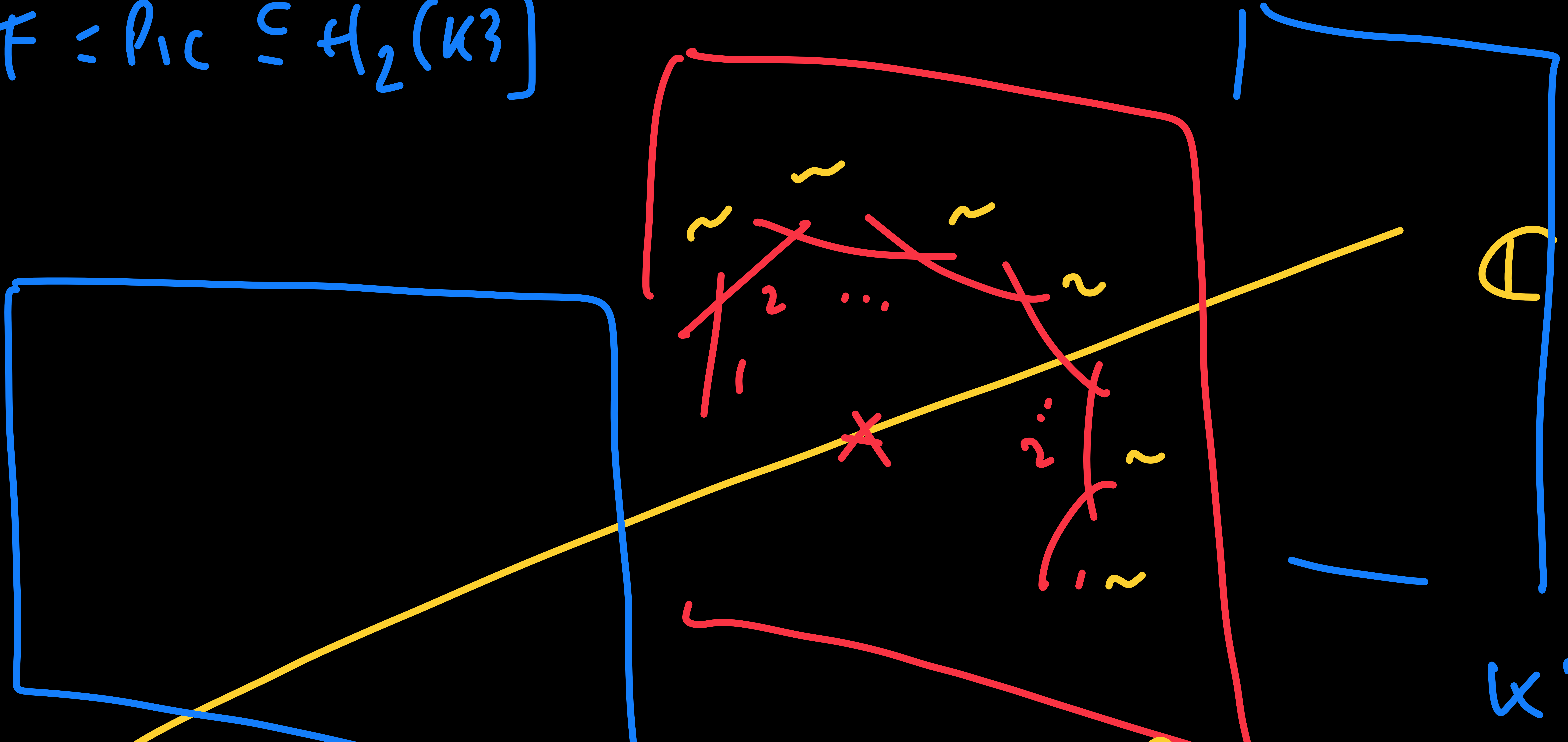
ANOMALY TFT_{D+1} | SYMMETRY TFT_{D+1} | $\widetilde{QFT}_D$

[DZ, OHMORI '20 ; OHMORI, TACHIKAWA '21]

$F = \mathrm{Pic} \subseteq H_2(K3)$



$\tilde{F} = \widetilde{\mathrm{Pic}} \subseteq H_2(K3)$



$\widetilde{K3}_F$