Light-cone sum rules for $B \to K\pi$ form factors and applications to rare decays

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based on arXiv:1908.02267 with A. Khodjamirian and J. Virto

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LPT Orsay

$B \to K\pi\ell\ell$

Generally discussed in term of $B \to K^*\ell\ell$ (+ S-wave contribution)

- K^* identified in a narrow experimental window
- treated as an (infinitely) narrow resonance in most LCSR and in latttice computations

Reasonable approximation as a start, but not enough at the level reached by current analyses

- What about the width of the K^* ?
- What impact from heavier resonances like $K^*(1430)$?

Issues already tackled in $B \to \pi\pi\ell\nu$
through $B \to \pi\pi$ form factors analysed using Light-Cone sum rules

[Cheng, Khodjamirian, Virto]

$\Longrightarrow$ Extension and application to $B \to K\pi\ell\ell$

[SDG, Khodjamirian, Virto]

$B \to K\pi$ form factors

$$i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(q+k)\rangle = F_\perp k_\perp^\mu + F_t k_t^\mu + F_0 k_0^\mu + F_\parallel k_\parallel^\mu$$
$$\langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu(1+\gamma_5)b|\bar{B}^0(q+k)\rangle = F_\perp^T k_\perp^\mu + F_0^T k_0^\mu + F_\parallel^T k_\parallel^\mu$$

with $k_t^\mu \propto q^\mu$ and $k_0^\mu, k_\parallel^\mu, k_\perp^\mu$ orthogonal basis

Partial-wave expansion where we focus on P wave ($\ell = 1$) using θ_K angle between $\vec{p}_\pi$ and $\vec{p}_B$ in $K\pi$ rest frame

$$F_{0,t}(k^2, q^2, q\cdot\bar{k}) = \sum_{\ell=0}^{\infty} \sqrt{2\ell+1}\, F_{0,t}^{(\ell)}(k^2, q^2)\, P_\ell^{(0)}(\cos\theta_K)$$
$$F_{\perp,\parallel}(k^2, q^2, q\cdot\bar{k}) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2, q^2)\, \frac{P_\ell^{(1)}(\cos\theta_K)}{\sin\theta_K}$$

where $\bar{k}^\mu = (k_1 - k_2)^\mu - (k_1 + k_2)^\mu \times (m_K^2 - m_\pi^2)/k^2$

LCSR with B-meson Distribution Amplitudes (1)

$$\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$$

where j_a interpolating current for $K\pi$ and j_b quark transition current

$$\mathcal{P}^{\mathrm{OPE}}_{ab}(k^2,q^2) = \frac{1}{\pi}\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{\mathrm{Im}\mathcal{P}_{ab}(s,q^2)}{s-k^2}\ .$$

- Hadronic ($k^2 > 0$): Im part from unitarity, by inserting set of states

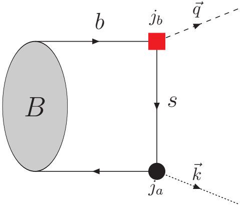
$$2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) = \sum_h \int d\tau_h\, \langle 0|j_a|h(k)\rangle\langle h(k)|j_b|\bar{B}^0(q+k)\rangle.$$

- OPE ($k^2 \ll 0$): Expansion near the light cone, with q^2, k^2 below hadronic thresholds

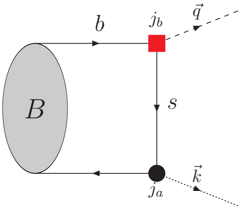
$$\mathcal{P}_{ab}(k^2,q^2) \simeq \mathcal{P}^{\mathrm{OPE}}_{ab}(k^2,q^2)$$

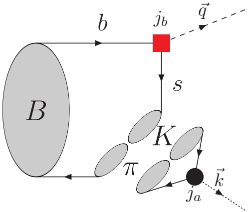
with B-meson Distribution Amplitudes of various twists (dim-spin)

LCSR with B-meson Distribution Amplitudes (2)



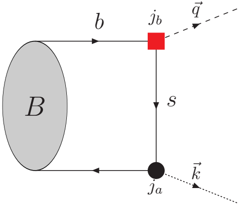
Correlator
Full information



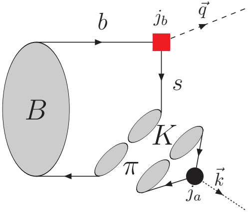


Correlator
Full information

Im part: $k^2 \geq 0$
Hadronic ff



Correlator
Full information



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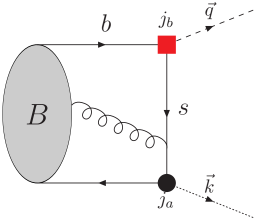
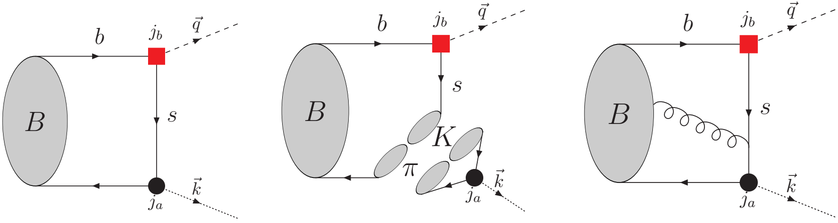


OPE: $k^2 \ll 0$
Hard $s \times B$-DA

LCSR with B-meson Distribution Amplitudes (2)

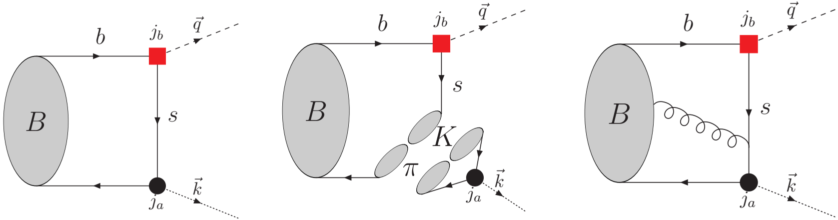


Correlator	Im part: $k^2 \geq 0$	OPE: $k^2 \ll 0$
Full information	Hadronic ff	Hard s $\times$ B-DA

- Quark-hadron duality: $\int_{s_0}^{\infty}$ OPE = sum over (many) states
- Borel tf: suppress $s \geq M^2$ to keep lightest state contribution

$$\frac{1}{\pi}\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2}\, \mathrm{Im}\mathcal{P}_{ab}(s,q^2) = \mathcal{P}_{ab}^{\rm OPE}(q^2,\sigma_0(s_0),M^2)$$

B-meson LCSR for $B \to K\pi$ form factors



Correlator
Full information

Im part: $k^2 \geq 0$
Hadronic ff

OPE: $k^2 \ll 0$
Hard $s \times B$-DA

$$\int_{s_{\rm th}}^{s_0} ds\, e^{-s/M^2} \omega_i(s,q^2)\, f_+^\star(s)\, F_i^{(T)(\ell=1)}(s,q^2) = \mathcal{P}_i^{(T),\rm OPE}(q^2,\sigma_0,M^2)$$

- $i = \{t, \perp, \|, -\}$; F_- comb of $F_\|$ and F_0, ω_i kinematic functions
- convolution of $B \to K\pi$ form factor F and $K\pi$ form factor f_+
- model to describe them with a few parameters to be fixed by LCSR

Model for the $K\pi$ form factor

$$\langle K(k_1)\pi(k_2)|\bar{s}\gamma^\mu d|0\rangle = \sum_{R,\eta} BW_R(k^2)\langle K\pi|R(k,\eta)\rangle\langle R(k,\eta)|\bar{s}\gamma^\mu d|0\rangle$$

with two resonances $R = K^*(892), K^*(1410)$ of polarisation η

$$f_+(s) = -\sum_R \frac{m_R\, f_R\, g_{RK\pi}\ e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

with a phase ϕ_R chosen so that all resonances yield the $K\pi$ phase

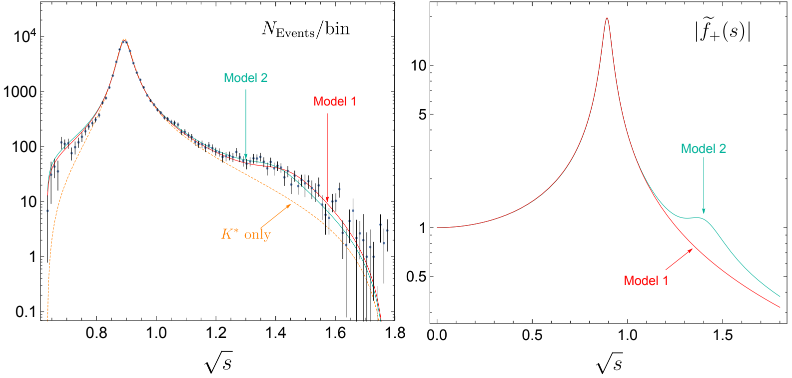
$$\tan\left[\delta_{K\pi}(s) - \phi_R(s)\right] = \frac{\sqrt{s}\,\Gamma_R(s)}{m_R^2 - s} \qquad f_+(s) = |f_+(s)|e^{i\delta_{K\pi}(s)}$$

Two models

From Belle analysis of $\tau \to K_S \pi \nu_\tau$ [0706.2231]

- Model 1: $K^*(890)$ only
- Model 2: $K^*(890) + K^*(1430)$

+ model for scalar contribution (used for Belle fit, not used here)



Model for the $B \to K\pi$ form factor

$$\langle K(k_1)\pi(k_2)|\bar{s}\Gamma b|B(q+k)\rangle = \sum_{R,\eta} BW_R(k^2)\langle K\pi|R(k,\eta)\rangle\langle R(k,\eta)|\bar{s}\Gamma b|B\rangle$$

$$F_i^{(T),(\ell=1)}(s,q^2) = \sum_R \frac{Y_{R,i}^{(T)}(s,q^2)\, g_{RK\pi}\, \mathcal{F}_{R,i}^{(T)}(q^2)\, e^{i\phi_R(s)}}{m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)}$$

- $i = \{t, \perp, \|, -\}$, $Y(s,q^2)$ and d kinematic form factors
- same ϕ_R as f_+, obeying unitarity $\mathrm{Im}[F_i^{(T),(\ell=1)}(s,q^2) f_+^*(s)] = 0$

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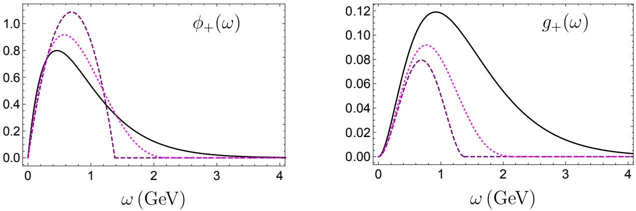
LCSR can be rewritten to constrain $\mathcal{F}_{R,i}^{(T)}(q^2)$

$$\sum_R d_{R,i}^{(T)}\, \mathcal{F}_{R,i}^{(T)}(q^2)\, I_R(s_0, M^2) = \mathcal{P}_i^{(T),\mathrm{OPE}}(q^2, \sigma_0, M^2)$$

with the overlap integral

$$I_R(s_0, M^2) = \frac{m_R}{16\,\pi^2}\int_{s_{\mathrm{th}}}^{s_0} ds\, e^{-s/M^2}\, \frac{g_{RK\pi}\,\lambda_{K\pi}^{3/2}(s)\,|f_+(s)|}{s^{5/2}\sqrt{(m_R^2 - s)^2 + s\,\Gamma_R^2(s)}}$$

Hadronic and OPE parameters



- *B*-meson LCDAs 2- and 3-particle, up to twist 4
  - OPE (including m_s) in agreement with [Gubernari, Kokulu, van Dyk]
  - 3 different models [Braun, Ji, Manashov]
  - ϕ_+ parametrised in term of $\lambda_B = 460 \pm 110$ MeV
  - higher twist models involve $R = \lambda_E^2/\lambda_H^2 = 0.4^{+0.5}_{-0.3}$

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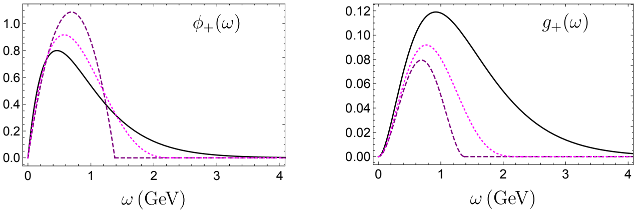

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- quark-hadron duality threshold s_0 [Khodjamirian, Mannel, Offen]
  - from QCD sum rule for $\langle 0|T(\bar{d}\gamma_\mu s)(\bar{s}\gamma_\nu d)|0\rangle$
  - around 1.3 GeV^2, lower than earlier estimates for K^* ($\simeq 1.7$ GeV^2)
  - above s_0, contrib from states with higher multiplicity ($K\pi\pi, K3\pi\ldots$)

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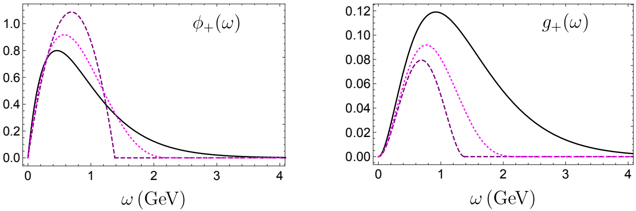

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  - around 1.3 GeV^2, lower than earlier estimates for K^* ($\simeq$ 1.7 GeV^2)
  - above s_0, contrib from states with higher multiplicity ($K\pi\pi, K3\pi\ldots$)
- Borel parameter M^2: 1-1.5 GeV^2 [Cheng, Khodjamirian, Virto] to limit in OPE
  - 3-particle contributions (less than 15% of 2-particle)
  - impact of quark-had duality (subtracted $\geq s_0$ less than 40% total)

Narrow-width limit

In the narrow width limit, the overlap integrals yield

$$I_R(s_0, M^2) \xrightarrow{\Gamma_R^{\rm tot}\to 0} 3\, m_R f_R\, \mathcal{B}(R \to K^+\pi^-)\, e^{-m_R^2/M^2} \ .$$

leading well-known expression for narrow-K^* form factors

$$3\, m_{K^*} f_{K^*}\, d^{(T)}_{K^*,i}\, \mathcal{F}^{(T)}_{K^*,i}(q^2)\, e^{-m_{K^*}^2/M^2}\, \mathcal{B}(K^* \to K^+\pi^-) = \mathcal{P}_i^{(T),\rm OPE}(q^2, \sigma_0, M^2)\ .$$

Form Factor	[This work]	[Khodj'06]	[Khodj'10]	[Gubernari'18]	[Straub'15]
$\mathcal{F}_{K^*,\perp}(0) = V^{BK^*}(0)$	0.26(15)	0.39(11)	0.36(18)	0.32(11)	0.34(4)
$\mathcal{F}_{K^*,\parallel}(0) = A_1^{BK^*}(0)$	0.20(12)	0.30(8)	0.25(13)	0.26(8)	0.27(3)
$\mathcal{F}_{K^*,-}(0) = A_2^{BK^*}(0)$	0.14(13)	0.26(8)	0.23(15)	0.24(9)	0.23(5)
$\mathcal{F}_{K^*,t}(0) = A_0^{BK^*}(0)$	0.30(7)	–	0.29(8)	0.31(7)	0.36(5)
$\mathcal{F}^T_{K^*,\perp}(0) = T_1^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,\parallel}(0) = T_2^{BK^*}(0)$	0.22(13)	0.33(10)	0.31(14)	0.29(10)	0.28(3)
$\mathcal{F}^T_{K^*,-}(0) = T_3^{BK^*}(0)$	0.13(12)	–	0.22(14)	0.20(8)	0.18(3)

Numerical difference wrt (Khodjamirian'06)

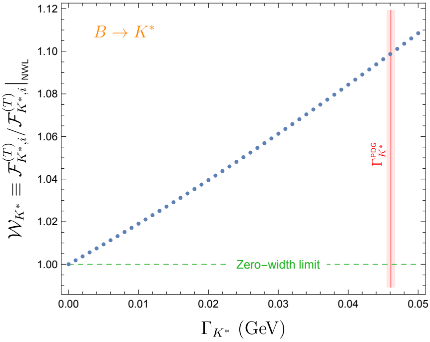
Results consistent within uncertainties, but our result is lower

- numerical inputs (decay constants...)
- lower value for the effective threshold s_0
- three-particle contributions
- twist-four two-particle contributions (in particular from g_+)

$\mathcal{F}^{BK^*}(q^2=0)$	V^{BK^*}	$A_1^{BK^*}$	$A_2^{BK^*}$	$A_0^{BK^*}$	$T_{1,2}^{BK^*}$	$T_3^{BK^*}$
[Khodjamirian'06]	0.39	0.30	0.26	–	0.33	–
[Khodjamirian'06] inputs, no g_+	0.38	0.29	0.26	0.31	0.33	0.25
[Khodjamirian'06] inputs, with g_+	0.27	0.21	0.14	0.31	0.24	0.14
Our inputs, $s_0 = 1.7\,\mathrm{GeV}^2$	0.33	0.26	0.17	0.38	0.29	0.17
Our inputs, our s_0, no g_+	0.36	0.28	0.25	0.30	0.31	0.23
Our inputs, our s_0, with g_+	0.26	0.20	0.14	0.30	0.22	0.13

Beyond the narrow-width approximation for K^*

LCSR with K^* only: $\mathcal{F}^{(T)}_{K^*,i}(q^2)\, d^{(T)}_{K^*,i}\, I_{K^*}(s_0, M^2) = \mathcal{P}_i^{(T)\text{OPE}}(q^2, \sigma_0, M^2)$.



Width induces universal shift in normalisation from overlap integral I_{K^*}

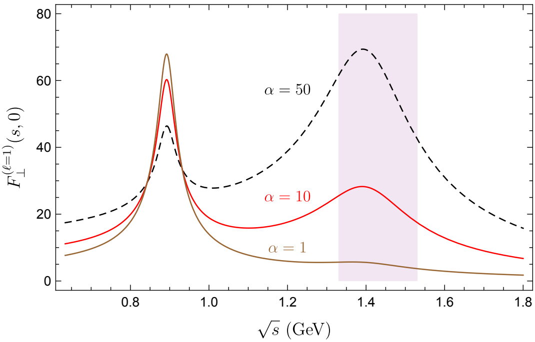
(effect depending mainly on s_0, M^2 and Γ_{K^*})

$$\mathcal{W}_{K^*} \equiv \frac{\mathcal{F}^{(T)}_{K^*,i}(q^2)}{\mathcal{F}^{(T)}_{K^*,i}(q^2)_{\text{NWL}}} = \frac{I_{K^*}(s_0, M^2)\big|_{\Gamma_{K^*}\to 0}}{I_{K^*}(s_0, M^2)} \simeq 1 + 1.9\,\frac{\Gamma_{K^*}}{m_{K^*}}$$

Correction $\simeq$ 10 % (increasing the discrepancy data-SM for $B \to K^*\mu\mu$)

Role of higher resonances in LCSR

- LCSRs mostly sensitive to $B \to K\pi$ form factors for $s \simeq m_{K^*} \pm \Gamma_{K^*}$
- But constrain combination of $K^*(892)$ and $K^*(1410)$ contributions with relative weights from $I_{K^*(1410)}/I_{K^*(892)} \simeq 0.03$
- $K^*(1410)$ with significant weight if its contribution $\mathcal{F}_{K^*(1410)}$ at least an order or magnitude larger than $\mathcal{F}_{K^*(892)}$



Impact ? assume $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$

- For $\alpha = 1$, $\mathcal{F}_{K^*,\perp}(0) = 0.28$ ($\simeq$ narrow width)
- (Much) larger α OK with LCSR, but reduce $\mathcal{F}_{K^*,\perp}(0)$

$B \to K\pi\ell\ell$ differential decay rate

Compute with $B \to K\pi$ factors

- Same angular structure as $B \to K^*(\to K\pi)\ell\ell$ decay rate (obvious since $K\pi$ in P-wave only)
- Angular coefficients J_i expressed as interferences among helicity amplitudes $\widehat{A}$ defined through

$$\widehat{A}_\perp^{L,R} = \frac{\sqrt{\lambda_{K\pi}}}{k^2}\mathcal{A}_\perp^{L,R(\ell=1)}\,, \qquad \widehat{A}_\parallel^{L,R} = \frac{\sqrt{\lambda_{K\pi}}}{k^2}\mathcal{A}_\parallel^{L,R(\ell=1)}\,,$$

$$\widehat{A}_0^{L,R} = -\mathcal{A}_0^{L,R(\ell=1)}/\sqrt{2}\,, \qquad \widehat{A}_t = -\mathcal{A}_t^{(\ell=1)}/\sqrt{2}\,.$$

with the P-wave of amplitudes involving $B \to K\pi$ form factors $F^{(T)}$

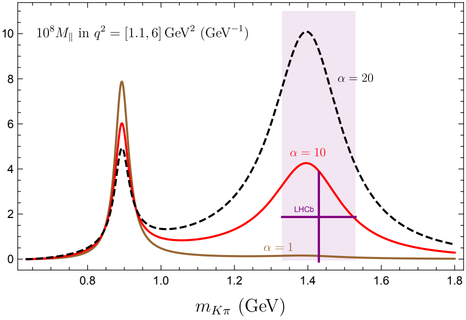
$$\mathcal{A}_i^{L,R} = \mathcal{N}\left[(C_9 \mp C_{10})F_i + \frac{2m_b}{q^2}\left\{C_7 F_i^T - i\frac{16\pi^2}{m_b}\mathcal{H}_i\right\}\right]\,, \quad i = \{\perp, \parallel, 0, t\}$$

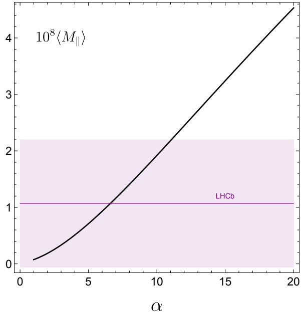
$\Longrightarrow$Can be evaluated, neglecting non-local $c\bar{c}$ contribution $\mathcal{H}_i$

Comparison with LHCb results (1609.04736)

- $B \to K\pi\ell\ell$ decay around $K^*(1430)$, for $\sqrt{k^2} \in [1.33, 1.53]\,\mathrm{GeV}$
- Angular coefficients for $q^2 \in [1.1, 6]\,\mathrm{GeV}^2$, BR in several other bins

- 4 combinations of angular coefficients only sensitive to P-wave
- BR sum of squares of partial waves, thus bound the P-wave part





$\Longrightarrow$upper bounds for α (here for combination of ang coeff), from 3 to 18
(still room for a decrease by $\sim$ 10% of the K^* contribution)

Outlook

$B \to K\pi$ form factors interesting tool to describe hadronic dynamics

- LCSR based on B-meson distribution amplitudes
- Recover structure of earlier narrow-width results for K^* only

Several applications of the LCSRs for $B \to K\pi$ form factors

- Numerical differences coming from nonperturbative inputs
- Universal effect from K^* width, enhancing form factors by 10%
- $K^*(1430)$ contribution bound by LHCb results, but not negligible
- Caution with systematic uncertainties in LCSR

Open questions

- Better understanding of numerical impact of twist-4 contribution
- Impact of other models for f_+ and $B \to K\pi$ form factors
- Inclusion of S wave (dedicated LCSR and form factors)

[SDG, Khodjamirian, Virto, Vos] in progress