Corrections to *Categories, Allegories*

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Aside from some spelling, punctuation and font errors please note the following corrections:

• **p12 [1.262] Last sentence of 1st paragraph:** Switch x and y:

For any $y \in Y$, $\{\ x \in X\ \mid\ fx = y\ \}$ is a discrete subset of X.

• **p46 [1.437] 3rd line down:** Reference should be [1.432] (not [1.423]).

• **p50 [1.461] 1st line:** The category composed of local homeomorphisms [1.262] is not cartesian; it does not have a terminator. It does have pullbacks. See *Amplifications* for p50.

• **p50 [1.462] 2nd line down:** References should be [1.422, 1.424. 1.428] (not [1.429]).

• **p52 [1.475] last sentence** is false: Any category of the form $1\backslash\mathbf{A}$ is strongly connected and it is an easy exercise that any strongly connected cartesian category is a one-valued special cartesian category (the identity functor is a retract of every functor of the form $A\times -$). For a one-valued non-special cartesian category see *Amplifications* for p52.

• **p55 [1.493] 4th line down** The rightmost comma should be a $<$. It should read:

define τ to be the class of tables $\langle T; f_1, f_2 \ldots f_n\rangle$ such that for $x < y \in T$ and

• **p69 [1.512] top sentence:** If the sentence were in italics—as it should be—we might have noticed that it needed a proof. And then we might have noticed that it needs a caveat. Covers need not be closed under composition if there aren't enough pullbacks. See *Amplifications* for p69 for a proof and a counterexample.

• **p104 [1.632] last line:** Reference should be [1.453] (not [1.442]).

• **p104 [1.633] last line before bottom diagram:** Reference should be [1.624] (not [1.631]).

• **p120 [1.725] 5th line down (3rd equation) should be:**

$$z \wedge (x \leftrightarrow y)\ =\ z \wedge [(z \wedge x) \leftrightarrow (z \wedge y)].$$

• **p139 [1.814] 3rd paragraph:** $\mathbf{A}$ should be the category of distributive lattices and $\mathbf{A}'$ the category of locales.

• **p143 [1.82(10)] 13th line down (1st line in italics)** should be:

A functor that preserves pre-limits preserves limits.

• **p167 [1.935] bottom line:** Insert the word "small"

Every small topos may be faithfully represented in a capital topos.

• **p172 [1.947]: 4th line down:** Formula should read $R\left(\bigcap F\right) \subset \bigcap F$

• **p192 [1.(10)31] 1st line down:** Insert the word "exact"

An exact retract of an exacting category is exacting.

• **p196 [2.11] bottom half:** Please note that we begin by saying that an allegory is a category. All equations of 1.1 are to be considered part of the definition of allegory.

• **p207–210 [2.158 small print]** (Thanks to Roger Maddux!): The sentence about graphs on p209, 8th line down, is false: "If one identifies any one or any two of the pairs of vertices, the resulting graph is not in $\overline{\mathbf{G}}$." The trouble is that if the vertices labeled s and t are identified the result is in $\overline{\mathbf{G}}$. Worse, the displayed formula *is* a consequence of the allegory equations (using distributivity and 2.124):

$$1 \cap \big((R_1 \cap R_4^\circ)(R_2 \cap R_3^\circ)(S_2 \cap S_3^\circ)(S_1 \cap S_4) \big) \subset 1 \cap \big((R_1R_2 \cap R_4^\circ R_3^\circ)(S_2^\circ S_1^\circ \cap S_3S_4) \big) =$$
$$\mathrm{Dom}\big((R_1R_2 \cap R_1^\circ R_3^\circ) \cap (S_2^\circ S_1^\circ \cap S_3S_4)^\circ \big) = \mathrm{Dom}\big((R_1R_2 \cap S_1S_2) \cap (R_3R_4 \cap S_3S_4)^\circ \big) =$$
$$1 \cap \big((R_1R_2 \cap S_1S_2)(R_3R_4 \cap S_3S_4) \big)$$

The subscripts in the complicated formula in the middle of p210 are remarkably wrong. They should be:

$$1 \cap (R_0 \cap R_{2n-1}^\circ) \left[\bigcap_{i=1}^{n-1} (R_{2i-1} \cap R_{2i}^\circ)(S_{2i-1}^\circ \cap S_{2i}) \right] (S_0^\circ \cap S_{2n-1}) \subset \prod_{i=0}^{n-1} (R_{2i}R_{2i+1} \cap S_{2i}S_{2i+1})$$

The argument that these formulas are not consequences of the allegory equations is OK for $n > 2$.

• **p217 [2.214] 5th line up:** The equation $u_2^\circ u_1 = 0$ should be $u_2 u_1^\circ = 0$ (which, it should be noticed, makes it equivalent to the previous equation).

• **p234 [2.357] 9th line down:** Of the two equations on this line the 1st is, of course, just a restatement of the definition of domain of simplicity. The 2nd, however, should be referenced with [2.124].

• **p236 [2.412] 12th line up:** The reference [2.357] is wrong. It should be [2.124].

• **p238 [2.418 small print]:** The last appearance of $[X]/E$ should be X/I.

• **p250 [2.446] 3rd paragraph:** the equal sign should be a containment sign.

$$R \subset R^\circ \text{ and } S \subset S^\circ \text{ imply } R \cup S \subset R^\circ \cup S^\circ.$$

Three lines below a missing virgule:

$$R \cup R \subset (\ni/R)\backslash\ni \subset ((R/\ni)(\ni/R))((\ni/R)\backslash\ni) \subset (R/\ni)\ni \subset R$$

• **p272 [B.211] last sentence:** There are two rules for existential quantification and the 2nd is needed for equality. The best correction seems to be simply to remove this last sentence (and, of course, the index entry for Horn logic).

• **p274 [B.229] bottom:** The rules for the commutativity and idempotence of existential quantifiers are given. The same rules for universal quantifiers should, of course, also be given.

• **p275–6 [B.3]:** The definition of DERIVED PREDICATE is correct but too terse. Be warned.

• **SUBJECT INDEX** entries to be added:

p287		ASSEMBLIES	2.153
p287		CARRIER	2.153
p288		CAUCUSES	2.153
p289	*	Effective Topos	2.418
p292		MODULUS	2.153
p295		stable (union)	1.752
p296		Wilson space	1.749

Finally, there should not be an asterisk on the index entry for SUBTERMINATOR on p295.

Available at
`http://www.math.upenn.edu/~pjf/corrections.pdf`
See also
`http://www.math.upenn.edu/~pjf/amplifications.pdf`