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Five Dimensional Locally Supersymmetric Theories with Branes

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Chapter 1

Introduction

The key-words for this thesis are 'extra dimensions' and 'supersymmetry'. Both of them are often used in recent articles on high energy physics. Nevertheless, the possibility that we live in a more than four dimensional universe may still sound exotic to some physicists. Also, supersymmetry is not visible in the spectrum of the up-to-now discovered particles and some authors (though surprisingly few) question its relevance to our world. Thus, a brief introduction which justifies the research in this field seems to be necessary. In the next paragraphs we explain what are the theoretical motivations for the search for extensions of the well-established and tested theories like the Standard Model (SM) and the General Relativity.

The contemporary theory of fundamental interactions is the Standard Model (SM). This theory describes three of the four known forces of nature (electromagnetic, weak and strong) and has been extremely successful in explaining phenomena of subnuclear physics up to currently accessible energies, which are of the order of 100 GeV. The quantitative predictions of the SM are in perfect agreement with the experimental data; sometimes the accuracy is incredible, just to mention the anomalous magnetic moment of the electron, for which the experimentally measured value agrees with the theoretical calculation within 10^{-13} precision.

The SM is a quantum field theory which respects the Poincaré invariance (Lorentz rotations + space-time translations). It is founded on the gauge principle: the particles are assigned to various representations of the gauge group, which is the local symmetry group of the theory. In the case of the SM the gauge group is $SU(3) \times SU(2) \times U(1)$. The SU(3) factor corresponds to strong interactions, while the $SU(2) \times U(1)$ factor corresponds to weak and electromagnetic interactions. The latter is spontanously broken to U(1), by the vacuum expectation value of the scalar field transforming as a doublet of SU(2). The consequence of this mechanism is the existence of a fundamental, scalar particle - the famous, though still not discovered Higgs boson.

The SM is a consistent theory. Although calculations of quantum corrections yield divergent results, the theory is renormalizable - the divergencies can be absorbed into redefinitions of the parameters in the lagrangian and one ends up with finite and well-defined predictions for the scattering cross-sections and other observables.

Despite its considerable success, most of the physicists tend to the opinion that the SM is not the ultimate theory of nature and that the effects of the underlying, more fundamental theory should become visible at higher energy scales. First, the SM contains many arbitrary parameters. For example, the SM does not predict the mass of the electron; it must be considered as the experimental input to the theory. The SM does not explain why quarks and leptons occur in three similar copies (generations) with the same quantum numbers. Also, one could imagine other groups as symmetries of the theory; the choice of the group and the representations is restricted only by cancellation of anomalies. Classical symmetries of field theories can be broken by quantum effects - just mentioned anomalies. If local symmetries are anomalous then theory is inconsistent because, first of all, it is not unitary. The SM is anomaly free, but the mechanism behind it is not fundamental, rather it holds due to a miraculous interplay of quantum numbers of the SM particles. Thus, the way the anomaly cancellation works in the SM is a very important hint pointing towards physics beyond the SM.

Next, the SM model suffers from the so-called 'hierarchy problem'. To retain the perturbativity of the theory (and thus to be able to perform calculations) the Higgs boson mass should not be much higher than 1TeV. But in general, in higher orders of the perturbation theory , masses of scalar particles receive quantum corrections proportional to the supposed ultraviolet cut-off of the theory set by the Plank scale (about 10^{19} GeV). Thus, to keep the physical Higgs boson mass to be below 1TeV, one must choose very special values of the parameters of the original Lagrangian, so that 'miraculous' cancellations in the calculation of the physical Higgs boson mass could hold. Such situation, usually described as the 'fine-tuning', is considered very unlikely by the physicists, so other mechanisms are proposed to explain why the value of the Higgs boson mass can be many orders of magnitude smaller than the Plank scale. The other way to state the hierarchy problem is to say that it is unnatural to have several different mass scales in one theory unless we have some symmetry to protect them. In the SM we have no explanation why the electroweak scale (10^3 GeV) is hugely different from the Planck scale.

However, the most important drawback of the SM is that it cannot consistently incorporate gravity, the fourth fundamental force of nature. Indeed, trying to marry the SM with Einstein's General Relativity inevitably leads to unrenormalizable theory. Calculation of quantum corrections yields infinities which cannot be absorbed into redefinition of the parameters, and the theory loses much of its predictive power.

It may seem strange, that the theory which fails to describe gravity is so successful. The reason is that the effects of gravity in the experiments performed at currently accessible energies are negligible due to the smallness of the gravitational coupling (Newton's constant). However, at energies compared to the Planck scale the strength of gravity becomes comparable to the strength of the other fundamental forces, and the effects of gravity can no longer be neglected. If this expectation turns out to be correct, the SM is only an effective theory valid in a restricted energy range. It is not clear what is the energy at which the SM will finally break down; it cannot be greater than the Planck scale, but most of the physicist expect new effects already at energies of the order of 1TeV.

It should be stressed that the above motivations for the search of a 'new physics' are rather of aesthetical nature. Apart from them, there are several open problems which may require serious modifications of the SM. The most spectacular problems of this kind are:

1. For the consistency of the SM it is crucial, that at least one elementary scalar field is present in nature. But, the so-called Higgs boson still evades its discovery. This does not ruin the foundations of the SM, as it is likely that the Higgs boson mass may be above 100 GeV and this particle is inaccesible in the present accelerators. The problem will really begin, if the Higgs boson is not found in the next generation of accelerators, which will probe physics up to a few TeV.

2. The recent discovery of neutrino oscillations requires extending the SM, because this phenomenon can occur only if the netrinos are massive. The experimental data are still insufficient to favour one of the several possible extensions. One trivial possibility is

that one adds a new set of particles (right-handed neutrinos) to the spectrum, and the neutrino masses are incorporated in the theory in a way analogous to the masses of the other fermions. However, it may turn out that to explain neutrino oscillation one has to add a non-renormalizable term to the SM lagrangian which would point towards a new physics at higher energy scale and confirm that the SM is merely an effective low energy theory. This issue is currently the subject of extensive experimental studies.

3. As noticed already by Einstein, the General Relativity admits the so-called cosmological term in the action without violating the general coordinate transformations invariance. This term can be interpreted as the energy density of the vacuum. Its presence modifies the solutions of the theory excluding the flat Minkowski space. Cosmological observations strongly constrain the magnitude of this hypothetical term and any fundamental theory should explain why the observed cosmological constant is zero or very small. The SM itself does not describe gravity, so one may think that this problem is not relevant. But when we try to combine the SM with the General Relativity, the energy of the zero-mode oscillations of the SM fields leads to the estimate of the cosmological constant more than 100 orders of magnitude bigger than the current limits!. This discrepancy is even more striking when compared to the fantastic precision of other predictions. So far no-one has given a satisfactory explanation of the cosmological constant problem which may turn out to be an important clue pointing towards a new physics.

Having enumerated the basic motivations to search for physics beyond the SM, we review some of the most popular directions. All the ideas listed below will be relevant to the content of this thesis.

One possible extension of the SM, which is known under the name GUT (Great Unified Theory), is to extend the local symmetry group [19]. Instead of having three different groups glued together one could have one group, which is then spontaneously broken to the SM group. Many explicit models of this kind were proposed, the most popular gauge groups being $SU(5)$, $SO(10)$, E_6. Apart from aesthetical reasons, an extension of this kind is suggested by the apparent unification (within 30 percent accuracy) of the SM gauge couplings at energies of order 10^{14} GeV. So far there has been no direct evidence for the relevance of GUT groups to our world. The most spectacular prediction of GUT theories is the proton decay, extensively searched for in some experiments.

Another interesting possibility of extending the symmetries of the SM is supersymmetry. At the mathematical level this is equivalent to replacing the Poincaré algebra of space-time symmetries with a superalgebra (graded Lie algebra). Physically, one introduces in this way a symmetry between bosons and fermions. Supersymmetry possesses many beautiful features which make a considerable number of physicist believe in its existence, despite the fact that for over 20 years from the theoretical discovery of supersymmetry there has been no experimental evidence in its favour.

Supersymmetry solves the hierarchy problem. More precisely, it does not explain the huge ratio of the electroweak and Planck scales but renders this ratio stable against radiative corrections. It is the symmetry between bosons and fermions which leads to 'miraculous' cancellations among quantum corrections to the Higgs boson mass. What is left are mild (logarithmic in the cut-off) divergencies, so the existence of a light scalar field in the supersymmetric theory is more plausible. An additional (and unexpected at the beginning) virtue of supersymmetry is the fact the simplest extension of the SM, known as the MSSM (Minimal Supersymmetric Standard Model) leads to the gauge coupling unification within 1 percent accuracy. In the coming years, with the running of the next generation of accelerators, physicists expect to gain evidences in favour or against supersymmetry. In particular, we do not see any supersymmetric pattern in the spectrum of the up-to-now discovered particles. If supersymmetry is relevant to our world then new, yet undiscovered particles must exist, which together with the SM particles, fit into representations of the super-Poincaré algebra.

Even more rich in consequences are models which embed the SM in a theory with the number of space-time dimensions higher than four. In fact, this idea dates back to early days of the quantum theory when Kaluza and Klein (KK) proposed that that we may live in a five dimensional universe [22]. In the models of the KK type the additional (usually more than one) space-like dimensions are compact and their characteristic size is very small compared to the length scales probed in experiments. It can be proven that isometries of the compact dimensions give rise to gauge symmetries of the effective four-dimensional theory. Besides, models of the KK type predict that moduli of the compactification (that is - deformations of the compact manifold which do not change the energy of the system) become dynamical fields in the effective theory. Originally, it was hoped that with the help of extra dimensions one can explain the abundance of various particles in our world by means of a simpler theory, maybe just a higher dimensional gravity. At the same time extra dimensions provide new possibilities for adressing the hierarchy problem as in these models the Planck scale is not a fundamental quantity but merely a derived scale depending e.g on the volume of the compact manifold.

The idea of a higher dimensional universe has been revived with the development of string theories (see e.g. [27]) which include the only known examples of consistent quantum theories that describe gravity. The idea behind string theories is to replace point particles with extended object of one spatial dimension. The observed particles correspond to various oscillation modes of the fundamental strings. Surprisingly enough, these theories turn out to be very constrained, merely by the demand of their consistency. To avoid tachyons in the particle spectrum one is forced to introduce supersymmetry and ends up with so-called Superstring Theories. We know of only five examples of consistent Superstring Theories, namely type I, IIA, IIB, heterotic $E_8 \times E_8$ and heterotic SO(32). The number of space-time dimension in which the strings live is not arbitrary; the Superstring Theories require ten dimensions. String theories contain no free parameters and, in principle, once we decide on one of the five above mentioned realisations the whole dynamics is in principle determined.

At low energies string theories reduce to quantum field theories. In the course of the years string vacua have been constructed, such that compactifications of string theories yield spectra imitating the SM. Up to recently, the compactification of the $E_8 \times E_8$ heterotic superstrings, in which six extra dimension curl up to form a manifold known to mathematicians as the Calabi-Yau manifold, appeared to be most promising. This theory has many virtues: in the process of compactification one of the E_8 factors breaks down to E_6. The latter is a good candidate for the GUT group; the supersymmetry is preserved by the compactification and is thus expected to be broken at energies comparable to the electroweak scale; the second factor E_8 can serve as the gauge group of the so-called 'hidden sector' (which is the preferred scenario of supersymmetry breaking in the concrete, phenomenological models of low energy supersymmetry); the parameters of the SM can be expressed in terms of the topological characteristics of the compact manifold.

Recently, the monopoly of the heterotic strings has been broken. New vacua of type I and type II superstrings have been constructed. At low energies these vacua lead to physics very close to the one derived from the SM. Although these constructions themselves are not of direct concern in this thesis, they introduce in a natural way the notion of branes (precisely, D-branes are hypersurfaces on which open string are allowed to end). These objects will be important in the following. In the nomenclature we use, p-branes are p+1 dimensional submanifolds in the higher dimensional space-time (note that a 4d space-time corresponds to a three-brane). They may host various gauge as well as matter fields, which are confined on these submanifolds, in the sense of not being allowed to propagate in the remaining transverse dimension.

Independently of the string theories, even more recent motivation for the study of higher dimensional theories was given by the Randall-Sundrum (RS) model [11]. Contrary to the Superstring Theories, the RS model is not a consistent, self-contained theory but rather an ad-hoc construction. The basic set-up consists of a five dimensional gravity with a cosmological constant, which allows for anti-de-Sitter solution. In addition, there are three-branes located at various points along the fifth dimension. Randall and Sundrum showed, that for some configurations of those branes observers living on one of the brane would find that the gravity obeys the ordinary 4d Newton's law, in spite of the fact that the world is five dimensional and none of the dimensions is compact. Thus, we may be living in a higher dimensional universe without simply noticing it!

The modern understanding of Superstring Theories is that they are not five distinct theories but rather different vacua of an underlying, still to some extent hypothetical, 11-dimensional theory named M-theory. This view has emerged from the study of non-perturbative relations, the so-called dualities, between various Superstring Theories. It turned out that dual to some of the Superstring Theories are not any of the other ten-dimensional string theories but rather some eleven-dimensional theory. Very little is known about the quantum formulation of M-theory (it is not even clear what M stands for). We know however that in the low energy limit (in this case below the Plank scale) it reduces to the 11 dimensional supergravity theory. Also, in addition to strings which have one spatial dimension, brane objects with p-spatial dimensions should be present in the M-theory.

We now come to the starting point of this thesis. Horava and Witten showed [2] that at low energies the strong coupling limit of the heterotic $E_8 \times E_8$ superstring theory reduces to eleven dimensional supergravity defined on the manifold which is the product of a smooth manifold M_{10} and the interval $S_1/\mathbf{Z}_2$. In addition to eleven dimensional supergravity multiplet, the spectrum of this model consists of ten dimensional gauge multiplets in the adjoint representation of E_8 confined to the boundaries of the manifold, in other words, to the fixed points of the $\mathbf{Z}_2$ symmetry. As mentioned earlier, $E_8 \times E_8$ superstring theory is phenomenologically interesting and so is its strong coupling limit, because to obtain the unification of gauge and gravitational couplings at one common scale, one must assume the string coupling much greater than 1. Thus, in the interesting region of the parameter space the fundamental theory may be described by the Horava-Witten model.

The question arises whether models containing fields living in different space-time dimensions can be supersymmetric. The answer should be positive in this case, as we consider the strong coupling limit of a supersymmetric theory but the question is non-trivial because the Horava-Witten model is not a complete, consistent description of the M-theory. Horava and Witten proved by a direct construction [3] that, in the framework of their model, supersymmetrization indeed can be completed, at least in the first order of the expansion in the gravitational constant. However, the required modifications appeared to be highly non-trivial and substantially affected the vacuum solution of the model [1].

To make contact with the real world one must compactify the Horava-Witten model to four dimensions. It turns out that in order to have unification of gauge and gravitational couplings, the length of the eleventh dimension must be an order of magnitude greater than the characteristic size of the remaining six compact dimensions [1]. Thus, at the intermediate energy scale, the universe should appear five-dimensional. Lukas *et al.* [4] managed to obtain the effective five-dimensional theory resulting from the compactification of the Horava-Witten model on the Calabi-Yau three-fold (a manifold with three complex , i.e six real dimensions). For the reason which will be explained in detail in the next sections, this effective theory is not a simple 5d supergravity, but its gauged version. Gauging of supergravities introduces profound changes in the theory. It implies the existence of potentials for the scalar fields and, generically, the flat space can no longer be a vacuum solution. Apart from the sector living in five dimensions (in the so-called bulk) the compactified Horava-Witten model contains two parallel 3-branes with gauge fields and matter content which depends on the specific form of the compactification. The supersymmetrization of the brane theory with the bulk theory (supersymmetrization of branes with bulk, in short) has not been performed before and one of the objectives of this thesis is to fill this gap.

In the following, we derive the supersymmetric coupling of fields confined to a 3-brane to 5d supergravity. Using the Noether procedure, we add new terms to the bulk and brane lagrangian, which are necessary to arrive at a locally supersymmetric action. More precisely, our set-up consists of a five-dimensional N=2 supergravity on the manifold $M_4 \times S_1/\mathbf{Z}_2$ coupled to $SU(2,1)/U(2)$ non-linear sigma model. Two parallel 3-branes are located at $x^5 = 0$ and $x^5 = \pi\rho$ and host a 4d gauge supermultiplet and a chiral matter supermultiplet. In the context of the compactified Horava-Witten model this corresponds to only one universal hypermultiplet in the bulk, which is equivalent to choosing a special Calabi-Yau three-fold with Hodge numbers $h_{1,1} = 1$ $h_{1,2} = 0$. However, the construction we present is far more general and can be utilised for constructing other supersymmetric 5d models with branes, even those which do not have stringy origin. In particular, the brane potential term for the bulk scalars which arises in the compactified Horava-Witten model can be replaced by a constant brane tension, which immediately leads to the supersymmetric version of the Randall-Sundrum model. Because supersymmetrization of the RS model is presently the subject of intensive study [15, 8], we present this extension in this thesis, although the RS model most probably cannot be obtained from the heterotic compactifications of superstring theories.

The outline of this thesis is as follows. In Section 2 we briefly review supergravity theories in various, relevant dimensions. In Section 3 the Horava-Witten model and its compactification to five dimensions is introduced. Then we begin the presentation of the original results of this thesis. In Section 4 we present a detailed derivation of the supersymmetric coupling of gauge and matter fields confined to 3-branes to 5d N=2 supergravity. Necessary modifications of the supersymmetry transformation laws of bulk and brane fields are also discussed. To make the process of supersymmetrization more transparent we start with a 4d gauge multiplet on the brane and then we successively add matter fields in the bulk and on the brane. Then we carefully analyse the role of brane potentials, and their connection to the cosmological terms in the bulk supergravity. In Section 5 we derive the effective 4d theory. We find a vacuum solution of the 5d theory which preserves N=1 supersymmetry and compactify our model on this background. We discuss various contributions to the effective 4d lagrangian coming from the moduli of the vacuum solution and from the fact that fields on the branes act as sources in the equations of motion of the bulk fields. We determine the precise form of the compactified theory in terms of the canonical 4d supergravity. In section 6 we perform the reduction of the 5d supersymmetry transformation law and finally, in Section 7, we comment on supersymmetry breaking in the five-dimensional framework.

Chapter 2

Supergravities in 11, 5 and 4 dimensions

Supersymmetry is a non-trivial extension of the Poincaré symmetry. According to the celebrated Coleman-Mandula no-go theorem [24], the Poincaré algebra is the largest possible Lie algebra of symmetries of a quantum field theory which acts non-trivially on space-time. Extending further the algebra of symmetries leads to a trivial S-matrix, that is to no interactions. Supersymmetry evades the limitations of the Coleman-Mandula theorem, because the mathematical concept behind it is a graded Lie algebra. If such an algebra is a symmetry algebra of a theory then, apart from standard commuting bosonic symmetries, we have anti-commuting fermionic symmetries. The supersymmetry charge Q commutes with the momentum operator P_μ and with generators of internal symmetries but does not commute with the generators of the Lorentz rotations $M_{\mu\nu}$. Thus, one-particle states in supermultiplets, which are obtained by acting successively with Q on a lowest weight state, have the same masses and internal quantum numbers but different spins. Supersymmetry predicts, therefore, that particles are accompanied by a number of superpartners with similar properties except for the spin. Size of supermultiplets, and thus a number of superpartners may vary depending on the chosen representation of the superalgebra and dimensionality of the space-time.

In this thesis we consider local supersymmetry, that is symmetry generated by parameters which depend on space-time coordinates. There are several reasons to prefer this option. From our experience with the SM we know that local (gauge) symmetries play a more fundamental role in the theory than global symmetries (like baryon or lepton number conservation). This view is supported by the no-hair theorem of quantum gravity which states that only local symmetries can be exact in the presence of gravitational effects. The reason specific for supersymmetry is that locally supersymmetric theories necessarily include gravity. This is easy to see from the supersymmetry algebra. The anticommutator of two supersymmetry charges $\{Q, \bar{Q}\}$ equals the momentum operator P and if the parameters on the left-hand side depend on space-time coordinates the right-hand side is a local translation which vary from point to point, in other words a general coordinate transformation. Thus we can expect that a theory invariant under local supersymmetry is also invariant under general coordinate transformations which is the symmetry of the General Relativity.

For our purpose we will not need the detailed mathematical formulation of supersymmetry. All we need to do is to represent the supersymmetry algebra on the fields of our lagrangians. We require that the lagrangian we consider is invariant up to a total derivative under infinitesimal local supersymmetry transformations. This is analogous to representing gauge symmetries in the way we know from the Standard Model. The only difference is that in the case of supersymmetry the infinitesimal parameter of the transformations is an anticommuting spinor. Given the field content, supersymmetry fixes the form of the lagrangian up to a few arbitrary functions. The possible supermultiplets that can be present in various space-time dimensions are determined by a more involved analysis [16].

2.1 Eleven dimensional supergravity

There are several reasons to start our survey from eleven dimensions:

1. This is the highest space-time dimension in which a consistent, interacting supergravity can be formulated.

2. The field content of the 11d supergravity is very simple and the supersymmetry fixes uniquely the form of the lagrangian.

3. Many supergravities in lower dimensions can be obtained by a truncation of the 11d supergravity. In particular, this is the case with 5d N=2 and 4d N=1 supergravities which will concern us further in this thesis.

In eleven dimensions the gravity multiplet consists of a vielbein e_I^m, one gravitino ψ_I and one three form C_{IJK}. I,J... are eleven dimensional vector indices equal 0..9, 11.
The vielbein formulation of gravity is equivalent to the more familiar metric formulation [23]. The connection between the two is given by $g_{IJ} = \eta_{mn} e_I^m e_J^n$ where η is the flat 11d Minkowski metric. As is well-known, at any single point of the Riemannian manifold, a general metric can be reduced to the flat Minkowski metric by the appropriate choice of a coordinate frame. Vielbeins can be considered as the basis vectors of this (locally inertial) frame at a given point. The upper index, is a vector index of SO(9,1) corresponding to the Lorentz symmetry of the Minkowski metric. The kinetic term for the vielbein is the standard Ricci curvature scalar, just like in the four-dimensional General Relativity.

The gravitino ψ_I is a vector-spinor field (spinor indices are suppressed). Spinors in odd D dimensional spaces have $2^{(D-1)/2}$ components [25], so in our case ψ has 32 complex components. However, in 11 dimension we can impose the Majorana condition and we effectively end up with 32 real components (in the real Majorana basis). The kinetic term is the Rarita-Schwinger action given in the first line of (2.1). In four dimensions gravitino describes a spin 3/2 elementary particle . Such particles has not been discovered, but they must be present in any locally supersymmetric theories. Therefore, if local supersymmetry is relevant to our universe, gravitinos must be either very heavy or light and very weakly interacting.

The field C is anti-symmetric in its 3 indices, hence its name three-form. The notion of n-form fields is generally known because in 4d n-forms do not introduce any new possibilities to describe physics: a 0-form is just a scalar-field, a 1-form is a gauge field (this is how gauge fields are presented in more geometrically oriented books) and a 2-form is equivalent to a (pseudo-)scalar by the Hodge duality. In D dimensions one can consider n-forms with n=0...D-2 as propagating fields. In $D > 4$ dimensions form fields describe essentially new objects. The kinetic term, similarly to the vector Abelian case, is proportional to the square of the external derivative dC.

The unique supergravity Lagrangian is [18]:

$$\mathcal{L}_{11} = \frac{1}{\kappa_{11}^2} e_{11} \Big(-\frac{1}{2}R - \frac{1}{2}\psi_I \Gamma^{IJK} D_J \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL}$$
$$-\frac{\sqrt{2}}{192}(\bar{\psi_I}\Gamma^{IJKLMN}\psi_N + 12\bar{\psi^J}\Gamma^{KL}\psi^M)G_{JKLM} - \frac{\sqrt{2}}{3456}\epsilon^{I_1..I_{11}} C_{I_1..I_3} G_{I_4..I_7} G_{I_8..I_{11}} + (4fermi)\Big) \quad (2.1)$$

In the above κ_{11} is a gravitational constant, e_{11} is the determinant of the 11d vielbein. The gamma matrices have dimension 32×32 and obey $\{\Gamma_I, \Gamma_J\} = 2g_{IJ}$. The anti-symmetrized products of matrices are defined as: $\Gamma^{I_1..I_n} = \Gamma^{[I_1}..\Gamma^{I_n]} = \frac{1}{n!}\Gamma^{I_1}...\Gamma^{I_n} \pm (permutations)$. The covariant derivative acting on the gravitino is $D_I \psi_J = \partial_I \psi_J + \frac{1}{4}\omega_{Imn}\Gamma^{mn}\psi_J$ and contains the spin connection ω defined by the formula:

$$\omega_{Imn} = \frac{1}{2}e_m^J(\partial_I e_{nJ} - \partial_J e_{nI}) - \frac{1}{2}e_n^J(\partial_I e_{mJ} - \partial_J e_{mI}) - \frac{1}{2}e_m^J e_n^K(\partial_J e_{pK} - \partial_K e_{pJ})e_I^p \quad (2.2)$$

The four-form field strength G is defined as $G_{IJKL} = 24\partial_{[I}C_{JKL]}$, in short $G = 6dC$. Obviously, G satisfies the Bianchi identity dG=0. Later, we shall see that coupling to YM fields defined on boundaries requires redefinition of G, so that the right-hand side of the Bianchi identity becomes non-trivial.
The four-fermion terms are also known, but we will not need them in further considerations. It is a common practice to skip them when possible to avoid lengthy mathematical formulae.

The 11d supergravity action is invariant under the following local supersymmetry transformations:

$$\begin{aligned}
\delta e_I^m &= \frac{1}{2}\eta\Gamma^m\psi_I \\
\delta\psi_I &= D_I\eta + \frac{\sqrt{2}}{288}(\Gamma_I^{JKLM} - 8g_I^J\Gamma^{KLM})\eta G_{JKLM} + (three - fermi) \\
\delta C_{IJK} &= -\frac{\sqrt{2}}{8}\bar{\eta}\Gamma_{[IJ}\psi_{K]}
\end{aligned} \quad (2.3)$$

Note the derivative of the spinor parameter η in the transformation law of gravitino, which can be interpreted, in analogy to the Yang-Mills case, that gravitino is the gauge field of supersymmetry. This is the justification of the previous statement that the gravitino must be present in locally supersymmetric theories. The number of conserved supersymmetry charges is 32 (counting each component of Q separately). From the 4d point of view this number corresponds to N=8 supersymmetry.

2.2 Supergravities in five dimension

The plural in the subtitle suggests that, contrary to the 11d case, 5d supergravity is not unique. Indeed, in 5d we have certain freedom in choosing the spectrum of matter fields, as well as the sigma model which governs their dynamics. We can also consider various numbers of supersymmetries. In this section we concentrate on the case of N=2 supersymmetry which corresponds to eight conserved supercharges [1].

Every locally supersymmetric 5d theory contains the gravity multiplet which consists of the metric $g_{\alpha\beta}$ (here we work with the vielbein e_α^a), two symplectic Majorana gravitinos ψ_α^A and a

[1] Some authors call it N=1 susy as it is the least possible number of supersymmetries in five dimensions. We prefer to keep the label in N=2 because of the similarity to N=2 supergravity in four dimension

vector field, in this context usually called the graviphoton $\mathcal{A}_\alpha$. The greek indices α β ... from the beginning of the alphabet are five dimensional and run over values 0..3,5. The reason we impose symplectic conditions is that in five dimensions it is impossible to satisfy the standard Majorana condition $\lambda^c \equiv C\bar{\lambda}^T = \lambda$, where C is a charge conjugation matrix, because this leads to a contradiction $\lambda = (\lambda^c)^c = -\lambda$. Instead one can arrange spinors into pairs by demanding $\bar{\lambda}^A = (\Omega^{AB}\lambda_B)^T C$, where C is the charge conjugation matrix satisfying $\gamma^{\mu\, T} = C\gamma^\mu C^{-1}$ and Ω is a symplectic matrix which squares to -1. In the case of gravitinos the index A runs from 1 to 2, and the symplectic matrix is just the antisymmetric tensor ϵ^{AB}.
The notation using symplectic spinors makes explicit another symmetry of the N=2 supergravity action. A theory with N supersymmetries possesses SU(N) R-symmetry, which transforms the supercharges into each other. This symmetry, or rather its $\mathbf{Z_2}$ subgroup, so-called R-parity, is familiar to those acquinted with the MSSM. For the case at hand, this symmetry is SU(2) and the gravitino index A tranforms in the fundamental representation of the R-symmetry group. This index is raised and lowered with ϵ^{AB}; SU(2) invariant contraction of spinors is $\overline{\lambda}^A\lambda_A \equiv \epsilon_{AB}\overline{\lambda}^A\lambda^B$, the conventions are $\epsilon^{12} = \epsilon_{12} = 1$. Note also somewhat unusual definition $\overline{\lambda}^A \equiv \overline{\lambda_A}$.

The lagrangian for the gravity multiplet alone takes the form:

$$\mathcal{L}_5 = e_5 \tfrac{1}{\kappa^2} \Big(-\tfrac{1}{2}R - \tfrac{1}{2}\overline{\psi_\alpha}^A\gamma^{\alpha\beta\gamma}D_\beta\psi_{\gamma\, A} - \tfrac{1}{2}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}$$
$$-\tfrac{1}{12\sqrt{2}}\epsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{A}_\alpha\mathcal{F}_{\beta\gamma}\mathcal{F}_{\delta\epsilon} + \tfrac{i}{4\sqrt{2}}(\overline{\psi_\gamma}^A\gamma^{\alpha\beta\gamma\delta}\psi_{\delta\, A} + 2\overline{\psi^\alpha}^A\psi^\beta_A)\mathcal{F}_{\alpha\beta} + (four - fermi) \Big) \tag{2.4}$$

The form of the above lagrangian resembles the one of 11d supergravity, e.g. the 'topological' term $\mathcal{AFF}$ is similar to the 11d CGG term. Thus, we can expect that 5d N=2 supergravity can be obtained as a compactification of 11d supergravity. This statement is almost correct, as we can compactify the 11d supergravity on the six-dimensional Calabi-Yau manifold leaving eight of thirty-two supercharges unbroken, which indeed leads to N=2 supergravity. However, this procedure yields additional scalars and fermion corresponding to the moduli of the compactification; e.g. one of the always present scalar moduli is the volume of the compact manifold. Because of that, it is necessary to consider a coupling of matter multiplets to the 5d gravity multiplet.

The gravity multiplet can be coupled to an arbitrary number of vector multiplets which consist of a vector field, two symplectic Majorana gauginos and a single real scalar field. At the same time, we can couple hypermultiplets with two symplectic Majorana hyperinos and four real scalar fields. It turns out that hypermultiplets and vector multiplets couple to the gravity multiplet only and not to one another. In a supersymmetric lagrangian containing hyper- and vector multiplets, lengthy polynomials of scalar fields appear, which are most convieniently characterized in terms of geometry on some Riemannian manifold. The arbitrariness lies in the freedom to choose one of those special geometries.

It should be stressed that in five dimensions there are no supermultiplets with chiral fermions. To introduce chiral matter charged under Yang-Mills symmetries, one must locate it on a 4d submanifold. The Yang-Mills vector fields can also be confined to the boundary and this is the case we study carefully in this thesis. At the same time we can have gauge symmetries in the bulk with vector fields of the vector multiplets and the graviphoton being the gauge fields. This possibility will also be studied in the following, rather not for the virtue of having gauge symmetries, but in order to introduce potential for the scalar fields. Otherwise, in ungauged 5d supergravities, scalar potentials are always absent.

In the next subsections we follow closely the Appendix B of reference [5]

2.2.1 Coupling of vector multiplets

Below we describe coupling of n_v Abelian vector multiplets to the gravitational multiplet of N=2 supergravity. We now have n_v vector fields $\mathcal{A}^i_\alpha$, $2n_v$ symplectic pairs of spinors (gauginos) λ^{Ax}, and n_v real scalars ϕ^x. The index A of the gauginos is the same as that of gravitino. It is convienient to group vectors with the graviphoton so that the index $i = 0, 1..n_v$. The kinetic terms of the scalars define the sigma model: $\mathcal{L}_{kin} = -\frac{1}{2}g_{xy}(\phi)\partial_\alpha\phi^x\partial^\alpha\phi^y$ If the vector multiplets are coupled in a supersymmetric way, then g_{xy} can be interpreted as a metric of a Riemannian manifold $\mathcal{M}_V$ with the very special geometry; in such the case the scalars ϕ^x can be intethe vector multiplets are coupled in a supersymmetric way, then g_{xy} can be interpreted as a metric of a Riemannian manifold $\mathcal{M}_V$ with the very special geometry; in such the case the scalars ϕ^x can be interpreted as coordinates on $\mathcal{M}_V$.

To see the structure of $\mathcal{M}_V$ one starts with a $n_v + 1$-dimensional space $\mathcal{C}$ with coordinates b^i and the metric:

$$G_{ij}(b) = -\frac{1}{2}\frac{\partial}{\partial b^i}\frac{\partial}{\partial b^j}ln\mathcal{K}(b) \tag{2.5}$$

where $\mathcal{K}$ is a homogenous polynomial of degree three:

$$\mathcal{K} = d_{ijk}b^i b^j b^k \tag{2.6}$$

One then takes $\mathcal{M}_V$ as the hypersurface $\mathcal{K} = 6$. Restricting ourselves to that submanifold we have $b^i = b^i(\phi^x)$ and we can write the induced metric as:

$$g_{xy}(\phi) = \frac{\partial b^i}{\partial \phi^x}\frac{\partial b^j}{\partial \phi^y}G_{ij}(b) \tag{2.7}$$

The rest of the lagrangian is detemined by the sigma model metric. We restrain from giving the lagrangian and the supersymmetry transformation laws until the subesection 2.2.4.

2.2.2 Coupling of hypermultiplets

In this subsection we review coupling of n_h hypermultiplets to the gravity multiplet. We are given $2n_h$ symplectic Majorana fermions (hyperinos) λ^a and $4n_h$ real scalars q^u. As in the previous case, the central object is the metric h of the sigma-model: $\mathcal{L}_{kin} = -h_{uv}(q)\partial_\alpha\phi^u\partial^\alpha\phi^v$ Again, to render the coupling possible, h_{uv} must have the interpretation of a metric of some Riemannian manifold $\mathcal{M}_H$ on which the scalars q^u are the coordinates. One finds that for N=2 supergravity $\mathcal{M}_H$ is a quaternionic manifold. Below we present basic facts about quaternionic geometry.

A quaternionic manifold can be thought of as a generalization of a complex manifold. The name is due to the three complex structures J^A_B, which satisfy the quaternionic algebra under matrix multiplication. It is endowed with a triplet of Kähler forms K^A_B satisfying:

$$dK + \omega \wedge K = 0 \tag{2.8}$$

ω^A_B is a SU(2) part of the spin-connection. As the holonomy group of a $4n_h$ dimensional quaternionic manifold is by definition the product $SU(2) \times Sp(2n_h)$, the corresponding spin connection decomposes into a sum of the SU(2) connection ω^A_B and the $Sp(2n_h)$ connection Δ^a_b. In the context of N=2 supersymmetry, SU(2) is interpreted as the R-symmetry group and the index A transforms in the same way as that of gravitino. Unlike the gauginos and gravitinos, the hyperinos λ^a are symplectic Majorana with respect to the $Sp(2n_h)$ connection, so the index 'a' runs over values $1..2n_h$.

2.2.3 Gauging universal hypermultiplets

If the manifold $\mathcal{M}_H$ admits isometries we can gauge them, modifying significantly the structure of 5d supergravity. The procedure of gauging isometries of scalar manifolds is similar to gauging global symmetries in order to obtain ordinary supersymmetric Yang-Mills theories. The derivatives acting on fields must be replaced with covariant derivatives involving gauge fields, and the potential for scalar fields must be added, which in the super-Yang-Mills case corresponds to the so-called D-terms. The gauge fields are provided by vector multiplets and the omni-present graviphoton from the gravity multiplet. Gauged isometries become local in the space-time sense.
In this subsection we consider only Abelian isometries as the general case is not given in the literature. We gauge only hypermultiplets; gauging of vector multiplets is also possible, but we do not utilize that construction in this thesis.

Isometries that preserve the quaternionic structure of $\mathcal{M}_H$ are generated by the Killing vectors satisfying the Killing equation $\nabla_u k_v + \nabla_v k_u = 0$, which can be solved in terms of a function $\mathcal{P}^A_B$ called the prepotential:

$$k_i^u K_{uv} = \partial_v \mathcal{P}_i + [\omega_v, \mathcal{P}_i] \tag{2.9}$$

Space-time derivatives acting on the hypermultiplet scalars must be replaced with covariant derivatives:

$$\partial_\alpha q^u \to D_\alpha q^u \equiv \partial_\alpha q^u + g\mathcal{A}^i_\alpha k^u_i \tag{2.10}$$

Derivatives acting on the fermions have to be modified as well, and those modifications are all summarized in the next subsection.

The most significant aspect of gauging is the fact that it introduces, otherwise absent, potential for the scalar fields:

$$V = -2G_{ij} tr\mathcal{P}^i\mathcal{P}^j + 4b_i b_j tr\mathcal{P}^i\mathcal{P}^j + \frac{1}{2}b^i b^j h_{uv} k^u_i k^v_j \tag{2.11}$$

In the absence of potentials the simplest solution to the equations of motion of the 5d supergravity is the flat Minkowski space. Compactification to 4d on such background is analogous to the standard Kaluza-Klein procedure. It does not break any of the supersymmetry and yields N=2 supergravity in four dimensions. Non-trivial potentials generically forbid flat space solutions. The simplest solution are then so-called BPS solutions which preseve exactly one half of supersymmetries. The solutions preserving 4d Poincaré invariance usually depend on the fifth, transverse coordinate; this is not compatible with the standard Kaluza-Klein ansatz and makes the process of compactification less straightforward. One specific example of such procedure will be thoroughly studied in section 5.

Another interesting aspect of gauging is that fermion mass-like terms appear in the lagrangian. At first sight, this may seem strange, for graviton remains massless and one of the common opinions about supersymmetry is that it requires the same masses for each member of a supermultiplet. But the above statement is true only for the case of supersymmetry in the flat space. Thus, the fermion mass terms are another indication that we should not expect flat space solutions in gauged supergravities.

2.2.4 The final form of the action and supersymmetry transformations

In this section we present the general action up to four-fermi terms and supersymmetry transformation up to three-fermi terms of five-dimensional N=2 gauged supergravity with gauged Abelian isometries of the hypermultiplet manifold .
The action is given by:

$$S = \int_{M_5} d^5x \frac{e_5}{\kappa^2}(\mathcal{L}_{kinetic} + \mathcal{L}_{fermi\,mass} + \mathcal{L}_{four\,fermi} - g^2 V) \tag{2.12}$$

$$\begin{aligned}\mathcal{L}_{kinetic} = &-\tfrac{1}{2}R - \tfrac{1}{2}\overline{\psi_\alpha}^A\gamma^{\alpha\beta\gamma}D_\beta\psi_{\gamma\,A} - \tfrac{1}{2}G_{ij}\mathcal{F}^i_{\alpha\beta}\mathcal{F}^{j\alpha\beta} - \tfrac{1}{12\sqrt{2}}d_{ijk}\epsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{A}^i_\alpha\mathcal{F}^j_{\beta\gamma}\mathcal{F}^j_{\delta\epsilon}\\ &-\tfrac{1}{2}G_{ij}\partial_\alpha b^i\partial^\alpha b^j - h_{uv}D_\alpha q^u D^\alpha q^v - \tfrac{1}{2}\overline{\lambda}^{Ax}\gamma^\alpha D_\alpha\lambda_{Ax} - \tfrac{1}{2}\overline{\lambda}^a\gamma^\alpha D_\alpha\lambda_a\\ &+\tfrac{i}{4\sqrt{2}}(\overline{\psi_\gamma}^A\gamma^{\alpha\beta\gamma\delta}\psi_{\delta\,A} + 2\overline{\psi^\alpha}^A\psi^\beta_A - \overline{\lambda}^{Ax}\gamma^{\alpha\beta}\lambda_{Ax} - \overline{\lambda}^a\gamma^{\alpha\beta}\lambda_a)b_i\mathcal{F}^i_{\alpha\beta} + \tfrac{1}{2\sqrt{2}}(\overline{\lambda}^A_x\gamma^\alpha\gamma^{\beta\gamma}\psi_{\alpha A})b^x_i\mathcal{F}^i_{\beta\gamma}\\ &-\tfrac{i}{8\sqrt{2}}(\overline{\lambda}^{Ax}\gamma^{\alpha\beta}\lambda_{Ax})d_{ijk}b^i_x b^j_y\mathcal{F}^k_{\alpha\beta} - \tfrac{i}{2}(\overline{\lambda}^A_x\gamma^\alpha\gamma^\beta\psi_{\alpha A})b^x_i\partial_\beta b^i + i(\overline{\lambda}_a\gamma^\alpha\gamma^\beta\psi_{\alpha A})V^{Aa}_u D_\beta q^u\end{aligned} \tag{2.13}$$

In the above formula n_v vector multiplet scalars ϕ^x appear through n_v+1 scalars b^i subject to the constraint $d_{ijk}b^ib^jb^k = 6$; b^i_x is short for $\frac{\partial b^i}{\partial\phi^x}$. V^{Aa}_u denotes the vierbein of the quaternionic manifold $\mathcal{M}_h$, which is connected to the metric h through the formula:

$$h_{uv} = V^{Aa}_u V^{Bb}_v \Omega_{ab}\epsilon_{AB} \tag{2.14}$$

The gauge covariant derivative acting on hypermultiplet scalars is $D_\alpha q^u \equiv \partial_\alpha q^u + g\mathcal{A}^i_\alpha k^u_i$. We do not gauge vector multiplets, so we have ordinary partial derivatives acting on the vector multiplet scalars contained in b^i fields. The covariant derivatives acting on fermion fields are:

$$\begin{aligned}D_\alpha\lambda^a &= \nabla_\alpha\lambda^a + D_\alpha q^u\Delta_u{}^a{}_b\lambda^b + g\mathcal{A}^i_\alpha\partial_u k^v_i V^{uAa}V_{vAb}\lambda^b\\ D_\alpha\lambda^{Ax} &= \nabla_\alpha\lambda^{Ax} + \partial_\alpha\phi^y\Gamma^x_{yz}\lambda^{Az} + D_\alpha q^u\omega_u{}^A{}_B\lambda^{Bx} + g\mathcal{A}^i_\alpha\mathcal{P}_i{}^A{}_B\lambda^{Bx}\\ D_\alpha\psi^A_\beta &= \nabla_\alpha\psi^A_\beta + D_\alpha q^u\omega_u{}^A{}_B\psi^B_\beta + g\mathcal{A}^i_\alpha\mathcal{P}_i{}^A{}_B\psi^B_\beta\end{aligned} \tag{2.15}$$

In these formulae, ∇ denotes an ordinary space-time covariant derivative including the space-time spin connection. The term involving vector fields $\mathcal{A}^i_\alpha$ is due to the gauging described in the previous subsection. The terms involving derivatives of scalars are to render the expression covariant on the scalar manifolds; these terms can be readily worked out by noting that the SU(2) and $Sp(2n_h)$ indices are contracted with the corresponding part of the spin connection , and the vector index x is contracted with the Christoffel connection on the vector multiplet manifold.

The fermion mass terms are:

$$\begin{aligned}\mathcal{L}_{fermi\,mass} = &-\tfrac{ig}{\sqrt{2}}b^i\mathcal{P}^{AB}_i\overline{\psi_\alpha}_A\gamma^{\alpha\beta}\psi_{\beta B} + g\sqrt{2}b^i_x\mathcal{P}^{AB}_i\overline{\lambda^x}_A\gamma^\alpha\psi_{\alpha B} + \tfrac{g}{\sqrt{2}}V^A_u{}_a b^i k^u_i\overline{\lambda}^a\gamma^\alpha\psi_{\alpha A}\\ &+ig(\tfrac{3}{\sqrt{2}}d_{ijk}b^{ix}b^{jy}\mathcal{P}^{kAB} + 3\sqrt{2}b^{ix}b^{jy}G_{ij}b_k\mathcal{P}^{kAB})\overline{\lambda_x}_A\lambda_{yB}\\ &+\tfrac{ig}{\sqrt{2}}V^{Aa}_u b^{ix}k^u_i\overline{\lambda}_a\lambda_{Ax} - \tfrac{ig}{4\sqrt{2}}V^{Aa}_u V^{Bb}_v\epsilon_{AB}b^i\nabla^{[u}k^{v]}_i\overline{\lambda}_a\lambda_b\end{aligned} \tag{2.16}$$

As usually, we skip all four-fermion terms. We recall that the potential V is given by:

$$V = -2G_{ij}tr\mathcal{P}^i\mathcal{P}^j + 4b_ib_jtr\mathcal{P}^i\mathcal{P}^j + \frac{1}{2}b^ib^jh_{uv}k^u_ik^v_j \tag{2.17}$$

Finally, we give the supersymmetry transformation laws. The supersymmetry parameter ϵ^A, like gravitino, is a Majorana symplectic spinor and carries the R-symmetry SU(2) index. The three-fermion terms in the transformation laws of fermions are omitted:

- Gravity multiplet

$$\begin{aligned}\delta e_\alpha^m &= \frac{1}{2}\bar{\epsilon}^A\gamma^m\psi_{\alpha A}\\ \delta\psi_\alpha^A &= D_\alpha\epsilon^A - \frac{i}{6\sqrt{2}}(\gamma_\alpha^{\ \beta\gamma} - 4\delta_\alpha^\beta\gamma^\gamma)a_i\mathcal{F}^i_{\beta\gamma}\epsilon^A + \frac{ig\sqrt{2}}{3}b^i\mathcal{P}_i^{AB}\gamma_\alpha\epsilon_B\end{aligned} \tag{2.18}$$

- Vector multiplet + graviphoton

$$\begin{aligned}\delta\mathcal{A}_\alpha^i &= -\frac{i}{2\sqrt{2}}b^i\overline{\psi_\alpha}^A\epsilon_A + \frac{1}{2\sqrt{2}}b_x^i\bar{\epsilon}_A\gamma_\alpha\lambda^{Ax}\\ \delta\lambda^{Ax} &= b_i^x(\frac{i}{2}\gamma^\alpha\partial_\alpha b^i + \frac{1}{2\sqrt{2}}\gamma^{\alpha\beta}\mathcal{F}^i_{\alpha\beta})\epsilon^A + g\sqrt{2}b_i^x\mathcal{P}^{iAB}\epsilon_B\\ \delta b^i &= -\frac{i}{2}b_x^i\bar{\epsilon}_A\lambda^{Ax}\end{aligned} \tag{2.19}$$

- Hypermultiplet

$$\begin{aligned}\delta q^u &= \frac{i}{2}V_{Aa}^u\bar{\epsilon}^A\lambda^a\\ \delta\lambda^a &= -iV_u^{Aa}\gamma^\alpha D_\alpha q^u\epsilon_A + g\frac{1}{\sqrt{2}}V_u^{Aa}b^ik_i^u\epsilon^a\end{aligned} \tag{2.20}$$

Note that only fermions receive corrections from gauging (always represented by the last term).

2.3 Supergravity in four dimensions

In this section we follow closely the reference [17]. In four dimensions the simplest (and the only phenomenologically viable) supergravity theory is N=1 supergravity with four supercharges. The gravity multiplet contains only two component fields: spin 2 metric $g_{\mu\nu}$ and spin 3/2 vector spinor ψ_μ, which in the customary formulation is subject to the Majorana condition. The greek indices μ ν ... from the middle alphabet are four-dimensional and run over 0..3.

As in five dimension, scalar fields and their superpartners can be coupled to the four dimensional gravity multiplet. In four dimensions a scalar multiplet contains a spin 0 complex scalar field z^i and its spin 1/2 fermion superpartner λ^i; the index i counts the number of scalar multiplets . The complete lagrangian is determined by the kinetic terms of the scalars which can be written in terms of a sigma model metric, also in this case having the geometrical interpretation. This time scalar fields parametrize a complex manifold of the Kähler type, and the kinetic terms are determined by the Kähler manifold metric. For our purpose it is important to know that this metric can be expressed in terms of a Kähler potential K:

$$g_{ij} = -\frac{\partial}{\partial z^i}\frac{\partial}{\partial z^j}K(z, z^*) \tag{2.21}$$

Even without gauging we can have a potential for scalar fields which can be described in terms of a holomorphic function W called the superpotential. It is usufel to define:

$$G = -K - ln(|W|^2) \tag{2.22}$$

In this subsection we put the 4d Planck scale equal to one. Contrary to the 5d case, in four dimensions we can can introduce a Yang-Mills supermultiplet which contains a spin 1 vector field A^a_μ and a spin 1/2 gaugino χ^a, both in the adjoint representation of the gauge group (a is the group index). Gauge multiplets can be coupled to 4d supergravity and the scalar multiplets transform in some representation of the gauge group. The basic function which determines the coupling is the gauge kinetic function f_{ab}. It is a holomorphic function of z. The kinetic terms of the gauge fields are:

$$\mathcal{L}_{gkin} = -\frac{1}{4} Ref_{ab} F^a_{\mu\nu} F^{b\mu\nu} \tag{2.23}$$

The 4d supergravity action consists of the following terms:

$$S_4 = \frac{1}{\kappa^2} \int d^4 x e_4 (\mathcal{L}_{Bkin} + \mathcal{L}_{pot} + \mathcal{L}_D + \mathcal{L}_{Fkin} + \mathcal{L}_{Fmass} + \mathcal{L}_{4fermi}) \tag{2.24}$$

The determinant of the 4d vierbein is denoted e_4. The bosonic kinetic terms are:

$$\mathcal{L}_{Bkin} = -\frac{1}{2} R + G^i_j D_\mu z_i D^\mu z^{*j} - \frac{1}{4} Ref_{ab} F^a_{\mu\nu} F^{b\mu\nu} - \frac{1}{4} Im f_{ab} F^a_{\mu\nu} \tilde{F}^{b\mu\nu} \tag{2.25}$$

The notation we use is $G_i = \frac{\partial G}{\partial z^i}, G^j = \frac{\partial G}{\partial (z^j)^*}$, and so on. Note the axion type couplings determined by the imaginary part of the gauge kinetic function f.

The potential part is:

$$\mathcal{L}_{pot} = exp(-G)(3 + G_k (G^{-1})^k_l G^l) \tag{2.26}$$

Whenever scalars are charged under gauge symmetries, the so-called D-terms arise.

$$\mathcal{L}_D = -\frac{1}{2} \frac{g^2}{Ref_{ab}} (G^i T^{aj}_i z_j)(G^k T^{bl}_k z_l) \tag{2.27}$$

The fermion kinetic part of the lagrangian is:

$$\begin{aligned}\mathcal{L}_{Fkin} = -\tfrac{1}{2}\overline{\psi_\mu}\gamma^{\mu\nu\rho} D_\nu \psi_\rho + G^i_j \overline{\lambda_i}\gamma^\mu D_\mu \lambda^j \\ +\tfrac{1}{4} Ref_{ab}(-\overline{\chi^a}\gamma^\mu D_\mu \chi^b + \tfrac{1}{2}\overline{\chi^a}\gamma^\mu\gamma^{\nu\rho}\psi_\mu F^b_{\nu\rho} - \tfrac{1}{2}\overline{\chi^a_R}\gamma^\mu \chi^b_R G^i D_\mu z_i) \\ +\tfrac{1}{8}\overline{\chi^a}\gamma_5\gamma^\mu\chi^b D_\mu Im f_{ab} - \tfrac{1}{4} f^i_{ab}\overline{\lambda_{Ri}}\gamma^{\mu\nu}\chi^b_L F^a_{\mu\nu} + \tfrac{1}{8}\overline{\psi_\mu}\gamma_5\gamma^{\mu\nu\rho}\psi_\nu G^i D_\rho z_i \\ -G^j_i \overline{\psi_{R\mu}}\gamma^\nu\gamma^\mu \lambda_{Lj} D_\nu z^{*i} - (G^{ij}_k + \tfrac{1}{2} G^i_k G^j)\overline{\lambda_{Ri}}\gamma^\mu\lambda^k_L D_\mu z_j\end{aligned} \tag{2.28}$$

The terms described as 'fermi mass' contain interactions bilinear in fermion fields and polynomial in scalar fields. They become real mass terms only when scalars develop vacuum expectation values:

$$\begin{aligned}\mathcal{L}_{Fmass} = \tfrac{1}{2} e^{-G/2}\overline{\psi_{L\mu}}\gamma^{\mu\nu}\psi_{R\nu} + \tfrac{1}{4} e^{-G/2} G^l (G^{-1})^k_l f^*_{abk}\overline{\chi^a_L}\chi^b_R \\ +e^{-G/2}(G^{ij} - G^i G^j - G^l (G^{-1})^k_l G^{ij}_k)\overline{\lambda_{Ri}}\lambda_{Lj} - e^{-G/2} G^i \overline{\psi_{L\mu}}\gamma^\mu\lambda_{Li} \\ -\tfrac{1}{2} ig G^i T^{aj}_i z_j \overline{\psi_{R\mu}}\gamma^\mu\chi^a_R + 2ig G^j_i T^{ak}_j z_k \overline{\chi^a_L}\lambda^i_R + \tfrac{1}{2} ig (Ref)^{-1}_{ab} f^{bck} G^i T^{aj}_i z_j \overline{\lambda_{Rk}}\chi^c_L\end{aligned} \tag{2.29}$$

The supersymmetry transformation laws are:

$$\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\overline{\eta}\gamma^m\psi_\mu \\
\delta\psi_{L\mu} &= D_\mu\epsilon_L + \frac{1}{2}e^{-G/2}\gamma_\mu\epsilon_R - \frac{1}{4}\epsilon_L(G^iD_\mu z_i - G_iD_\mu z^{*i}) - \frac{1}{16}(2g_{\mu\nu} - \gamma_{\mu\nu})\epsilon_L\overline{\chi^a}\gamma_5\gamma^\nu\chi^b f_{ab} + \dots \\
\delta A_\mu^a &= -\frac{1}{2}\overline{\epsilon}\gamma_\mu\chi^a \\
\delta\chi_R^a &= -\frac{1}{4}\gamma^{\mu\nu}\epsilon_R F_{\mu\nu}^a \frac{ig}{2} Ref_{ab}^{-1} G^i T_i^{bj} z_j\epsilon_R + \dots \\
\delta z_i &= \overline{\epsilon_R}\lambda_{Li} \\
\delta\lambda_{Li} &= \frac{1}{2}\gamma^\mu D_\mu z_i\epsilon_R - \frac{1}{2}e^{-G/2}(G^{-1})_i^j G_j\epsilon_L - \frac{1}{8}\epsilon_L(G^{-1})_i^k f_{abk}^*\overline{\chi_L^a}\chi_R^a \qquad (2.30)
\end{aligned}$$

We skipped all three-fermi terms except for those involving gaugino bilinears which will be important in further discussions.

Chapter 3

Horava-Witten model

At the time when Horava and Witten constructed their model, the common opinion was that the only phenomenologically viable string theory is the $E_8 \times E_8$ heterotic superstring theory. To understand the motivation of the authors we must first briefly recall the basic features of low energy theories derived from the heterotic superstring theories.

In the low energy limit (that is below the Planck scale) the weakly coupled $E_8 \times E_8$ heterotic string theory reduces to the 10d Type I supergravity coupled to one $E_8 \times E_8$ Yang-Mills multiplet. To make contact with the real world this theory is further compactified to four dimensions on the background $M_4 \times K$, where M_4 is the non-compact space where we live, and K is a compact six dimensional manifold. The number of supercharges of the Type I 10d supergravity is 16 and corresponds to N=4 supersymmetry in 4d. However, if supersymmetry is relevant to our TeV scale world it can be at most N=1 supersymmetry, so the compactification must somehow break the remaining supersymmetries. This can be achieved by choosing for K a complex Kähler Ricci-flat manifold of SU(3) holonomy known as the Calabi-Yau three-fold. Once we decide on the Calabi-Yau manifold, we obtain a number of remarkable predictions concerning the four-dimensional effective theory:

1. If we want (for cosmological reasons) the non-compact part of the background M_4 to be maximally symmetric, then by field equations it is necessarily the Minkowski flat space (de Sitter and anti-de Sitter spaces are excluded); thus string theory can in principle provide us with the explanation of the observed flatness of the universe.

2. The simplest choice of the vacuum expectation value for the gauge fields which satisfies the equations of motions (precisely - the Bianchi identity of the two-form field) breaks the E_8 gauge group to E_6. The exceptional group E_6 was proposed for the Grand Unified Group long before the advent of the string phenomenolgy. It has a complex representation **27** which can accomodate one generation of the Standard Model fields. The compactification predicts a number of supermultiplets in this representsation. Moreover, E_6 contains SO(10) and SU(5) as its subgroups so it can be broken to more standard and thoroughly investigated GUT groups.

3. Generically, we get more than one copy of massless **27** (which become massive only after supersymmetry breakdown and their masses are of the order of the electroweak scale). Thus, we have a natural explanation of the existence of generations in the SM. The predicted number of generations in the low energy world is one half of the Euler characteristic of the Calabi-Yau three-fold and so the actual number of generations can be understood on strictly topological grounds.aaaaaa

4. Once we choose to place the SM matter in the representations of E_6 we automatically get extra matter from the second E_8 sector which couples only gravitationally to the observable particles; in other words we automatically get the so-called hidden sector - the most popular mechanism of supersymmetry breaking in realistic model-building

5. The compactification universally yields also an axion so, in principle, we are able to solve the strong CP problem.

The impresive success of the heterotic string theory was shadowed by one disturbing fact: the gravitational and gauge coupling did not unify at the GUT scale but rather at a scale an order of magnitude higher. Imposing the unification at the GUT scale required the string coupling constant much bigger than one and thus out of the range in which perturbative calculations in string theory could make any sense (note that string theories are formulated only perturbatively).

Shortly before the Horava-Witten model was constructed the unexpected relations (dualities) between various string theories had been discovered. It became clear that taking the limit of large (aproaching infinity) string coupling constant could lead to another theory which was not necessarily a string theory. At that time it was known e.g. that the strong coupling limit of the type IIA superstring theory is an eleven dimensional theory provisionally named M-theory.

Horava and Witten took seriously the message stemming from the (lack of) gauge-gravitational unification and considered the strong coupling limit of the $E_8 \times E_8$ heterotic superstring theory. They conjectured [2] that in this limit one got M-theory compactified on $M_{10} \times S_1/Z_2$, where M_{10} is a smooth 10d manifold and S_1/Z_2 is equivalent to the interval. In the low energy limit M-theory reduces to 11d supergravity. Thus, the strong coupling limit of the $E_8 \times E_8$ heterotic string theory should correspond to eleven dimensional supergravity on a manifold with boundaries. This conjecture can be the starting point for phenomenological considerations.

What happens to gauge group present in the weakly coupled limit? Contrary to the ten-dimensional case, there is no Yang-Mills supermultiplet in eleven dimensions, but in the Horava-Witten model we still have ten-dimensional boundaries of the interval at our disposal. Horava and Witten found [3] that the consistency of the model (precisely - the anomaly cancellation) requires one YM supermultiplet in the adjoint of E_8 at each end of the interval. In a sense, the $E_8 \times E_8$ of the weakly coupled limit is cut in two parts. The size $\pi\rho$ of the eleventh dimension can be shown to correspond to the strength of the string coupling; taking the limit $\rho \to 0$ reduces the Horava-Witten model back to the ten-dimensional description of the weakly coupled case and the two E_8 factors merge together.

Though it was not clear from the begining whether a theory in which part of the fields resided on a lower-dimensional manifold could be consistently supersymmetrized, Horava and Witten showed [3] by the direct construction, that supersymmetrization was possible. One can expect the supersymmetry of the string theory to survive in the strong coupling limit, which makes supersymmetrization of the Horava-Witten model a non-trivial test of the consistency of the entire set-up. In the case of supergravities on smooth manifolds we can classify possible theories and field representations by means of the so-called tensor calculus. In the case of the Horava-Witten model, due to the presence of boundaries the commutation relations become singular and the tensor calculus does not work. So far a general formulation of locally supersymmetric theories with matter residing on submanifolds has not appeared.

The procedure applied by Horava and Witten in order to couple the Yang-Mills supermultiplet is known as the Noether method. The idea is to start with a globally supersymmetric theory lagrangian. To promote this symmetry to a local one, new terms are iteratively added to the lagrangian and to the transformation laws. At each step the lagrangian is varied and the modifications of the lagrangian and supersymmetry transformations needed to cancel the variation are guessed. It is not guaranteed that the procedure ends in finite time, but if the coupling is possible one usually needs only a few steps.

3.1 11d supergravity on $M_{10} \times S_1/Z_2$

Before proceeding we must first define our 11d supergravity on the manifold of which one dimension (say, the eleventh) is an interval. $M_{10} \times S_1/Z_2$ is essentially a manifold with boundary and we should specify the appropriate boundary conditions for the eleven-dimensional fields. However, there is a more convenient way to deal with this problem. In the following, we work with fields defined on the smooth manifold $M_{10} \times S_1$ and impose a $\mathbf{Z_2}$ symmetry on the fields of 11d supergravity.

We parametrize the circle S_1 with the coordinate x^{11} which extends from $-\pi\rho$ to $\pi\rho$ and we identify the endpoints. The $\mathbf{Z_2}$ parity acts by $x^{11} \to -x^{11}$. The fixed points of this symmetry operation are ten-dimensional hypersurfaces $x^{11} = 0$ and $x^{11} = \pi\rho$ where the gauge fields are located. Eleven dimensional fields can be even ($\phi(x^{11}) = \phi(-x^{11})$) or odd ($\phi(x^{11}) = -\phi(-x^{11})$) under $\mathbf{Z_2}$. Note that the odd fields must either vanish or be discontinuous at the fixed points, hence they are not dynamical fields on the submanifolds where the gauge fields live. $\mathbf{Z_2}$ takes ∂_{11} into $-\partial_{11}$ so the eleventh derivative reverses the parity assignments.

We require $\mathbf{Z_2}$ to be the symmetry of the eleven dimensional action. In the following we single out ten dimensonal indices (0..9) which are denoted with latin letters from the beginning of the alphabet A,B,... We define g_{AB} to be even so that the ten-dimensional part of the metric is dynamical at the fixed points; all the subsequent parity assignments follow from this choice. The Ricci scalar R contains the eleventh derivative of g_{A11} so those components must be odd. The similar reasoning leads to $g_{11\,11}$ being even: R contains either two or no eleventh derivatives of this component of the metric. Equivalently, in the vielbein language e^a_A and e^{11}_{11} are even and e^{11}_A and e^a_{11} are odd. In summary, the metric components which contain odd number of '11' are odd.

The parity assignments of the three-form field C follow from the 'topological' term in the action $\epsilon^{I_1..I_{11}} C_{I_1..I_3} G_{I_4..I_7} G_{I_8..I_{11}}$. Let us suppose $I_1 = 11$ (thus the remaining indices are ten dimensional; otherwise the Levi-Civita tensor is zero). The two field strengths G_{ABCD} multiplied by each other are even, whatever parity is chosen for a single G_{ABCD}. Then the whole expression is $\mathbf{Z_2}$ invariant only when we chose C_{11AB} even and it follows that G_{11ABC} must be even. Next, the invariance of the kinetic term $G_{11ABC}G^{11ABC}$ together with the fact that g^{11A} is odd requires that G_{ABCD} is odd. In summary, an odd number of '11' in C or G means that this component is even.

A little less straightforward is the action of $\mathbf{Z_2}$ on gravitinos. Consider the interaction term $\overline{\psi}^J \Gamma^{KL} \psi^M G_{JKLM}$. From the previously obtained $\mathbf{Z_2}$ assignments of G it follows that $\overline{\psi}^A \Gamma^{B11} \psi^C$ is even and $\overline{\psi}^A \Gamma^{BC} \psi^D$ is odd. This is possible only if $\psi_A(x^{11}) = \Gamma^{11}\psi_A(-x^{11})$. Then the former expression:

$$\overline{\psi}^A(x^{11})\Gamma^{B11}\psi^C(x^{11}) = \overline{\Gamma^{11}\psi}^A(-x^{11})\Gamma^{B11}\Gamma^{11}\psi^C(-x^{11}) = -\overline{\psi}^A(-x^{11})\Gamma^{11}\Gamma^{B11}\Gamma^{11}\psi^C(-x^{11})$$
$$= \overline{\psi}^A(-x^{11})\Gamma^{B11}\psi^C(-x^{11})$$

is indeed even as one must anti-commute once with Γ^B to anihilate two Γ^{11}'s. Similarly the latter expression is odd as one must anti-commute twice. Analogous reasoning leads to $\psi_{11}(x^{11}) =$

$-\Gamma^{11}\psi_{11}(-x^{11})$.

If we want the supersymmetry transformations to commute with $\mathbf{Z_2}$ we must also asign the correct $\mathbf{Z_2}$ parity to the supersymmetry transformation parameter η. From the gravitino transformation law $\delta\psi_A = D_A\eta + ...$ we can read off that the parity assignment of η must be the same as that of the ten-dimensional components of the gravitino: $\Gamma^{11}\eta(x^{11}) = \eta(-x^{11})$

One can easily check that with this assignments the rest of the terms in the 11d supergravity lagrangian as well as the supersymmetry transformation laws are $\mathbf{Z_2}$ invariant. Below we summarize the $\mathbf{Z_2}$ properties of the 11d fields:

$$\begin{array}{cc} even & odd \\ e^a_A, e^{11}_{11} & e^{11}_A, e^a_{11} \\ C_{11AB}, G_{11ABC} & C_{ABC}, G_{ABCD} \end{array} \tag{3.1}$$

$$\begin{aligned} \Gamma^{11}\psi_A(x^{11}) &= \psi_A(-x^{11}) \\ \Gamma^{11}\psi_{11}(x^{11}) &= -\psi_{11}(-x^{11}) \\ \Gamma^{11}\eta(x^{11}) &= \eta(-x^{11}) \end{aligned} \tag{3.2}$$

3.2 Coupling 10d Yang-Mills supermultiplet to 11d supergravity

In this subsection we review the Horava-Witten construction following the reference [3]. We start with the 11d supergravity lagrangian given in (2.1). We know that this lagrangian possesses local supersymmetry and the supersymmetry transformations are given in (2.3). Next a perturbation consisting of a 10d vector supermultiplet in the adjoint representation of E_8 at each fixed point is added. In the folowing, the gauge group will not be important and the supersymmetric coupling is possible for any group. We concentrate only on the brane at $x^5 = 0$; the modifications required on the second brane are identical. Following the standard terminology we will call the interior of the 11d space the 'bulk' and the boundaries will be described as the 'branes'.

Ten dimensional gauge supermultiplet contains gauge fields A^a_A and gauginos χ^a. The latin indices A,B,... are ten dimensional and run over values 0..9. a is a group index which we often suppress (it should not be confused with the $Sp(2n_h)$ index of 5d symplectic Majorana spinors). In ten dimensions we can define spinors which satisfy both the Majorana and the Weyl conditions. The gaugino χ is such a Majorana-Weyl spinor with definite chirality and satisfies $\Gamma^{11}\chi = \chi$. We add to the 11d supergravity action the kinetic terms for the gauge multiplet:

$$\begin{aligned} S_{YM} &= \frac{1}{\lambda^2}\int_{M_{11}} d^{11}x e_{11}\delta(x^{11})\mathcal{L}_{YM} \\ \mathcal{L}_{YM} &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2}\overline{\chi}^a\Gamma^A D_A\chi^a \end{aligned} \tag{3.3}$$

Classically, λ is a free parameter - the gauge coupling of E_8. However gravitational and gauge anomalies cancel out only if λ is related to the gravitational coupling by the formula [3]:

$$\lambda^2 = 2\pi(4\pi\kappa^2)^{2/3} \tag{3.4}$$

Note that the bulk action is multiplied by $1/\kappa^2$ and the boundary action by $1/\lambda^2 \sim 1/\kappa^{4/3}$. Thus the boundary action can be considered a first order perturbation in $\kappa^{2/3}$. The relation 3.4 can be qualitatively established on the basis of the dimensional analysis, as λ has dimension $(mass)^{-3}$ and κ - $(mass)^{-9/2}$; the anomaly cancellation analysis gives just the precise form of this relation.

The delta function is defined covariantly:

$$\int_{M_{11}} d^{11}x e_{11}\delta(x^{11}) = \int_{M_{10}} d^{10}x e_{10} \tag{3.5}$$

where M_{10} is the hypersurface $x^{11} = 0$ in M_{11} and e_{10} is built from the ten dimensional components of the vielbein. The above super-Yang-Mills lagrangian possesses global supersymmetry and the supersymmetry transformations are:

$$\begin{aligned} \delta A^a_A &= \frac{1}{2}(\overline{\eta}\Gamma_A\chi^a) \\ \delta\chi^a &= -\frac{1}{4}\Gamma^{AB}\eta F^a_{AB} \end{aligned} \tag{3.6}$$

Interestingly enough, the super-Yang-Mills action exists only in 3, 4, 6 and 10 dimensions. The form of the action is always the same (but of course the dimensionality of spinors and gamma matrices must be appropriate for a given space-time dimension).

The spinor parameter η is Majorana-Weyl. It is crucial for the whole construction to identify η with the parameter of the local supersymmetry transformations in 11d supergravity. Although in eleven dimensions we cannot impose the Weyl condition (such condition would not be invariant under general coordinate transformations as Γ^{11} is one of the matrices of the 11d Clifford algebra), enforcing the $\mathbf{Z_2}$ symmetry $\Gamma^{11}\eta(x^{11}) = \eta(-x^{11})$ has the effect that at the fixed point the 11d parameter η indeed satisfies the Weyl condition $\Gamma^{11}\eta = \eta$.

We compute the variation of the lagrangian (3.3) using the transformations (3.6) with a general x-dependent parameter η. The variation of the gauge kinetic term F^2 yields $-\frac{e}{2}F^{AB}D_{[A}(\overline{\eta}\Gamma_{B]}\chi)$ while the variation of the gaugino kinetic term yields $\frac{e}{4}F_{AB}(\overline{\eta}\Gamma^{AB}\Gamma^C D_C\chi)$. Together they sum to

$$\begin{aligned} \delta\mathcal{L}_{YM} &= -\tfrac{e}{4}F_{AB}[D_C(\overline{\eta}(-\Gamma^{AB}\Gamma^C + \Gamma^{ABC})\chi) + (\overline{\eta}\Gamma^{AB}\Gamma^C D_C\chi)] \\ &= \tfrac{e}{4}[F_{AB}(\overline{D_C\eta}\Gamma^{AB}\Gamma^C\chi) + F_{AB}D_C(\overline{\eta}\Gamma^{ABC}\chi)] \\ &= \tfrac{e}{4}F_{AB}(\overline{D_C\eta}\Gamma^{AB}\Gamma^C\chi) \end{aligned} \tag{3.7}$$

In the first line the gamma matrices identity $-\Gamma^{AB}\Gamma^C + \Gamma^{ABC} = -2g^{A[C}\Gamma^{B]}$ was used. In the last line we integrated by parts (this is allowed as the invariance of the action requires the lagrangian to be invariant up to a total derivative) and used the Bianchi identity for the field strength $D_{[A}F_{BC]} = 0$.

The variation (3.7) can be cancelled by adding a new term to the boundary lagrangian:

$$\mathcal{L}_N = -\frac{e}{4}(\overline{\psi}_A\gamma^{BC}\gamma^A\chi)F_{BC} \tag{3.8}$$

The part of the gravitino variation proportional to $D_A\eta$ in $\mathcal{L}_N$ cancells the variation of $\mathcal{L}_{YM}$. Note that in in the language of the 11d action this term is multiplied by the delta function.

This term is usually called the Noether term. In fact, what we did was to couple the Noether curent (supercurrent) of a globally supersymmetric lagrangian to the gravitino, which is the gauge field of supersymmetry [17], in analogy to what one does in locally symmetric Yang-Mills theories.

It turns out that more modifications are needed. The hint is given by considering the vartiations of the form $\eta\psi F^2$. These come from varying the vielbein in the gauge kinetic term and from varying gauginos in the Noether term. After some tedious manipulations one finds that these variations do not cancel by themselves. What is left is:

$$\delta\mathcal{L} = \frac{e}{16}\overline{\psi_A}\Gamma^{ABCDE}\eta F_{BC}F_{DE} \tag{3.9}$$

The situation is reminescent of what we encounter in 10d supergravity. There, the identical calculations yield the same result and to cancel the variations of the form $\eta\psi F^2$ one is forced to modify the Bianchi identity for the three-form field strength. In the framework of the Horava-Witten model we do not have any form fields on the boundary, but we have the four-form field strength G in the bulk. Therefore, the natural idea is to cancel the above variation by modifying the Bianchi identity for G. It turns out that the correct solution is to replace G in the bulk lagrangian with:

$$\hat{G}_{11ABC} = G_{11ABC} + \frac{\kappa^2}{\sqrt{2}\lambda^2}\delta(x^{11})\omega_{ABC} \tag{3.10}$$

where ω is the Chern-Simons form satisfying:

$$\partial_{[A}\omega_{BCD]} = 6F^a_{[AB}F^a_{CD]} \tag{3.11}$$

Another way to describe the above corrections is to say that the Bianchi identity for the modified four-form field strength reads:

$$(dG)_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{\lambda^2}\delta(x^{11})F^a_{[AB}F^a_{CD]} \tag{3.12}$$

What is the mechanism to cancel (3.9)? When we vary the 11d bulk lagrangian, we must check, in particular, if the variations of the form $\eta\psi G$ cancel. Varying the gravitino kinetic term and considering the part of the gravitino transformation law proportional to G, we get an expression of the form:

$$\delta\mathcal{L} = \overline{\psi_A}(Gamma's)D_B(\eta G_{11CDE}) \tag{3.13}$$

When the derivative acts on the spinor η the variations cancel with the variations of gravitno proportional to $D_A\eta$ in the $\psi^2 G$ terms. In the pure 11d supergravity, the part with the derivative acting on G is identically zero due to the Bianchi identity dG=0. If the Bianchi identity is modified as in (3.12) the derivative acting on G contributes to the variation and precisely cancels (3.9).

Redefinition of the field strength G must be supplemented by the modification of the supersymmetry transformation law of G by a term:

$$+\delta\hat{G}_{11ABC} = \frac{3\kappa^2}{\sqrt{2}\lambda^2}\delta(x^{11})\overline{\eta}\Gamma_{[A}\chi F_{BC]} \tag{3.14}$$

It should be stressed that the modification of the Bianchi identity is just a convienent and compact way of saying that we add new boundary couplings. In the case at hand we couple the bulk field C to the polynomial built of the boundary gauge fields.

Another necessary boundary coupling can be determined by considering variations of the form $\eta\chi GF$ which originate from variation of ψ proportional to G in the Noether term and from variation of $\hat{G}$ proportional to F in the kinetic term of G. These two variations do not cancel and the left-over is:

$$\delta\mathcal{L} = -\frac{\sqrt{2}e}{96}\overline{\eta}\Gamma^{BC}\Gamma^{DEF}\chi F_{BC}G_{DEF11} \tag{3.15}$$

It is easy to see that to cancel the above a new term in the boundary lagrangian is needed:

$$+\mathcal{L} = \frac{\sqrt{2}e}{48}\overline{\chi}\Gamma^{DEF}\chi G_{DEF11} \tag{3.16}$$

The YM variation (3.6) of the gauginos is enough to ensure that variatons of the form $\eta\chi FG$ indeed cancel out.

The remaining corrections are four-fermi terms in the boundary lagrangian and three-fermi terms in the supersymmetry transformation laws. Although, to have confidence in the theory it is very important to show that supersymmetic variation of the action indeed vanishes after adding these corrections, the actual calculations is tedious and not very spectacular. Therefore, we concentrate only on a few interesting aspects of this calculation, which will be important in the following sections.

First, we shall take a closer look at modifications of the gravitino transformation law. They can be determined from the study of 4-fermi variations of the form $D\psi\eta\chi\chi$. They do not cancel by themselves and the gravitino transformation law must be supplemented with:

$$\begin{aligned} +\delta\psi_A &= -\frac{\kappa^2}{288\lambda^2}\delta(x^{11})(\overline{\chi}\Gamma_{BCD}\chi)(\Gamma_A{}^{BCD} - 6g_A^B\Gamma^{CD})\eta \\ +\delta\psi_{11} &= -\frac{\kappa^2}{288\lambda^2}\delta(x^{11})(\overline{\chi}\Gamma^{ABC}\chi)\Gamma_{ABC}\eta \end{aligned} \tag{3.17}$$

The object of interest here is the delta function. It must be present, since the variations we want to cancel are on the boundary and we vary the gravitino kinetic term which lives in the bulk. But this may cause troubles. We have already gravitno interactions on the boundary (the Noether term) and such terms in the 11d action are already proportional to the delta function. If we vary gravitino entering the boundary terms (3.17) we get singular variations which are formally proportional to the delta squared. Even worse, such singular variations do not cancel out. Since, such variations are proportional to $\frac{\kappa^4}{\lambda^4}$ (w.r.t to the bulk action) we have to admit that the Horava-Witten model is valid only to the first order in perturbation in $\frac{\kappa^2}{\lambda^2} \sim \kappa^{2/3}$.

Nevertheless, we can try to cancel at least some of the variations of order $\frac{\kappa^4}{\lambda^4}$. For example (3.17) in the Noether term and variation of $\hat{G}$ in the $G\chi\chi$ interaction yield:

$$\delta S = -\int d^{11}x e_{11}\delta^2(x^{11})\frac{\kappa^2}{1536\lambda^4}(\overline{\chi}\Gamma_{ABC}\chi)(\overline{\chi}\Gamma^{ABC}\Gamma^{DE}\eta)F_{DE} \tag{3.18}$$

which can be easily cancelled by varying the gaugino in the new singular, quartic in gauginos interaction:

$$+S = -\int d^{11}x e_{11}\delta^2(x^{11})\frac{\kappa^2}{1536\lambda^4}(\overline{\chi}\Gamma_{ABC}\chi)(\overline{\chi}\Gamma^{ABC}\chi) \tag{3.19}$$

An interesting observation is that this singular term is a part of a 'perfect square'. The situation is similar to what one encounters in ten-dimensional supergravity, where gauginos group into a perfect square $(H + \chi\chi)^2$ with the three-form field strength (of course, in 10d there are no singular terms). In the Horava-Witten model, gauginos combine into the perfect square with the four-form field strength G. The bulk kinetic term of G, the boundary interaction $G\chi\chi$ and the singular term χ^4 can be written as:

$$S_{ps} = -\int d^{11}x e_{11} \frac{1}{48}(G_{ABC11} - \frac{\sqrt{2}\kappa^2}{8\lambda^2}\delta(x^{11})\overline{\chi}\Gamma^{ABC}\chi)^2 \tag{3.20}$$

This form suggests, that we could formally get rid of the divergent term by redefining the field strentgh G. Thus, we can trust, that in spite of the singularities, the Horava-Witten model is a sensible theory. Later we will see that singularities indeed drop out from the effective four-dimensional theory.

3.3 Higher derivative corrections

The derivations of the previous subsection were limited to terms which are at most second order in derivatives. However, plenty of physics is contained in higher derivative interactions. To avoid gravitational and gauge anomalies we must include terms proportional to R^4 and F^4 in the action [3]. The precise form of these terms will not concern us in this thesis but there are two correcions to the lagrangian which will be important in the following discussions.

First, we found that the Bianchi identity for the four-form field strength G must be modified modified. Anomaly cancellation analysis introduces further, higher order in derivatives corrections, so that the Bianchi identity reads (up to $\frac{\kappa^4}{\lambda^4}$ terms):

$$(dG)_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{\lambda^2}[(F^{(1)}_{[AB}F^{(1)}_{CD]} - \frac{1}{2}tr(R_{[AB}R_{CD]})\delta(x^{11}) + (F^{(2)}_{[AB}F^{(2)}_{CD]} - \frac{1}{2}tr R_{AB}R_{CD})\delta(x^{11} - \pi\rho)] \tag{3.21}$$

Note the factors 1/2 appearing in front of the traces of the curvature tensor, which will be crucial in the subsequent discussion, because they forbid vacuum solutions with G=0.

The second modification we mention is the boundary term involving the curvature, so that the bosonic part of the boundary lagrangian reads:

$$\mathcal{L}_{YM} = -\frac{e}{4\lambda^2}(F^{(1)}_{AB}F^{(1)AB} - \frac{1}{2}R_{ABCD}R^{ABCD}) \tag{3.22}$$

and similar terms are added on the second brane. This modification will be helpful to determine the boundary scalar potential in the low energy theory.

3.4 Compactification to five dimensions

If theory is to describe our physical world, it has to reduce to a four dimensional effective field theory at low energies. But the compactification does not have to proceed in one step; there may exist some intermediate scale, at which the theory can effectively be formulated in more than four dimensions. This is the case with the Horava-Witten model in an interesting region of its parameter space. To obtain the unification of gauge and gravitational couplings, the size of the eleventh (orbifold) dimension must be about an order of magnitude larger that the characteristic length of the remaining six compact dimensions. Thus, just below the Planck scale, the Horava-Witten model is described by a five dimensional theory.

Generally, to obtain a low energy effective theory one has to perform the Kaluza-Klein reduction. The extensive introduction to KK reduction in the context of string theories can be found in [27] and here we present only the most important key-words. We assume that the background manifold on which we compactify is a direct product $M \times K$, where M is non-compact and parametrized with coordinates x and K is a compact manifold with coordinates y. We write the fields Ψ of the original theory as a sum $\Psi(x, y) = \sum_n \phi_n(x)\chi_n(y)$. The equations of motion $\Delta\Psi = 0$ of the original theory split into:

$$(\Delta_M + \Delta_K)\phi(x)\chi(y) = 0 \tag{3.23}$$

The laplacian Δ has a different meaning depending on the context; if Ψ is a scalar field it is an ordinary laplacian, if Ψ is a spinor it is the Dirac operator. We demand that the fields χ are eigenvectors of the laplacian Δ_K on the manifold K with eigenvalue m_n^2. Then the equations of motion take the form:

$$(\Delta_M + m_n^2)\phi_n = 0 \tag{3.24}$$

The linear independence of eigenvectors of the laplacian was used. This is the equation of motion for a field ϕ_n with mass m_n. Thus, in the effecive theory we get a tower of fields with masses corresponding to the eigenvalues of Δ_K. On dimensional grounds we expect that generic masses are proportional to 1/R where R is the characteristic length scale of K. For phenomenologically viable values of R, these masses are huge and the corresponing fields decouple from the low energy theory. The only fields that remain in the effective description correspond to $m_n = 0$, in other words, to the zero-modes of Δ_K. Usually, massless ϕ_n are also called zero-modes. Below we determine only the zero-modes of bosonic fields; if our background preserves supersymmetry, the zero modes of fermionic fields must fit in supermultiplets.

As was already mentioned, the Horava-Witten model is the strong coupling limit of the heterotic $E_8 \times E_8$ string theory. Phenomenologically promising compactifications of that string theory are obtained on backgrounds of which the compact component is a six-dimensional Calabi-Yau manifold. It is then reasonable to compactify the Horava-Witten model on a Calabi-Yau three-fold. A Calabi-Yau three-fold breaks exactly one fourth of the supersymmetries. In the case of heterotic strings we have a ten dimensional theory with 16 supercharges, so the effective theory is four dimensional and possesses N=1 supersymmetry (4 supercharges). The Horava-Witten model is eleven dimensional and has 32 supercharges, so its compactification on a Calabi-Yau three-fold yields a five dimensional theory with 8 supercharges. Such theory is called N=2 5d supergravity and was described in the section 2.2.

The precise form of the five dimensional effective theory was found in [5]. The background metric is given by:

$$ds^2 = V^{-2/3} g_{\alpha\beta}(x) dx^\alpha dx^\beta + g_{ij}(y) dy^i dy^j \tag{3.25}$$

g_{ij} is the metric on the Calabi-Yau and V is the Calabu-Yau volume defined by $V = \int_{CY} \sqrt{det(g_{ij})}$. The factor $V^{-2/3}$ is to ensure that the five-dimensional metric $g_{\alpha\beta}$ has the canonical Einstein-Hilbert action (that is, the kinetic term of the metric is $-\frac{1}{2}R$). We have changed the notation, so that the original eleventh dimension has become the fifth.

Having decided on the compact Calabi-Yau manifold we still have certain freedom in choosing its parameters. The equations of motion do not restrict these parameters so they correspond to massless scalar fields in the effective theory. They are called the moduli of the compactification.

Every Calabi-Yau manifold is endowed with the Kähler form ω. This is a closed two-form ($d\omega = 0$, usually we chose $d^* w = 0$, so that it is also harmonic) with one holomorphic and one anti-holomorphic index, that is a (1,1) form in the terminology of complex manifolds. The number of independent harmonic (1,1) forms on a Calabi-Yau three-fold is arbitrary and is characterized by the Hodge number $h_{1,1}$. The Kähler form is thus a linear combination of $h_{1,1}$ forms:

$$\omega_{a\bar{b}} = a^i \omega_{ia\bar{b}} \tag{3.26}$$

Another characteristic parameter of a Calabi-Yau three-fold is the Hodge number $h_{2,1}$. In the usual aproach to compactification it is assumed that $h_{2,1} = 0$. If this number is non-zero, a number of hypermultiplets in the 5d theory appears, but their structure is independent of the specific features of the Horava-Witten model. The Calabi-Yau three-folds have no other independent Hodge numbers. We have $h_{00} = h_{30} = h_{03} = 1$ $h_{22} = h_{11}$ and the remainig h_{ab} are zero.

The a^i's become the dynamical fields in the effective theory. However they are not independent of V, as the Calabi-Yau volume can be expressed as $V = \frac{1}{6}\int \omega \wedge \omega \wedge \omega$. We have the relation

$$6V = d_{ijk} a^i a^j a^k \tag{3.27}$$

where the Calabi-Yau intersection numbers are defined as $d_{ijk} = \int \omega_i \wedge \omega_j \wedge \omega_k$. Thus, the fields a^i together with V describe only $h_{1,1}$ independent degrees of freedom.

We must also determine the zero-modes corresponding to the three-form field C. In the first order in $\frac{\kappa^2}{\lambda^2}$ its equations of motion are $dG = d^*G = 0$ which are trivially satisfied by G=0=dC. The 11d three-form field C survives in the effective 5d theory as a one 5d three form field (which by duality corresponds to one real pseudoscalar), $h_{1,1}$ vector fields $\mathcal{A}^i_\alpha$ and one complex scalar ξ. If C is harmonic its various components can be decomposed in the following way:

$$\begin{aligned} &C_{\alpha\beta\gamma}(x) \\ C_{\alpha a\bar{b}} &= \frac{1}{6}\mathcal{A}^i_\alpha(x)\omega_{ia\bar{b}} \\ C_{abc} &= \frac{1}{6}\xi(x)\Omega_{abc} \\ C_{\overline{abc}} &= \frac{1}{6}\bar{\xi}(x)\Omega_{\overline{abc}} \end{aligned} \tag{3.28}$$

In the first line we used $h_{00} = 1$ (the unique harmonic (0,0) form is just a constant), while the last two lines result from $h_{30} = h_{03} = 1$ and Ω is the unique harmonic (3,0) form on Calabi-Yau.

Let us summarize the bosonic spectrum of the five dimensional effective theory obtained by the compactification of the bulk action. We have the 5d metric $g_{\alpha\beta}$, $h_{1,1}$ vector fields $\mathcal{A}^i_\alpha$, $h_{1,1}$ real scalars a^i (which are subject to the constraint (3.27)) , three scalars V, ξ, and $\bar{\xi}$ and a three-form $C_{\alpha\beta\gamma}$. Our task is to interpret them as componenents of 5d supermultiplets.

Obviously, the metric belongs to the gravitational multiplet. Due to the definition (3.25 it has the correct Einstein-Hilbert kinetic term $-\frac{1}{2}R$. To complete the bosonic part we need a graviphoton. We have the vectors fields $\mathcal{A}$ and we expect that the graviphoton is their linear combination. The precise formula is $\frac{2}{3}b_i\mathcal{A}^i_\alpha$ but it is not so important as the formulation of 5d supergravity we gave in the previous section places all vector fields on equal footing.

Of course, we can have only one gravitational multiplet, so the remaining vector fields must fit in $h_{1,1} - 1$ vector multiplets. To complete the vector multiplets we have scalars a^i. If we define:

$$b^i = V^{-1/3} a^i \tag{3.29}$$

then the fields b^i represent $h_{1,1}-1$ degrees of freedom and are subject to constraint:

$$\mathcal{K}(b) \equiv d_{ijk} b^i b^j b^k = 6 \tag{3.30}$$

This is exactly compatible with the formulation of dynamics of scalars belonging to vector multiplets we presented in subsection 2.2.1. In the general formulation the symmetric tensor d is arbitrary, while in the compactified theory it acquires an interpretation of the Calabi-Yau intersection numbers.

The metric of the sigma model describing scalars b can be explicitly expressed in terms of the harmonic forms on the Calabi-Yau manifold:

$$G_{ij} = \frac{1}{2V} \int_{Calabi-Yau} \omega_i \wedge (*\omega_j) \tag{3.31}$$

The functions $\mathcal{K}$ and G_{ij} given above are sufficient to recover the coupling of the vector multiplets to 5d supergravity, as described in subsection 2.2.1

We are left with three scalars and one three form which is equivalent to a scalar. As we have no vectors left, the natural guess is that these four scalar degrees of freedom belong to a hypermultiplet. This multiplet is usually called the universal hypermultiplet as the above mentioned moduli arise in any compactication of M-theory on Calabi-Yau. After dualizing the three form to a scalar σ by $G_{\alpha\beta\gamma\delta} = \frac{1}{\sqrt{2}} \epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon} (\partial_\epsilon \sigma - i(\xi \partial_\epsilon \bar{\xi} - \bar{\xi} \partial_\epsilon \xi))$ the kinetic terms of the hypermultiplet scalars read:

$$\begin{array}{c} S_{kin} = - \int d^5 x e_5 \frac{1}{2\kappa^2} \left(\frac{1}{2V^2}(\partial_\alpha V \partial^\alpha V + \partial_\alpha \sigma \partial^\alpha \sigma) + \frac{2}{V} \partial_\alpha \xi \partial^\alpha \bar{\xi} \right. \\ \left. + \frac{i}{2V^2}(\xi \partial_\alpha \bar{\xi} \partial^\alpha \sigma - \bar{\xi} \partial_\alpha \xi \partial^\alpha \sigma) - \frac{1}{2V^2}((\xi \partial_\alpha \bar{\xi})^2 + (\bar{\xi} \partial_\alpha \xi)^2 - |\bar{\xi} \partial_\alpha \xi|^2) \right) \end{array} \tag{3.32}$$

In the languag used in the section 2.2 this sigma model corresponds to the Kähler potential:

$$\begin{array}{c} K = -ln(S + \bar{S} - 2\xi\bar{\xi}) \\ S = V + \xi\bar{\xi} + i\sigma. \end{array} \tag{3.33}$$

If the compactification of the Horava-Witten model were the standard KK reduction, this would be the whole story. But in the consistent reduction we are not allowed to neglect the background value of the four-form field strength G. The reason is that we must satisfy the Bianchi identity (3.21), and $G = 0$ does not solve it. Thus, compactification with $G = 0$ is not consistent as the solutions the theory compactified with $G = 0$ would not be the solutions of the original theory.

In the case of the heterotic $E_8 \times E_8$ string theory the situation is much simpler. The Bianchi identity for the three-form field strength H reads $(dH)_{ABCD} \sim F^{(1)}_{[AB} F^{(1)}_{CD]} + F^{(2)}_{[AB} F^{(2)}_{CD]} - tr R_{AB} R_{CD}$. As the spin connection and the curvature on Calabi-Yau are SU(3) matrices, we can put them equal to the SU(3) subgroup of, say, the first E_8 and the demand that the vevs of the second E_8 sector are equal to zero. This is what is usually referred to as the standard embedding. Then the Bianchi identity reduces to dH=0, the solution H=0 is perfectly legitimate, and the compactification is the standard KK reduction.

In the case of the Horava-Witten model, because of the unfortunate factor 1/2, there is no possibility to cancel the right-hand side of the Bianchi identity. However we can still keep the standard embedding:

$$\begin{array}{c} tr F^{(1)} \wedge F^{(1)} = tr R \wedge R \\ F^{(2)} = 0 \end{array} \tag{3.34}$$

The Bianchi identity reduces in this case to

$$(dG)_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{2\lambda^2}[trR_{[AB}R_{CD]}\delta(x^5) - trR_{[AB}R_{CD]}\delta(x^5 - \pi\rho)] \tag{3.35}$$

Its right-hand side has non-zero delta function sources supported by the boundaries. The equations of motion and the Bianchi identity for G are now solved by

$$G_{abcd} = \frac{1}{4V}\epsilon_{\bar{a}\bar{b}\bar{c}\bar{d}}{}^{e\bar{f}}\omega_{i\,e\bar{f}}\alpha^i\epsilon(x^5) \tag{3.36}$$

where the constants α^i are defined by the integrals:

$$\alpha^i := -\frac{\kappa^2}{\lambda^2}\int_{C_i} trR\wedge R \tag{3.37}$$

over four-cycle C_i corresponding to the harmonic ω_i. In consequence, α^i are proportional to the Pontryagin index of Calabi-Yau and are quantized. The step function $\epsilon(x^5)$ takes values $+1$ for $x^5 \in (0, \pi\rho)$ and -1 for $x^5 \in (-\pi\rho, 0)$.

Taking into acount the non-zero backround value of G essentially changes the effecive 5d theory. Instead of the simple 5d supergravity we obtain its gauged version. Below we argue why the effective theory should be a gauged supergravity

1. The non-zero G in the kinetic term G^2 in the 11d lagrangian leads in the effecive theory the potential term $-\frac{1}{4V^2}G^{ij}\alpha_i\alpha_j$. This term depends on the scalar V and on the scalars of the vector multiplets (through the metric G_{ij}). However, potentials for scalar fields are generally forbidden in ungauged 5d supergravities.

2. Also on the boundaries, when we substitute the kinetic terms $-\frac{e}{4\lambda^2}(trF^{(1)}_{AB}F^{(1)AB} - \frac{1}{2}trR_{AB}R^{AB})$ with their background values, we get the boundary potential $\frac{\sqrt{2}}{V}\alpha_i b^i$. In the next section we show, that supersymmetrization of such background potentials is possible only when the supergravity in the bulk is a gauged one.

3. Reduction of the topological term $C_{\epsilon ef}G_{\alpha\beta\gamma\delta}G_{abcd}$ yields the coupling of the form:

$$\frac{1}{V^2}\alpha_i\mathcal{A}^i_\alpha\partial^\alpha\sigma \tag{3.38}$$

   In ungauged supergravities vector fields do not couple in this manner to scalars but in the gauged version we recognize in (3.38) a part of the kinetic term $(D_\alpha\sigma)^2$ with the partial derivatives substituted with the covariant derivatives. Hence, we see that the vector fields are the gauge fields and that it is the field σ of the universal hypermultiplet which becomes gauged. From the kinetic terms (3.32) we see that the sigma-model possesses a translational U(1) symmetry $\sigma \to \sigma + const$, and in fact it is this symmetry which is gauged.

More detailed calculation proves that indeed all terms in the effective lagrangian fit into the framework presented in subsection 2.2.3. The functions which describe the precise form of the gauged lagrangian are:

- Killing vector

$$k^u = (0, -2, 0, 0) \tag{3.39}$$

- Prepotential

$$g\mathcal{P}_i{}^A{}_B = \begin{pmatrix} -\frac{1}{4V} i\epsilon(x^{11})\alpha_i & 0 \\ 0 & \frac{1}{4V} i\epsilon(x^{11})\alpha_i \end{pmatrix} \tag{3.40}$$

If instead of the standard embedding we used other solutions, the precise form of gauging and the above functions would change, but the general features of the effective theory would stay intact.

This completes the description of the effective bulk theory. We also have zero-modes of the 10d boundary gauge fields. We expect that they yield four dimensional gauge supermultiplets and some scalar supermultiplets - their number and representation depends on the choice of the embedding. In the next section we determine the boundary theory using the Noether method, in the similar way as it was done in the original paper of Horava and Witten. One could try to obtain the boundary theory directly from the reduction, but the method we use can be extended to more general five-dimensional theories with matter residing on branes, including theories which do not follow from the compactification of a higher dimensional theory.

Chapter 4

Coupling of 5-dimensional supergravity to boundaries

The purpose of this section is to repeat the Horava-Witten construction of supergravity coupled in a supersymmetric way to matter fields on the boundaries, but this time, in the framework of five-dimensional supergravity defined on the $M_4 \times S_1/\mathbf{Z}_2$ manifold and with YM multiplets living on two 3-branes located at the $\mathbf{Z}_2$ fixed points. We restrict ourselves to a specific non-linear sigma model, namely the $SU(2,1)/(SU(2)\times U(1))$, coupled to 5d supergravity. The system describes the dynamics of the universal moduli of the M-theory compactification on a Calabi-Yau three-fold. In this section we do not consider vector multiplets. It is not difficult to modify this construction to include other multiplets.

In the bulk theory we have the gravitational multiplet $(e^a_\alpha, \psi^A, \mathcal{A}_\alpha)$, and the universal hypermultiplet $(\lambda^a, V, \sigma, \xi, \overline{\xi})$. We quote the kinetic part of the 5d action and write explicitly the sigma-model metric and the symplectic index of the Majorana spinors :

$$
\begin{aligned}
S = -\int d^5x e_5 \tfrac{1}{\kappa^2}\big(&-\tfrac{1}{2}R + \tfrac{3}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \tfrac{1}{2\sqrt{2}}\epsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{A}_\alpha\mathcal{F}_{\beta\gamma}\mathcal{F}_{\delta\epsilon} + \tfrac{1}{4V^2}(\partial_\alpha V\partial^\alpha V + \partial_\alpha\sigma\partial^\alpha\sigma) \\
&+\tfrac{1}{V}\partial_\alpha\xi\partial^\alpha\bar{\xi} + \tfrac{i}{4V^2}(\xi\partial_\alpha\bar{\xi}D^\alpha\sigma - \bar{\xi}\partial_\alpha\xi D^\alpha\sigma) - \tfrac{1}{4V^2}((\xi\partial_\alpha\bar{\xi})^2 + (\bar{\xi}\partial_\alpha\xi)^2 - |\bar{\xi}\partial_\alpha\xi|^2) \\
&+(\tfrac{1}{2}\overline{\psi^1_\alpha}\gamma^{\alpha\beta\gamma}D_\beta\psi^1_\gamma + 1\to 2) + (\tfrac{1}{2}\overline{\lambda^1}\gamma^\alpha D_\alpha\lambda^1 + 1\to 2)\ \big)
\end{aligned}
\tag{4.1}
$$

The supersymmetry transformation laws are:

$$
\begin{aligned}
\delta e^m_\alpha &= \tfrac{1}{2}\overline{\epsilon^1}\gamma^m\psi^1_\alpha + (1\to 2) \\
\delta\psi^1_\alpha &= D_\alpha\epsilon^1 - \tfrac{i}{4\sqrt{2}}(\gamma_\alpha^{\ \beta\gamma} - 4\delta^\beta_\alpha\gamma^\gamma)\mathcal{F}_{\beta\gamma}\epsilon^1 + \tfrac{i}{4V}D_\alpha\sigma\epsilon^1 + \tfrac{1}{4V}(\xi\partial_\alpha\bar{\xi} - \bar{\xi}\partial_\alpha\xi)\epsilon^1 - \tfrac{1}{\sqrt{V}}\partial_\alpha\xi\epsilon^2 \\
\delta\psi^2_\alpha &= D_\alpha\epsilon^2 - \tfrac{i}{4\sqrt{2}}(\gamma_\alpha^{\ \beta\gamma} - 4\delta^\beta_\alpha\gamma^\gamma)\mathcal{F}_{\beta\gamma}\epsilon^2 - \tfrac{i}{4V}D_\alpha\sigma\epsilon^2 - \tfrac{1}{4V}(\xi\partial_\alpha\bar{\xi} - \bar{\xi}\partial_\alpha\xi)\epsilon^2 + \tfrac{1}{\sqrt{V}}\partial_\alpha\bar{\xi}\epsilon^1 \\
\delta\mathcal{A}_\alpha &= -\tfrac{i}{2\sqrt{2}}\overline{\psi^1_\alpha}\epsilon^1 + (1\to 2)
\end{aligned}
\tag{4.2}
$$

$$
\begin{aligned}
\delta V &= \tfrac{i}{\sqrt{2}}V(\overline{\epsilon^1}\lambda^1) - (1\to 2) \\
\delta\sigma &= +\tfrac{1}{\sqrt{2}}V(\overline{\epsilon^1}\lambda^1) + (1\to 2) + \sqrt{\tfrac{V}{2}}(\xi\overline{\epsilon^1}\lambda^2 - \bar{\xi}\overline{\epsilon^2}\lambda^1) \\
\delta\xi &= -\tfrac{i\sqrt{V}}{\sqrt{2}}(\overline{\epsilon^2}\lambda^1) \quad \delta\bar{\xi} = -\tfrac{i\sqrt{V}}{\sqrt{2}}(\overline{\epsilon^1}\lambda^2) \\
\delta\lambda^1 &= -\tfrac{i}{2\sqrt{2}V}(\partial\!\!\!/(V+i\sigma) - \bar{\xi}\partial\!\!\!/\xi + \xi\partial\!\!\!/\bar{\xi})\epsilon^1 + \tfrac{i}{\sqrt{2V}}\partial\!\!\!/\xi\epsilon^2 \\
\delta\lambda^2 &= +\tfrac{i}{2\sqrt{2}V}(\partial\!\!\!/(V-i\sigma) + \bar{\xi}\partial\!\!\!/\xi - \xi\partial\!\!\!/\bar{\xi})\epsilon^2 + \tfrac{i}{\sqrt{2V}}\partial\!\!\!/\bar{\xi}\epsilon^1\,.
\end{aligned}
\tag{4.3}
$$

The $\mathbf{Z_2}$ projection is defined in such a way that bosonic fields $(e^m_\mu, e^5_5, \mathcal{A}_5, V, \sigma)$ are even w.r.t the orbifold dimension, and $(e^m_5, e^5_\mu, \mathcal{A}_\mu, \xi)$ are odd. The action of $\mathbf{Z_2}$ on fermion fields and on parameter ϵ of supersymmetry transformations is defined as:

$$\gamma_5\psi^A_\mu(x^5) = (\sigma^3)^A{}_B\psi^B_\mu(-x^5) \quad \gamma_5\psi^A_5(x^5) = -(\sigma^3)^A{}_B\psi^B_5(-x^5)$$
$$\gamma_5\lambda^a(x^5) = -(\sigma^3)^a{}_b\lambda^b(-x^5) \quad \gamma_5\epsilon^A(x^5) = (\sigma^3)^A{}_B\epsilon^B(-x^5) \tag{4.4}$$

where $\gamma_5 = \left(\begin{smallmatrix}-\mathbf{1} & 0\\ 0 & \mathbf{1}\end{smallmatrix}\right), \sigma^3 = \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$ and $A, a = 1, 2$. Symplectic Majorana spinors in 5d satisfy $\bar{\chi}^A = (\chi^A)^T C_5$ with $C_5 = i\gamma^2\gamma^2\gamma^5$ in 4d chiral representation. At the $\mathbf{Z_2}$ fixed points one half of the degrees of freedom is eliminated, which means, that the number of supercharges is reduced to one half. This leaves 4 supercharges corresponding to N=1 supersymmetry in 4d. Thus effectively, at $x^5 = 0, x^5 = \pi\rho$, the second supersymmetry is killed by $\mathbf{Z_2}$, and we can locate 3-branes with (chiral) matter content characteristic for N=1 supersymmetry.

It is convenient to combine two symplectic Majorana spinors into one Majorana (in a four dimensional sense) spinor even w.r.t to the fifth coordinate. We define:

$$\psi_\mu = \begin{pmatrix} \psi^2_{L\mu} \\ \psi^1_{R\mu} \end{pmatrix} \quad \psi_5 = \begin{pmatrix} -\psi^1_{L5} \\ \psi^2_{R5} \end{pmatrix} \quad \lambda = \sqrt{2}V \begin{pmatrix} -i\lambda^1_L \\ i\lambda^2_R \end{pmatrix}. \tag{4.5}$$

$$\epsilon = \begin{pmatrix} \epsilon^2_L \\ \epsilon^1_R \end{pmatrix}. \tag{4.6}$$

These are the combinations which couple to 4-dimensional spinors on the boundary. Using the above definitions we can re-express the five dimensional lagrangian (4.1) involving fermions in terms of even (and odd) fermion combinations. For example, the gravitino kinetic term can be expressed as $-\frac{1}{2}(\overline{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho)+(odd)$. Since, as already discussed in chapter 3, the odd fields do not couple to the boundary, and we are interested in finding supersymmetric coupling to the boundary, we can neglect the odd spinor combinations and fields in subsequent formulas. The supersymmetry transformations of the even bulk fields expressed in terms of variables defined in (4.5) read:

$$\delta e^a_\mu = \tfrac{1}{2}(\overline{\epsilon}\gamma^a\psi_\mu)$$
$$\delta e^5_5 = \tfrac{1}{2}(\overline{\epsilon}\psi_5)$$
$$\delta\psi_\mu = D_\mu\epsilon - \tfrac{i}{2\sqrt{2}}(\gamma^\nu_\mu - 2g^\nu_\mu)e^5_5\gamma^5\epsilon\mathcal{F}_{\nu 5} - \tfrac{i}{4V}\partial_\mu\sigma\gamma^5\epsilon$$
$$\delta\psi_5 = \partial_5\epsilon^- - \tfrac{i}{\sqrt{2}}\gamma^\mu\gamma^5\epsilon\mathcal{F}_{\mu 5} + \tfrac{1}{\sqrt{V}}(\partial_5\xi\epsilon_L + \partial_5\bar{\xi}\epsilon_R)$$
$$\delta\mathcal{A}_5 = \tfrac{i}{2\sqrt{2}}(\overline{\psi_5}\gamma^5\epsilon)$$
$$\delta V = \tfrac{1}{2}(\overline{\epsilon}\lambda)$$
$$\delta\sigma = \tfrac{i}{2}(\overline{\epsilon}\gamma^5\lambda)$$
$$\delta\lambda = \tfrac{1}{2}\partial\!\!\!/(V + i\gamma^5\sigma)\epsilon + \sqrt{V}\tfrac{1}{e^5_5}(\partial_5\xi\epsilon_L + \partial_5\bar{\xi}\epsilon_R) \tag{4.7}$$

Recall, that the fifth derivative of an odd field is even. Thus, in the above transformation the fifth derivatives of ξ and $\epsilon^-_L \equiv -\epsilon^1_L$, $\epsilon^-_R \equiv \epsilon^2_R$ should not be neglected.

Our task is to couple gauge and matter fields on the boundary in such a way, that local supersymmetry is preserved. The strategy is similar to the one employed for the Horava-Witten model; we start with a globally supersymmetric lagrangian and succesively add new couplings to make the supersymmetry local. It is impossible to give all the calculations leading to the final results in this thesis. Below we will present some sample calculations just to give a flavour of what is going on but often we limit ourselves to presenting the final formulae without a discussion. The final form of the lagrangian can be found in the appendix.

4.1 Pure 5d supergravity coupled to Yang-Mills supermultiplets on the brane

We now add matter on the boundary. It is convenient to proceed in several steps. First, we consider only pure 5d supergravity and a Yang-Mills multiplet on the boundary. As long as we do not adress the problem of anomalies, the gauge group is arbitrary. In the Horava-Witten model compactified by using the standard embedding, the gauge group is E_6 on one brane and E_8 on the other.

The action of the four-dimensional super-Yang-Mills theory reads:

$$S = \int d^5x \delta(x^5) e_5 \frac{1}{g^2} \mathcal{L}_{YM}$$
$$\mathcal{L}_{YM} = f[-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{2} \overline{\chi}^a \not{D} \chi^a] \tag{4.8}$$

For the time being we keep the factor f multiplying the kinetic term unspecified. This action is known to possess global supersymmetry and the transformations are:

$$\begin{aligned} \delta A^a_\mu &= -\frac{1}{2}(\overline{\epsilon}\gamma_\mu \chi^a) \\ \delta \chi^a &= \frac{1}{4}\gamma^{\mu\nu}\epsilon F^a_{\mu\nu} \end{aligned} \tag{4.9}$$

(Note the sign difference with respect to the convention used in Chapter 3. We identify the spinor ϵ parametrizing the Yang-Mills supersymmetry transformations with an even combination of 5d spinors, as defined in (4.5).

The next few steps are completely analogous to the Horava-Witten model. If the above transformations are made local, the supersymmetric variation of the YM action is non-zero and is proportional to $D_\mu \epsilon$ is:

$$\delta \mathcal{L}_{YM} = -\frac{e}{4} F_{\nu\rho} (\overline{D_\mu \epsilon} \gamma^{\nu\rho} \gamma^\mu \chi) \tag{4.10}$$

To cancel it, one is forced to add the so-called Noether term to the boundary action:

$$+\mathcal{L} = \frac{fe}{4} (\overline{\psi}_\mu \gamma^{\nu\rho} \gamma^\mu \chi) F_{\nu\rho} \tag{4.11}$$

When the gravitino in the Noether term is varied, the part of the gravitino transformation law equal to $D_\mu \epsilon$ cancels $\delta \mathcal{L}_{YM}$. Since we identified ϵ with the even combination of 5d spinors, it is precisely the even combination of 5d gravitinos ψ_μ given by (4.5) which appears here.

There is only one more term bilinear in fermion fields that has to be added. Another part of the gravitino transformation law $\delta\psi_\mu = -\frac{i}{2\sqrt{2}}(\gamma^\nu_\mu - 2g^\nu_\mu)\mathcal{F}_{\mu 5}$, when applied to the Noether term yields:

$$\delta \mathcal{L}_N = \frac{ief}{8\sqrt{2} e_5^5} \overline{\epsilon} \gamma_5 (\gamma^{\sigma\mu} - 2g^{\sigma\mu}) \gamma^{\nu\rho} \gamma_\mu \chi \mathcal{F}_{\sigma 5} F_{\nu\rho} = -\frac{3ief}{8\sqrt{2} e_5^5} \overline{\epsilon} \gamma_5 \gamma^{\nu\rho} \gamma^\sigma \chi \mathcal{F}_{\sigma 5} F_{\nu\rho} \tag{4.12}$$

The identities $\gamma^{\sigma\mu} = \gamma^\sigma\gamma^\mu - g^{\sigma\mu}$ and $\gamma^\mu\gamma^{\nu\rho}\gamma_\mu = 0$ were used. This variation can be cancelled only if we add a new term to to the boundary lagrangian.

$$+\mathcal{L} = \frac{3i}{4\sqrt{2}}\frac{ef}{e_5^5}(\overline{\chi}\gamma^5\gamma^\mu\chi)\mathcal{F}_{\mu5} \tag{4.13}$$

Varying the gaugino $\delta\chi \sim \gamma^{\nu\rho}F_{\nu\rho}$ cancels (4.12). Note that e_5^5 appears explicitly in the boundary lagrangian. It always accompanies the bulk fields carrying the fifth world index, so that the action is covariant also from the 5d point of view.

At this stage, one profound difference with respect to the Horava-Witten model appears. In the Horava-Witten model one finds that after adding the Noether term, there exists a term in the variation of the Lagrangian proportional to $F^2(\overline{\psi}\Gamma^{ABCDE}\epsilon)$, which does not cancel. Its presence was the motivation to modify the Bianchi identity for the 4-form G. Below we perform the corresponding calculation in the 5d framework and we find that the corresponding variations do cancel in the 4d case. We need to check cancellations of the variations proportional to $F^2\epsilon\psi$. These originate from:

1. Variation of the metric in the gauge kinetic term:

$$\mathcal{L}_{gkin} = -\frac{ef}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} \tag{4.14}$$

Variations of the determinant and variations of the inverse metric are:

$$\delta e = e\delta e_\mu^a e_a^\mu = \frac{e}{2}\overline{\epsilon}\gamma^\mu\psi_\mu \quad \delta g^{\mu\nu} = -\overline{\epsilon}\gamma^{(\mu}\psi^{\nu)} \tag{4.15}$$

Hence the variation of the gauge kinetic term yields:

$$\delta\mathcal{L}_{gkin} = -\frac{ef}{4}(\frac{1}{2}\overline{\epsilon}\gamma^\rho\psi_\rho F_{\mu\nu}F^{\mu\nu} - 2\overline{\epsilon}\gamma^{(\mu}\psi^{\nu)}F_{\mu\rho}F_\nu{}^\rho) \tag{4.16}$$

2. Variation of the gaugino $\delta\chi = \frac{1}{4}\gamma^{\nu\rho}F_{\nu\rho}\epsilon$ in the Noether term, which yields:

$$\delta\mathcal{L}_N = \frac{fe}{16}(\overline{\psi}_\mu\gamma^{\nu\rho}\gamma^\mu\gamma^{\delta\sigma}\epsilon)F_{\nu\rho}F_{\delta\sigma} \tag{4.17}$$

To be able to compare it with the previous variation one has to decompose the product of gamma matrices:

$$\gamma^{\nu\rho}\gamma^\mu\gamma^{\delta\sigma} = \\ 2\gamma^{\nu\rho\sigma}g^{\mu\delta} + 4\gamma^{\rho\mu\sigma}g^{\nu\delta} + 2\gamma^{\nu\delta\sigma}g^{\mu\rho} + 2\gamma^\mu g^{\rho\delta}g^{\nu\sigma} + 4\gamma^\nu g^{\mu\delta}g^{\rho\sigma} + 4\gamma^\sigma g^{\nu\delta}g^{\rho\mu} \tag{4.18}$$

The gamma matrices with three indices vanish when contracted with the gauge field strength tensors: those with coefficient 2 cancel against each other and the one with coefficient four vanishes when contracted with the symmetric combination of the two gauge field strength tensors. The remainining three terms with a single gamma matrix yield:

$$\delta\mathcal{L}_N = \frac{fe}{16}\overline{\psi_\mu}(-2\gamma^\mu F^{\rho\nu}F_{\rho\nu} + 4\gamma^\nu F_{\nu\sigma}F^{\mu\sigma} + 4\gamma^\sigma F_\sigma^\nu F_\nu{}^\mu)\epsilon = \frac{fe}{4}\overline{\psi_\mu}(-\frac{1}{2}\gamma^\mu F^{\rho\nu}F_{\rho\nu} + 2\gamma^\nu F_{\nu\sigma}F^{\mu\sigma})\epsilon \tag{4.19}$$

This indeed cancels with the variation of the gauge kinetic term if the Majorana spinors identity $\overline{\epsilon}\gamma_\mu\psi_\nu = -\overline{\psi_\nu}\gamma_\mu\epsilon$ is used.

Note that in this case the calculation is almost the same as in the Horava-Witten model, but gamma matrices antysymmetrized in five indices are trivially zero. Because of that fact, there is no uncancelled variations proportional to F^2 left. Thus, having added the Noether term (4.11) and the term (4.12) we arrive at the lagrangian which is already supersymmetric up to 4-fermi variations.

For supersymmetry variations to close, one needs to add a collection of four-fermi term, as well as 3-fermi corrections to the supersymmetry variations of fermions. The analysis is often parallel to the case of the ordinary 4d supergravity, as described in [10]. The calcalution is very tedious so instead of going through complicated algebra we concentrate only on the most interesting aspects:

1. The correction to the gravitino transformation law:

$$\delta\psi_\mu = \delta(x^5)\frac{g^2}{\kappa^2}\frac{f}{8}(g^{\mu\rho} - \frac{1}{2}\gamma^{\mu\rho})\gamma^5\epsilon\,(\overline{\chi^a}\gamma^5\gamma_\rho\chi^a) \tag{4.20}$$

   has the delta function in front. This is necessary, because the gravitino kinetic term lives in five dimension, and this correction must cancel the supersymmetric variation restricted to the boundary.

2. One has to check the cancellation of variations proportional to the fifth derivative of an odd bulk spinor (which can be non-zero on the boundary). For example we have:

$$\begin{aligned} \delta\mathcal{A}_\mu &= -\tfrac{i}{2\sqrt{2}}\overline{\psi^1_\mu}\epsilon^1 + (1\to 2) \\ \delta\mathcal{F}_{\mu 5} &= \tfrac{i}{2\sqrt{2}}\overline{\psi^1_\mu}\partial_5\epsilon^1 + (1\to 2) + \ldots \\ \delta\mathcal{L} = \tfrac{3i}{4\sqrt{2}}\tfrac{ef}{e_5^5}(\overline{\chi}\gamma^5\gamma^\mu\chi)\delta\mathcal{F}_{\mu 5} + \ldots &= -i\tfrac{3}{16}\tfrac{ef}{e_5^5}(\overline{\chi}\gamma^5\gamma^\mu\chi)(\overline{\psi_{\mu R}}\partial_5\epsilon^1 + \overline{\psi_{\mu L}}\partial_5\epsilon^2) \end{aligned} \tag{4.21}$$

   To cancel (4.21) a term proportional to ψ_5 has to be added:

$$+\mathcal{L} = \frac{3}{16}\frac{f}{e_5^5}(\overline{\psi}_\mu\gamma^5\psi_5)(\overline{\chi}\gamma^5\gamma^\mu\chi) \tag{4.22}$$

   Varying $\delta\psi_{5L} = -i\delta\psi_5^1 = -i\partial_5\epsilon^1$, $\delta\psi_{5R} = i\delta\psi_5^2 = i\partial_5\epsilon^2$ cancels (4.21).

   One can also check that variations proportional to the fifth derivative of the odd combination of the gravitino cancel.

3. The four-gaugino term (present in 4d supergravity with the same numerical coefficient) is proportional to $\delta^2(x^5)$. This is because it should cancel the gravitino variation proportional to the gaugino fields multiplied by the delta function, in the Noether term, which as a boundary term, is already proportional to the delta function. Using (4.20) we can calculate

$$\begin{aligned} \delta\mathcal{L}_N &= -\tfrac{ef^2\kappa^2}{32g^4}\delta(0)\overline{\epsilon}\gamma^5(g^{\mu\rho} - \tfrac{1}{2}\gamma^{\rho\mu})\gamma^{\nu\sigma}\gamma_\mu\chi F_{\nu\sigma}(\overline{\chi}\gamma^5\gamma_\mu\chi) \\ &= -\tfrac{3ef^2\kappa^2}{64g^4}\delta(0)\overline{\epsilon}\gamma^{\nu\sigma}\gamma^5\gamma^\rho\chi F_{\nu\sigma}(\overline{\chi}\gamma^5\gamma_\mu\chi) \end{aligned} \tag{4.23}$$

   Thus, if we insist on supersymmetry in order $(\frac{\kappa}{g})^4$ we must add a singular term to the lagrangian:

$$\mathcal{L}_{\chi^4} = -\frac{3ef^2\kappa^2}{64g^4}\delta(0)(\overline{\chi}\gamma^5\gamma_\mu\chi)(\overline{\chi}\gamma^5\gamma^\mu\chi) \tag{4.24}$$

However, one can formally get rid of this singular term by redefining the field strength of the graviphoton:

$$\hat{\mathcal{F}}_{\mu 5} = \mathcal{F}_{\mu 5} - \frac{if}{4\sqrt{2}} \frac{\delta(x^5)}{e_5^5} (\overline{\chi}\gamma^5\gamma_\mu\chi) \tag{4.25}$$

Replacing $\mathcal{F}$ with $\hat{\mathcal{F}}$ in the bulk action reproduces $\mathcal{F}\chi^2$ coupling as well as the singular χ^4 term. As we will see later in section 5, due to the fact, that singular terms always combine into the perfect square structures, the singular terms will dissapear from the four-dimensional effective action. Also, the gaugino bilinear in the transformation law of ψ_μ matches the perfect square structure of $\mathcal{F}$. This is not the case for $\delta\psi_5$, which has a term proportional to $\mathcal{F}_{\mu 5}$, but no pieces bilinear in gaugino fields. The deviation of $\delta\psi_5$ from the perfect square structure was also noted in the 11d framework in [13], and has important consequences for supersymmetry breaking.

The lagrangian obtained at this stage (including terms not discussed here) is given in Appendix A as $\mathcal{L}_{YM}$ in equation (A.2).

4.2 Sigma model in the bulk

We now couple the $SU(2,1)/U(2)$ non-linear sigma-model to 5d supergravity . In the bulk we have therefore four real scalar fields $(V, \sigma, \xi, \bar{\xi})$. Their fermion superpartner is a symplectic Majorana spinor λ^a, called hyperino. We define the even Majorana spinor λ as in (4.5). We also specify the gauge kinetic function f, which appeared in the previous subsection, to be $f = V$. This choice is motivated by the fact, that such a kinetic term appears in the compactified Horava-Witten theory. Supersymmetric coupling is possible for more general gauge kinetic functions, but it has not been worked out in this thesis. The presence of sigma model fields affect the boundary Lagrangian in the following ways:

1. The supersymmetry variation of the non-standard gauge kinetic term produces a term proportional to $(\overline{\epsilon}\lambda)F^2$. To cancel it, two new boundry terms are needed:

$$+\mathcal{L} = \frac{e}{g^2}[-\frac{1}{4}\sigma F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}(\overline{\lambda}\gamma^{\nu\rho}\chi)F_{\nu\rho})] \tag{4.26}$$

   We see, that the sigma field aquires an axion-type coupling.

2. Supersymmetry variations of the bulk fermions ψ_μ and λ contain derivatives of the hypermultiplet scalars. When we vary these fermions in the boundary action (e.g. ψ_μ in the Noether term (4.11), or λ in (4.26), we get new uncancelled variations. It turns out that the following terms are needed:

$$+\mathcal{L} = \frac{e}{g^2}[-\frac{i}{8}(\overline{\chi}^a\gamma^5\gamma^\mu\chi^a)\partial_\mu\sigma - \frac{\sqrt{V}}{2e_5^5}[(\overline{\chi}_L\chi_R)\partial_5\overline{\xi} + (\overline{\chi}_R\chi_L)\partial_5\xi] \tag{4.27}$$

   Note, that the odd field ξ now appears explicitly in the boundary Lagrangian through its fifth derivative which is even.

Again, 4-fermi terms in the boundary Lagrangian and 3-fermi terms in the supersymmetry transformation laws are needed to render the action supersymmetric. They are all given in Appendix A as $\mathcal{L}_H$ in eq. (A.3). Here, we concetrate on those, which uncover the 'perfect square structure'. We get a bilinear in gaugino fields correction to the hyperino transformation law:

$$+\delta\lambda = \delta(x^5)\frac{\kappa^2}{g^2}\frac{V^2}{2}[\epsilon_L(\overline{\chi_L}\chi_R) + \epsilon_R(\overline{\chi_R}\chi_L)] \tag{4.28}$$

A singular, quartic in gaugino term is needed, too:

$$+\mathcal{L} = -e\frac{\kappa^2}{4g^4}\delta^2(x^5)V^2(\overline{\chi_L}\chi_R)(\overline{\chi_R}\chi_L) \tag{4.29}$$

As before, we can formally get rid of this singularity. We define a new variable:

$$\partial_5\hat{\xi} = \partial_5\xi + \frac{\kappa^2}{g^2}\delta(x^5)\frac{V^{3/2}}{2}(\overline{\chi_L}\chi_R) \tag{4.30}$$

and replace $\partial_5\xi$ with $\partial_5\hat{\xi}$ in the bulk action. This procedure reproduces the $\partial_5\xi\chi^2$ coupling as well as the singular χ^4 term. Bilinear gaugino term in the transformation law of the hyperino λ also matches the perfect square structure of $\partial_5\hat{\xi}$, but there is no gaugino bilinear in the transformation law of ψ_5 to complete the perfect square with $\partial_5\xi\chi^2$ (We noted in the previous subsection, that also $\mathcal{F}$ does not combine into the perfect square in $\delta\psi_5$).

4.3 Scalar multiplets on the boundary

To make contact with the phenomenology, we should introduce scalar multiplets living on the boundary, which can provide us with known matter fields such as quarks and leptons. As in the case of the Yang-Mills multiplet, we begin with a globally supersymmetric action for a multiplet that consists of a complex scalar C, and a Majorana spinor ζ:

$$S = \int d^5x e_5 \delta(x^5)\mathcal{L}_S$$
$$\mathcal{L}_S = -D_\mu C D^\mu \overline{C} - \overline{\zeta}\not{D}\zeta \tag{4.31}$$

(Note, that we use a different normalization of C, than the reference [5])

Global supersymmetry transformation laws are:

$$\delta C = (\overline{\epsilon}_R\zeta_L)$$
$$\delta\zeta_L = \frac{1}{2}\not{D}C \tag{4.32}$$

If the transformations (4.32) become local, variation proportional to $D_\mu\epsilon$ appears, and must be cancelled by the gravitino variation in the new term:

$$+\mathcal{L} = (\overline{\psi}_{R\mu}\not{D}\overline{C}\gamma^\mu\zeta_L + h.c.) \tag{4.33}$$

The origin of this term is similar to the Noether term (4.11),and as in subsection 4.1, the presence of ψ_μ causes new uncancelled variations, which must be cancelled by adding new terms to the boundary Lagrangian. The part of the gravitinovariation proportional to $\partial_\mu\sigma$ is cancelled by the variation of ζ in:

$$+\mathcal{L} = -\frac{i}{4V}\partial_\mu\sigma(\overline{\zeta}\gamma^5\gamma^\mu\zeta) \tag{4.34}$$

The variation proportional to $\mathcal{F}_{\mu 5}$ requires more profound modifications. Not only the new terms of the form:

$$+\mathcal{L} = \frac{i\mathcal{F}_{\mu 5}}{\sqrt{2}e_5^5}(CD^\mu\overline{C} - \overline{C}D^\mu C) - \frac{i\mathcal{F}_{\mu 5}}{2\sqrt{2}e_5^5}(\overline{\zeta}\gamma^5\gamma^\mu\zeta) \tag{4.35}$$

have to be added to the boundary lagrangian. One must also modify the supersymmetry transformation law of the graviphoton:

$$\tilde{\delta}\mathcal{A}_5 = \frac{i\kappa^2}{6g^2}\delta(x^5)(\overline{\epsilon}_R\zeta_L + h.c.)\bar{C} \tag{4.36}$$

These modification can be summarized by the redefinition of the graviphoton field-strength:

$$\hat{\mathcal{F}}_{\mu 5} = \mathcal{F}_{\mu 5} + \frac{iV}{4\sqrt{2}}\frac{\delta(x^5)}{e_5^5}(\overline{\chi}\gamma^5\gamma_\mu\chi) + \frac{i\kappa^2}{6g^2}(CD^\mu\overline{C} - \overline{C}D^\mu C), \tag{4.37}$$

(The first term in (4.37) was determined in subsection 4.1).

The supersymmetry transformation law of $\hat{\mathcal{F}}_{\mu 5}$, apart from standard 5d piece, receives a correction:

$$\tilde{\delta}\hat{\mathcal{F}}_{\mu 5} = \frac{i\kappa^2}{3g^2}\delta(x^5)(\overline{\epsilon}_R\zeta_L + h.c.)\bar{C} \tag{4.38}$$

These modifications are analogous to those required in the Horava-Witten model for the case of the four-form field strength G. This could have been expected, because in the context of M-theory the 5d graviphoton field strength F comes from the reduction of G.

To cancel the variation of ζ in (4.33), we must add corrections proportional to $D_\mu C$ to the gravitino transformation law. It turns out that these correction can be obtained by replacing $\mathcal{F}_{\mu 5}$ with $\hat{\mathcal{F}}_{\mu 5}$ in the transformation laws $\delta\psi_\mu$ and $\delta\psi_5$.

The rest of the corrections to the boundary Lagrangian are 4-fermi terms, and are given in Appendix A as $\mathcal{L}_S$ in eq. (A.4).

If we want to introduce a superpotential W for scalar fields C, further modifications of the boundary lagrangian are necessary. The derivation is fairly straightforward, and the results are given in Appendix A as $\mathcal{L}_W$ in eq. (A.6). The interesting aspect of this construction the appearance of yet another perfect square structure. It turns out that the $W\partial_5\xi$ coupling has to be added, as well as singular terms $\delta^2(x^5)W\bar{W}$ and $\delta^2(x^5)W\chi^2$. This can be summarized by the redefintion of the 'ξ field strength':

$$\partial_5\hat{\xi} = \partial_5\xi + \frac{\kappa^2}{e_5^5 g^2}\delta(x^5)\frac{V^{3/2}}{4}(\overline{\chi_L}\chi_R) + \frac{2\kappa^2}{e_5^5 g^2}\bar{W}\delta(x^5) \tag{4.39}$$

This replacement of $\partial_5\xi$ with $\partial_5\hat{\xi}$ in the bulk ξ kinetic term reproduces all the above mentioned couplings. Also the λ and W parts of ψ_5 (but not the gaugino part as noted earlier) transformation laws match the perfect square structure of $\partial_5\xi$

4.4 Supersymmetrizing bulk and boundary potentials

In this section we supersymmetrize potentials that are defined on the brane, but are functions of the bulk scalars (this case is different from that considered in the previous subsection, where we supersymmetrized the potential W for the brane scalar fields). We know that such terms arise in the compactifications of the Horava-Witten model, but in this section we consider a wider class of potentials, which do not neccesarily originate from M-theory.

We assume a scalar potential $\delta(x^5)\frac{e}{\kappa^2}(-\Lambda+\frac{\sqrt{2}\alpha}{V})$ localized on the first brane (note the delta function). The parameters α and Λ are constants, while V is one of the bulk hypermultiplet field. The motivation for the constant (Λ) part of this expression is that it will finally lead us to the Randall-Sundrum exponential solutions. At the same time we allow for 'cosmological potential' α/V for the hypermultiplet scalar; this particular form is motivated by the M-theory example and is a natural extension in the presence of hypermultiplets. More general potentials are possible, but σ-dependent terms in the potential break the translational U(1) symmetry $\sigma \to \sigma + const$ which is useful when it comes to solving the strong CP problem, while ξ cannot appear in the boundary potential because of parity assignments. We will be able to supersymmetrize this action by modifying the bulk action only (thus, our construction is alternative to the one presented in [15]). We initially put $\alpha = 0$ and assume that only the gravity multiplet is present in the bulk. Consider a cosmological term of the form:

$$\mathcal{L}_B = -\delta(x^5)\frac{e}{\kappa^2}\Lambda \tag{4.40}$$

e is the determinant built of the metric induced on the brane. We want to supersymmetrize this term. The supersymmetry variation of $\mathcal{L}_B$ arises from varying e_4:

$$\delta\mathcal{L} = \frac{1}{2}\delta(x^5)e\Lambda(\overline{\psi^1_\mu}\gamma^\mu\epsilon^1 + (1\to 2)) \tag{4.41}$$

We observe that, without further modification of the boundary action, we can cancel this variation by modifying the gravitino transformation law:

$$\begin{aligned} +\delta\psi^1_\alpha &= +\frac{\Lambda}{12}\epsilon(x^5)\gamma_\alpha\epsilon^1 \\ +\delta\psi^2_\alpha &= -\frac{\Lambda}{12}\epsilon(x^5)\gamma_\alpha\epsilon^2 \end{aligned} \tag{4.42}$$

With this modification, when ψ is varied in the gravitino kinetic term, the fifth derivative acting on the step function produces an expression multiplied by the delta function, which precisely cancels (4.41):

$$\begin{aligned} \mathcal{L}_{kin} &\supset -\tfrac{e_5}{2\kappa_5^2}\overline{\psi^1_\mu}\gamma^{\mu5\nu}\partial_5\psi^1_\nu + (1\to 2) \\ \delta\mathcal{L} = \tfrac{e_5}{\kappa_5^2}(\overline{\psi^1_\mu}\gamma^{\mu\nu}\gamma^5\partial_5(\tfrac{\Lambda}{12}\epsilon(x^5)\gamma_\nu\epsilon^1) - (1\to 2) &= -\tfrac{e_5}{2\kappa_5^2}\Lambda\delta(x^5)(\overline{\psi^1_\mu}\gamma^\mu\gamma^5\epsilon^1) - (1\to 2) + \ldots \\ &= -\tfrac{e_5}{2\kappa_5^2}\Lambda\delta(x^5)(\overline{\psi^1_\mu}\gamma^\mu\epsilon^1) + (1\to 2) + \ldots \end{aligned} \tag{4.43}$$

In the first line we used $\gamma^{\mu\nu}\gamma_\nu = 3\gamma^\mu$ and $\partial_5\epsilon(x^5) = 2\delta(x^5)$ while in the second line we used the fact that spinors have definite chirality on the boundary. In fact, to cancel (4.41) we need to modify only $\delta\psi_\mu$, but we modify $\delta\psi_5$ as well so as to maintain the 5d covariance.

Note that these corrections are compatible with the $\mathbf{Z}_2$ symmetry defined by (4.4); for example:

$$\gamma^5\delta\psi^1_\mu(x^5) = -\frac{\Lambda}{12}\epsilon(x^5)\gamma_\mu\gamma^5\epsilon^1(x^5) = \frac{\Lambda}{12}\epsilon(-x^5)\gamma_\mu\epsilon^1(-x^5) = \delta\psi^1_\mu(-x^5) \tag{4.44}$$

But as soon as we add (4.42) the bulk theory is no longer supersymmetric. In addition to the boundary term (4.43) the variations of the gravitino kinetic term resulting from (4.42) yield:

$$\begin{aligned} \delta\mathcal{L}_{kin} = &-\frac{e_5}{\kappa_5^2}\frac{\Lambda}{12}\epsilon(x^5)(\overline{\psi^1_\alpha}\gamma^{\alpha\beta\gamma}D_\beta\gamma_\gamma\epsilon^1) - (1\to 2) \\ &-\frac{e_5}{\kappa_5^2}\frac{\Lambda}{4}\epsilon(x^5)(\overline{\psi^1_\alpha}\gamma^{\alpha\beta}D_\beta\epsilon^1) - (1\to 2) \end{aligned} \tag{4.45}$$

The above variation can be cancelled by adding a 'gravitino mass term':

$$\mathcal{L}_{\psi^2} = +\frac{e_5}{8\kappa^2}\Lambda\epsilon(x^5)(\overline{\psi^1_\alpha}\gamma^{\alpha\beta}\psi^1_\beta - \overline{\psi^2_\alpha}\gamma^{\alpha\beta}\psi^2_\beta) \tag{4.46}$$

The gravitino variation $\delta\psi^A_\alpha = D_\alpha\epsilon^A$ in (4.46) cancels (4.45), but now (4.42) yields the variation of the mass term (4.46) proportional to Λ^2:

$$\begin{aligned}\delta\mathcal{L}_{\psi^2} &= +\frac{e_5}{4\kappa^2}\Lambda\epsilon(x^5)(\overline{\psi^1_\alpha}\gamma^{\alpha\beta}\frac{\Lambda}{12}\epsilon(x^5)\gamma_\beta\epsilon^1) + (1\to 2)\\ &= \frac{e_5}{12\kappa^2}\Lambda^2(\overline{\psi^1_\alpha}\gamma^\alpha\epsilon^1) + (1\to 2)\end{aligned} \tag{4.47}$$

which can be cancelled by varying the determinat in the new 'cosmological term':

$$\mathcal{L}_C = \frac{e_5}{6\kappa^2}\Lambda^2 \tag{4.48}$$

Moreover, in our framework, $\epsilon(x^5)$ has another discontinuity at $x^5 = \pi\rho$, so the fifth derivative in the gravitino kinetic term yields an additional variation multiplied by $\delta(x^5 - \pi\rho)$. This variation can be cancelled by adding a cosmological term confined to that brane:

$$\mathcal{L}_{B'} = \delta(x^5 - \pi\rho)\frac{e}{\kappa^2}\Lambda \tag{4.49}$$

(The minus sign relative to (4.40) appears here because $\epsilon(x^5)$ has a 'step down' at $x^5 = \pi\rho$). Note that the cosmological term (4.48) appeared with a plus sign. The relevant part of the bulk action now reads $S = -\frac{1}{2}\int(R - \frac{1}{3}\Lambda^2)$ which admits the anti-de-Sitter solutions. In fact, the coefficient of (4.48) is precisely the one we need to obtain the Randall-Sundrum scenario, as we will show shortly.
The above mentioned corrections are still not sufficient to supersymmetrize the bulk lagrangian. To achieve this goal we also need the coupling of the graviphoton to the gravitino:

$$\mathcal{L}_A = -\frac{ie_5}{4\sqrt{2}\kappa^2}\epsilon(x^5)\Lambda\left((\overline{\psi^1}_\alpha\gamma^{\alpha\beta\gamma}\psi^1_\gamma)\mathcal{A}_\beta - (1\to 2)\right). \tag{4.50}$$

If we switch on the hypermultiplets, one can infer that to achieve cancellation of variations of the form $\Lambda\epsilon(x^5)(\lambda\epsilon)\partial_\alpha V$ the hyperino mass term is needed:

$$\mathcal{L}_{\lambda^2} = +\frac{e_5}{8\kappa^2}\epsilon(x^5)\Lambda\left(\overline{\lambda^1}\lambda^1 - (1\to 2)\right) \tag{4.51}$$

The presence of the hyperino mass term indicates that to arrive at a fully supersymmetric action we must gauge some isometry of the hypermultiplet sigma model but this is worked out elsewhere [9].

In addition, a graviphoton dependent correction to the gravitino transformation law appears:

$$\delta\psi^A_\alpha = -\frac{i}{2\sqrt{2}}\epsilon(x^5)\Lambda(\sigma^3)^A{}_B\epsilon^B\mathcal{A}_\alpha. \tag{4.52}$$

Note that that the presence of the step function could potentially produce another delta function in the variation of the bulk lagrangian (more precisely, in the variation of the gravitino kinetic term, similarly as in (4.43)). But this variation has the form $\delta\mathcal{L} \sim \delta(x^5)\Lambda\mathcal{A}_\mu\bar{\epsilon}\psi$ and vanishes, because $\mathcal{A}_\mu$, being odd, is zero on the brane.

Furthermore, we need 4-fermi terms in the bulk action to complete the supersymmetrization, but these are not given in this thesis. The action we arrive at fits in the framework of 5d gauged supergravity without matter. The gauged group is the U(1) subgroup of the R-symmetry SU(2) group. One can check that the terms found above containing the graviphoton can be arranged into the covariant derivatives. The difference to the standard case is that the charge $\epsilon(x^5)\Lambda$ has opposite sign on the two sides of the brane [1]. The prepotential which describe this gauging is piecewise constant and takes the form:

$$g\mathcal{P}_B^A = \begin{pmatrix} \frac{1}{4\sqrt{2}} i\epsilon(x^5)\Lambda & 0 \\ 0 & -\frac{1}{4\sqrt{2}} i\epsilon(x^5)\Lambda \end{pmatrix} \tag{4.53}$$

Let us now assume $\Lambda = 0$ and re-introduce the hypermultiplet in the bulk. Consider the boundary term:

$$\mathcal{L} = \delta(x^5)\frac{e}{\kappa^2}\frac{\sqrt{2}\alpha}{V}\,. \tag{4.54}$$

The variation of the determinant can be canceled by modifying $\delta\psi$, similarly to the previous case:

$$\begin{aligned} \delta\psi_\alpha^1 &= -\frac{\sqrt{2}}{12}\frac{\alpha}{V}\epsilon(x^5)\gamma_\alpha\epsilon^1 \\ \delta\psi_\alpha^2 &= +\frac{\sqrt{2}}{12}\frac{\alpha}{V}\epsilon(x^5)\gamma_\alpha\epsilon^2\,. \end{aligned} \tag{4.55}$$

We must also vary the hypermultiplet modulus V in (4.54) ($\delta V = \frac{iV}{\sqrt{2}}(\overline{\epsilon^1}\lambda^1 - \overline{\epsilon^2}\lambda^2)$) and this yields:

$$\delta\mathcal{L} = -i\delta(x^5)e\frac{\alpha}{V}(\overline{\epsilon^1}\lambda^1 - (1 \to 2)) \tag{4.56}$$

This variation can be cancelled by modifying supersymmetry transformation law of the hyperino λ:

$$\begin{aligned} \delta\lambda^1 &= \frac{i}{2V}\alpha\epsilon(x^5)\epsilon^1 \\ \delta\lambda^2 &= \frac{i}{2V}\alpha\epsilon(x^5)\epsilon^2\,. \end{aligned} \tag{4.57}$$

A similar mechanism works: in the variation of the hyperino kinetic term the fifth derivative acts on the step function which leads to a term which precisely cancels (4.56). Note that it is only the potential α/V which causes the corrections to the hyperino transformation law. As before, we need to supersymmetrize further. Two-fermi terms and, consequently, a cosmological potential is necessary:

$$\mathcal{L} = \frac{e_5}{2V\kappa^2}\alpha\epsilon(x^5)\left(-\frac{\sqrt{2}}{4}(\overline{\psi_\alpha^1}\gamma^{\alpha\beta}\psi_\beta^1 - (1 \to 2)) + i(\overline{\lambda^1}\gamma^\alpha\psi_\alpha^1 + (1 \to 2)) + i\frac{3\sqrt{2}}{4}(\overline{\lambda^1}\lambda^1 - (1 \to 2))\right) \tag{4.58}$$

$$\mathcal{L}_C = -\frac{e_5}{6\kappa^2}\frac{\alpha^2}{V^2} \tag{4.59}$$

However, this time a minus sign relative to that of (4.48) appears, and anti-de-Sitter solution is not allowed. Moreover, contrary to the previous case, the 2-fermi and cosmological terms

[1] In the recent reference [28] the gauge charge is promoted to a supersymmetry singlet field

are not enough to render the bulk lagrangian supersymmetric. Closer inspection shows, that terms of the form $\alpha(\epsilon\psi)\partial_\alpha\sigma$ do not cancel and the bulk lagrangian must be supplemented with the coupling $\alpha\partial_\beta\sigma\mathcal{A}^\beta$. In the context of 5d supergravity this means that the translations of the pseudoscalar σ from the hypermultiplet are gauged, with the graviphoton being the gauge field. To recapitulate, starting with the boundary term (4.54) we are led to 5d gauged supergravity similar to that studied in [4]. The gauging can be described by the prepotential 3.40

One could also imagine other powers of V occuring in (4.54), or more generally, some function $f(V)$. But then supersymmetrization is possible only if the bulk sigma model quaternionic metric is found. In some simple cases one can appropriately redefine Re(S) and end up in the same sigma model, however in general one has to search for new sigma models with quaternionic kinetic metric that allow for gauging, which is beyond the scope of this paper.

The interesting question is if we can join both schemes discussed in this section and introduce in a supersymmetric way a boundary term $\mathcal{L}_B = \delta(x^5)\frac{e}{\kappa^2}(-\Lambda + \frac{\sqrt{2}\alpha}{V})$. The answer is yes and the necessary steps are given in [9].

Chapter 5

Compactification to 4 dimensions

In order to investigate the phenomenological consequences of theories formulated in $D > 4$ space-time dimensions, one has to go to the effective four-dimensional description, that is one has to compactify. If the additional compact dimension are assumed to be small (which is the case with the fifth dimension in our model), one keeps only massless Kaluza-Klein modes (zero modes) in the effective description; heavy Kaluza-Klein excitation decouple from the effective theory. But in the case at hand the compactification is not a straightforward task because one cannot simply truncate the five-dimensional theory. By 'truncation' we mean ignoring the dependence of the fields on the fifth coordinate; the integral over the compact dimensions in the action yields just the volume which can be absorbed into the definition of the 4d gravitational constant. In the standard case of Kaluza-Klein compactification on flat backgrounds it can be shown, that truncation is equivalent to ignoring the heavy Kaluza-Klein modes. But we shall see, that in the model discussed in section 4 the flat space is not a solution to to the equations of motion. The vacuum solution we will find, will depend on the fifth coordinate and the zero modes will be the x^5 independent excitations around this vacuum solution. In such cases, simple truncation is not consistent. Instead we have to carefully integrate out the x^5 dependence from the action.

The background solutions to the equations of motion depend dramatically on the choice of the potential in the bulk (and on the boundary since the two are connected by supersymmetry). In this section we assume the more general potential introduced in subsection 4.4, since for this choice, in certain limits, we can obtain the pure M-theoritical solution, while in other limits the interesting solution of the Randall-Sundrum type can be obtained. We want to compactify our model down to 4d and we demand that the effective theory has N=1 supersymmetry. Thus, we must search for the background solution which preserves exactly four supercharges, that is half of the 5d supersymmetry. The solutions, which leave some portion of supersymmetry unbroken, fit into a very special class of supersymmetric objects, called BPS states. It is generally believed that they are stable, since they minimize energy for a given charge. The best way to find such BPS solutions is to consider first the supersymmetry transformation laws. We will see that the configuration preserving an unbroken N=1 supersymmetry (which are quite easy to find), automatically satisfies the equations of motion.

For brevity, some formulae presented in the subsequent section are written as if both Λ and α/V parts of the boundary potential were present although, as discussed at the end of the last chapter, the theory is supersymmetric only for $\Lambda = 0$ or for $\alpha = 0$.

5.1 BPS solution

The supersymmetry transformation laws of fermions, including modifications found in the previous paragraphs are:

$$\delta\psi_\alpha^A = D_\alpha\epsilon^A - \epsilon(x^5)\frac{1}{12}(-\Lambda + \frac{\sqrt{2}\alpha}{V})\gamma_\alpha(\sigma^3)^A{}_B\epsilon^B$$
$$\delta\lambda^a = -\frac{i}{2\sqrt{2}V}\partial_5 V\gamma^5(\sigma^3)^a{}_B\epsilon^B + \alpha\epsilon(x^5)\frac{i}{2V}\epsilon^a\,. \quad (5.1)$$

In the above formulas we neglected terms with 4d derivatives ∂_μ in order to preserve the 4d Poincaré invariance of the background we seek. We also put $\sigma = \mathcal{A}_5 = 0$ since these fields do not occur in the potential, so setting them to zero is consistent with the equations of motion. Finally, we neglected $\partial_5\xi$ since, as we show later in this thesis, non-zero expectation value of this term generically breaks all supersymmetries.

The ansatz for a static solution is:

$$\begin{aligned} ds^2 &= a(x^5)dx^\mu dx^\nu\eta_{\mu\nu} + b(x^5)(dx^5)^2 \\ V &= V(x^5) \end{aligned} \quad (5.2)$$

The relevant supersymmetry transformation laws evaluated for this ansatz are (a prime denotes ∂_5 and the world indices now refer to the 4d Minkowski metric $\eta_{\mu\nu}$):

$$\delta\psi_\mu^A = \tfrac{a'}{4\sqrt{ab}}\gamma_\mu\gamma_5\epsilon^A - \epsilon(x^5)\tfrac{\sqrt{a}}{12}(-\Lambda + \tfrac{\sqrt{2}\alpha}{V})\gamma_\mu(\sigma^3)^A{}_B\epsilon^B$$
$$\delta\psi_5^A = \partial_5\epsilon^A - \epsilon(x^5)\tfrac{\sqrt{b}}{12}(-\Lambda + \tfrac{\sqrt{2}\alpha}{V})\gamma_5(\sigma^3)^A{}_B\epsilon^B$$
$$\delta\lambda^a = -\tfrac{i}{2\sqrt{2b}V}V'(\sigma^3)^a{}_B\gamma_5\epsilon^B + \alpha\epsilon(x_5)\tfrac{i}{2V}\epsilon^a. \quad (5.3)$$

The conditions for unbroken supersymmetry are equivalent to the requirement that the above variations of fermionic fields vanish for vacuum configurations. This leads to the following conditions:

$$\tfrac{a'}{a} = \tfrac{1}{3}(-\Lambda + \tfrac{\sqrt{2}\alpha}{V})\epsilon(x^5)\sqrt{b}$$
$$V' = \sqrt{2}\alpha\epsilon(x^5)\sqrt{b}$$
$$\partial_5\epsilon^A = \tfrac{\sqrt{b}}{12}(-\Lambda + \tfrac{\sqrt{2}\alpha}{V})\epsilon(x^5)\epsilon^A. \quad (5.4)$$

In addition we need thechirality conditions for the spinorial supersymmetry transformation parameters , which break N=2 supersymmetry down to N=1:

$$\gamma_5\epsilon^1 = \epsilon^1 \quad \gamma_5\epsilon^2 = -\epsilon^2 \quad (5.5)$$

The chirality conditions arise because of the σ^3 Pauli matrices multiplying ϵ^A in (5.3). Their presence causes sign difference between the A=1 and A=2 components of the supersymmetry transformation, which must be compensated for by (5.5) if we want satisfy both $\delta\psi^1 = 0$ and $\delta\psi^2 = 0$.

First, we check that if the parameters a, b, V of our ansatz satisfy the conditions (5.4), they automatically satisfy the equations of motion (with delta sources). To do this, it will be convienent to work with $\phi \equiv lnV$.

Einstein's equations are $\frac{1}{2}Rg_{\alpha\beta} - R_{\alpha\beta} = T_{\alpha\beta}$, where $T_{\alpha\beta} \equiv \frac{1}{e_5}\frac{\partial \mathcal{L}_{matter}}{\partial g_{\alpha\beta}}$. Taking its trace and substituting back for R we can express it in the equivalent form:

$$R_{\alpha\beta} = -(T_{\alpha\beta} - \frac{1}{3}T^{\gamma}{}_{\gamma} g_{\alpha\beta}) \equiv -S_{\alpha\beta} \tag{5.6}$$

For the ansatz (5.2) the components of the Ricci tensor are:

$$\begin{aligned} R_{\mu\nu} &= (\tfrac{a''}{2b} - \tfrac{a'b'}{4b^2} + \tfrac{(a')^2}{2ab})\eta_{\mu\nu} \\ R_{55} &= \tfrac{2a''}{a} - \tfrac{a'b'}{ab} - \tfrac{(a')^2}{a^2} \\ R_{\mu 5} &= 0 \end{aligned} \tag{5.7}$$

For our lagrangian the tensor S defined in (5.6) takes the form:

$$\begin{aligned} S_{\mu\nu} &= [(\tfrac{a}{3}\alpha^2 e^{-2\phi} - \tfrac{a}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2))\eta_{\mu\nu} - \tfrac{a}{3\sqrt{b}}(-\Lambda + \sqrt{2}\alpha e^{-\phi})(\delta(x^5) - \delta(x^5 - \pi\rho))] \\ S_{55} &= [\tfrac{b}{3}\alpha^2 e^{-2\phi} - \tfrac{b}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2 + \tfrac{1}{2}(\phi')^2 - \tfrac{4\sqrt{b}}{3}(-\Lambda + \sqrt{2}\alpha e^{-\phi})(\delta(x^5) - \delta(x^5 - \pi\rho))] \\ S_{\mu 5} &= 0 \end{aligned} \tag{5.8}$$

Note the delta functions originating from the boundary potential. Though the T_{55} components of the energy-momentum tensor vanish on the boundary, S_{55} is non-zero as it contains contributions from $T_{\mu\nu}$. The $(\mu 5)$ components of the Einstein's equations are trivially satisfied.

The remaining components of the Einstein's equation together with the equation of motion for ϕ take the form:

$$\begin{aligned} &\frac{a''}{2b} - \frac{a'b'}{4b^2} + \frac{(a')^2}{2ab} + \frac{a}{3}\alpha^2 e^{-2\phi} - \frac{a}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2 \\ &\qquad = \frac{a}{3\sqrt{b}}(-\Lambda + \sqrt{2}\alpha e^{-\phi})[\delta(x^5) - \delta(x^5 - \pi\rho)] \\ &\frac{2a''}{a} - \frac{a'b'}{ab} - \frac{(a')^2}{a^2} + \frac{b}{3}\alpha^2 e^{-2\phi} - \frac{b}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2 + \frac{1}{2}(\phi')^2 \\ &\qquad = \frac{4\sqrt{b}}{3}(-\Lambda + \sqrt{2}\alpha e^{-\phi})[\delta(x^5) - \delta(x^5 - \pi\rho)] \\ &\frac{1}{2}\phi'' + \frac{a'}{a}\phi' - \frac{1}{4}\frac{b'}{b}\phi' + b\alpha^2 e^{-2\phi} - \frac{b}{3}\sqrt{2}\alpha e^{-\phi}(-\Lambda + \sqrt{2}\alpha e^{-\phi}) \\ &\qquad = \sqrt{2}b\alpha[\delta(x^5) - \delta(x^5 - \pi\rho)] \end{aligned} \tag{5.9}$$

To check if these equations are satisfies, we re-write the relations (5.4) in the form which is more convienient for our purpose. Dividing the first relation (5.4) by the second we get:

$$\frac{a'}{a} = \frac{1}{3}\phi'(1 - \frac{\Lambda}{\sqrt{2}\alpha}e^{\phi}) \tag{5.10}$$

We can also obtain a useful relation for b:
$\frac{b'}{b} = \frac{2}{\sqrt{b}}\frac{b'}{2\sqrt{b}} = \frac{2(\sqrt{2}\alpha e^{\phi} - \Lambda}{3\frac{a'}{a}} = \frac{2a''}{a'} - \frac{2a'}{a} + \frac{2\sqrt{2}\alpha\phi'}{\sqrt{2}\alpha - \Lambda e^{\phi}} = \frac{2a''}{a'} - \frac{2a'}{a} + \frac{6\frac{a'}{a}}{(1 - \frac{\Lambda}{\sqrt{2}\alpha}e^{\phi})^2}$
So finally:

$$\frac{b'}{b} = \frac{2a''}{a'} - \frac{2a'}{a} + \frac{6\frac{a'}{a}}{(1 - \frac{\Lambda}{\sqrt{2}\alpha}e^{\phi})^2} \tag{5.11}$$

We show how this works on the example of the first equation in (5.9). Away from the boundary, where the delta functions do not contribute, we have:

$$l.h.s. = \frac{a''}{2b} - \frac{a'b'}{4b^2} + \frac{(a')^2}{2ab} + \frac{a}{3}\alpha^2 e^{-2\phi} - \frac{a}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2$$

$$= \frac{1}{b}\left[\frac{a''}{2} - \frac{a'}{4}(\frac{2a''}{a'} - \frac{2a'}{a} + \frac{6\frac{a'}{a}}{(1-\frac{\Lambda}{\sqrt{2}\alpha}e^{\phi})^2}) + \frac{(a')^2}{2a}\right] + \frac{a}{3}\alpha^2 e^{-2\phi} - \frac{a}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2$$

$$= \frac{1}{b}[\{-\frac{3(a')^2}{2a}\frac{1}{(1-\frac{\Lambda}{\sqrt{2}\alpha}e^{\phi})^2} + \frac{ab}{3}\alpha^2 e^{-2\phi}\} + \{\frac{(a')^2}{a} - \frac{a}{9}(-\Lambda + \sqrt{2}\alpha e^{-\phi})^2)\}] = 0 \quad (5.12)$$

In the second line we used (5.11) and in the last line (5.10). We must still satisfy the delta functions on the right-hand side of the equations. From the $\mathbf{Z_2}$ properties of the metric and of the hypermultiplet field V we know that functions a,b,ϕ are even (and continuous), their fifth derivatives are odd and so can be discontinous across the boundaries. Thus, their second derivatives can have delta function singularities. The coefficients of the delta function is equal to the discountinuity of the first derivative or, equivalently, twice the boundary value of the first derivative.

We again consider the example of the first equation of (5.9). We need to satisfy

$$\frac{a''}{2b} = \frac{a}{3\sqrt{b}}(-\Lambda + \sqrt{2}\alpha e^{-\phi})\delta(x^5) \quad (5.13)$$

in the vicinity of the first brane. But the above consideration allow to re-express this equation as:

$$\frac{a'}{b} = \frac{a}{3\sqrt{b}}(-\Lambda + \sqrt{2}\alpha e^{-\phi})\epsilon(x^5) \quad (5.14)$$

which is again the relation (5.10) if the equality $\sqrt{b} = \frac{\phi' e^{-\phi}}{\sqrt{2}\alpha\epsilon(x^5)}$ is used. Thus, we have indeed shown that the first of the Einstein's equations (5.9) is satisfied for the BPS configuration (5.4). The remaining equations can be checked in the similar way.

We can now solve the conditions (5.4). This can be easily done in the coordinate frame in which $b = R_0^2$. The vacuum solution is:

$$\begin{aligned} V &= V_0 + \alpha\sqrt{2}\, R_0\, (|x^5| - \tfrac{\pi\rho}{2}) \\ g_{\mu\nu} &= a_0 \left(1 + \alpha\sqrt{2}\tfrac{R_0}{V_0}(|x^5| - \tfrac{\pi\rho}{2})\right)^{1/3} e^{\frac{-R_0\Lambda}{3}|x^5|}\eta_{\mu\nu} \\ g_{55} &= R_0^2 \end{aligned} \quad (5.15)$$

The 4d effective theory for the general potential is difficult to obtain (and it is not clear if integrating out the fifth dimension makes sense in the general case). In the following we determine the effective theory only in the M-theoretical ($\Lambda = 0$) limit. In this limit it is customary to work in a different coordinate frame in which $g_{55} \neq 0$. Then the solution is:

$$\begin{aligned} g_{\mu\nu} &= \tfrac{1}{R_0} H \bar{g}_{\mu\nu} \\ g_{55} &= R_0^2 H^4 \\ V &= V_0 H^3 \\ H &:= 1 + \alpha\tfrac{\sqrt{2}R_0}{3V_0}(|x^5| - \tfrac{\pi\rho}{2}) \end{aligned} \quad (5.16)$$

From the view-point of the effective 4d theory the integration constants R_0, V_0 and $\bar{g_{\mu\nu}}$ become the dynamical fields (moduli). They are defined in such way that $\bar{g}_{\mu\nu}$ is the 4d metric with the standard Einstein-Hilbert kinetic term, and $V_0 =< V >$,$R_0 =< \sqrt{g_{55}} >$, up to $\mathcal{O}(\alpha^2)$ corrections ($< \ldots >$ denotes averaging by integrating over the fifth dimension) .

The formulae (5.16) describe the vacuum solution with vanishing all boundary fields. Since in the full lagrangian the bulk fields couple to gauge fields on the boundary, allowing for non-zero boundary fields changes also the field configuration in the bulk (in their equation of motion this manifests itself as delta funtion sources). As mentioned earlier, we cannot simply ignore this back-reaction. Neglecting all quantum corrections, we can account for the dependence of bulk fields on the boundary dynamics by replacing the bulk fields in the 5d action with the solutions of their equations of motion. Having done this we integrate over the fifth dimension.

Due to the complicated non-linear sigma model in the bulk, the quest for the exact solution is a hopeless task. Instead, we can simplify the problem by taking a specific limit. We will assume that: $\partial_5\phi >> \partial_\mu\phi$. This corresponds to the limit of small 4d momenta compared to the momentum along x^5. In the following we will simply neglect ∂_μ.

One more assumption turns out to be very helpful. The boundary action is suppresed by a parameter $\frac{\kappa^2}{g^2}$, and consequently, the sources for the bulk fields are suppressed by this parameter. Thus, we can write down the solution as a series in $\frac{\kappa^2}{g^2}$. We will be able to solve the equations of motion in the first order in $\frac{\kappa^2}{g^2}$.

5.2 Solving equations for the even fields

In our model, the even bosonic fields in the bulk are: $(g_{\mu\nu}, g_{55}, \mathcal{A}_5, V, \sigma)$. There are no $(\partial_5)\mathcal{A}_5$ terms in the bulk so $\mathcal{A}_5$ is not excited in the limit we consider.

The procedure of extracting gauge field dependence of even bulk fields was described in [7]. Here, we quote only basic results. The detailed form of the solution is not important to us, because, as we show in the next subsection, to the order we perform the calculations the effective theory depends only on the background value of the even fields (with the exception of the $\alpha\mathcal{A}_5$ dependence of σ_B).

We write a generic even bulk field ϕ as a sum: $\phi = \phi_{vac}+\phi_B$, where ϕ_{vac} is the corresponding background solution given by (5.16). Then ϕ_B satisfies an equation of the form:

$$\partial_5\partial_5\phi_B = J_1\delta(x^5) + J_2\delta(x^5 - \pi\rho) - \frac{1}{2\pi\rho}(J_1 + J_2) \tag{5.17}$$

where J_i are boundary sources for ϕ. The part of the r.h.s without the delta function comes from integrating ϕ_{vac} out of the equation of motion. It yields the $(x^5)^2$ dependence of the solution. The delta functions provide the boundary conditions for the fifth derivative. Recall, that the fifth derivative of the even field is odd, and in principle can be discontinuous at $\mathbf{Z}_2$ fixed points. The coefficient of the delta function equals this discontinuity, so the boundary value of $\partial_5\phi$ equals precisely one half of this coefficient. Moreover, we require that $< \phi_B >$ vanishes. The detailed calculation shows that to the first order in $\frac{\kappa^2}{g^2}$ and α we can write:

$$V_B = \frac{\kappa^2 R_0^3 V_0^2}{2\pi\rho\sqrt{\bar{g}}}[J_{1V}((x^5)^2 - 2\pi\rho x^5 + \frac{2}{3}(\pi\rho)^2) + J_{2V}((x^5)^2 - \frac{1}{3}(\pi\rho)^2)\frac{\kappa^2 R_0 V_0^2}{2\pi\rho\sqrt{\bar{g}}}]$$
$$\sigma_B = 2\alpha(|x^5| - \pi\rho)\mathcal{A}_5 + \frac{\kappa^2 R_0^3 V_0^2}{2\pi\rho\sqrt{\bar{g}}}[J_{1\sigma}((x^5)^2 - 2\pi\rho x^5 + \frac{2}{3}(\pi\rho)^2) + J_{2\sigma}((x^5)^2 - \frac{1}{3}(\pi\rho)^2)]$$

$$(g_B)_{\mu\nu} = \frac{\kappa^2 R_0^3}{2\pi\rho\sqrt{g}}[(J_{1g})_{\mu\nu}((x^5)^2 - 2\pi\rho x^5 + \frac{2}{3}(\pi\rho)^2) + (J_{2g})_{\mu\nu}((x^5)^2 - \frac{1}{3}(\pi\rho)^2)] \quad (5.18)$$

where $J_{i\phi}$ denotes a derivative of the i-th boundary lagrangian with respect to the field ϕ. The $\alpha\mathcal{A}_5$ dependence of the solution for σ_B arises because of the gauge covariant derivative $D_\alpha\sigma = \partial_\alpha\sigma + 2\alpha\epsilon(x^5)\mathcal{A}_\alpha$ acting on σ in the lagrangian; the fifth derivative acting on the step function accompanying α yields a delta function, which effectively acts as a source in the equation of motion.

5.3 Solving equations for the odd fields

We begin with ξ. It couples to the boundary theory through its fifth derivative, and thus acquires a non-trivial gauge field dependence. The relevant terms in the Lagrangian are:

$$-\frac{e_5}{\kappa^2 V}g^{55}\partial_5\xi\partial^5\bar{\xi} + \frac{e_5}{g^2 e_5^5}\partial_5\xi[\delta(x^5)(-\frac{\sqrt{V}}{2}(\overline{\chi}_R\chi_L)^1 + \frac{2}{V}W) + \delta(x^5-\pi\rho) - \frac{\sqrt{V}}{2}(\overline{\chi}_R\chi_L)^2] \quad (5.19)$$

We ignored ξ^4 terms in the bulk since they contribute only at order $(\frac{\kappa^2}{g^2})^3$. As justified before, we also neglected derivatives other than ∂_5.

The equation of motion for ξ is:

$$\partial_5(\frac{e_5}{\kappa^2 V}g^{55}\partial^5\bar{\xi}) = \frac{\kappa^2}{g^2}\partial_5[e_5\delta(x^5)(-\frac{\sqrt{V}}{2}(\overline{\chi}_R\chi_L)^1 + \frac{2}{V}W) - e_5\delta(x^5-\pi\rho)\frac{\sqrt{V}}{2}(\overline{\chi}_R\chi_L)^2] \quad (5.20)$$

Substituting for the bulk fields their vacuum solutions (5.16), and integrating twice, we obtain:

$$\frac{\partial^5\bar{\xi}}{H^{-3}} = \frac{\kappa^2}{2g^2}(V_0R_0)^{3/2}[\delta(x^5)(-\chi_1^2 + \frac{4}{(V_0R_0)^{3/2}}WH^{-3}(0)) - \delta(x^5-\pi\rho)\chi_2^2] + f \quad (5.21)$$

$$\epsilon(x^5)\bar{\xi} = \frac{1}{4\alpha'}H^4 f + h \quad (5.22)$$

We defined:

$$\chi_1^2 := (\overline{\chi}_R\chi_L)_1\left(\frac{H(0)}{R_0}\right)^{3/2} \quad \chi_2^2 := (\overline{\chi}_R\chi_L)_2\left(\frac{H(\pi\rho)}{R_0}\right)^{3/2} \quad (5.23)$$

The integration constants f,h can be calculated using boundary conditions. Matching delta funtions in (5.21) requires that $\bar{\xi}$ has discontinuities at $x^5 = 0$ and $x^5 = \pi\rho$. One half of this discontinuity is the boundary value for $\overline{\xi}$. One can calculate:

$$\frac{f}{4\alpha'} = \frac{\kappa^2}{4g^2}(V_0R_0)^{3/2}\frac{-\chi_1^2H^3(0) - \chi_2^2H^3(\pi\rho) + 4(V_0R_0)^{-3/2}W}{H^4(0) - H^4(\pi\rho)} \quad (5.24)$$

$$h = \frac{\kappa^2}{4g^2}(V_0R_0)^{3/2}\frac{-\chi_1^2H^3(0)H^4(\pi\rho) - \chi_2^2H^3(\pi\rho)H^4(0) + 4(V_0R_0)^{-3/2}WH^4(\pi\rho)}{H^4(0) - H^4(\pi\rho)} \quad (5.25)$$

In the same way we solve the equation of motion for the 4d components of the graviphoton, which also couples to the boundary through its fifth derivative. The result is:

$$\begin{aligned}\partial_5\mathcal{A}_\mu - \partial_\mu\mathcal{A}_5 = \\ \tfrac{\kappa^2}{3R_0g^2}[\delta(x^5)(-\tfrac{3i}{4\sqrt{2}}V_0R_0\chi_{1\,\mu}^2H^2(0) - \tfrac{i}{\sqrt{2}}(CD_\mu\bar{C} - \bar{C}D_\mu C) + \tfrac{i}{2\sqrt{2}}\zeta_\mu^2) + \\ \delta(x^5-\pi\rho)(-\tfrac{3i}{4\sqrt{2}}V_0R_0\chi_{2\,\mu}^2H^2(\pi\rho))] + Hf_\mu\end{aligned} \quad (5.26)$$

$$\mathcal{A}_\mu = H^2 \frac{f_\mu}{2\alpha'} + \partial_\mu \mathcal{A}_5 x^5 + g_\mu \tag{5.27}$$

$[\chi^2_{1\ \mu} := (\overline{\chi}\gamma^5\gamma_\mu\chi)_1(\frac{H(0)}{R_0})^{3/2} \quad \chi^2_{2\ \mu} := (\overline{\chi}\gamma^5\gamma_\mu\chi)_2(\frac{H(\pi\rho)}{R_0})^{3/2} \ \zeta^2_\mu := (\overline{\zeta}\gamma^5\gamma_\mu\zeta)(\frac{H(0)}{R_0})^{1/2}]$

As before f_μ and g_μ are specified by the boundary conditons:

$$\begin{aligned}
\frac{f_\mu}{2\alpha'} &= \frac{\frac{\kappa^2}{6g^2}(-\frac{3i}{4\sqrt{2}}V_0 R_0(\chi^2_{1\ \mu}H^2(0)+\chi^2_{2\ \mu}H^2(\pi\rho)-\frac{i}{\sqrt{2}}(CD_\mu\bar{C}-\bar{C}D_\mu C)+\frac{i}{2\sqrt{2}}\zeta^2_\mu)+\partial_\mu\mathcal{A}_5\pi\rho}{H^2(0)-H^2(\pi\rho)} \\
g_\mu &= \frac{\frac{\kappa^2}{6g^2}(-\frac{3i}{4\sqrt{2}}V_0 R_0(\chi^2_{1\ \mu}+\chi^2_{2\ \mu})H^2(0)H^2(\pi\rho)-\frac{i}{\sqrt{2}}(CD_\mu\bar{C}-\bar{C}D_\mu C)H^2(\pi\rho)+\frac{i}{2\sqrt{2}}\zeta^2_\mu H^2(\pi\rho))}{H^2(0)-H^2(\pi\rho)} \\
&\quad + \frac{\partial_\mu\mathcal{A}_5\pi\rho H^2(\pi\rho)}{H^2(0)-H^2(\pi\rho)}
\end{aligned} \tag{5.28}$$

Note, that there is no arbitrary integration constant (they are all specified in terms of the matter fields on the boundary) in the solutions for ξ and $\mathcal{A}_\mu$. Thus, there will be no zero modes corresponding to these fields in the effective 4d theory.

The other odd fields in the bulk do not couple to the boundary, so they are not excited.

5.4 First order compactification

We briefly review the compactification in the $(\frac{\kappa^2}{g^2})^0$ and $(\frac{\kappa^2}{g^2})^1$ order. This step is well-known since the effect of the non-trivial background is visible only at $(\frac{\kappa^2}{g^2})^2$ (thus, to up to this order we can simply truncate the 5d action). Gravity enters at $(\frac{\kappa^2}{g^2})^0$. The definition of 4d moduli in (5.16) is chosen such, that the Ricci scalar R built out of $\bar{g}_{\mu\nu}$ is canonically normalized in this order. The 4d gravitational constant $\frac{1}{\kappa_4^2}$ can be expressed in terms of its 5d counterpart as:

$$\frac{1}{\kappa_4^2} = \frac{2\pi\rho}{\kappa_5^2} \tag{5.29}$$

The superpartner of the graviton is the gravitino ψ_μ which originates from the even part of the 5d gravitino. To give the correct normalization to the gravitino kinetic term we must rescale: $\psi_\mu \to (R_0)^{-1/4}\psi_\mu$. We have also kinetic terms for the moduli V_0, σ_0 which are zero-modes of the corresponding hypermultiplet scalars, as well as for the moduli $R_0, \mathcal{A}_5$ which are zero-modes of g_{55} and the fifth component of the graviphoton, respectively:

$$\kappa_4^2 \mathcal{L}_{KIN} = \sqrt{\bar{g}}[-\frac{1}{4V_0^2}(\partial_\mu V_0 \partial^\mu V_0 + \partial_\mu\sigma_0\partial^\mu\sigma_0) - \frac{3}{4R_0^2}(\partial_\mu R_0 \partial^\mu R_0 + 2\partial_\mu\mathcal{A}_5\partial^\mu\mathcal{A}_5)] \tag{5.30}$$

The boundary action enters in $(\frac{\kappa^2}{g^2})^1$ order. We have two gauge sectors: $(A_\mu;\chi)_1$ and $(A_\mu;\chi)_2$ with kinetic terms:

$$g^2\mathcal{L}_{gaugekin} = \sqrt{\bar{g}}\sum_{n=1}^{2}[-\frac{1}{4}V_0(F_{\mu\nu}F^{\mu\nu})_n - \frac{1}{4}\sigma_0(F_{\mu\nu}\tilde{F}^{\mu\nu})_n] \tag{5.31}$$

We get also a kinetic term for the scalar C:

$$g^2\mathcal{L}_{KIN} = -\frac{1}{R_0}D_\mu C D^\mu \bar{C} \tag{5.32}$$

There are no $(\alpha)^1$ corrections to bulk kinetic terms, because $\int_0^{\pi\rho} d^5x\ \alpha(y-\frac{\pi\rho}{2})=0$. Thus, in the first order$\frac{\kappa^2}{g^2}$ and α , the 4d effective supergravity can be described by the Kähler potential G and the gauge kinetic functions f_i:

$$G = ln(S+\bar{S}) + 3ln(T+\bar{T} - \frac{2}{3}\frac{\kappa_4^2}{g^2}C\bar{C}) - ln(64W\bar{W}) \tag{5.33}$$

$$f_1 = f_2 = S \tag{5.34}$$

The numerical factor coming with the superpotential W can be read off from the bilinear fermionic terms as given in $\mathcal{L}_W$ in Appendix A. The $W\bar{W}$ term enters at $(\frac{\kappa^2}{g^2})^2$, and we will obtain it after integrating out ξ (so far $W\bar{W}$ occurs as a singular term in the boundary Lagrangian). The moduli fields S and T are defined as:

$$\begin{aligned} S &= V_0 + i\sigma_0 \\ T &= R_0 - \frac{\kappa^2}{3g^2}C\bar{C} + i\sqrt{2}\mathcal{A}_5 \end{aligned} \tag{5.35}$$

The superpartners of the modulus C is the boundary fermion ζ, and of the modulus S the even part of bulk hyperino λ. To have fermion kinetic terms normalized as in [17], we rescale: $\lambda \to (R_0)^{1/4}\lambda$, $\zeta \to (R_0)^{1/4}\zeta$. The supersymmetry transformation laws suggest, that the suparpartner of T is ψ_5, but as yet, it has no kinetic term. To obtain the kinetic term we use the fact, that the gravitino kinetitic term in the bulk mixes ψ_5 with ψ_μ. To diagonalize it, and to obtain a legitimate kinetic term of ψ_5 we must redefine the 4d gravitino:

$$(\psi_\mu)_{4d} := \psi_\mu + \frac{i}{2R_0}\gamma_\mu\gamma^5\psi_5 \tag{5.36}$$

We can define the fermion superpartner of the modulus T:

$$\Lambda_L^T = (R_0)^{-1/4}[\psi_{L\,5} + \frac{2\kappa_4^2}{3g^2}\bar{C}\zeta_L] \tag{5.37}$$

The ζ dependent correction is necessary here, because of the terms involving C in the definition of ReT.

5.5 Higher order corrections

Having solved the equation of motion we can proceed with finding α^2 ,$\alpha\frac{\kappa^2}{g^2}$ and $(\frac{\kappa^2}{g^2})^2$ corrections to the effective 4d theory.

First, we should comment on the cosmological potential. In the 5d bulk theory, we have the cosmological term $\frac{\alpha^2}{V^2}$. What happens in 4d effective theory? There is a general argument, that such a cosmological term should be absent. Indeed, the background solution (5.16) was obtained under the assumption of N=1 supersymmetry and vanishing expectation value of the superpotential W. This, in turn, is equivalent to vanishing of the potential energy at its minimum (although potential in 4d supergravity is given by $-3exp(-G)$ which is not semi-positive definided, this expression is zero if $< W >= 0$). It is reassuring to see that the 4d cosmological potential vanishes if we explicitly calculate it our framework.

In the 5d bulk there are three terms which contribute to the 4d vacuum energy: the curvature scalar R, the kinetic term of V and the original 5d potential. We can simplify the calculations using the relations (5.10, 5.11) which for $\Lambda = 0$ reduce to:

$$\frac{a'}{a} = \frac{1}{3}\frac{V'}{V} \tag{5.38}$$

$$\frac{b'}{b} = \frac{2a''}{a'} + \frac{4a'}{a} \tag{5.39}$$

Using (5.39) we can re-write the curvature tensor given by (5.7) in the form $R_{\mu\nu} = -\frac{(a')^2}{2ab}\eta_{\mu\nu}$, $R_{55} = -\frac{5(a')^2}{a^2 b}$, so the contribution from the Ricci scalar is:

$$-\frac{1}{2}R = -\frac{1}{2}(g^{\mu\nu}R_{\mu\nu} + g^{55}R_{55}) = \frac{7(a')^2}{2a^2 b} \tag{5.40}$$

Using (5.38), the kinetic term of V contributes:

$$-\frac{1}{4}g^{55}(\frac{V'}{V})^2 = -\frac{9(a')^2}{4a^2 b} \tag{5.41}$$

while the 5d cosmological potential can be re-written using the first relation of (5.4) $\frac{1}{V} = \frac{3a'}{\sqrt{2}\alpha\sqrt{b}a}$ and contributes:

$$-\frac{\alpha^2}{6V^2} = -\frac{3(a')^2}{4a^2 b} \tag{5.42}$$

These three contributions sum to $\frac{(a')^2}{2a^2 b}$. Inserting this in the solution (5.16) and integrating over the fifth dimension yields the 4d effective potential:

$$\begin{aligned} V_{bulk} &= \textstyle\int_0^{2\pi\rho} dx^5 a^2\sqrt{b}\frac{(a')^2}{2a^2 b} \\ &= 2\textstyle\int_0^{\pi\rho} dx^5 \frac{\alpha^2}{9V_0^2}\frac{1}{R_0 H^2} = \frac{2}{9}\frac{\alpha^2}{R_0 V_0^2}\int_0^{\pi\rho}\frac{dx^5}{1+\frac{\sqrt{2}\alpha R_0}{3V_0}(x^5-\frac{\pi\rho}{2})} \\ &= \frac{\sqrt{2}\alpha}{3R_0^2 V_0}(\frac{1}{H(0)} - \frac{1}{H(\pi\rho)}) = \frac{2\alpha^2\pi\rho}{9R_0 V_0^2}\frac{1}{1-(\frac{\sqrt{2}\alpha R_0\pi\rho}{6V_0})^2} \end{aligned} \tag{5.43}$$

We should not forget about the delta functions in R. The singular part of the second derivative of the metric is:

$$a'' = \frac{2\sqrt{2}\alpha}{3V_0}(\delta(x^5) - \delta(x^5 - \pi\rho)) \tag{5.44}$$

From (5.7) we see that the singularities of the Ricci tensor are

$$\begin{aligned} R_{\mu\nu} &\sim \frac{\sqrt{2}\alpha}{3bV_0}(\delta(x^5) - \delta(x^5-\pi\rho))\eta_{\mu\nu} \\ R_{55} &\sim \frac{4\sqrt{2}\alpha}{3aV_0}(\delta(x^5) - \delta(x^5-\pi\rho)) \\ \Rightarrow R &\sim \frac{8\sqrt{2}\alpha}{3abV_0}(\delta(x^5) - \delta(x^5-\pi\rho)) \end{aligned} \tag{5.45}$$

Putting this into the action yields an additional contribution to the potential:

$$\begin{aligned} V_{sing} &= -\tfrac{1}{2}\textstyle\int_0^{2\pi\rho} dx^5 a^2\sqrt{b}\frac{8\sqrt{2}\alpha}{3abV_0}(\delta(x^5) - \delta(x^5-\pi\rho)) = -\frac{4\sqrt{2}\alpha}{3R_0^2 V_0}(\frac{1}{H(0)} - \frac{1}{H(\pi\rho)}) \\ &= -\frac{4\sqrt{2}\alpha}{3R_0^2 V_0}\frac{\alpha\sqrt{2}R_0\pi\rho}{3V_0}\frac{1}{1-(\frac{\sqrt{2}\alpha R_0\pi\rho}{6V_0})^2} = -\frac{8\alpha^2\pi\rho}{9R_0 V_0^2}\frac{1}{1-(\frac{\sqrt{2}\alpha R_0\pi\rho}{6V_0})^2} \end{aligned} \tag{5.46}$$

The last contribution comes from the boundary potentials.

$$V_{bound} = \sqrt{2}\alpha(\tfrac{a^2}{V}(0) - \tfrac{a^2}{V}(\pi\rho)) =$$
$$\frac{\sqrt{2}\alpha}{R_0^2 V_0}(\frac{1}{H(0)} - \frac{1}{H(\pi\rho)}) = \frac{2\alpha^2\pi\rho}{3R_0V_0^2}\frac{1}{1-(\frac{\sqrt{2}\alpha R_0\pi\rho}{6V_0})^2} \tag{5.47}$$

Thus, we see that:

$$V_{4d} = V_{bulk} + V_{sing} + V_{bound} = 0 \tag{5.48}$$

and no tree level cosmological potential appears in the 4d effective lagrangian. The cancellation works as well even if there is no second brane and the fifth dimension is infinite. Such a situation is equivalent to neglecting $1/H(\pi\rho)$; various contributions cancel in the same way as previously.

Of course we cannot claim that the cosmological constant problem is solved as there is nothing to prevent the cosmological potential to appear at the one-loop level after supersymmmetry breaking (which must necessarily occur if the model is to describe the physical world). The analysis can be repeated for the case of more general bulk/boundary potential we considered before. The result is the same but the calculations are a little bit more tricky, because for the solution (5.4) we cannot do the integrations over x^5 explicitly.

Another point of view on this issue is given in [14], in which conditions for vanishing of the cosmological constant derived from the requirement of consistency of the Einstein's equations are discussed.

Next, we consider corrections to the 4d effective action coming from the non-trivial x^5 dependence of V. We can represent V as a sum:$V = V_{vac} + V_B$ where V_{vac} is the vacuum expectation value of V as given in (5.16), and V_B takes into account back-reaction of the boundary; it is given to first order in $\frac{\kappa^2}{g^2}$ in (5.18). There are three sources of corrections to the effective lagrangian, that contain no more than two space-time derivatives:

1. Integrating out V_B in the kinetic term of V;
2. Expanding the boundary term $e_5\frac{\sqrt{2}\alpha}{V}(\delta(x^5) - \delta(x^5 - \pi\rho))$ to the first order in V_B;
3. Substituting V with V_{vac} in the rest of the boundary Lagrangian.

The contribution from 1. is:

$$+\mathcal{L}_{EFF} = -\frac{1}{4\kappa^2}\int_0^{2\pi\rho} dx^5 e_5 g^{55}(\frac{\partial_5(V_{vac}+V_B)}{V_{vac}+V_B})^2 = -\frac{1}{4\kappa^2 R_0^3}\int_0^{2\pi\rho} dx^5\sqrt{\bar{g}}(\frac{3V_0H^2H'+V_B'}{V_0H^3+V_B})^2 =$$
$$-\frac{1}{2\kappa^2}\int_0^{\pi\rho} dx^5\sqrt{\bar{g}}[\frac{9(H')^2}{H^2}(1 - \frac{2}{V_0H^3V_B}) + \frac{6H'}{V_0H^4}V_B'] + (\sim V_B^2) \tag{5.49}$$

The zeroth order term contributes to the cosmological potential, which we calculated before. The term proportional to $V_B(H')^2$ without the fifth derivative is of order $(\alpha)^2\frac{\kappa^2}{g^2}$. Thus we are left with:

$$+\mathcal{L}_{EFF} = -\sqrt{\bar{g}}\frac{\sqrt{2}\alpha}{\kappa^2V_0^2R_0^2}\int_0^{\pi\rho} dx^5\frac{V_B'}{H^4} = -\sqrt{\bar{g}}\frac{\sqrt{2}\alpha}{\kappa^2V_0^2R_0^2}[V_B(\pi\rho) - V_B(0)] + \mathcal{O}(\alpha^2) \tag{5.50}$$

while the contribution from 2. is:

$$+\mathcal{L}_{EFF} = +\frac{\sqrt{2}\alpha}{\kappa^2V_0^2R_0^2}[V_B(\pi\rho) - V_B(0)] + \mathcal{O}(\alpha^2) \tag{5.51}$$

The contributions (5.49) and (5.50) cancel against each other, so in the first order in α the effective theory does not depend on the form of KK modes of V. If we wanted to go beyond the first order approximation, the contribution from the non-trivial gauge dependence of V_B would affect the effective theory.

The same situation occurs in the case of the contribution from the metric. The contribution originating from expansion of the curvature R cancels in order $(\alpha)^1$ against the contribution coming from expanding the the determinant in the boundary term $\frac{\alpha}{V}$.

Situation is different in the case of the σ field. The solution of its equation of motion is:

$$\sigma = \sigma_0 + 2\alpha\mathcal{A}_5(|x^5| - \frac{\pi\rho}{2}) + (gauge) \tag{5.52}$$

where $(gauge)$ denotes the gauge fields dependence, which is relevant only for higher derivative terms. Thus, the covariant derivative $D_5\sigma_B = \partial_5\sigma_B - 2\alpha\epsilon(x^5)\mathcal{A}_5$ vanishes, and we are left with the contribution from inserting the solution for σ in the boundary action, as well as from the D_μ part of the covariant derivative in the bulk action.

We have not determined yet those terms in the effective Lagrangian which result from integrating out the odd fields in the bulk. The equations of motion have been solved explicitly in subsection 5.3. First, let us consider ξ. This field occurs in our action as a 'perfect square':

$$S_\xi = -\int d^5x e_5 \frac{1}{V}\partial_\alpha\hat{\xi}\partial^\alpha\overline{\hat{\xi}} \tag{5.53}$$

Where the hat denotes the modified fifth derivative (4.39) which we repeat here:

$$\partial_5\hat{\xi} = \partial_5\xi + \frac{\kappa^2}{e_5^5 g^2}\delta(x^5)\frac{V^{3/2}}{2}(\overline{\chi_L}\chi_R) + \frac{2\kappa^2}{e_5^5 g^2}\bar{W}\delta(x^5) \tag{5.54}$$

(We have neglected terms in the equation (5.53) proportional to $\xi^2 D\sigma$ and ξ^4, as the yield only higher order corections).

Inserting the solution (7.2) and neglecting all 4d derivatives yields:

$$\begin{aligned} +\mathcal{L}_{EFF} &= -\tfrac{\sqrt{\bar{g}}}{\kappa^2 V_0 R_0^3}\textstyle\int_0^{2\pi\rho} dx^5 H^3|f|^2 \\ &= -\tfrac{1}{4}\sqrt{\bar{g}}\tfrac{\kappa_4^2 V_0^2}{g^4}| - \chi_1^2 H^3(0) - \chi_2^2 H^3(\pi\rho) + 4W(V_0R_0)^{-3/2}|^2 + \mathcal{O}(\alpha^2) \end{aligned} \tag{5.55}$$

Similarly, integrating out the kinetic term of the graviphoton yields:

$$+\mathcal{L}_{EFF} = -\frac{3}{2\kappa_4^2 R_0^2}\sqrt{\bar{g}}[\partial_\mu\mathcal{A}_5 + \frac{\kappa_4^2}{3\sqrt{2}g^2}(-\frac{3i}{4}V_0R_0(\chi_1^2{}_\mu H^2(0) - \chi_2^2{}_\mu H^2(\pi\rho)) - i(CD_\mu\bar{C} - \bar{C}D_\mu C) + \frac{i}{2}\zeta_\mu^2)]^2 \tag{5.56}$$

Note, that all the delta functions have cancelled out. Although the original 5d theory had singularities of the δ^2 type, the effective theory is perfectly well defined (supposedly, to all orders in α). The singularities have cancelled precisely, due to the perfect square structure of the odd fields.

Having identified possible sources of corrections to the effective theory, we can now interpret the resulting Lagrangian in terms of functions G, f defined sin subesection (2.3) , which unambigously describe 4d supergravity. We expect non-trivial corrections to the kinetic functions of gauge fields. As explained previously, the only contribution comes from inserting the background solution into the boundary lagrangian, which results in:

$$\begin{aligned} +\mathcal{L}_{EFF} &= -\frac{1}{4}\sqrt{\bar{g}}V_0[H^3(0)(F_{\mu\nu}F^{\mu\nu})^{(1)} - [H^3(\pi\rho)(F_{\mu\nu}F^{\mu\nu})^{(2)}] \\ &= -\frac{1}{4}\sqrt{\bar{g}}[(V_0 - \alpha\frac{\sqrt{2}}{2}\pi\rho R_0)(F_{\mu\nu}F^{\mu\nu})^{(1)} + (V_0 + \frac{\sqrt{2}}{2}\alpha\pi\rho R_0)(F_{\mu\nu}F^{\mu\nu})^{(2)}] + \mathcal{O}(\alpha^2) \end{aligned} \tag{5.57}$$

This implies the following modification of the gauge kinetic functions:

$$f_1 = S - \frac{\sqrt{2}}{2}\alpha\pi\rho T$$
$$f_2 = S + \frac{\sqrt{2}}{2}\alpha\pi\rho T \tag{5.58}$$

For consistency, the imaginary part of the moduli fields S,T should have axionic couplings. They are provided by inserting the solution for σ into the boundary axionic term:

$$+\mathcal{L}_{EFF} = -\frac{1}{4}\sqrt{\bar{g}}[(\sigma_0 + \sigma_B(0))F_{\mu\nu}\tilde{F}^{\mu\nu})^{(1)} + (\sigma_0 + \sigma_B(\pi\rho))F_{\mu\nu}\tilde{F}^{\mu\nu})^{(2)}]$$
$$= -\frac{1}{4}\sqrt{\bar{g}}[(\sigma_0 - \alpha\pi\rho\mathcal{A}_5)F_{\mu\nu}\tilde{F}^{\mu\nu})^{(1)} + (\sigma_0 - \alpha\pi\rho\mathcal{A}_5)F_{\mu\nu}\tilde{F}^{\mu\nu})^{(2)}] \tag{5.59}$$

exactly as required by (5.58) (Note $\sqrt{2}$ in the definiton of Im T).

According to the results of subsection (2.3) the gaugino kinetic term should be multiplied by $-\frac{1}{2}Ref$. Instead, in the effective theory we obtain:

$$+\mathcal{L}_{EFF} = -\frac{1}{2}\sqrt{\bar{g}}(R_0)^{-3/2}[(\overline{\chi}\not{D}\chi)^{(1)}H^{9/2}(0) + (\overline{\chi}\not{D}\chi)^{(2)}H^{9/2}(\pi\rho)] \tag{5.60}$$

The gauge kinetic functions (5.58) require H^3 instead of $H^{9/2}$, so to have gauginos correctly normalized, the following rescaling has to be performed:

$$\chi^{(1)} \to \left(\frac{H(0)}{R_0}\right)^{-3/4}\chi^{(1)}$$
$$\chi^{(2)} \to \left(\frac{H(\pi\rho)}{R_0}\right)^{-3/4}\chi^{(2)} \tag{5.61}$$

Likewise, the correct normalization of the Noether term, as well as of the coupling of the gaugino to the hyperino, requires the rescaling:

$$\psi_\mu \to \left(\frac{H}{R_0}\right)^{1/4}\psi_\mu$$
$$\lambda \to \left(\frac{H}{R_0}\right)^{-1/4}\lambda \tag{5.62}$$

Another modification due to the αx^5 dependence of the background solution appears in the kinetic term of matter field C. Inserting the background (5.16) into the corresponing boundary term yields:

$$+\mathcal{L}_{EFF} = -\sqrt{\bar{g}}\frac{H(0)}{R_0}D_\mu C D^\mu \bar{C} = \sqrt{\bar{g}}(-\frac{1}{R_0} + \alpha\frac{\sqrt{2}}{6V_0}\pi\rho)D_\mu C D^\mu \bar{C} \tag{5.63}$$

Thus we have to modify both the Kähler potential and the definition of the modulus S:

$$ln(S+\bar{S}) \to ln(S + \bar{S} + \frac{\kappa_4^2}{3g^2}\alpha\sqrt{2}\pi\rho C\bar{C})$$
$$ReS \to V_0 - \frac{\kappa_4^2}{6g^2}C\alpha\sqrt{2}\pi\rho C\bar{C} \tag{5.64}$$

Similarly to the case of other fermions, bringing the coefficient multiplying the kinetic term of ζ (superpartner of C) to the correct form given in subsection 2.3 requires the rescaling:

$$\zeta \rightarrow \left(\frac{H(0)}{R(0)}\right)^{-1/4} \zeta \tag{5.65}$$

Finally, according to subsection 2.3 a term: $-\frac{1}{4} f_{,i} (\Lambda^i \gamma^{\mu\nu} \chi) F_{\mu\nu}$ should appear. In our boundary Lagrangian we find instead:

$$-\delta(x^5) e_5 \frac{1}{4} (\lambda \gamma^{\mu\nu} \chi) F_{\mu\nu} \tag{5.66}$$

and no other terms of this form appear in the course of the compactification. Therofore, we must define the superpartner of S as:

$$\Lambda^S = < \lambda - \alpha\sqrt{2}(|x^5| - \pi\rho)\psi_5 > \tag{5.67}$$

The same result can be obtained by solving the equation of motion for λ^a and identyfying the S fermion with the zero mode of this solution.

The Kähler potential given by the first order solution (5.33) contained the superpotenial W, and the formula for G was derived from the 2-fermi terms in the effective lagrangian. The $|W|^2$ term appears in the effective action (with the correct coefficient) after integrating out the ξ field. Generally, integrating out the odd fields provides us with higher order terms required by the Kähler potential derived from the first order reduction (another example of this kind are quartic gaugino terms).

To summarize, we collect below the results obtained in this section for the Kähler potential G, the gauge kinetic function f, and the definitions of the moduli fields.

$$G = ln(S + \bar{S} + \frac{\kappa_4^2}{3g^2}\alpha\sqrt{2}\pi\rho C\bar{C}) + 3ln(T + \bar{T} - \frac{2}{3}\frac{\kappa_4^2}{g^2}C\bar{C}) - ln(64W\bar{W}) \tag{5.68}$$

$$f_1 = S - \frac{\sqrt{2}}{2}\alpha\pi\rho T \quad f_2 = S + \frac{\sqrt{2}}{2}\alpha\pi\rho T \tag{5.69}$$

$$\begin{array}{cc}
S = V_0 - \frac{\kappa_4^2}{6g^2}\alpha\sqrt{2}\pi\rho C\bar{C} + i\sigma & \Lambda^S = < \left(\frac{H}{R_0}\right)^{1/4} (\lambda - \alpha\sqrt{2}(|x^5| - \pi\rho)\psi_5) > \\
T = R_0 - \frac{\kappa^2}{3g^2}C\bar{C} + i\sqrt{2}\mathcal{A}_5 & \Lambda^T = < \left(\frac{H}{R_0}\right)^{1/4} [\psi_5 + \frac{2\kappa_4^2}{3g^2}(\bar{C}\zeta_L + C\zeta_R)] > \\
C & \Lambda^C = \left(\frac{H(0)}{R_0}\right)^{1/4} \zeta \\
A_\mu^{(1)} & \tilde{\chi}^{(1)} = \left(\frac{H(0)}{R_0}\right)^{3/4} \chi^{(1)} \\
A_\mu^{(2)} & \tilde{\chi}^{(2)} = \left(\frac{H(\pi\rho)}{R_0}\right)^{3/4} \chi^{(2)}
\end{array} \tag{5.70}$$

Chapter 6

Reduction of the supersymmetry transformation laws

In the preceding sections we have determined the form of the 4d effective theory by direct compactification of the 5d lagrangian. Although the functions G and f are sufficient to reconstruct the rest of the supergravity lagrangian, an interesting consistency check would be to obtain explicitly the complete 4d lagrangian by integrating out the fifth dimension. This is rather difficult as, e.g., the 4-fermi terms receive contributions that are of higher order in α. Another approach is to reduce 5d supersymmetry transformation laws to 4d, and check if they are consistent with the results (5.68). This has the advantage that corrections from the non-trivial α dependence of the functions G and f can be seen at lower order in the expansion in α and κ^2. As an example we present how to determine the gauge kinetic functions from the transformation laws of the superpartners of the moduli scalars.

First, we need to determine what is the 4d parameter of supersymmetry in terms of its 5d counterpart. To this end, we need to solve the Killing eqution for 5d spinor which is just the condition $\delta\psi_5^A = 0$ in (5.3). This condition can be easily solved with the general potential:

$$\begin{aligned}\epsilon_R^1 &= e^{-\frac{\Lambda R_0|x^5|}{12}}\left(1+\alpha\sqrt{2}\frac{R_0}{V_0}(|x^5|-\frac{\pi\rho}{2})\right)^{1/12} a_0^{-1/4}\eta_R \\ \epsilon_L^2 &= e^{-\frac{\Lambda R_0|x^5|}{12}}\left(1+\alpha\sqrt{2}\frac{R_0}{V_0}(|x^5|-\frac{\pi\rho}{2})\right)^{1/12} a_0^{-1/4}\eta_L \end{aligned} \quad (6.1)$$

Since $\epsilon_L^2 = -i\sigma^2\epsilon_R^{1*}$ (the 5d Majorana condition) the spinor η is Majorana in the 4d sense. The appearance of the factor a_0 in (6.1) yields canonical form of the reduced 4d supersymmetry transformation law of the gravity multiplet. Then η depends only on x^μ and has the interpretation of the parameter of supersymmetry transformations in the 4d theory.

We again put $\Lambda = 0$ and choose the coordinate frame $g_{55} = R_0^2 H^4$. The Killing spinor is then $\epsilon_R^1 = \frac{H}{R_0}^{1/4}\eta_R$. First we determine gaugino bilinears in the transformation law of the modulus T superpartner Λ^T. As in (5.68) it is defined as $\Lambda^T =< (\frac{H}{R_0})^{1/4}[\psi_5 + \frac{2\kappa_4^2}{3g^2}(\bar{C}\zeta_L + C\zeta_R)] >$. The $C\zeta$ part is unimportant as there are no gauginos in the transformation law of the C superpartner. The relevant part in the transformation law of ψ_5 reads:

$$\delta\psi_{5L} = \frac{1}{\sqrt{V}}\partial_5\xi\epsilon_L \quad (6.2)$$

Note that here $\partial_5\xi$ does not appear as a perfect square. Inserting in this expression the solution for the Killing spinor as well as the solution for ξ and other bulk fields yields:

$$\delta\psi_{5L} = \frac{\kappa^2}{g^2}\left(\frac{H}{R_0}\right)^{-1/4}\frac{V_0R_0}{2}(-H^2(0)(\chi_1)^2\delta(x^5)-H^2(\pi\rho)(\chi_2)^2\delta(x^5-\pi\rho))\eta_L-\frac{1}{\sqrt{V_0R_0}}H^2\left(\frac{H}{R_0}\right)^{-1/4}f\eta_L \tag{6.3}$$

where f is defined as:

$$\frac{f}{4\alpha'} = \frac{\kappa^2}{4g^2}(V_0R_0)^{3/2}\frac{-\chi_1^2H^3(0)-\chi_2^2H^3(\pi\rho)}{H^4(0)-H^4(\pi\rho)} \tag{6.4}$$

Thus the transformation law of Λ^T is:

$$\begin{aligned}\delta\Lambda^T &= \tfrac{1}{2\pi\rho}\textstyle\int_0^{2\pi\rho}\left(\tfrac{H}{R_0}\right)^{1/4}\delta\psi_5\\ &= \tfrac{1}{2\pi\rho}\tfrac{2}{3\alpha'\sqrt{V_0R_0}}(H^3(\pi\rho)-H^3(0))f\eta_L-\tfrac{1}{2\pi\rho}\tfrac{\kappa^2}{g^2}\tfrac{V_0R_0}{2}(H^2(0)(\chi_1)^2+H^2(\pi\rho)(\chi_2)^2)\eta_L\\ &= \tfrac{2\kappa_4^2}{3g^2}V_0R_0(-\chi_1^2H^3(0)-\chi_2^2H^3(\pi\rho))\tfrac{H^3(\pi\rho)-H^3(0)}{H^4(0)-H^4(\pi\rho)}\eta_L\\ &\quad-\tfrac{\kappa_4^2}{g^2}\tfrac{V_0R_0}{2}(H^2(0)(\chi_1)^2+H^2(\pi\rho)(\chi_2)^2)\eta_L\end{aligned} \tag{6.5}$$

The second part of the above expression would be absent if $\delta\psi_5$ respected the perfect square structure. The gaugino bilinears would the enter in the zeroth order in α violating the canonical form of the supersymmetry transformation law. Instead, after expanding in α the result to the first order is:

$$\delta\Lambda_L^T = -\frac{\kappa_4^2}{12g^2}R_0^2\alpha\sqrt{2}\pi\rho\left(\chi_1^2-\chi_2^2\right)\eta_L\,. \tag{6.6}$$

Similarly, we can calculate the supersymmetry transformation law of the superpartner of the modulus S:

$$\delta\Lambda_L^S = \frac{\kappa_4^2}{2g^2}V_0^2\left(\chi_1^2+\chi_2^2\right)\eta_L \tag{6.7}$$

Recalling form subsection (2.3) that in 4d supergravity, scalar gaugino condensates in the transformation law of the fermions Λ^S,Λ^T are multiplied by $\frac{1}{8}f_{,S}\,(G^{-1})^S_S$ and $\frac{1}{8}f_{,T}\,(G^{-1})^T_T$, respectively, the result (6.6,6.7) indeed agrees with (5.68). We stress that the agreement is due to the perfect square structure in $\delta\lambda$ and the lack thereof in $\delta\psi_5$. Thus, when we calculate $\delta\Lambda^T$ the linear part of the solution for $\partial_5\xi$ cancels to zeroth order in α with the delta functions occuring in this solution, leading to the correct form of $f_{,T}$. Note also, that the admixture of ψ_5 in the definition of Λ^S is crucial to obtain the correct form of $f_{,S}$.

From the transformation laws (6.6,6.7) it can be read off that presence of gaugino condensates breaks supersymmetry in the 4d effective theory. Although one can adjust $\chi_1^2=-\chi_2^2$ so that the condesates cancel in the regular part of the solution (7.2) for $\partial_5\xi$ and in consequence in $\delta\Lambda^S$, but then the non-zero condensate contribution appears in $\delta\Lambda^T$ due to the above mentioned lack of the perfect square structure in $\delta\psi_5$. However, if we allow for boundary scalar fields, appropriately adjusting their superpotentials we have the possibility to cancel the contribution of the condensates.

Chapter 7

Gaugino condensation and supersymmetry breaking in five dimensions

Similarly to models derived from the heterotic string theory, in the theory formulated in section 4 there is the possibility to break supersymmetry by gaugino condensation on the hidden and/or visible branes. The supersymmetry breaking is communicated from one brane to another by the expectation value of the hypermultiplet field ξ. This mechanism works because ξ, although odd, couples to gauginos on the boundaries through its fifth derivative (a toy-model of this kind is studied in [21]). The equation of motion for the ξ field in the presence of the gaugino condensates on the branes is:

$$\frac{1}{\kappa^2}\partial_5(\frac{e_5 g^{55}}{V}\partial_5\xi) = \partial_5\left(-\frac{e_4\sqrt{V}}{2g^2 e_5^5}(\delta(x^5)(\overline{\chi}_L\chi_R)_1 + \delta(x^5-\pi\rho)(\overline{\chi}_L\chi_R)_2)\right). \tag{7.1}$$

We are interested in the solution for $\partial_5\xi$ (and not for ξ alone) because it is just this expression which enters the relevant formulae. For $\Lambda = 0$ we obtain the solution:

$$\frac{\partial^5\xi}{H^{-3}}\frac{\kappa^2}{2g^2}(V_0R_0)^{3/2}\left(-\delta(x^5)\chi_1^2 - \delta(x^5-\pi\rho)\chi_2^2\right) + C \tag{7.2}$$

$$C = \frac{\kappa^2}{3g^2}\alpha\sqrt{2}\pi\rho(V_0R_0)^{3/2}\frac{-\chi_1^2H^3(0) - \chi_2^2H^3(\pi\rho)}{H^4(0) - H^4(\pi\rho)} \tag{7.3}$$

It is worth noting, that in the 5d theory gaugino condensates break the supersymmetry In the presence of the condensates we have no way to satisfy simultanously $\delta\psi_\mu^A = 0$ with any other of the remaining conditions for unbroken supersymmetry. Indeed, neither $\partial_5\xi$ nor the condensates do not alter the transformation law of ψ_μ, so in particular, the conditions resulting from $\delta\psi_\mu^A = 0$ include the chirality conditions (5.5). But in such a case, the condensates in the formulae for $\delta\lambda^a$ and $\delta\psi_5^A$ multiply the supersymmetry parameter ϵ, which is of the chirality opposite to other ϵ's occuring in these transformation laws. Thus, conditions $\delta\psi_5^A = 0$ and $\delta\lambda^a = 0$ cannot be satisfied.

Chapter 8

Conclusions

Let us summarize the content of this thesis. Having prepared the necessary background in sections 2 and 3, in section 4 we presented the five dimensional construction analogous to the Horava-Witten construction in eleven dimensions. Precisely, we derived a locally supersymmetric lagrangian which consists of two sectors: the 5d bulk supergravity coupled to one 'universal' hypermultiplet, the 4d chiral matter and the YM fields on the brane. This thesis does not describe general compactification of the Horava-Witten model on Calabi-Yau threefolds. Realization of such programm would require considering an arbitrary number of hyper- and vector multiplets in the bulk. Instead, we concentrated on some general features of 5d locally supersymmetric theories containing chiral matter confined to 3-branes. The class of potentials we considered was wider than those obtained in the compactification of the Horawa-Witten model. We show that gauge and matter fields residing on the brane can be supersymmetrized by modifying the brane action only but one has to modify the supersymmetry transformation laws of both brane and bulk fields. On the other hand we show that the boundary potential terms for bulk scalars can be reconciled with supersymmetry by modifying the bulk action and the supersymmetry transformation laws of the bulk fields. The coupling of the 4d Yang-Mills and matter fields to the bulk fields does not depend neither on the boundary nor on the bulk potentials. In particular, the 'visible' brane action would have the same form in the supersymmetric version of the Randall-Sundrum scenario. In the original RS model, the interactions of the bulk with the brane fields yield specific experimental signatures [26] which may be seen at the Tevatron and LHC. In the supersymmetric version of this model the phenomenological consequences may be even richer, as e.g. the gravitino and its massisve KK modes interact with the SM fields. The action obtained in this thesis can be the starting point for phenomenology in the framework of the supersymmetric RS model or modifications thereof.

In section 5 the supersymmetry preserving compactification to four dimensions of the Horava-Witten model is studied. Generally, our results confirm the conclusions of reference [6], where the effective theory was obtained by direct compactification from eleven dimensions down to four. We analyze contributions to the effective action and interpret them in terms of the canonical form of 4d supergravity as given in [17]. For example, we study the cancellation of various contributions to the 4d tree-level cosmological constant; as advocated, its vanishing is necessary for the consistency of the compactification. In addition, we express the canonically normalized fermion fields of the 4d theory in terms of their 5d counterparts. In section 6 we point out that the effective theory can also be consistently deduced from the reduction of the 5d supersymmetry transformation laws.

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Appendix A

This appendix contains the full Langrangian of five-dimensional N=2 gauged supergravity on $M_4 \times S_1/\mathbf{Z}_2$ coupled to non-linear sigma model $SU(2,1)/U(2)$ and to YM multiplets (A^a_μ, χ^a) on two parallel branes placed at $x_5 = 0$ and at $x_5 = \pi\rho$. Matter multiplet (C, ζ) on the "visible" brane, transforming under gauge group, is included. This Langrangian contains the following parts:

$$S = \int d^5 x e_5 [\mathcal{L}_{BULK} + \mathcal{L}_{YM1}\delta(x^5) + \mathcal{L}_{YM2}\delta(x^5 - \pi\rho)$$
$$+\mathcal{L}_{H1}\delta(x^5) + \mathcal{L}_{H2}\delta(x^5 - \pi\rho) + \mathcal{L}_{27}\delta(x^5) + \mathcal{L}_W\delta(x^5) + \mathcal{L}_\alpha\delta(x^5) - \mathcal{L}_\alpha\delta(x^5 - \pi\rho)] \quad \text{(A.1)}$$

$\mathcal{L}_{BULK}$ is given by eq. 4.1. $\mathcal{L}_{YM}$ and $\mathcal{L}_H$ part contains gauge fields living on the brane. Of course, gauge fields should have an appropriate index corresponding to its location, e.g. $A^{a\,(1)}_\mu$ for the part of the action multiplied by $\delta(x^5)$. Only the even part of bulk fermions appears here; ψ_μ, λ is defined in terms of five-dimensional symplectic Majorana spinors ψ^A_μ, λ^a in (4.5).

$$\begin{aligned}
g^2\mathcal{L}_{YM} = \;& -\tfrac{V}{4}F^a_{\mu\nu}F^{a\mu\nu} - \tfrac{V}{2}\overline{\chi^a}\not{D}\chi^a + \tfrac{V}{4}(\overline{\psi}_\mu\gamma^{\nu\rho}\gamma^\mu\chi^a)F^a_{\nu\rho} + \tfrac{3i}{4\sqrt{2}}\tfrac{V}{e_5^5}(\overline{\chi}^a\gamma^5\gamma^\mu\chi^a)\mathcal{F}_{\mu 5} \\
& +\tfrac{V}{32}(\overline{\psi}_\mu\gamma^\mu\gamma^\nu\psi_\nu)(\overline{\chi^a}\chi^a) + \tfrac{V}{32}(\overline{\psi}_\mu\gamma^\mu\gamma^5\gamma^\nu\psi_\nu)(\overline{\chi^a}\gamma^5\chi^a) - \tfrac{V}{8}(\overline{\psi}_\mu\psi^\mu)(\overline{\chi^a}\chi^a) \\
& +\tfrac{V}{8}(\overline{\psi}_\mu\gamma^5\psi^\mu)(\overline{\chi^a}\gamma^5\chi^a) + \tfrac{V}{32}(\overline{\psi}_\mu\gamma^5\gamma_\rho\psi^\mu)(\overline{\chi^a}\gamma^5\gamma^\rho\chi^a) \\
& -\tfrac{V}{16}(\overline{\psi}_\mu\gamma^\mu\gamma^5\psi_\rho)(\overline{\chi^a}\gamma^5\gamma^\rho\chi^a) + \tfrac{3}{16}\tfrac{1}{e_5^5}(\overline{\psi}_\mu\gamma^5\psi_5)(\overline{\chi^a}\gamma^5\gamma^\mu\chi^a) \\
& +\tfrac{3\kappa^2}{64g^2}\delta(x^5)V^2(\overline{\chi^a}\gamma^5\gamma^\rho\chi^a)(\overline{\chi^b}\gamma^5\gamma_\rho\chi^b)
\end{aligned} \quad \text{(A.2)}$$

$$\begin{aligned}
g^2\mathcal{L}_H = \;& -\tfrac{1}{4}\sigma F^a_{\mu\nu}\tilde{F}^{a\mu\nu} - \tfrac{1}{4}(\overline{\lambda}\gamma^{\nu\rho}\chi^a)F^a_{\nu\rho} - \tfrac{i}{8}(\overline{\chi}^a\gamma^5\gamma^\mu\chi^a)\partial_\mu\sigma - \tfrac{\sqrt{V}}{2e_5^5}[(\overline{\chi^a}_L\chi^a_R)\partial_5\overline{\xi} + (\overline{\chi^a}_R\chi^a_L)\partial_5\xi] \\
& -\tfrac{1}{4}(\overline{\lambda}\gamma^{\nu\rho}\chi^a)(\overline{\psi}_\nu\gamma_\rho\chi^a) - \tfrac{1}{8}(\overline{\psi}_\mu\gamma^\mu\lambda)(\overline{\chi^a}\chi^a) \\
& -\tfrac{1}{8}(\overline{\psi}_\mu\gamma^\mu\gamma^5\lambda)(\overline{\chi^a}\gamma^5\chi^a) - \tfrac{i}{8e_5^5}(\overline{\psi}_5\gamma^5\lambda)(\overline{\chi^a}\chi^a) - \tfrac{i}{8e_5^5}(\overline{\psi}_5\lambda)(\overline{\chi^a}\gamma^5\chi^a) \\
& +\tfrac{1}{64V}(\overline{\chi^a}\gamma^5\gamma^\rho\chi^a)(\overline{\lambda}\gamma^5\gamma_\rho\lambda) + \tfrac{3}{64V}(\overline{\chi^a}\gamma^5\chi^a)(\overline{\lambda}\gamma^5\lambda) + \tfrac{3}{64V}(\overline{\chi^a}\chi^a)(\overline{\lambda}\lambda) \\
& -\tfrac{\kappa^2}{16g^2}\delta(x^5)V^2[(\overline{\chi^a}\chi^a)(\overline{\chi^b}\chi^b) - (\overline{\chi^a}\gamma^5\chi^a)(\overline{\chi^b}\gamma^5\chi^b)]
\end{aligned} \quad \text{(A.3)}$$

If we include matter on the visible brane, we have to add following couplings:

$$\begin{aligned}
g^2\mathcal{L}_S = \;& -D_\mu C^p D^\mu\overline{C_p} - \overline{\zeta}\not{D}\zeta \\
& +(\overline{\psi}_{R\mu}\not{D}\overline{C_p}\gamma^\mu\zeta^p_L + h.c.) + \tfrac{i\mathcal{F}_{\mu 5}}{\sqrt{2}e_5^5}(C^pD^\mu\overline{C_p} - \overline{C_p}D^\mu C^p) - \tfrac{i\mathcal{F}_{\mu 5}}{2\sqrt{2}e_5^5}(\overline{\zeta}\gamma^5\gamma^\mu\zeta) \\
& -\tfrac{i}{4V}\partial_\mu\sigma(\overline{\zeta}\gamma^5\gamma^\mu\zeta) + \tfrac{1}{e_5^5}(\overline{\psi}_\mu\psi_5)[-\tfrac{i}{8}(\overline{\zeta}\gamma^5\gamma^\mu\zeta) + \tfrac{i}{4}(C^pD^\mu\overline{C_p} - \overline{C_p}D^\mu C^p)] \\
& +(\overline{\psi}_\mu\gamma^{\mu\rho\nu 5}\psi_\rho)[-\tfrac{1}{16}(\overline{\zeta}\gamma^5\gamma_\nu\zeta) - C^pD_\nu\overline{C_p} - \overline{C_p}D_\nu C^p)] - \tfrac{1}{8}(\overline{\psi}_\mu\gamma^5\gamma^\rho\psi^\mu)(\overline{\zeta}\gamma^5\gamma_\rho\zeta)
\end{aligned}$$

$$+\tfrac{1}{16V^2}(\overline{\lambda}\gamma^5\gamma_\mu\lambda)[-\tfrac{1}{2}(\overline{\zeta}\gamma^5\gamma^\mu\zeta)-(C^pD^\mu\overline{C_p}-\overline{C_p}D^\mu C^p)]-\tfrac{1}{2V}(\overline{C}T^aC)(\overline{C}T^aC)$$
$$+\tfrac{1}{2}(\overline{\psi}_\mu\gamma^\mu\chi^a)(\overline{C}T^aC)+(2i\overline{\chi^a_R}\overline{C}T^a\zeta_L)+h.c.)-\tfrac{i}{2V}(\overline{\lambda}\chi^a)(\overline{C}T^aC)$$
$$-\tfrac{\kappa^2}{g^2}\delta(x^5)[-\tfrac{1}{16}(\overline{\zeta}\gamma^5\gamma^\rho\zeta)(\overline{\chi^a}\gamma^5\gamma_\rho\chi^a)+\tfrac{1}{24}(\overline{\zeta}\gamma^5\gamma^\rho\zeta)(\overline{\zeta}\gamma^5\gamma_\rho\zeta)$$
$$+\tfrac{1}{24}(C^pD_\mu\overline{C_p}-\overline{C_p}D_\mu C^p))(C^pD^\mu\overline{C_p}-\overline{C_p}D^\mu C^p))-\tfrac{1}{12}(\overline{\zeta}\gamma^5\gamma^\mu\zeta)(C^pD_\mu\overline{C_p}-\overline{C_p}D_\mu C^p)] \quad (A.4)$$
$$(A.5)$$

$$g^2\mathcal{L}_W=-\tfrac{2}{V}\tfrac{\partial W}{\partial C^p}\tfrac{\partial\overline{W}}{\partial\overline{C_p}}-\tfrac{2}{\sqrt{V}}\tfrac{\partial^2W}{\partial C^p\partial C^q}(\overline{\zeta}^p_R\zeta^q_L)+\tfrac{2}{\sqrt{V}}\tfrac{\partial W}{\partial C^p}(\overline{\psi}_{L\mu}\gamma^\mu\zeta^p_L)$$
$$+\tfrac{1}{\sqrt{V}}W(\overline{\psi}_{L\mu}\gamma^{\mu\nu}\psi_{R\nu})+\tfrac{2}{V^{3/2}}\tfrac{\partial W}{\partial C^p}(\overline{\lambda}_R\zeta^p_L)-\tfrac{1}{V^{3/2}}W(\overline{\psi}_{L\mu}\gamma^\mu\lambda_L)$$
$$-\tfrac{2}{Ve_5^5}W\partial_5\xi+\tfrac{i}{V^{3/2}e_5^5}W(\overline{\psi}_{R5}\lambda_L)$$
$$+\tfrac{\kappa^2}{g^2}\delta(x^5)[-\tfrac{4}{V}\overline{W}W-\sqrt{V}\overline{W}(\overline{\chi^a_R}\chi^a_L)]+h.c. \quad (A.6)$$

The boundary cosmological term must appear if five-dimensional supergravity is gauged:

$$\kappa^2\mathcal{L}_\alpha=\sqrt{2}\frac{\alpha}{V} \quad (A.7)$$

The supersymmetry transformation laws of the YM and matter multiplets are:

$$\begin{aligned}
\delta A^a_\mu &= -\frac{1}{2}(\overline{\eta}\gamma_\mu\chi^a)\\
\delta\chi^a &= \frac{1}{4}\gamma^{\mu\nu}\eta\,[F^a_{\mu\nu}+(\overline{\psi}_\mu\gamma_\nu\chi^a)]+\frac{1}{8V}\gamma^5\chi^a\,(\overline{\eta}\gamma^5\lambda)+\frac{1}{4V}\gamma^5\eta\,(\overline{\chi^a}\gamma^5\lambda)-\frac{i}{2V}\eta\,(\overline{C}T^aC)\\
\delta C^p &= (\overline{\eta}_R\zeta^p_L)\\
\delta\zeta^p_L &= \frac{1}{2}[D_\rho C^p-(\overline{\psi}_{R\rho}\zeta^p_L)]\,\gamma^\rho\eta_R+\frac{1}{8V}\zeta^p_L\,(\overline{\eta}\gamma^5\lambda)-\frac{1}{\sqrt{V}}\frac{\partial\overline{W}}{\partial\overline{C_p}}\eta_L
\end{aligned} \quad (A.8)$$

One has to modify supersymmetry transformation laws of the even combinations of the bulk fermions:

$$\frac{\kappa^2}{g^2}\delta\psi_\mu=\mathcal{M}_{\mu\,YM1}\delta(x^5)+\mathcal{M}_{\mu\,YM2}\delta(x^5-\pi\rho)+\mathcal{M}_{\mu\,27}\delta(x^5) \quad (A.9)$$

$$\begin{aligned}
\mathcal{M}^\mu_{YM} &= \frac{V}{8}(g^{\mu\rho}-\frac{1}{2}\gamma^{\mu\rho})\gamma^5\eta\,(\overline{\chi^a}\gamma^5\gamma_\rho\chi^a)\\
\mathcal{M}^\mu_{27} &= (g^{\mu\rho}-\frac{1}{2}\gamma^{\mu\rho})\gamma^5\eta\,[\frac{1}{6}(C^pD_\rho\overline{C_p}-\overline{C_p}D_\rho C^p)-\frac{1}{12}(\overline{\zeta}\gamma^5\gamma_\rho\zeta)]
\end{aligned} \quad (A.10)$$

$$\frac{\kappa^2}{g^2}\delta\psi_5=+\mathcal{N}_{27}\delta(x^5)+\mathcal{N}_W\delta(x^5) \quad (A.11)$$

$$\begin{aligned}
\mathcal{N}_{27} &= e_5^5\gamma^\rho\eta\,[\frac{i}{6}(C^pD_\rho\overline{C_p}-\overline{C_p}D_\rho C^p))+\frac{i}{6}(\overline{\zeta}\gamma^5\gamma_\rho\zeta)]\\
\mathcal{N}_W &= \frac{2ie_5^5}{\sqrt{V}}(W\eta_L-\overline{W}\eta_R)
\end{aligned} \quad (A.12)$$

$$\frac{\kappa^2}{g^2}\delta\lambda=\mathcal{P}_{YM1}\delta(x^5)+\mathcal{P}_{YM2}\delta(x^5-\pi\rho)+\mathcal{P}_W\delta(x^5) \quad (A.13)$$

$$\begin{aligned}
\mathcal{P}_{YM} &= \frac{V^2}{4}[\eta(\overline{\chi}^a\chi^a) - \gamma^5\eta(\overline{\chi}^a\gamma^5\chi^a)] \\
\mathcal{P}_{W} &= -2\sqrt{V}(W\eta_L + \overline{W}\eta_R)
\end{aligned} \tag{A.14}$$

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