C Project description

Project title: Subtle symmetries and the refined Monster

Aims and background

This program is about *categorical groups*, a.k.a. *2-groups* or *group-like categories*. The notion of categorical group is a refinement of the notion of group, in which the symmetries themselves are related by symmetries. A number of important groups naturally occur in the form of 2-groups, and we plan to show that studying their previously ignored categorical aspects leads to simplification of some important and difficult mathematics. Preliminary work suggests that our program leads to an entirely new and illuminating development of the representations of affine Lie algebras, and we also propose to give a simple construction of the conjectural *Monster 2-group*, which sheds light on the most difficult aspects of the Monster, including triality.

We judge our success by how simple and natural we can keep our constructions: in the ideal scenario, we would like them to turn out simpler than their known, non-categorical counterparts. Our first results in [Gan] are very encouraging, giving rise to a significant simplifiaction of a fundamental definition in the seminal book of Pressley and Segal on loop groups [PS86].

We are guided by the philosophy that, just as a group is often best understood as the symmetries of a naturally occuring object, a categorical group is often best understood as the *refined symmetries* of a higher categorical object, i.e., an object whose symmetries have symmetries of their own.

From this perspective, we find ourselves studying the symmetries of a theory of strings moving on some target space, and we take the opportunity to rewrite the foundations of such a theory. Our key contribution is the definition of the *thin bordism chain complex*, a tool that allows us to translate all the categorical information of that string theory into relatively simple, 1950ies style homological algebra. At the same time, we avoid all technical boundary conditions on the worldsheets. In doing so, we hope to contribute to a formulation of string theory that fully lives up to its original promise of elegance and simplicity.

The program falls into two parts: 1. constructing categorical groups and 2. studying their actions as refined symmetries.

(1) We plan to give new constructions of some prominent categorical groups. Our approach is constructive, low-tech, and deliberately avoids obstruction theoretic arguments. Projects:
   (a) Construct the *String 2-groups* as the refined symmetries of string theories, namely of the Wess-Zumino-Witten models. This is mostly done, there is a pre-release preprint, joint with Matthew Ando. Our methodology is different from Fiorenza, Rogers and Schreiber's. The most important idea is the *thin bordism chain complex* of a smooth manifold M, providing a strikingly simple model for the cobordism 2-groupoid of an extended sigma-model with target space M.
   (b) In recent work [Gan], we gave three very simple, hands-on constructions of *2-group extensions of tori*. Work in progress gives another, even simpler, formulation in terms of crossed modules.

(c) Give a simple construction of the conjectural *Monster 2-group* as a refined symmetry group. In previous work, we encountered strong empirical evidence for this conjecture of Mason's. Work in progress constructs such a refinement of $2^{1+24} \cdot Co_0$ as the symmetries of a representation involving the categorical Leech torus, see Point (2a).

(d) Construct categorical extensions of all extraspecial 2-groups. Here *2-group* is used in its traditional meaning of p-group with the prime $p = 2$. In particular, the subgroup of the Monster that is usually denoted 2^{1+24} is an extraspecial 2-group.

(e) Publish and further develop the theory of *Platonic 2-groups* explored in N. Epa's Honour's thesis under the supervision of CI Ganter. These categorical extensions of the finite subgroups of $SU(2)$ are expected to play a role in the (conjectural) categorification of McKay correspondence. Joint with Epa.

(2) We plan to systematically develop the representation and character theory of categorical groups. Others have attempted to approach this question via loop group representations. We emphasise that our formalism works without ever referring to loops or infinite dimensional manifolds. In fact, we use the opposite approach and work from categorical groups to loop groups.

Our preliminary results strongly suggest that the representation theory of Lie 2-groups gives an entirely new geometric counterpart to the representation theory of affine Lie algebras, which is analytically much less intricate than loop group representations.

(a) Representations of categorical tori. In work in progress, we show that the categorical tori of Point (1b) are realized quite naturally as functors and natural transformations acting on $\mathcal{C}oh_{T_{\mathbb{C}}}$, the category of coherent sheaves on the complexified torus. Point (1c) refers to the symmetries of (an extension of) this basic representation in the case of the Leech torus.

(b) Class functions on categorical tori. Preliminary results: starting only from the idea of examining what a class function (in the sense of Bartlett-Ganter-Kapranov-Willerton) of a torus 2-group looks like, we recover key features of the representation theory of loop groups, namely the Looijenga line bundle, the theta function formalism and, conjectureally, the Verlinde fusion product. Their derivation is straigh-forward from our results in (1b).

(c) Representation and character theory of finite categorical groups. This was the subject of the 2013 Masters thesis of Ganter's student Robert Usher. In joint work with Usher, we are planning to further develop this theory and to study applications, e.g., to generalized moonshine.

(d) Weyl 2-groups and the representations and characters of general Lie 2-groups. Planning stage. We have the relevant definitions in place. There is a straightforward approach to analyzing these objects, building on our results in Part (1b). This project is likely to have applications to Kapranov's conjectural *super-duper symmetry* formalism.

Categorical tori and their representations. The starting point for the majority of our constructions is given by the following data:

- a lattice L (We use this term in the sense of *'free $\mathbb{Z}$-module'*.)
- an even symmetric bilinear form I on L,

$$I(m,m) \ \in \ 2\,\mathbb{Z} \qquad \text{for } m \in L$$

- a finite group W of linear isometries of (L, I).

From these we can form

- the torus

$$T \ = \ L \otimes_{\mathbb{Z}} (\mathbb{R}/\mathbb{Z})$$

  with Lie algebra

$$\mathfrak{t} \ = \ L \otimes_{\mathbb{Z}} \mathbb{R},$$

- the degree 4 cohomology class

$$I \ \in \ H^4(BT;\mathbb{Z})^W,$$

  corresponding to our bilinear form. It is well-known that such an I classifies
- the Looijenga line bundle L_I on $T \times T$,
- a 2-group extension $\mathcal{T}$ of T,
- a central extension $\widetilde{\mathcal{L}T}$ of the loop group of T by $U(1)$,
- a central extension $\widetilde{T[2]}$ of the points of order two in T by $\{\pm 1\}$,
- twists of T-equivariant elliptic cohomology.

Our first key construction is an explicit model for $\mathcal{T}$. This part is complete and has now appeared in the online preprint [Gan].

Construction 1. Given L and I as above, choose an integer-valued bilinear form J on L satisfying

$$I(m,n) \ = \ J(m,n) + J(n,m).$$

J need not be symmetric. Let $\mathfrak{t}$ act on $L \times U(1)$ by

$$x\colon (m,z) \ \longmapsto \ \left(m, z e^{2\pi i J(m,x)}\right),$$

for $x \in \mathfrak{t}$ and $m \in L$ and $z \in U(1)$. The *categorical torus* $\mathcal{T}$ is the strict monoidal groupoid

$$\mathcal{T} = \quad \begin{array}{c} \mathfrak{t} \ltimes (L \times U(1)) \\ {\scriptstyle pr_1}\downarrow\;\downarrow \\ \mathfrak{t} \end{array} \qquad\qquad \begin{array}{c} (x,m,z) \\ \downarrow \\ x+m. \end{array}$$

Writing $x \xrightarrow{z} x+m$ for the arrow (x,m,z), composition of arrows is the obvious one (multiply lables) and the monoidal structure is given by group multiplication. Explicitly, multiplication of arrows is:

$$(x \xrightarrow{z} x+m) \bullet (y \xrightarrow{w} y+n) \quad = \quad (x+y \xrightarrow{\quad zwe^{2\pi i J(m,y)} \quad} x+y+m+n).$$

This is one of the the simple and concrete constructions of $\mathcal{T}$ that we referred to in Point (1b) above. The following examples give a flavour of the far-reaching applications of this first key construction.

Example 2. $L \subseteq \mathbb{R}^{24}$ is the Leech lattice, I the symmetric bilinear form on $\mathbb{R}^{24}$ making L an even unimodular lattice, and W is the *Conway group* $W = Co_0$. In this situation we will speak about the *(categorical) Leech torus* $\mathcal{T}_{Leech}$.

Example 3. L is one of the Niemeyer lattices A_1^{24} or A_2^{12}, and I is the symmetric bilinear form on $\mathbb{R}^{24}$ making L an even unimodular lattice. W is the *Mathieu group* $W = M_{24}$ or $W = M_{12}$. Categorical extensions of the Mathieu groups by the circle have been classified and play an important role in Mathieu Moonshine.

Example 4. G is a compact connected Lie group with maximal torus T and W is the corresponding Weyl group. L is the coweight lattice

$$L \quad = \quad ker\left(\mathfrak{t} \xrightarrow{exp} T\right).$$

The form I is a multiple of the Killing form. If G is simple and simply connected, then

$$H^4(BG;\mathbb{Z}) \;\cong\; H^4(BT;\mathbb{Z})^W \;\cong\; \mathbb{Z},$$

so I also classifies a 2-group $String_k(G)$, a central extension of $\mathcal{L}G$ and an extended sigma-model with target space G. We will refer to extended sigma models of this form as 'WZW-models'.

Project 5 (Conway, Mathieu, and Weyl 2-groups)**.** In a very recent development, we found the 1-automorphism group of the categorical torus $\mathcal{T}$ to be a categorical extension of the group of isometries $O(L, I)$. This observation results in a construction of Conway, Mathieu and Weyl 2-gruops. We plan a detailed study of these objects and their applications.

Construction 6 (Categorical extensions of extraspecial 2-groups)**.** In work in progress, we construct the categorical extensions of extraspecial 2-groups, promised in Point (1d) as the categorical fixed points $\mathcal{T}^{\{\pm 1\}}$ of an involution ι on $\mathcal{T}$. More precisely, ι sends the object x of $\mathcal{T}$ to $-x$ and the arrow

$$x \xrightarrow{z} x+m \qquad \text{to} \qquad -x \xrightarrow{z} -x-m.$$

This is the correct categorification of the involution of the torus T that sends each element to its inverse, important for the process of "orbifolding" in the moonshine literature. By *categorical fixed points* we mean a category of equivariant objects, as defined by Grothendieck.

Our approach is to build the orbifoldization in with our categorical torus:

Construction 7. Similarly to Construction 1, we construct the "semi-direct product" $\mathcal{T} \rtimes \{\pm 1\}$ as the strict monoidal groupoid

$$\mathcal{T} \rtimes \{\pm 1\} = \quad \begin{array}{ccc} (\mathfrak{t} \rtimes \{\pm 1\}) \ltimes (L \times U(1)) & & (x, \varepsilon, m, z) \\ pr_1 \Big\downarrow \; \Big\downarrow & & \Big\downarrow \\ \mathfrak{t} \rtimes \{\pm 1\} & & (x + \varepsilon m, \varepsilon), \end{array}$$

where the action of $\mathfrak{t}$ is as before and -1 acts on everything by ι.

The following is our second key construction.

Construction 8 (basic representations of categorical tori)**.** In a very recent development, we are now able to construct the basic representation of $\mathcal{T}$ and $\mathcal{T} \rtimes \{\pm 1\}$ on the category of coherent sheaves on the complexified torus $T_{\mathbb{C}}$. More precisely, we construct a strict monoidal functor

$$\mathcal{T} \rtimes \{\pm 1\} \longrightarrow 1Aut(\mathcal{C}oh(T_{\mathbb{C}})),$$

where the target is the category of autoequivalences of $\mathcal{C}oh(T_{\mathbb{C}})$ and natural isomorphisms between them.

The status of this project is quite advanced. This, and a number of similar "level k" constructions are the content of a manuscript in progress. There, we also prove the following theorem, which is the first stepping stone to the proposed construction of the refined Monster.

Theorem 9. *The 1-automorphisms of the basic representation of $\mathcal{T} \rtimes \{\pm 1\}$ form the categorical extension of an extraspecial 2-group as in Construction 6. To be more precise, the extraspecial 2-group in question is the extension of*

$$T[2] \;\; L \;\; 2L$$

classified by the Arf invariant

$$\phi(m) \; := \; \frac{1}{2} I(m, m) \mod 2.$$

Character theory. In [Bar11] [GK08], we considered the situation where a (finite) group G acts by functors on a category, or more generally, by 1-morphisms in a bicategory. We set out to describe the character of such a (linear) 2-representation. It turned out that, in this context, there are two natural notions of character:

- **the categorical character:** this is a bundle on the *inertia groupoid*

  $$\Lambda G \; := \; G /\!\!/ G^{\text{conj}}$$

  (the action groupoid of G acting on itself by conjugation), much like characters of classical representations are class functions, i.e., invariant functions on ΛG;
- **the 2-character:** this is a function on pairs of commuting elements of G, invariant under simultaneous conjugation, i.e., an invariant function on $\Lambda\Lambda G$.

Bartlett and we independently arrived at the same definitions of characters. We take this as evidence that our choices were sensible. Now we propose to investigate the characters of actions of categorical groups (2-groups): a categorical group is a monoidal groupoid all of whose objects are weakly invertible. A 2-representation ϱ of a categorical group $\mathcal{G}$ may be viewed as projective (or 'gerbal') 2-representation of the group

$$G \; := \; \text{ob}(\mathcal{G})/\cong$$

(see [Ush13]). Assume that the conjugation action of G on the abelian group $A = aut(1)$ is trivial.

Definition 10. A categorical class function on $\mathcal{G}$ is a bundle on the *inertia groupoid* of $\mathcal{G}$,

$$\Lambda\mathcal{G} \quad := \quad Bifun(\mathrm{pt}/\!\!/\mathbb{Z}, \mathrm{pt}/\!\!/\mathcal{G}) \,/\, \text{2-isos}.$$

Here $\mathrm{pt}/\!\!/\mathcal{G}$ is the one-object bicategory with $\mathcal{G}$ as 1-endomorphisms. A 2-class function on $\mathcal{G}$ is an invariant function on $\Lambda\Lambda\mathcal{G}$.

The case where G is finite was treated in Robert Usher's 2013 Masters thesis (supervised by Ganter).

Theorem 11 ([Ush13])**.** *If G is finite, the categorical character of any representation ϱ of $\mathcal{G}$ is a categorical class function on $\mathcal{G}$. The 2-character of ϱ is a 2-class function on $\mathcal{G}$.*

More precisely, we can show that if $\mathcal{G}$ is classified by a 3-cocycle α (the associator) on the finite group G, then $\Lambda\mathcal{G}$ is equivalent to the central extension of ΛG classified by the 2-cocycle

$$\tau(\alpha)\left(g \xrightarrow{s} h \xrightarrow{t} k\right) \quad = \quad \frac{\alpha(k,t,s)\cdot\alpha(t,s,g)}{\alpha(t,h,s)},$$

$h = sgs^{-1}$ and $k = tht^{-1}$ (transgression of α). So, a reformulation of Theorem 11 is

Theorem 12. *If G is finite, the categorical character of any representation ϱ of $\mathcal{G}$ is a module over the twisted Drinfeld double of G.*

Here *twisted* means twisted by α.

Of central importance for the character theory of Lie 2-groups is the case where $G = T$ is a torus and $aut(1) = U(1)$. We have the following preliminary result (unpublished):

Theorem 13. *Let $\mathcal{T}$ be a categorical torus as in Construction 1. Then*

- *the inertia groupoid $\Lambda\mathcal{T}$ is equivalent to a Lie groupoid with objects T and arrows a $U(1)$-torsor over $T \times T$;*
- *the 2-characters of $\mathcal{T}$ are theta functions.*

More precisely,

- *the torsor corresponds to the Looijenga line bundle (see Construction 1 below),*
- *the theta functions transform like the corresponding loop group characters, and*
- *we conjecture that the composition of arrows in $\Lambda\mathcal{T}$ induces the multiplication in the Verlinde algebra.*

Project 14 (Categorified class functions on $\mathcal{T}$)**.** There is a second description of the groupoid $\Lambda\mathcal{T}$ in Theorem 13, taking into account that the automorphisms of any object t form a central extension of the centralizer T of t. This description is well-suited for our study of bundles on $\Lambda\mathcal{T}$, a.k.a. categorical class functions on $\mathcal{T}$. The precise picture is dictated by the definitions we have already made. In preliminary work, we prove: let V be a representation space of $T \times U(1)$, and let

$$\psi_m\colon V \quad \xrightarrow{\cong} \quad V \qquad m \in L$$

be a family of linear operators satisfying

$$\psi_m\psi_n \quad = \quad \psi_{m+n},$$

and

$$\psi_m \circ (z, y) \;=\; \left(z, z^{I^\sharp(m)} \cdot y\right) \circ \psi_m,$$

for $z \in T$, $y \in U(1)$, where $z^{I^\sharp(m)}$ is our notation for the image of z under the character

$$e^{I(m,-)} \colon T \;\longrightarrow\; U(1).$$

Then $\mathfrak{t} \times V \;/\; \sim$ with

$$(x + m, v) \;\sim\; (x, \psi_m(v))$$

is a bundle over the groupoid $\Lambda\mathcal{T}$. We conjecture that every categorified class function on $\mathcal{T}$ is isomorphic to one of this form.

In the situation of Project (14), Decompose V into

$$V \;\cong\; \bigoplus_{\lambda \in \Lambda} V_\lambda,$$

where T acts with weight λ on the summand V_λ. Assuming that the central circle $U(1)$ acts with weight 1 on V, we have isomorphisms

$$\psi_m \colon V_\lambda \;\xrightarrow{\cong}\; V_{\lambda + I^\sharp(m)},$$

$m \in L$. If $U(1)$ acts with weight k, replace $I^\sharp(m)$ with $k \cdot I^\sharp(m)$. In particular, if V is non-trivial, it is infinite dimensional.

If G is a compact connected Lie group with maximal torus T and (finite) Weyl group W, then the Looijenga line bundle is W-equivariant. In this situation, we strongly expect the following picture:

Conjecture 15. *Let $\mathcal{G}$ be a Lie 2-group extension of G, and let $\mathcal{T}$ be its restriction to the maximal torus. Then the 2-class functions on $\mathcal{G}$ are W-invariant theta functions as in Theorem 13.*

String 2-groups and the refined Monster.

Project 16 (String 2-groups)**.** The subject of our work in progress [AG] is to describe the 2-group $String_k(G)$ as symmetry 2-group of the WZW-model on G. The key idea is rather straight-forward, namely to consider the symmetries of the relevant Deligne cocycle. It has now appeared in independent work of Fiorenza, Rogers, and Schreiber. The more serious work is the string-theoretic interpretation, where our formalism is quite different. The main novelties we introduce are the thin bordism chain complex and the interpretation of the Gomi-Terashima transfer [GT01] as map of double complexes.

Project 17 (Attempt at the full refined Monster)**.** It is well-known that the Monster is generated by a subgroup isomorphic to

$$\widetilde{T[2]} \rtimes Co_1,$$

$Co_1 = Co_0/\{\pm 1\}$, together with one more element. This element is part of an S_3-symmetry, referred to as *triality* in [FLM88, Chapter 11]. Combining Project 5 and Construction 6, we strongly expect to interpret a categorical extension of $\widetilde{T[2]} \rtimes Co_0$ as a normalizer of the basic representation of $\mathcal{T}_{Leech} \rtimes \{\pm 1\}$.

In the known constructions of the Monster, the real challenge lies always in realizing the triality symmetry. Its constructions in [Tit85] and [FLM88] are so intricate, that 30 years on, there is barely any secondary literature on this famous subject. There has been some work in the physics literature, notably [DGH88] and [DGM96], see also [CGS09], [Tai11]. We will use [DGM96] as the blueprint for our constructions.

We can be more precise about our plan of attack on triality: both [FLM88] and [DGM96] start with a baby example, namely $SU(2)$-triality, and from there, build up to the full triality. To understand this example, it is useful to identify $SU(2)$ with the group $\mathbb{S}^3$ of unit quaternions. We make the convention that the (rank 1) maximal torus passes through k. The non-trivial element of the Weyl group is the reflection

$$w \ := \ jT.$$

Inside $\mathbb{S}^3$ sits the binary octahedral group, as the normalizer of the quaternion group

$$\{\pm 1, \pm i, \pm j, \pm k\}.$$

The quotient of the binary octahedral group by the quaternion group is isomorphic to the symmetric group S_3. Now the crucial point is that since $\mathbb{S}^3$ is a simply laced group, it acts on the basic representation of the loop group of its maximal torus. In other words, the basic representation of $\widetilde{\mathcal{L}T}$ has some unexpected symmetries. In particular, the binary octahedral group acts on it, mixing up the roles of j (Weyl group) and k (maximal torus).

A similar picture is emerging in the context of the basic representation of $\mathcal{T}$. Recall that that basic representation acts on the category of coherent sheaves on the complexified torus

$$T_{\mathbb{C}} \ = \ spec\, \mathbb{C}[\widehat{T}].$$

Here $\widehat{T} = Hom(T, U(1))$ is the character ring. In the case where T is the maximal torus of $SU(2)$,

$$\mathbb{C}[\widehat{T}] \ = \ \mathbb{C}[z, z^{-1}]$$

carries an action by the full $SU(2)$. In the general case, the ring $\mathbb{C}[\widehat{T}]$ does turn up in the triality picture.

Project 18 (Basic 2-representation of $String(3)$)**.** Our approach to triality suggests to look for an action of $SU(2) = Spin(3)$ on the basic representation of the maximal torus of $String(3)$. If this is successful, it is natural to try to extend it to all of $String(3)$.

Project 19 (Basic 2-representations of simply laced groups)**.** Extend the Project 18 to arbitrary simply laced groups.

Time-line. As with all creative work, it is difficult to predict a precise time-line for a program in pure mathematics. We can with some confidence break the program into four stages:

Stage 1: During the first year, we plan to complete the work on the various manuscripts in progress mentioned above. These are (working titles):

(1) Representations of categorical tori
(2) Class functions on categorical tori
(3) Platonic 2-groups
(4) The thin bordims chain complex
(5) The symmetries of string theory.

It is realistic to expect four of these to be ready for submission by the end of 2016.

Stage 2: The first half of 2017 will be dedicated to the dissemination of the results so far. With the publications in place, we will prepare more seriously for Project 17 and plan to seek the input by some of the world's experts (see Part E.1 for details).

Stage 3: During the second half of 2017, we plan to carry out Project 17 and to systematically work through the various follow-up questions for the points in Stage 1, as discussed above.

Stage 4: In the final year, we will work to connect our new theoretical framework with the interests of existing communities. We have already made some key contacts for this endeavor, most notably D. Persson (fundamental physics).

Role of personnel

CI Ganter brings in her expertise on 2-group representations and Moonshine. The program brings together many themes of Ganter's past research. PI Ando initiated the program for the string 2-groups. He has a very successful history of collaborating with physicists. His expertise in penetrating the physics literature and translating it into mathematics will be essential for the project. We are hoping to hire a research associate whose expertise complements ours. An ideal candidate would have a background in vertex operator algebras. The associate will become especially important during the final stage fo the program. (S)he will be the one responsible for translating the results into a language the VOA community can relate to and to link our results to the current research trends and open questions in the field.

Research environment

CI Ganter is part of the relatively new and very international research group of representation theory at the University of Melbourne. With its wide range of seminars, ranging from guest lectures by fields medallists to series of focused research talks for Melbourne based colleagues and students, the pure math group at Melbourne provides an unusually stimulating and highly collaborative research environment. All student supervision so far has involved projects with high relevance to Ganters research program. A new PhD-student, Matthew Spong, has just joined her team. While no funds are requested in this proposal to support him, he is likely to contribute to part of the program. The pure math group at Melbourne is about to be joined by five new members (Professors Haesemeyer and Vilonen and Lecturers Robertson, Xue and Murphet).

PI Ando is part of the strong homotopy theory group at UIUC, whose math library is proud to host one of the largest and richest collection of research books in North America.

Communication of results

We will continue to publish our results in high quality journals as well as the online arXiv. We will continue to present our results in international conferences and in research seminars around the world. We plan for the Research Associate to help create a strong online presence.

Management of data

Not applicable.

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