# Progress Toward Covariant Formulation of All D = 4 GS-type $\sigma$-model Actions

ABSTRACT

[1]

The initial steps toward the development of the general formalism for the description of four dimensional Green-Schwarz superstrings coupled to massless background fields are discussed. A number of open problems are described.

## I. Introduction

In order to use string and superstring theories [1] for four dimensional physics, the concept of compactification developed along diverse lines [2]. However, the net result of all of these seemingly different compactification techniques is the specification of a four dimensional internal symmetry group and a spectrum of ordinary fields which form representations of the internal symmetry group. Thus, a conceptually simpler way to formulate these string and superstring theories is as *ab initio* four dimensional theories but in which the internal space arises in the manner of the conventional fiber bundle viewpoint [3].

In this way, the symmetries (both spacetime and internal) represented by fundamental particles all have their origins in the Kac-Moody algebras of the string theories. String theories treat configuration space and 'isotopic' space in almost the same manner. The only differences between configuration space and 'isotopic' space arise because of the different treatment of zero modes. With this view in

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mind, I will discuss progress toward the formulation of four dimensional superstring theories wherein all symmetries are manifest at the level of the corresponding two-dimensional actions. In particular, for spacetime supersymmetry this implies that a Green-Schwarz type description is our ultimate goal.

## II. The Ultimate D = 4 Superstring $\sigma$-model Problem

We seek to formulate all D = 4 superstring nonlinear $\sigma$-models describing the propagation of D = 4 superstrings in a complete background of massless modes. Such $\sigma$-models should possess an explicit "$\beta$-function coupling constant" (equivalent to vertex operators) for every massless mode of the superstring theory. Thus we are attempting to extend our previous works[4] in NSR $\sigma$-models which cover the bosonic fields only. In the present problem we want to introduce the *superfield* coupling constants listed below in our first table. The coupling constants should appear in

| $\sigma$-model coupling tensor | string massless supermultiplet |
|---|---|
| $E_{\underline{A}}{}^{\underline{M}}(\Theta, X)$ | *superfield supergravity multiplet* |
| $B_{\underline{AB}}(\Theta, X)$ | *axion multiplet* |
| $\Gamma_{\underline{M}}{}^{\hat{\alpha}}(\Theta, X)$ | *gravi-photon multiplets* |
| $\Gamma_{\underline{M}}{}^{\hat{I}}(\Theta, X)$ | *right gauge-group vector multiplets* |
| $\Phi_{\hat{\alpha}\hat{I}}(\Theta, X)$ | *scalar multiplets* |

Table 1: Background Superfields of the GS $\sigma$-model

a two dimensional world-sheet action $\mathcal{S}(E_{\underline{A}}{}^{\underline{M}}, B_{\underline{AB}}, \Gamma_{\underline{A}}{}^{\hat{\alpha}}, \Gamma_{\underline{A}}{}^{\hat{I}}, \Phi_{\hat{\alpha}\hat{I}})$ where $\mathcal{S}$ should satisfy the condition that for $E_{\underline{A}}{}^{\underline{M}} = B_{\underline{AB}} = \Gamma_{\underline{A}}{}^{\hat{\alpha}} = \Gamma_{\underline{A}}{}^{\hat{I}} = \Phi_{\hat{\alpha}\hat{I}} = 0$ it describes the action for a free four dimensional heterotic string. This is an ambitious undertaking of which we are not yet completely capable. Having defined the problem this way, we will in the remainder of this talk describe the present state of the art in reaching this goal.

## III. The D = 10 Paradigm

As a preliminary illustration of the use of manifest realization of internal and spacetime symmetries in a string theory, we may use the D = 10 heterotic string[5] as an example. A proposed action for $\mathcal{S}(E_{\underline{A}}{}^{\underline{M}}, B_{\underline{AB}}, \Gamma_{\underline{A}}{}^{\hat{I}})$ is given by $\mathcal{S} = S_{GS} + S_R + S_N$, where the terms refer to a Green-Schwarz type action, a non-abelian righton action[4] and a noton action[6], respectively. The Green-Schwarz action has the familiar form

$$
\begin{aligned}
S_{GS} = & \int d^2\sigma V^{-1}\Big[-\tfrac{1}{2}e^{-\Phi}\Pi_{\ddagger}{}^{\underline{a}}\Pi_{=}{}_{\underline{a}} \; + \; \int_0^1 dy \hat{\Pi}_y{}^{\underline{C}}\hat{\Pi}_{\ddagger}{}^{\underline{B}}\hat{\Pi}_{=}{}^{\underline{A}}\hat{G}_{\underline{ABC}}\Big], \\
\Pi_{\ddagger}{}^{\underline{A}} \;&=\; V_{\ddagger}{}^{m}(\partial_m Z^{\underline{M}})E_{\underline{M}}{}^{\underline{A}} \;\; , \;\; \Pi_{=}{}^{\underline{A}} \;=\; V_{=}{}^{m}(\partial_m Z^{\underline{M}})E_{\underline{M}}{}^{\underline{A}} \;\; , \\
\hat{Z}^{\underline{M}} \;&=\; Z^{\underline{M}}(\sigma,\tau,y) \;\; , \;\; \hat{\Pi}_y{}^{\underline{A}} \;=\; (\partial_y \hat{Z}^{\underline{M}})E_{\underline{M}}{}^{\underline{A}}(\hat{Z}) \;\; , \;\; \hat{G}_{\underline{ABC}} = G_{\underline{ABC}}(\hat{Z}) \; . \qquad (3.1)
\end{aligned}
$$

where $Z^{\underline{M}}(\tau,\sigma)$ is the superstring coordinate $(Z^{\underline{M}}(\tau,\sigma) \equiv (\Theta^{\mu}(\tau,\sigma), X^{\underline{m}}(\tau,\sigma))$, $G_{\underline{ABC}}$ is the field strength supertensor for a super 2-form $B_{\underline{AB}}(Z)$. The "hatted" coordinates are the usual extensions used in the Vainberg construction[7].

The next term in $\mathcal{S}$ introduces the internal degrees of freedom in a manifest way. For this purpose nonabelian rightons $\phi_R^{\hat{I}}(\tau,\sigma)$ are used

$$
\begin{aligned}
S_R = -\frac{1}{4\pi}\int d^2\sigma V^{-1} Tr\{ \; & (\mathbf{D}_{\ddagger}U^{-1})(\mathbf{D}_{=}U) - \lambda_{=}{}^{\ddagger}(U^{-1}D_{\ddagger}U)^2 \\
& + \int_0^1 dy(\tilde{U}^{-1}\frac{d}{dy}\tilde{U})[\; (\mathbf{D}_{\ddagger}\tilde{U}^{-1})(\mathbf{D}_{=}\tilde{U}) - (\mathbf{D}_{=}\tilde{U}^{-1})(\mathbf{D}_{\ddagger}\tilde{U}) \;] \qquad (3.2) \\
& -2\Pi_{\ddagger}{}^{\underline{B}}\Gamma_{\underline{B}}{}^{\hat{I}} t_{\hat{I}}(U^{-1}\mathbf{D}_{=}U) \; \} \;\; ,
\end{aligned}
$$

with $D_{\ddagger}U \equiv \mathbf{D}_{\ddagger}U - i\Pi_{\ddagger}{}^{\underline{B}}\Gamma_{\underline{B}}{}^{\hat{I}}Ut_{\hat{I}}$. The quantity $U \equiv exp[i\phi_R^{\hat{I}}(\tau,\sigma)t_{\hat{I}}]$ is an element of an arbitrary group. The matrices $t_{\hat{I}}$ generate a compact Lie algebra for the right-gauge group $\mathcal{G}_R$ where $\hat{I} = 1,\ldots,d_G$, $[t_{\hat{I}},\; t_{\hat{J}}] = if_{\hat{I}\hat{J}}{}^{\hat{K}}t_{\hat{K}}$, $f_{\hat{I}\hat{J}\hat{K}}f^{\hat{I}\hat{J}}{}_{\hat{L}} = c_2\delta_{\hat{K}\hat{L}}$, and $Tr(t_{\hat{I}}t_{\hat{J}}) = 2k\delta_{\hat{I}\hat{J}}$. Above and in the following discussion, we use the notation $\mathbf{D}_{\pm\pm}$ to denote the world-sheet two-dimensional gravitationally covariant derivative.

The final term of $\mathcal{S}$ is a noton action which is required for the covariant removal of the Siegel anomaly by use of the noton fermions $\rho_+^{\hat{\imath}}$ (with $\hat{\imath} = 1, ..., 20$)

$$
S_N = -i\tfrac{1}{2}\int d^2\sigma V^{-1}\delta_{\hat{\imath}\hat{\jmath}}[\; \rho_+^{\hat{\imath}}\mathbf{D}_{=}\rho_+^{\hat{\jmath}} + \lambda_{=}{}^{\ddagger}\rho_+^{\hat{\imath}}\mathbf{D}_{\ddagger}\rho_+^{\hat{\jmath}} \;] \;\; . \qquad (3.3)
$$

These notons do not introduce any physical degrees of freedom into the model. The actions in (3.2) and (3.3) possess the local gauge invariance known as Siegel symmetry[8]. The would-be anomaly in this symmetry is precisely cancelled between the contributions coming from (3.2) and (3.3). This is the Hull mechanism[6]. Although it is sufficient for the genus zero theory, it has not been studied for $g > 0$

two-surfaces. Thus, our present approach is well suited to only calculate string tree-level contributions to the effective action.

A conceptual advantage of such a $\sigma$-model action is that it clearly demonstrates the relation of the closed GS string theory and particle field theory approaches to the introduction of internal symmetries. In the particle theory approach, we would simply specify that the Yang-Mills matter multiplet, $\Gamma_{\underline{B}}{}^{\hat{I}}(Z)$, should transform as the adjoint representation of the right-gauge group $\mathcal{G}_R$, and thus construct a principal fiber bundle over D = 10 superspace. The string takes this process one step further by actually "coordinate-izing" the Lie algebra of the fiber with $\phi_R^{\hat{I}}$. Thus, the non-abelian rightons may be regarded as providing maps from the world-sheet into isotopic charge space.

The $\sigma$-model we have described so far is actually inconsistent for arbitrary choices of the right-gauge group. In fact, if we demand the absence of anomalies, only the well-known $E_8 \otimes E_8$ or $SO(32)/Z_2$ groups are found to lead to consistent theories. This illustrates how the kinematical description of a candidate string theory is divorced from questions of its anomaly-freedom.

An important feature of the action $\mathcal{S}$ is that it possesses $\kappa$-symmetry[9]. This is most easily seen if we set the background fields to zero. A direct calculation shows $\delta_\kappa \mathcal{S} = 0$ under the variations

$$\begin{aligned}
&\delta_\kappa Z^{\underline{M}} = i(\sigma_{\underline{c}})^{\alpha\beta}(\kappa_{=\alpha}\Pi_{\ddagger}{}^{\underline{c}})E_\beta{}^{\underline{M}} \quad , \\
&\delta_\kappa h_{=}{}^{\ddagger} = \kappa_{=\alpha}\Pi_{=}{}^{\alpha}e^{2l}[\, 1 - h_{=}{}^{\ddagger}h_{\ddagger}{}^{=}\,] \quad , \\
&\delta_\kappa \lambda_{=}{}^{\ddagger} = -\kappa_{=\alpha}\Pi_{=}{}^{\alpha} \quad , \quad \delta_\kappa h_{\ddagger}{}^{=} \; = \; \delta_\kappa \phi_R^{\hat{I}} \; = \; \delta_\kappa \rho_{+}^{\hat{i}} = 0 \quad , \\
&\delta_\kappa \psi^{-1} = -\tfrac{1}{2}\kappa_{=\alpha}\Pi_{=}{}^{\alpha}e^{2l}h_{\ddagger}{}^{=} \quad , \quad \delta_\kappa l = \tfrac{1}{2}\kappa_{=\alpha}\Pi_{=}{}^{\alpha}e^{2l}h_{\ddagger}{}^{=} \quad ,
\end{aligned} \tag{3.4}$$

where we have used the Beltrami decomposition of the zweibein $V_a{}^m$

$$\begin{aligned}
V_{\ddagger}{}^{m}\partial_m &\equiv \psi^{-1}e^{l}(\partial_{\ddagger} + h_{\ddagger}{}^{=}\partial_{=}) \quad , \\
V_{=}{}^{m}\partial_m &\equiv \psi^{-1}e^{-l}(\partial_{=} + h_{=}{}^{\ddagger}\partial_{\ddagger}) \quad ,
\end{aligned} \tag{3.5}$$

into its fundamental parts. In particular, global information about the 2-surface is carried by the left Beltrami field, $h_{=}{}^{\ddagger}$, and the right Beltrami field, $h_{\ddagger}{}^{=}$ along with the transition functions required to patch the surface together. (Taken together (3.4) and (3.5) imply $\delta_\kappa V_{=}{}^{m} = \kappa_{=\alpha}\Pi_{=}{}^{\alpha}V_{\ddagger}{}^{m}$ , $\delta_\kappa V_{\ddagger}{}^{m} = 0$.)

It is clear from (3.5) that the left and right Beltrami fields are treated nonsymmetrically. The $\kappa$-symmetry transformation is only nontrivial on the left Beltrami field. If we let the ordered pair $(N_L, N_R)$ denote the number of nontrivial

$\kappa$-symmetry transformations realized on the left and right Beltrami fields respectively, then the heterotic theory is a (1,0) theory. The fact that the $\kappa$-symmetry is nontrivial in the (1,0) theory only on the left Beltrami field means that there must be a deep connection between the "left" topology of the 2-surface and $\kappa$-symmetry.

The intrinsic advantage of the action $\mathcal{S}$ is that it permits all of the massless states of the D = 10 heterotic string to be represented by the superfield coupling functions $E_{\underline{M}}{}^{\underline{A}}$, $B_{\underline{AB}}$ and $\Gamma_{\underline{A}}{}^{\hat{I}}$ that explicitly appear in a world-sheet action. In other words, every massless state has an explicit representation among the component field expansions of $E_{\underline{M}}{}^{\underline{A}}$, $B_{\underline{AB}}$ and $\Gamma_{\underline{A}}{}^{\hat{I}}$. The superfield "equations of motion" for these quantities, when considered to be fields in super-spacetime, are determined from the principle that the $\beta$-functions calculated from $\mathcal{S}$ must vanish [10] as was first noted by Friedan. Alternately, these "equations of motion" should be derivable

| Vanishing $\beta$-function | Geometrically Constrained Superfield |
|---|---|
| $\beta(E_{\underline{A}}{}^{\underline{M}})$ | $R_{\underline{AB}de}$ , $T_{\underline{AB}}{}^{\underline{C}}$ |
| $\beta(B_{\underline{AB}})$ | $G_{\underline{ABC}}$ |
| $\beta(\Gamma_{\underline{A}}{}^{\hat{I}})$ | $F_{\underline{AB}}{}^{\hat{I}}$ |

Table 2: Beta-functions & Superfield Equations of Motion

from an action principle. It is possible to directly derive this action by use of the "c-theorem" [11] combined with the calculation of the "averaged" anomaly [12]. Thus, the $\sigma$-model approach is seen to be sufficient to describe the complete massless string effective action in D = 10 superspace. This discussion also shows why the GS $\sigma$-model approach must ultimately prove to be superior to the NSR $\sigma$-model approach. In the latter, we can represent the bosonic states but not the fermionic states as "coupling constants" of a d = 2 $\sigma$-model. The "coupling constants" of a d = 2 $\sigma$-model are equivalent to the construction of vertex operators for the emission or absorption of massless states from the the superstring.

## IV. The "Kernel" of D = 4 GS Actions

In the last section, we saw that the notion of a manifest and covariant action which represented all of the symmetries of D = 10 heterotic superstrings has an explicit realization. This suggests a general philosophy for treating all superstring theories, especially those in four dimensions. Namely, it should be possible to construct four dimensional GS-type actions which necessarily include some degrees of freedom associated with the presence of internal symmetries. Furthermore, our "discovery" that the D = 10 heterotic theory is a (1,0) theory implies that it should be possible to describe four dimensional theories which have an arbitrary $(N_L, N_R)$ set of $\kappa$-symmetries. Such theories would be the GS analogs of the type (p,q) theories [13] known in the NSR formalism. To this end we first introduce the notion of the "kernel" of all D = 4 GS-type actions.

Let $Z^{\underline{M}} \equiv (\Theta^{\mu\ i}, \Theta^{\mu\ i'}, \bar{\Theta}^{\dot{\mu}}{}_i, \bar{\Theta}^{\dot{\mu}}{}_{i'}, X^{\mu\dot{\mu}})$ define the supercoordinate of the string. We introduce the four dimensional bosonic string coordinates in the form of a two by two hermitian matrix,

$$X^{\mu\dot{\mu}}(\tau,\sigma) = \begin{pmatrix} X^0 + X^3 & X^1 - iX^2 \\ X^1 + iX^2 & X^0 - X^3 \end{pmatrix} \quad , \tag{4.1}$$

where the $\mu$ and $\dot{\mu}$ indices each take on two values. This will ultimately facilitate the derivation of low energy four dimensional results expressed in the notation of two component Weyl spinors. Our previous experience in D = 4, N = 1 superspace has amply demonstrated the convenience of such a notation.

The fermionic coordinates are also introduced in the form of two component spinors which carry additional "isospin" indices $i$ and $i'$,

$$\begin{aligned} &\Theta^{\mu i}(\tau,\sigma) \qquad i = 1, ..., N_L \ (\text{4D} - \text{Weyl spinor}) \quad , \\ &\Theta^{\mu i'}(\tau,\sigma) \qquad i' = 1, ..., N_R \ (\text{4D} - \text{Weyl spinor}) \quad . \end{aligned} \tag{4.2}$$

We also introduce the symbol $\hat{G}_{\underline{ABC}}$ defined by

$$\hat{G}_{\underline{ABC}} = i\tfrac{1}{2} C_{\alpha\gamma} C_{\dot{\beta}\dot{\gamma}} \left\{ \begin{array}{ll} \delta_i{}^j & : if\ \underline{A} = \alpha\ i\ ,\ \underline{B} = \dot{\beta}\ j\ ,\ \underline{C} = \gamma\dot{\gamma} \\ & \quad \text{or any even permutation,} \\ -\ \delta_i{}^j & \quad : \text{for any odd permutation,} \\ -\ \delta_{i'}{}^{j'} & : if\ \underline{A} = \alpha\ i'\ ,\ \underline{B} = \dot{\beta}\ j'\ ,\ \underline{C} = \gamma\dot{\gamma} \\ & \quad \text{or any even permutation,} \\ \delta_{i'}{}^{j'} & \quad : \text{for any odd permutation,} \\ 0 & \quad : \quad \text{otherwise.} \end{array} \right\} \quad . \tag{4.3}$$

Next we note the same action given in (3.1) (except with $\Phi = 0$) can be defined here for a D = 4 theory. We define $\kappa$-symmetry variations by

$$
\begin{aligned}
\delta_\kappa Z^{\underline{N}} = &\; i\Pi_{\ddagger}{}^{\alpha\dot\alpha}(\bar\kappa_{=\ \dot\alpha}{}^{i} E_{\alpha\ i}{}^{\underline{N}} \ + \ \kappa_{=\ \alpha\ i} E_{\dot\alpha}^{i\ \underline{N}}) \\
&+ i\Pi_{=}{}^{\alpha\dot\alpha}(\bar\kappa_{\ddagger\ \dot\alpha}{}^{i'} E_{\alpha i'}{}^{\underline{N}} \ + \ \kappa_{\ddagger\ \alpha\ i'} E_{\dot\alpha}^{i'\ \underline{N}}) \quad , \\
\delta_\kappa h_{=}{}^{\ddagger} = &\; \kappa_{=\ i\alpha}\Pi_{=}{}^{\alpha i} e^{2l}[\ 1 - h_{\ddagger}{}^{=} h_{=}{}^{\ddagger}\ ] + \text{h.c.} \quad , \\
\delta_\kappa h_{\ddagger}{}^{=} = &\; \kappa_{\ddagger\ i'\alpha}\Pi_{\ddagger}{}^{\alpha i'} e^{-2l}[\ 1 - h_{\ddagger}{}^{=} h_{=}{}^{\ddagger}\ ] + \text{h.c.} \quad , \\
\delta_\kappa \psi = &\; -\tfrac{1}{2}[\ \kappa_{\ddagger\ i'\alpha}\Pi_{\ddagger}{}^{\alpha i'} e^{-2l} h_{=}{}^{\ddagger} \ + \ \kappa_{=\ i\alpha}\Pi_{=}{}^{\alpha i} e^{2l} h_{\ddagger}{}^{=}\ ] + \text{h.c.} \quad , \\
\delta_\kappa l = &\; \tfrac{1}{2}[\ \kappa_{\ddagger\ i'\alpha}\Pi_{\ddagger}{}^{\alpha i'} e^{-2l} h_{=}{}^{\ddagger} \ - \ \kappa_{=\ i\alpha}\Pi_{=}{}^{\alpha i} e^{2l} h_{\ddagger}{}^{=}\ ] + \text{h.c.} \quad ,
\end{aligned}
\tag{4.4}
$$

and remarkably enough the action is invariant for arbitrary integers $N_L$ and $N_R$! The case of N = 1 spacetime supersymmetry is clearly the special case of $N_L = 1$ and $N_R = 0$ in our conventions.

This kernel can be used in several different ways. First we can impose the light-cone gauge condition on it. Then additional conformal field theories may be added to it so as to insure anomaly cancellation. In this way one would arrive at the light-cone formulation of (presumably all) D = 4 superstring theories! It seems likely that the condition of anomaly cancellation will imply that only theories with $1 \le N_L + N_R \le 8$ will be free of anomalies. This would provide a stringy reason why the maximally supersymmetric four dimensional theory would be one with eight spacetime supersymmetries. Although only the case of $N_L = 1,\ N_R = 0$ is phenemologically interesting, presumably the other theories provide all possible stringy extensions of D = 4 supergravity theories.

## V. SUSY Augmentation of the Kernel

Our eventual aims are more lofty than the construction of light-cone gauge four dimensional superstring theories. We are interested in the covariant realization of both super-spacetime and internal symmetries for four dimensions along the lines demonstrated for the ten dimensional theories. This is presently beyond our reach. But it seems to require that we augment the kernel in two ways:

(a.) supersymmetry augmentation,

(b.) internal symmetry augmentation.

The reason such augmentation is required is that the action given purely by the kernel $S_{GS}^{(K)}$ is anomalous for all values of $N_L$ and $N_R$. This brings us to the notoriously difficult problem of the covariant quantization of the GS action.

Despite the recent efforts [14] at covariant quantization of the GS action, it remains an unsolved problem! We are in a similar position with the covariant quantization of the GS formulation of superstring theories as was the case with QED between the time it was first formulated in the 1920's and its successful quantization and renormalization in the late 1940's. This problem poses quite a challenge to the progress of really constructing a theory of superstrings as opposed to the collection of facts that passes under the name of superstring theory for now.

Presently, the best hope to meet this challenge seems to lie in the idea of augmentation of the kernel. The only principle which presently seems to be available is that all augmentations of the kernel must be consistent with $\kappa$-symmetry. An example, of augmentation in four dimensions can be constructed by a slight modification of the model proposed by Siegel [15]. Consider the action given by

$$
\begin{aligned}
S_{GSS} = \int d^2\sigma V^{-1}\Big[ & -\tfrac{1}{2}\Pi_{\ddagger}{}^{\underline{a}}\Pi_{=}{}_{\underline{a}} + \int_0^1 dy \hat{\Pi}_y{}^{C}\hat{\Pi}_{\ddagger}{}^{B}\hat{\Pi}_{=}{}^{A}\hat{G}_{\underline{ABC}} \\
& + \Pi_{=}{}^{\alpha i}\mathsf{d}_{\ddagger\alpha i} + \Pi_{=}{}^{\dot\alpha}{}_{i}\bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i} + \Pi_{\ddagger}{}^{\alpha i'}\mathsf{d}_{=\alpha i'} + \Pi_{\ddagger}{}^{\dot\alpha}{}_{i'}\bar{\mathsf{d}}_{=\dot\alpha}{}^{i'} \\
& + \lambda_{[-4]}{}^{\alpha\dot\alpha}\mathsf{d}_{\ddagger\alpha i}\bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i} + \lambda_{[4]}{}^{\alpha\dot\alpha}\mathsf{d}_{=\alpha i'}\bar{\mathsf{d}}_{=\dot\alpha}{}^{i'} \\
& + i\Pi_{\ddagger}{}^{\alpha\dot\alpha}[\, \bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i}\psi_{[-4]\alpha i} + \mathsf{d}_{\ddagger\alpha i}\bar{\psi}_{[-4]\dot\alpha}{}^{i} \,] \\
& + i\Pi_{=}{}^{\alpha\dot\alpha}[\, \bar{\mathsf{d}}_{=\dot\alpha}{}^{i'}\psi_{[4]\alpha i'} + \mathsf{d}_{=\alpha i'}\bar{\psi}_{[4]\dot\alpha}{}^{i'} \,] \\
& + \phi_{[-6]}{}^{\alpha\dot\alpha}[\, \mathsf{d}_{\ddagger\alpha i}\mathbf{D}_{\ddagger}\bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i} - \bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i}\mathbf{D}_{\ddagger}\mathsf{d}_{\ddagger\dot\alpha i} \,] \\
& + \phi_{[6]}{}^{\alpha\dot\alpha}[\, \mathsf{d}_{=\alpha i'}\mathbf{D}_{=}\bar{\mathsf{d}}_{=\dot\alpha}{}^{i'} - \bar{\mathsf{d}}_{=\dot\alpha}{}^{i'}\mathbf{D}_{=}\mathsf{d}_{=\dot\alpha i'} \,] \Big] \quad .
\end{aligned}
\tag{5.1}
$$

This action includes several new (two-dimensional) fields that are not present in the original GS action. These new fields are $\mathsf{d}_{\ddagger\alpha i}$, $\mathsf{d}_{=\alpha i'}$, $\psi_{[-4]\alpha i}$, $\psi_{[4]\alpha i'}$, $\lambda_{[4]}{}^{\alpha\dot\alpha}$, $\lambda_{[-4]}{}^{\alpha\dot\alpha}$, $\phi_{[6]}{}^{\alpha\dot\alpha}$ and $\phi_{[-6]}{}^{\alpha\dot\alpha}$ where I have used the notation device $\psi_{[-4]\alpha i} \equiv \psi_{==\alpha i}$ etc. to simplify the appearance of these fields.

The astute reader may at this point wonder why this augmentation process is necessary. A simple answer to this question is that in covariant gauges, the GS action suffers from a serious drawback; it does not define a propagator for the $\Theta$-variables! This is a different problem from that usually encountered in a gauge theory. For example, the QED action, before gauge-fixing, contains terms quadratic in the photon field. The operator between these terms, however, is not invertible. Thus, the need for gauge-fixing. In the GS action there are simply no such terms at all for the Grassmann coordinates of the superstring. In the light-cone gauge quantization of the GS theory, this difficulty is overcome by performing a certain redefinition which, in the language of two dimensional field theory, turns a would-be three point function into a two-point function. There are a number of indications [16] that performing a similar such redefinition in covariant gauges leads to a new type of anomaly. The way the augmentation would solve this problem is seen by noting that the action $S_{GSS}$ contains terms of the form $\Pi_{=}{}^{\alpha}\mathsf{d}_{\ddagger\alpha}$. It is envisioned that these lead to a new two-point function involving $\Theta$ after gauge fixing $< 0|\Theta^{\alpha}\mathsf{d}_{\ddagger\beta}|0 >\sim \delta_{\beta}{}^{\alpha}p_{\ddagger}/p^2$. It has been suggested that such propagators are absolutely essential in type-II GS theories. Suggestive evidence has been found [17] that such propagators are required for renormalizability of the type-II $\sigma$-model. However, adding only new $\Pi_{=}{}^{\alpha}\mathsf{d}_{\ddagger\alpha}$-terms to the old GS action is not sufficient because $\kappa$-symmetry invariance is broken. Since $\kappa$-symmetry is to strings in the GS formulation as world-sheet supersymmetry is to the NSR formulations, one must find a way to restore $\kappa$-symmetry in the presence of the new terms. This is precisely the role of the new fields which occur in the modification of the GS action suggested by Siegel. Such extensions of the GS action are not unique [2]

An extended and modified version of the $\kappa$-symmetry transformations may

[2]Other extensions have also been suggested (see *Introduction to String Field Theory* by W. Siegel (World Scientific,1988) p. 100).

be defined with the goal of yielding an invariant action under the $\kappa$-symmetry,

$$
\begin{aligned}
\delta_\kappa Z^{\underline{N}} =&\ i\Pi_{\ddagger}{}^{\alpha\dot\alpha}\bar\kappa_{=}{}_{\dot\alpha}{}^{i}E_{\alpha}{}_{i}{}^{\underline{N}} \ +\ i\tfrac{1}{2}\kappa_{=}{}_{\alpha i}\bar{\mathsf{d}}_{\ddagger\dot\alpha}{}^{i}E^{\alpha\dot\alpha}{}^{\underline{N}} + \mathsf{h.c.} \\
&+ i\Pi_{=}{}^{\alpha\dot\alpha}\bar\kappa_{\ddagger}{}_{\dot\alpha}{}^{i'}E_{\alpha}{}_{i'}{}^{\underline{N}} \ +\ i\tfrac{1}{2}\kappa_{\ddagger}{}_{\alpha i'}\bar{\mathsf{d}}_{=\dot\alpha}{}^{i'}E^{\alpha\dot\alpha}{}^{\underline{N}} + \mathsf{h.c.} \quad , \\
\delta_\kappa \mathsf{d}_{\ddagger\alpha i} =&\ -2\kappa_{=\alpha i}[\ \Pi_{\ddagger}{}^{\beta j}\mathsf{d}_{\ddagger\beta j} + \Pi_{\ddagger}{}^{\dot\beta}{}_{j}\bar{\mathsf{d}}_{\ddagger\dot\beta}{}^{j}\ ] \\
&+ \Pi_{\ddagger}{}^{\dot\beta}{}_{i}[\ \kappa_{=\alpha j}\bar{\mathsf{d}}_{\ddagger\dot\beta}{}^{j} - \bar\kappa_{=\dot\beta}{}^{j}\mathsf{d}_{\ddagger\alpha j}\ ] \quad , \\
\delta_\kappa \mathsf{d}_{=\alpha i'} =&\ -2\kappa_{\ddagger\alpha i'}[\ \Pi_{=}{}^{\beta j'}\mathsf{d}_{=\beta j'} + \Pi_{=}{}^{\dot\beta}{}_{j'}\bar{\mathsf{d}}_{=\dot\beta}{}^{j'}\ ] \\
&+ \Pi_{=}{}^{\dot\beta}{}_{i'}[\ \kappa_{\ddagger\alpha j'}\bar{\mathsf{d}}_{=\dot\beta}{}^{j'} - \bar\kappa_{\ddagger\dot\beta}{}^{j'}\mathsf{d}_{=\alpha j'}\ ] \quad , \\
\delta_\kappa \psi_{[-4]\alpha i} =&\ -\mathbf{D}_{=}\kappa_{=\alpha i} + 2\psi_{[-4]\alpha i}[\ \Pi_{\ddagger}{}^{\beta j}\kappa_{=\beta j} + \Pi_{\ddagger}{}^{\dot\beta}{}_{j}\bar\kappa_{=\dot\beta}{}^{j}\ ] \\
&- 2\kappa_{=\alpha i}[\ \Pi_{\ddagger}{}^{\beta j}\psi_{[-4]\beta j} + \Pi_{\ddagger}{}^{\dot\beta}{}_{j}\bar\psi_{[-4]\dot\beta}{}^{j}\ ] \\
&+ \Pi_{\ddagger}{}^{\dot\beta}{}_{i}[\ \kappa_{=\alpha j}\bar\psi_{[-4]\dot\beta}{}^{j} + \bar\kappa_{=\dot\beta}{}^{j}\psi_{[-4]\alpha j}\ ] \quad , \\
\delta_\kappa \psi_{[4]\alpha i'} =&\ -\mathbf{D}_{\ddagger}\kappa_{\ddagger\alpha i'} + 2\psi_{[4]\alpha i'}[\ \Pi_{=}{}^{\beta j'}\kappa_{\ddagger\beta j'} + \Pi_{=}{}^{\dot\beta}{}_{j'}\bar\kappa_{\ddagger\dot\beta}{}^{j'}\ ] \\
&- 2\kappa_{\ddagger\alpha i'}[\ \Pi_{=}{}^{\beta j'}\psi_{[4]\beta j'} + \Pi_{=}{}^{\dot\beta}{}_{j'}\bar\psi_{[4]\dot\beta}{}^{j'}\ ] \\
&+ \Pi_{=}{}^{\dot\beta}{}_{i'}[\ \kappa_{\ddagger\alpha}{}^{j'}\bar\psi_{[4]\dot\beta}{}^{j'} + \bar\kappa_{\ddagger\dot\beta}{}^{j'}\psi_{[4]\alpha j'}\ ] \quad , \\
\delta_\kappa \lambda_{[-4]}{}^{\alpha\dot\alpha} =&\ \tfrac{1}{96}[\ \kappa_{=}{}^{\alpha}{}_{i}\mathbf{D}_{\ddagger}\bar\psi_{[-4]}{}^{\dot\alpha i} - \bar\kappa_{=}{}^{\dot\alpha i}\mathbf{D}_{\ddagger}\psi_{[-4]}{}^{\alpha}{}_{i}\ ] \\
&- \tfrac{1}{96}[\ (\mathbf{D}_{\ddagger}\kappa_{=}{}^{\alpha}{}_{i})\bar\psi_{[-4]}{}^{\dot\alpha i} - (\mathbf{D}_{\ddagger}\bar\kappa_{=}{}^{\dot\alpha i})\psi_{[-4]}{}^{\alpha}{}_{i}\ ] \\
&- \tfrac{1}{24}\phi_{[-6]}{}^{\alpha\dot\beta}[\ (\mathbf{D}_{\ddagger}\bar\kappa_{=(\dot\beta}{}^{i})\Pi_{\ddagger}{}^{\dot\alpha)}{}_{i} + \mathsf{h.c.}\ ] \\
&- \lambda_{[-4]}{}^{\alpha\dot\alpha}[\ \kappa_{=\beta i}\Pi_{\ddagger}{}^{\beta i} + \bar\kappa_{=\dot\beta}{}^{i}\Pi_{\ddagger}{}^{\dot\beta}{}_{i}\ ] \\
&- i\tfrac{1}{2}\lambda_{[-4]}{}^{\alpha\dot\beta}\bar\kappa_{=(\dot\beta}{}^{i}\Pi_{\ddagger}{}^{\dot\alpha)i} + \mathsf{h.c.} \quad , \\
\delta_\kappa \lambda_{[4]}{}^{\alpha\dot\alpha} =&\ \tfrac{1}{96}[\ \kappa_{\ddagger}{}^{\alpha}{}_{i'}\mathbf{D}_{=}\bar\psi_{[4]}{}^{\dot\alpha i'} - \bar\kappa_{\ddagger}{}^{\dot\alpha i'}\mathbf{D}_{=}\psi_{[4]}{}^{\alpha}{}_{i'}\ ] \\
&- \tfrac{1}{96}[\ (\mathbf{D}_{=}\kappa_{\ddagger}{}^{\alpha}{}_{i'})\bar\psi_{[4]}{}^{\dot\alpha i'} - (\mathbf{D}_{=}\bar\kappa_{\ddagger}{}^{\dot\alpha i'})\psi_{[4]}{}^{\alpha}{}_{i'}\ ] \\
&- \tfrac{1}{24}\phi_{[6]}{}^{\alpha\dot\beta}[\ (\mathbf{D}_{=}\bar\kappa_{\ddagger(\dot\beta}{}^{i'})\Pi_{=}{}^{\dot\alpha)}{}_{i'} + \mathsf{h.c.}\ ] \\
&- \lambda_{[4]}{}^{\alpha\dot\alpha}[\ \kappa_{\ddagger\beta i'}\Pi_{=}{}^{\beta i'} + \bar\kappa_{\ddagger\dot\beta}{}^{i'}\Pi_{=}{}^{\dot\beta}{}_{i'}\ ] \\
&- i'\tfrac{1}{2}\lambda_{[4]}{}^{\alpha\dot\beta}\bar\kappa_{\ddagger(\dot\beta}{}^{i'}\Pi_{=}{}^{\dot\alpha)i'} + \mathsf{h.c.} \quad , \\
\delta_\kappa\phi_{[-6]}{}^{\alpha\dot\alpha} =&\ i\tfrac{1}{4}\kappa_{=}{}^{\alpha}{}_{i}\bar\psi_{[-4]}{}^{\dot\alpha i} + \mathsf{h.c.} \quad , \quad \delta_\kappa\phi_{[6]}{}^{\alpha\dot\alpha} = i\tfrac{1}{4}\kappa_{\ddagger}{}^{\alpha}{}_{i'}\bar\psi_{[4]}{}^{\dot\alpha i'} + \mathsf{h.c.} \quad , \\
\delta_\kappa V_{=}{}^{m} =&\ -2[\ \kappa_{=\alpha i}\Pi_{=}{}^{\alpha i} + \bar\kappa_{=\dot\alpha}{}^{i}\Pi_{=}{}^{\dot\alpha}{}_{i}\ ]V_{\ddagger}{}^{m} \quad , \\
\delta_\kappa V_{\ddagger}{}^{m} =&\ -2[\ \kappa_{\ddagger\alpha i'}\Pi_{\ddagger}{}^{\alpha i'} + \bar\kappa_{\ddagger\dot\alpha}{}^{i'}\Pi_{\ddagger}{}^{\dot\alpha}{}_{i'}\ ]V_{=}{}^{m} \quad .
\end{aligned}
\tag{5.2}
$$

But even with the extension in (5.1) it turns out [18] that still we cannot covariantly quantize the GS action! So it remains an unsolved question how to proceed.

## VI. Internal Augmentation of the Kernel

Leaving aside the difficulties with the covariant realization of spacetime supersymmetry in D = 4 $\sigma$-model actions, we can also spend some deliberations on the situation for the covariant and manifest realization of internal symmetries. Here we have a simple answer to the question of the need for such augmentation. All D = 4 supergravity theories that arise from the zero-slope limit of a superstring theory possess spin-one gauge fields. Following our philosophy that every massless state should be associated with an explicit renormalizable coupling constant in a $\sigma$-model[3] implies that such constants should occur as

$$S_{Spin\ 1} \sim \int d^2\sigma V^{-1} \Pi_{\pm\pm}{}^{\underline{B}} \Gamma_{\underline{B}}{}^{\tilde{a}}(Z) J^{\tilde{a}}_{\mp\mp} \qquad (6.1)$$

where $J^{\tilde{a}}_{\mp\mp}$ is some explicit current which is realized in the action. The kernel possesses no fields from which these internal symmetries currents can be constructed.

But here too $\kappa$-symmetry is expected to play a role. Note in our D = 10 paradigm, even in the presence of the internal symmetry currents, the total action possessed an invariance under a $\kappa$-transformation. This suggests that the same should be true for any D = 4 theory also. We have been able to find thus far only one example in which this is true, so as an end to this talk, this model will be presented. Consider an action obtained by taking the GS kernel action in the case of $N_L = 4,\ N_R = 0$ and add three terms to it, i.e. $S = S^{(K)}_{GS} + S_1 + S_2 + S_3$. The first of these three is simply the action of (3.2) for the case of $\mathcal{G}_R = U(1)^{16}$. The second is given by the action in (3.3). The final term has the explicit form

$$\begin{aligned} S_3 = \int d^2\sigma V^{-1} \Big[ & - \tfrac{1}{4} \mathcal{P}_{\ddagger}{}^{[ij]} \mathcal{P}_{=}{}^{[ij]} + \int_0^1 dy \hat{\mathcal{P}}_y{}^{[ij]} \hat{\Pi}_{\ddagger}{}^{\underline{B}} \hat{\Pi}_{=}{}^{\underline{A}} \hat{\mathcal{B}}_{\underline{AB}[ij]} \\ & + \tfrac{1}{2} \int_0^1 dy \hat{\Pi}_y{}^{\underline{B}} (\hat{\Pi}_{\ddagger}{}^{\underline{A}} \hat{\mathcal{P}}_{=}{}^{[ij]} - \hat{\Pi}_{=}{}^{\underline{A}} \hat{\mathcal{P}}_{\ddagger}{}^{[ij]}) \hat{\mathcal{B}}_{\underline{AB}[ij]} \ \Big] \quad , \end{aligned} \qquad (6.2)$$

$$\mathcal{P}_{\pm\pm}{}^{[ij]} \equiv (\mathbf{D}_{\pm\pm} \chi^{[ij]}) - \Pi_{\pm\pm}{}^{\underline{A}} \Gamma_{\underline{A}}{}^{[ij]} \quad , \quad \hat{\mathcal{B}}_{\underline{AB}[kl]} \equiv (\ C_{\alpha\beta} C_{ijkl}\ ,\ C_{\dot{\alpha}\dot{\beta}} \delta_k{}^{[i} \delta_l{}^{j]}\ ) \quad , \qquad (6.3)$$

where $\chi^{[ij]}$ are six $(i, j = 1, ..., 4)$ world-sheet scalar fields. The quantities $\Gamma_{\underline{A}}{}^{[ij]}$ are N = 4 superspace gauge connections whose field strengths satisfy the conditions,

$$F_{\alpha i \beta j}{}^{[kl]} \equiv C_{\alpha\beta} \delta_i{}^{[k} \delta_j{}^{l]} \quad , \quad F_{\dot{\alpha}\dot{\beta}}^{ij}{}^{[kl]} \equiv C_{\dot{\alpha}\dot{\beta}} C^{ijkl} \quad . \qquad (6.4)$$

[3]This supposition is certainly supported by our experience in both the NSR formulation of superstring theories and the GS formulation of the D = 10 heterotic theory.

These conditions are known to be satisfied by the gravi-photon supergauge connections [19] in the D = 4, N = 4 superspace supergravity theory.

Now the encouraging feature of this model is that it is a D = 4 theory which introduces internal degrees of freedom via $S_2$ and $S_3$ and simultaneously possesses an invariance with respect to $\kappa$-symmetry. For the fields $\chi^{[ij]}$, we take their variations to be given by

$$\delta_\kappa \chi^{[kl]} = i \Pi_{\ddagger}{}^{\alpha \dot{\alpha}} [ \kappa_{= \alpha i} \Gamma^{i}_{\dot{\alpha}}{}^{[kl]} + \bar{\kappa}_{= \dot{\alpha}}{}^{i} \Gamma_{\alpha i}{}^{[kl]} ] \quad . \tag{6.5}$$

It is then a simple but tedious task to show that the entire action for this D = 4 model is invariant. We have at least shown that the suggestion of internal symmetry augmentation of the kernel action is realizable in an explicit construction.

We end our talk here. As we have shown there are many, many unsolved or incomplete problems that remain. However, we are optimistic that progress in our understanding of superstring theories will shed further light on these.

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