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Feynman's proof of the Maxwell equations

Freeman J. Dyson
Institute for Advanced Study, Princeton, New Jersey 08540

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Feynman's proof of the Maxwell equations, discovered in 1948 but never published, is here put on record, together with some editorial comments to put the proof into its historical context.

I. THE PROOF

As I mentioned in my talk at the Feynman Memorial Session of the AAAS meeting in San Francisco,[1] Feynman showed me in October 1948 a proof of the Maxwell equations, assuming only Newton's law of motion and the commutation relation between position and velocity for a single nonrelativistic particle. In response to many enquiries, I here publish the proof in a form as close as I can come to Feynman's 1948 exposition. Unfortunately, I preserved neither Feynman's manuscript nor my original notes. What follows is a version reconstructed at some unknown time from notes which I discarded.

Assume a particle exists with position x_j ($j = 1,2,3$) and velocity $\dot{x}_j$ satisfying Newton's equation

$$m\ddot{x}_j = F_j(x,\dot{x},t), \tag{1}$$

with commutation relations

$$[x_j,x_k] = 0, \tag{2}$$

$$m[x_j,\dot{x}_k] = i\hbar\,\delta_{jk}. \tag{3}$$

Then there exist fields $E(x,t)$ and $H(x,t)$ satisfying the Lorentz force equation

$$F_j = E_j + \epsilon_{jkl}\dot{x}_k H_l, \tag{4}$$

and the Maxwell equations

$$\operatorname{div} H = 0, \tag{5}$$

$$\frac{\partial H}{\partial t} + \operatorname{curl} E = 0. \tag{6}$$

Remark: The other two Maxwell equations,

$$\operatorname{div} E = 4\pi\rho, \tag{7}$$

$$\frac{\partial E}{\partial t} - \operatorname{curl} H = 4\pi j, \tag{8}$$

merely define the external charge and current densities ρ and j.

Proof: Equations (1) and (3) imply

$$[x_j,F_k] + m[\dot{x}_j,\dot{x}_k] = 0. \tag{9}$$

The Jacobi identity

$$[x_l,[\dot{x}_j,\dot{x}_k]] + [\dot{x}_j,[\dot{x}_k,x_l]] + [\dot{x}_k,[x_l,\dot{x}_j]] = 0 \tag{10}$$

with (3) and (9) implies

$$[x_l,[x_j,F_k]] = 0. \tag{11}$$

Equation (9) also implies

$$[x_j,F_k] = -[x_k,F_j], \tag{12}$$

and therefore we may write

$$[x_j, F_k] = -(i\hbar/m)\epsilon_{jkl}H_l. \tag{13}$$

Equation (13) is the definition of the field H, which would in general depend on x, $\dot{x}$, and t. But Eq. (11) says

$$[x_l, H_m] = 0, \tag{14}$$

which means that H is a function of x and t only.

Next we satisfy (4) by assuming it to be the definition of the field E. Again, E will in general depend on x, $\dot{x}$, and t, but Eqs. (3), (13), and (14) imply

$$[x_m, E_j] = 0, \tag{15}$$

which says that E is a function of x and t only.

The definition (13) of H may be written

$$H_l = -(im^2/2\hbar)\epsilon_{jkl}[\dot{x}_j, \dot{x}_k] \tag{16}$$

by virtue of (9). Another application of the Jacobi identity gives

$$\epsilon_{jkl}[\dot{x}_l, [\dot{x}_j, \dot{x}_k]] = 0. \tag{17}$$

Equations (16) and (17) imply

$$[\dot{x}_l, H_l] = 0, \tag{18}$$

which is equivalent to (5). It remains to prove the second Maxwell equation (6).

Take the total derivative of Eq. (16) with respect to time. This gives

$$\frac{\partial H_l}{\partial t} + \dot{x}_m \frac{\partial H_l}{\partial x_m} = -\frac{im^2}{\hbar}\epsilon_{jkl}[\ddot{x}_j, \dot{x}_k]. \tag{19}$$

Now by (1) and (4), the right side of (19) becomes

$$\begin{aligned} &-(im/\hbar)\epsilon_{jkl}[E_j + \epsilon_{jmn}\dot{x}_m H_n, \dot{x}_k] \\ &= -(im/\hbar)(\epsilon_{jkl}[E_j, \dot{x}_k] + [\dot{x}_k H_l, \dot{x}_k] - [\dot{x}_l H_k, \dot{x}_k]) \\ &= \epsilon_{jkl}\frac{\partial E_j}{\partial x_k} + \dot{x}_k \frac{\partial H_l}{\partial x_k} - \dot{x}_l \frac{\partial H_k}{\partial x_k} \\ &\quad + (im/\hbar)H_k[\dot{x}_l, \dot{x}_k]. \end{aligned} \tag{20}$$

On the right side of Eq. (20), the last term is zero by symmetry because of (16), the third term is zero because of (5), and the second term is equal to the second term on the left of (19). The remaining terms in Eqs. (19) and (20) give

$$\frac{\partial H_l}{\partial t} = \epsilon_{jkl}\frac{\partial E_j}{\partial x_k}, \tag{21}$$

which is equivalent to (6). End of proof.

II. EDITORIAL COMMENT

When I show this proof to young physicists educated in the 1980s, their response is usually disparaging. They say the result is trivial and the proof unnecessarily complicated. It is therefore incumbent on me to explain why the result is not trivial and why Feynman chose to prove it the hard way. To understand the motivation for the proof, it is essential to put it into a historical context. The young physicists of today are as far removed from the Feynman of 1948 as Feynman was then removed from Planck and Einstein.

The argument of the young physicists is simple.[2] We know, they say, the commutation relation between position and momentum:

$$[x_j, p_k] = i\hbar\,\delta_{jk}. \tag{22}$$

If we define a vector potential A_k by

$$p_k = m\dot{x}_k + A_k, \tag{23}$$

then the two commutation relations (3) and (22) together give

$$[x_j, A_k] = 0. \tag{24}$$

Therefore, the vector potential A_k is independent of velocity, and depends only on x and t.

We also know, they say, that the momentum and velocity of a particle are related by the equations of Lagrange:

$$p_k = \frac{\partial L}{\partial \dot{x}_k}, \tag{25}$$

$$\dot{p}_k = \frac{\partial L}{\partial x_k}, \tag{26}$$

where

$$L = L(x, \dot{x}, t) \tag{27}$$

is the Lagrangian. If we integrate (25) using (23), the result is

$$L = \tfrac{1}{2}m\dot{x}_k\dot{x}_k + \dot{x}_k A_k + \varphi, \tag{28}$$

where φ is also independent of velocity. The scalar potential φ is defined by (28). If we now differentiate (23) using (26) and (28), the result is Newton's equation (1) with the Lorentz force (4), the fields E and H being defined by the standard expressions

$$H = \operatorname{curl} A, \quad E = \operatorname{grad} \varphi - \frac{\partial A}{\partial t}. \tag{29}$$

The Maxwell equations (5) and (6) follow trivially from (29). End of proof. So, the young physicists say, what is the big deal? From a modern point of view, the assumption of Feynman's commutation rule (3) implies immediately the existence of a vector potential, and as soon as you have a vector potential you also have a Maxwell field.

Feynman's point of view was quite different. In 1948 he was still doubting all the accepted dogmas of quantum mechanics. He was exploring possible alternatives to the standard theory. His motivation was to discover a new theory, not to reinvent the old one. He was well aware that, if he assumed the existence of a momentum p_k satisfying the commutation rule (22) in addition to (3), he would only recover the standard formalism of electrodynamics. That was not his purpose. His purpose was to explore as widely as possible the universe of particle dynamics. He wanted to make as few assumptions as he could. In particular, he wanted to avoid assuming the existence of momentum and Lagrangian related by (25) and (26). He chose his starting assumptions (1), (2), and (3) because they appeared to be less restrictive than the standard assumptions (22), (25), and (26). He hoped that by going along this road he might be led to new physics. He hoped to find physical models that would not be describable in terms of ordinary Lagrangians and Hamiltonians.

Feynman in 1948 was not alone in trying to build theories outside the framework of conventional physics. At that time many of the greatest physicists, including Yukawa,[3] Born,[4] and Heisenberg,[5] were pursuing programs for the radical reform of physics. All these radical programs, including Feynman's, failed. But Feynman was the only one who thoroughly tested his program before rushing into print. His proof of the Maxwell equations was a demonstration that his program had failed. The proof showed him that his assumptions (1), (2), and (3) were not leading to new physics. The road that he had been exploring was a dead end. From Feynman's point of view, the proof was a failure, not a success. That is why he was not interested in publishing it.

I venture to disagree with Feynman now, as I often did while he was alive. I still believe that his proof is worth publishing. It is not only a historical relic of a failed program. It also raises some new questions. The Maxwell equations are relativistically invariant, while the Newtonian assumptions (1), (2), and (3), which Feynman used for his proof, are nonrelativistic. The proof begins with assumptions invariant under Galilean transformations and ends with equations invariant under Lorentz transformations. How could this have happened? After all, it was the incompatibility between Galilean mechanics and Maxwell electrodynamics that led Einstein to special relativity in 1905. Yet here we find Galilean mechanics and Maxwell equations coexisting peacefully. Perhaps it was lucky that Einstein had not seen Feynman's proof when he started to think about relativity.

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[2] I am grateful to Professor Pierre Ramond of the University of Florida for a letter presenting the argument which I follow here.

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Nitrogen temperature superconducting ring experiment

Fuhan Liu,[a] Rochelle R. Tucker, and Peter Heller
Department of Physics, Brandeis University, Waltham, Massachusetts 02254

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A student experiment is described for studying persistent currents in a commercially obtained ring of the "123" superconducting material at liquid-nitrogen temperature. The currents are easily detected with a standard analog Hall probe. From observations extended over a 3-week period, an upper limit on the possible resistance of one such ring was set at about 2×10^{-16} Ω. For the rings studied, the induced current saturated at about 2 A as the applied flux change was increased. An ac technique for checking the continuity of the superconducting path around the ring is also described. These experiments provide an interesting supplement for topics in first-year electricity and magnetism. The effects are striking and easily discussed at an introductory level. For example, the current induced by turning the ring over in the Earth's field is readily seen.

I. INTRODUCTION

The superconducting ring experiment of H. Kamerlingh Onnes[1] is a landmark in the physics of the last 100 years. With the discovery[2,3] of the new high-T_c superconductors, the experiment is easily adapted for classroom use. The "persistent current" effect is certainly the most sensitive indicator of the perfect conductivity—a fact which can be well appreciated by first-year students. It is a useful supplement to basic treatments of electromagnetism as it emphasizes fundamental principles such as Faraday induction and Lenz' law, conductivity, inductance, and the Biot–Savart law. At the same time, it is exciting, as it deals with materials and to an extent with issues currently under study around the world.

The ring experiment has the advantage of *not* requiring an extensive background in superconductivity, although for those who wish to learn more, many general references are available such as the books by Schoenberg[4] and Tinkham.[5] The technical literature dealing with the new materials has also been reviewed recently.[6]

Our philosophy has been to provide an approach that is as simple and generally doable as possible. The "ring" (with its drilled hole) was provided to us commercially[7] out of the "123" ceramic ($Y_1Ba_2Cu_3O_{7-\delta}$). The ring dimensions[7] (0.82-in. outer diameter, 0.26-in. hole diameter) were dictated by the practical requirement that it be possible to drill the hole without cracking the outside. Hence, the radial ring width was about equal to the hole diameter. In most of our experiments, the "ring current" was detected through the magnetic field it produced at a point 7.7 mm below the ring center. Since the form of the current distribution over the ring was not known, the ratio of the measured field to the total ring current could not be calculated very accurately, although it could be estimated rather well. The technique should then be described as "semiquantitative."

Section II describes qualitative observations of the ring current. These experiments are striking, easily followed at an introductory level, and can be done either as lecture demonstrations or by small student groups in the laboratory. This provides an exciting accompaniment to standard

